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Research Paper

Displacements in Mechanical Systems Due to Random Inputs in a Mine Hoist Installation

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Conveyances, cages and skips are typical components of hoisting installations in mines. The Authors emphasises the fact that already at the stage of design utmost care must be taken to reduce the skip's own mass, at the same time improving its reliability through controlling the deformations of its structural elements. That requires, however, a thorough analysis of the state of stress in the conveyance's load-bearing elements due to real loads acting both during the normal duty cycles and under the emergency conditions.

Real loads acting upon the load bearing elements of the conveyance were determined following the analysis of displacements of selected points of the conveyance's structure. For the purpose of calculation procedure, the conveyance is substituted by a triple mass model whose dynamic motion equations are based on an assumption that horizontal displacements of the skip mass are induced by random irregularities and misalignments of guiding strings. These equations are solved analytically, yielding the spectral densities of displacements of selected masses. Thus derived formulas and their graphical representations allow for selecting the system's parameters such that the displacements of the selected model points should not exceed the admissible levels set forth in relevant mining regulations [5].

Key words: mine hoisting installations, dynamics.

1. Introduction

Conveyances, cages and skips are typical components of hoisting installations in mines. Because of their important functions in the mines, they need to satisfy very strict requirements as to their design solutions, engineering materials and manufacturing quality. Proper shaping of the skip, suspensions, the guiding system and application of highly resistant materials allows for reducing the skip's own mass, at the same time retaining its load bearing capacity. Reducing the skip's mass allows the dimensions of other key components to be reduced as well: the diameter and mass of hoisting ropes and tail ropes, Keope pulleys and the winder machine, and that lowers the investment costs as well as and operation and maintenance costs.

Efforts should be made already at the stage of design to reduce the skip's own mass at the same time improving its reliability features. However, that requires a thorough analysis of the state of stress in the skip's structural components due to real loads acting both during the normal duty cycles and under the emergency conditions. This study is focused on finding the loads acting upon the skip during the ride at fixed velocity V_0. The conveyance is substituted by a triple mass model whose dynamic motion equations are based on an assumption that horizontal displacements of the modelled lumped masses are induced by random irregularities of guiding strings. These equations are solved analytically, yielding the spectral densities of displacements of selected masses in the system and giving the interaction forces between the shaft steelwork and the skip head and between the shaft steelwork and the lower frame. This study is restricted to the analysis of the skip head displacements.

2. Analysis of the skip dynamics

The analysis of the skip's dynamic behaviour when in operating conditions (at fixed hoisting velocity $V_0 = \text{const}$) relies on the model shown in Fig. 1 [4].

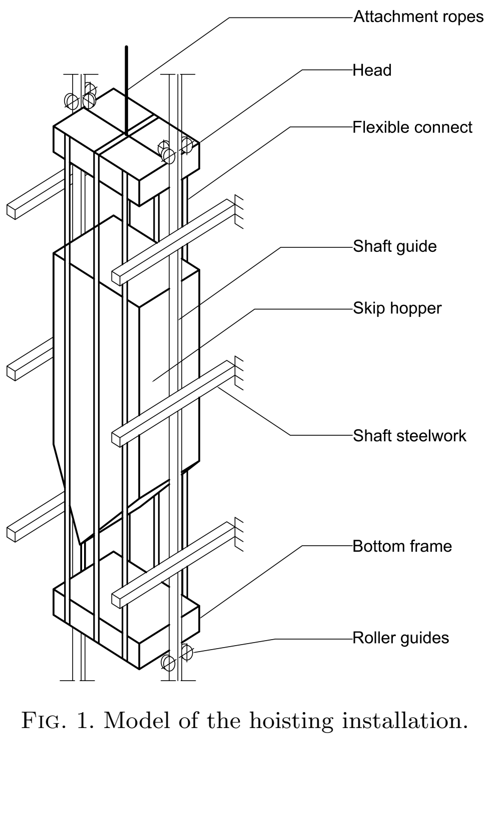

Fig. 1. Model of the hoisting installation.

The model comprises three lumped masses: a skip head, a bottom frame and a skip hopper, connected via weightless, laterally elastic and longitudinally non-deformable rods. Underlying the dynamic analysis are the following assumptions:

- elasticity profiles of roller shoes are linear,
- conveyance displacements are so small that the sliding shoes receive no impacts,
- in the plane determined by the guide shoe front face imperfections of opposite guide faces are parallel $x_1(t) = x_2(t) = x(t)$ and there are no operational clearances or pre-thrust between the roller shoes and the skip guides such that during the ride each shoe should remain in contact with the guide.

In practice all these conditions guaranteeing that the conveyance-shaft steelwork system could be treated as linear can be satisfied over short sections of the conveyance path only. Despite this limitation, the analysis of a linear three-mass system offers us a good insight into conveyance-shaft steelwork interactions, highlighting those processes that are entirely omitted in a single-mass model [4].

In the light of these assumptions, particularly the one regarding the parallelism of displacements of opposite guide string faces, the model shown schematically in Fig. 1 can be substituted by that given in Fig. 2.

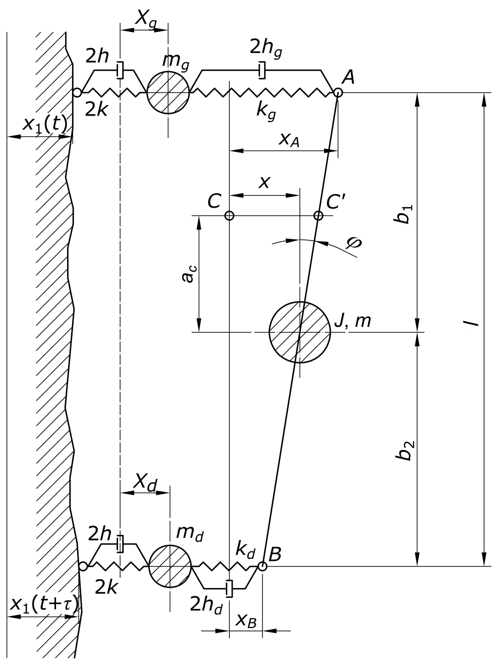

FIG. 2. Mechanical model of the hoisting installation.

Underlying the equations of motion of the system are the Lagrange equations of the second type. Accordingly, we get [4]:

$$\frac{mb_2^2+I}{I^2}\ddot{x}_A+\frac{mb_1b_2-I}{I^2}\ddot{x}+k_g(x_A-x_g)+2h_g(\dot{x}_A-\dot{x}_g)=0, \tag{2.1}$$

$$\frac{mb_1^2+I}{I^2}\ddot{x}_B+\frac{mb_1b_2-I}{I^2}\ddot{x}+k_d(x_B-x_d)=0, \tag{2.2}$$

$$m_g\ddot{x}_g+2k\left(x_g-x_1(t)\right)-k_g(x_A-x_g)-2h_g(\dot{x}_A-\dot{x}_g)+2h\left(\dot{x}_g-\dot{x}_1(t)\right)=0, \tag{2.3}$$

$$m_d\ddot{x}_d-k_d(x_B-x_d)+2k\left(x_d-x_1(t+\tau)\right)-2h_d(\dot{x}_B-\dot{x}_d)+2h\left(\dot{x}_d-\dot{x}_1(t+\tau)\right)=0, \tag{2.4}$$

where $x_A=x+b_1\varphi$, $x_B=x+b_2\varphi$, φ – angle of skip hopper rotation around its centre of gravity (c.o.g), x – horizontal displacement of a hopper mass, m – mass of a loaded skip hopper, I – inertia moment of a loaded skip hopper, m_g – mass of the skip head, m_d – mass of the bottom frame, $2h$, k – (linear) damping and elasticity factors of sliding and rolling shoes, $2h_g$, k_g – damping and elasticity factors of (lateral) flexible connector between the head and hopper, $2h_d$, k_d – damping and elasticity factors of (lateral) flexible connectors between the bottom frame and skip hopper, x_g, x_d – horizontal displacements of the face of the skip head and bottom frame, respectively, $x_1(t)$, $x_1(t+\tau)$ – function defining the imperfections of cage guides' front surfaces, τ – time of conveyance ride along the path equal to the distance between the front shoes (on the head and the bottom frame), b_1, b_2 – distance between the hopper c.o.g and front shoes on the top and at the bottom, respectively, l – distance between top and bottom shoes.

3. Inputs

Conveyances are subjected to acting inputs due to unevenness or irregularities of the guide strings, mostly attributable to misalignment of vertical guides between subsequent buntons [1] (within the specified tolerance limits [1]), imperfections of guide joints, the presence of faults at the guide joints and unpredictable geological features in the shaft lining.

Statistics of guide unevenness are obtained by analysing the irregularities of two parallel guide strings $x_1(t)$ and $x_2(t)$. The magnitudes of the guide string irregularities and misalignment were chosen such as to comply with the relevant regulations currently in force in Poland and such that their deviations from the vertical at the level of shaft buntons should follow the normal distribution. For thus defined guide string irregularities, their spectral densities were obtained for variable speed of conveyance travel [1].

Spectral densities of guide string irregularities were also measured on the hoist installations whilst in service [4].

These densities, however, must not be treated as "standard" when analysing the deviations from the vertical at the level of shaft buntons because with different configuration of those irregularities and for different velocities of the skip travel V, the spectral density pattern would change. For that reason the concept of spectral density of "general guide string irregularities" is propounded and its value should not exceed the spectral density values of all other irregularities satisfying all the requirements relative to the shaft steelwork installation [1].

Spectral density of the "general guide string irregularities" is assumed to be the envelope of the spectral densities [1], approximated by:

$$S_{x_1} = S_{x_2} = S_x = \frac{0.8}{32 + \omega^2} \quad [\mathrm{mm}^2\mathrm{s}], \tag{3.1}$$

and in the generalised form:

$$S_x = \frac{2D_x\alpha}{\alpha^2 + \omega^2} \quad [\mathrm{m}^2\mathrm{s}], \tag{3.2}$$

where ω – frequency, $D_x = 7 \cdot 10^{-8}\ \mathrm{m}^2$, $\alpha = 5.66\ \mathrm{s}^{-1}$.

When the condition $I = mb_1b_2$ is satisfied (when the skip head and bottom frame vibrate independently), we get two uncoupled systems of Eqs. (2.1)–(2.4):

$$\begin{cases} m_A\ddot{x}_A + k_g(x_A - x_g) + 2h_g(\dot{x}_A - \dot{x}_g) = 0, \\ m_g\ddot{x}_g + 2k(x_g - x_1(t)) - k_g(x_A - x_g) \\ \qquad -2h_g(\dot{x}_A - \dot{x}_g) + 2h(\dot{x}_g - \dot{x}_1(t)) = 0, \end{cases} \tag{3.3}$$

$$\begin{cases} m_B\ddot{x}_B + k_d(x_B - x_d) + 2h_d(\dot{x}_B - \dot{x}_d) = 0, \\ m_d\ddot{x}_d + 2k(x_d - x_1(t+\tau)) - k_d(x_B - x_d) \\ \qquad -2h_d(\dot{x}_B - \dot{x}_d) + 2h(\dot{x}_d - \dot{x}_1(t+\tau)) = 0, \end{cases} \tag{3.4}$$

where

$$m_A = \frac{I + mb_2^2}{l^2}, \qquad m_B = \frac{I + mb_1^2}{l^2}.$$

4. Displacement variance

In order to find the variances of the solution it is required that transmittances W_{x_A}, W_{x_g}, W_{x_B}, W_{x_d}, should be obtained first. For the systems of Eqs. (3.3) and (3.4) these are given as:

$$
\begin{aligned}
X_A(s) &= W_{x_A}(s) \cdot X_1(s),\\
X_g(s) &= W_{x_g}(s) \cdot X_1(s),\\
X_B(s) &= W_{x_B}(s) \cdot X_1(s) \cdot e^{-s\tau},\\
X_d(s) &= W_{x_d}(s) \cdot X_1(s) \cdot e^{-s\tau},
\end{aligned} \tag{4.1}
$$

where $\tau = \frac{l}{V_0}$ is the time required by the conveyance to travel the distance l equal to the distance between the top and bottom guide shoes, moving with the speed V_0.

Recalling the dimensionless equation $t_1 = p_{10} \cdot t$ in the system of equations (3.3) where p_{10} is the natural vibration frequency of the mass m_A (for $m_g = 0$), we obtain:

$$
\begin{aligned}
&\ddot{X}_A + \frac{2h_g}{m_A p_{10}} \cdot \dot{X}_A + X_A - \frac{2h_g}{m_A p_{10}} \cdot \dot{X}_g - X_g = 0,\\
&\ddot{X}_g + \frac{2(h+h_g)}{m_g \cdot p_{10}} \dot{X}_g + \frac{k_g + 2k}{m_g p_{10}^2} X_g - \frac{2h_g}{m_g p_{10}} \dot{X}_A - \frac{k_g}{m_g p_{10}^2} X_A\\
&\qquad = \frac{2h}{m_g p_{10}} X_1(t_1) + \frac{2k}{m_g p_{10}^2} \dot{X}_1(t_1).
\end{aligned} \tag{4.2}
$$

Assuming that

$$h = m_g \cdot p_{10} \cdot n_2,$$

(where n_2 is a dimensionless parameters) and recalling

$$\frac{h_g}{h} \ll 1, \qquad h + h_g \cong h \qquad \text{and} \qquad h_g = 0,$$

we get:

$$
\begin{aligned}
&\ddot{X}_A + X_A - X_g = 0,\\
&\ddot{X}_g + 2n_2 \dot{X}_g + (1 + 2n_3) n_1 X_g - n_1 X_A = 2n_1 \cdot n_3 \cdot X_1(t_1) + 2n_2 \dot{X}_1(t_1,)
\end{aligned} \tag{4.3}
$$

where

$$n_3 = \frac{k}{k_g} \qquad \text{and} \qquad n_1 = \frac{m_A}{m_g}.$$

Finally, the transmittances are given as follows. For example, $W_{x_g}(s)$ becomes:

$$W_{x_g} = \frac{2\left[n_2 s^3 + n_1 \cdot n_3 \cdot s^2 + n_2 s + n_1 \cdot n_3\right]}{s^4 + 2n_2 s^3 + \left[1 + n_1(1 + 2n_3)\right] s^2 + 2n_2 s + 2n_1 n_3}. \tag{4.4}$$

Spectral density of displacement x_g is expressed as [3]:

$$S_{x_g}(\omega) = \left|W_{x_g}\right|^2 \cdot S_{x_1} \tag{4.5}$$

and the amplitude variance is given by the formula:

$$D_{x_g} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left|W_{x_g}\right|^2 \cdot S_{x_1}(\omega) d\omega. \tag{4.6}$$

Spectral density of "general irregularities" given by Eq. (3.2) expressed in terms of dimensionless frequency $\omega' = \omega/p_{10}$ becomes:

$$S_{x_1} = S_x = \frac{2D_x\alpha'}{\left[(\alpha')^2 + (\omega')^2\right] \cdot p_{10}}, \tag{4.7}$$

where $\alpha' = \alpha/p_{10}$.

Assuming that $\alpha' = n$, where n is a dimensionless parameter, we obtain:

$$S_x = \frac{2D_x \cdot n}{p_{10}\left[n^2 + (\omega')^2\right]}. \tag{4.8}$$

Rearranging Eq. (4.6) to account for Eq. (4.4), substituting $s = i\omega'$ into Eq. (4.7) and assuming that

$$d\omega = p_{10} \cdot d\omega',$$

we obtain:

$$D_{x_g} = \frac{8D_x \cdot n}{2\pi} \int_{-\infty}^{\infty} \frac{a^*}{\left|b^*\right|^2 (n^2 + \omega'^2)} d\omega, \tag{4.9}$$

where

$$a^* = \left[-n_2^2(i\omega')^6 + \left[(n_1 \cdot n_3)^2 - 2n_2^2\right](i\omega')^4 + \left[2(n_1 \cdot n_3)^2 - n_2^2\right](i\omega')^2 + (n_1 \cdot n_3)^2\right],$$

$$b^* = \left[(i\omega')^4 + 2n_2(i\omega')^3 + \left[1 + n_1(1 + 2n_3)\right](i\omega') + 2n_2(i\omega') + 2n_1 \cdot n_3\right].$$

Rearranging, we get the expression most convenient for integration procedures [3], [2]:

$$D_{x_g} = \frac{4D_x \cdot n}{\pi} \int_{-\infty}^{\infty} \frac{G(i\omega')}{\left|A(i\omega')\right|^2} d\omega', \tag{4.10}$$

where

$$G\left(i\omega'\right) = -n_2^2(i\omega')^6 + \left[(n_1 \cdot n_3)^2 - 2n_2^2\right](i\omega')^4 + \left[2\,(n_1 \cdot n_3)^2 - n_2^2\right](i\omega')^2 + (n_1 n_3)^2,$$

$$A\left(i\omega'\right) = (i\omega')^5 + (2n_2 + n)(i\omega')^4 + \left[1 + n_1\,(1 + 2n_3) + 2n_2 \cdot n\right]\left(i\omega'\right)^3 + \left[2n_2 + n + n_1 n\,(1 + 2n_3)\right]\left(i\omega'\right)^2 + 2\,(n_1 n_3 + n_2 \cdot n)\left(i\omega'\right) + 2n_1 \cdot n_3 \cdot n.$$

The integration procedure yield:

$$D_{x_g} = \frac{4D_x \cdot n}{\pi} \cdot \frac{\pi(-1)^6 \cdot M_5}{a_0 \Delta_5} = n\frac{4D_x M_5}{a_0 \Delta_5}, \tag{4.11}$$

where

$$M_5 = -a_0\left\{a_5\left[b_1\,(a_3 \cdot a_4 - a_2 a_5) + b_2\,(a_0 a_5 - a_1 \cdot a_4) + b_3(a_1 \cdot a_2 - a_0 a_3)\right] + b_4\left[a_1\,(a_1 \cdot a_4 - a_2 \cdot a_3) + a_0(a_3^2 - a_1 a_5)\right]\right\},$$

$$\Delta_5 = -a_5\left\{a_0^2 \cdot a_5^2 - 2a_0 a_1 \cdot a_4 \cdot a_5 - a_0 a_2 a_3 a_5 + a_0 \cdot a_3^2 a_4 + a_1^2 a_4^2 + a_1 \cdot a_2^2 \cdot a_5 - a_1 \cdot a_2 \cdot a_3 \cdot a_4\right\},$$

$$a_0 = 1, \qquad a_1 = 2n_2 + n, \qquad a_2 = 1 + n_1\,(1 + 2n_3) + 2n_2 n,$$

$$a_3 = 2n_2 + n + n_1 \cdot n\,(1 + 2n_3), \qquad a_4 = 2(n_1 n_3 + n_2 \cdot n), \qquad a_5 = 2n_1 n_3 n,$$

$$b_0 = 0, \qquad b_1 = -n_2^2, \qquad b_2 = (n_1 n_3)^2 - 2n_2^2,$$

$$b_3 = 2(n_1 n_3)^2 - n_2^2, \qquad b_4 = (n_1 n_3)^2.$$

The expression (4.11) can be rewritten as:

$$D_{x_g} = 4D_x \cdot n I_5 \tag{4.12}$$

where

$$I_5 = \frac{M_5}{a_0 \Delta_5}. \tag{4.13}$$

The product $D_x \cdot n$ expresses the random input parameters whilst the expression (4.13) is associated with the features of the investigated object.

Figure 3 shows the variability of I_5 depending on $n_1 = m_A/m_g$ for two values of n_2 ($n = \frac{\alpha}{p_{10}} = \alpha\sqrt{\frac{m_A}{k_g}} = 0.1$ – associated with the applied input and for $n_3 = \frac{k}{k_g} = 0.01$ associated with the features of the investigated object).

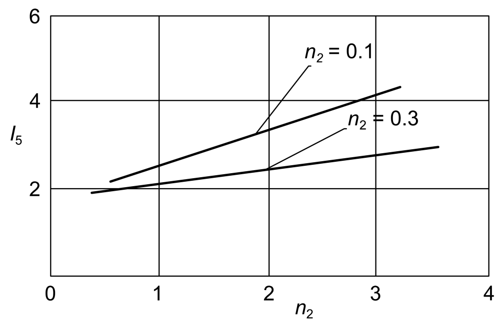

FIG. 3. Variation of I_5 depending on $n_1 = \frac{m_A}{m_g}$ for $n = 0.1$ and $n_3 = 0.01$.

Variability range of parameters n_1, n_2, n_3, n was chosen such that they should apply to typical hoisting installations currently designed and operated in Poland [5].

Basing on Fig. 3, the parameters of the hoisting installation can be chosen such that for the random characteristic of the acting input being known, the admissible displacement limits should not be exceeded. In the case considered here, the standard deviation of displacement of the skip head (modelled as mass m_g in Fig. 2) can be derived from the formula:

$$\sigma\left(x_g\right) = \sqrt{D_{x_g}}. \tag{4.14}$$

Probability of the absolute displacement not exceeding $\xi\sigma(x_g)$ becomes:

$$P\left\{|x_g| \le \xi\sigma(x_g)\right\} = \Phi(\xi), \tag{4.15}$$

where

$$\Phi\left(\xi\right) = \frac{2}{\sqrt{2\pi}}\int\limits_0^{\xi} e^{\frac{-x^2}{2}}dx \tag{4.16}$$

is the integral of the probability function.

5. Conclusions

The dynamic behaviour of hoisting installation was investigated during the normal duty cycle (conveyance travel with the fixed velocity $v = \text{const}$) and basing on the assumption that horizontal displacements of lumped masses in the modelled system are induced by random irregularities and misalignments of the guide strings. Spectral densities of displacement of selected masses were obtained accordingly. The analytical formula is provided expressing spectral density of the skip head mass displacement. Thus derived formula and its graphic interpretation allow the system's parameters to be chosen such that the displacements of modelled points should not exceed the admissible levels specified in applicable mining regulations.

The procedure yields the variances of interaction forces between the shaft steelwork and the skip, which is a necessary condition for effective control of the skip's profile in the context of its endurance parameters. This issue will be addressed in more detail in a separate study.

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