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The Reliability Analysis of Sheet Pile Wall Located in Soil with Random Properties Based on CPTu Results

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The paper deals with reliability analysis of sheet pile wall located in soil with strong spatial variability of strength properties. The purely frictional soil with the strength governed with Mohr-Coulomb criterion has been considered. Spatial variability of friction has been described using random fields theory. Implementation of Local Average Subdivision (LAS) algorithm has been used for generation of individual realizations of the field. Mean and variance as well as fluctuation scale of friction angle has been estimated for assumed exponential correlation model using the results obtained with CPTu testing in natural non-cohesive soil. Using the generated unidirectional field of friction the probabilistic analysis of sheet pile wall has been performed with finite difference method (FLAC). In the final step of the calculation, reliability analysis have been applied. The obtained results prove usefulness of the presented methodology for reliability based design of sheet pile walls.

Key words: sheet pile wall, random fields theory, structural reliability.

1. Introduction

In the last decades, there is a growing need of reliability analysis for newly designed engineering structures. So-called reliability based design become one of the design guidance according to modern codes of practice (e.g. code [4]). The reliability approach seem to be particularly important in geotechnics, where the designed construction occurs in the direct contact with natural soil medium which is characterized by strong spatial variability of properties [2, 6].

In the recent years, in probabilistic modeling of soil medium the methods which combines random fields theory together with Monte Carlo simulations are often used (e.g. RFEM [1] is particularly popular). The common problems with these methods is that the field identification random need large series of data available with conventional soil testing. From the other hand large data series are available in CPTu surveys. This data is usually sufficient for complete identification of random field. The present study shows a complete methodology for assessing the reliability of structure located in the soil medium. The procedure consist of the identification of the random field based on CPTU results followed by Monte-Carlo simulations and reliability assessment. Numerical calculations were performed for the problem of cantilever sheet pile wall in non-cohesive soil. Such structures are often prone to failure, and many factors such as the limited lifetime of the structure, or the consequences caused by its damage often makes the acceptable probability of failure for such a structure higher than in other cases.

2. Statistic measures of a random field

Description of the random field consist of two sets of statistic information: point value statistics as well as the autocorrelation function. If sufficient data series is available all the information on point statics can be obtained from the histogram. For the limited data series the distribution of random variable is often assumed (usually normal or lognormal distribution for the soil properties) and mean and variance, are calculated using formulas from their unbiased estimators:

$$\widehat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad \widehat{\sigma}^2 = \frac{1}{n+1}\sum_{i=1}^{n}\left(X_i - \widehat{\mu}\right)^2, \tag{2.1}$$

where X_i is i-th observation in the data series, n is number of observations.

The autocorrelation function provides information on spatial correlation of observations spaced by distance τ. In geotechnics exponentially fading function is the most popular correlation model. For unidirectional case it is defined as:

$$\rho(\tau) = \exp\left\{\frac{-2\left|\tau\right|}{\theta}\right\}, \tag{2.2}$$

where θ denotes scale of fluctuation which is a basic parameter of the model. Since CPTu test gives quasi-continuous results along the soil profile, the data series allows the estimation of the autocorrelation function. Since the quality of the experimental function depends strongly on number of available data series usually only the value of θ is identified by fitting the theoretical correlation model in the experimental function. For testing points spaced uniformly with distance $\Delta\tau$ the latter can be expressed as:

$$\widehat{\rho}\left(\tau_j\right) = \frac{1}{\widehat{\sigma}^2(n-j)}\sum_{i=1}^{n-j}\left(X_i - \widehat{\mu}\right)\left(X_{i+j} - \widehat{\mu}\right). \tag{2.3}$$

Algorithm for determining the value of fluctuation scale based on CPT data series has been described in literature, e.g. [5]. As has been shown in the cited work, if the field data show a strong trend usually linear de-trending is applied and the fluctuation scale is obtained based on de-trended, normalized observations.

3. Identification of random field based on CPTu results

The CPTu data has been obtained from 9 surveys which has been conducted in south-eastern Poland, on a regular grid with size 15 × 15 m. For the identification of random field the results from selected uniform non-cohesive layer at the depth from 9.5 to 13.5 m has been used.

It has been assumed that the most important property for considered limit state is soil friction angle. This parameter has been chosen to be random variable. Since the values measured in the field with CPTu device are cone resistance q_c and sleeve friction f_s and not directly a friction angle the value of the latter has been estimated using correlation between those quantities. Correlation obtained by Robertson and Campanella [7], has been used:

$$\phi' = \arctan\left[0.1 + 0.38 \cdot \log\left(\frac{q_t}{\sigma'_{vo}}\right)\right], \tag{3.1}$$

where $q_t = q_c + (1 - a_{net})u_2$ is cone resistance corrected for pore pressure, σ'_{vo} is effective vertical overburden stress. The above formula has been applied for the results obtained in all testing points resulting in new data series for the friction. For the assumed normal distribution the mean and variance of the friction angle has been obtained using formulas (2.1). The histogram of data has been presented in Fig. 1d. Interestingly, although the measured values of q_c and f_s showed strong linear trend the values of ϕ' obtained from correlation was constant over the entire thickness of the layer.

All three sets of data (namely q_c, f_s and ϕ') have been subsequently used to identify the fluctuation scales. The correlation scale has been derived by fitting experimental correlation function obtained for de-trended normalized observations with theoretical model. Exponential correlation model (2.2) has been assumed. Since the distance between testing points was too large to estimate horizontal scale of fluctuation only the vertical one has been identified. The obtained values of the scales for cone resistance, sleeve friction and friction angle were very similar, namely $\theta_{qc} = 0.438$ m, $\theta_{fs} = 0.421$ m, $\theta_{\phi} = 0.432$ m, respectively. The values of ϕ' over the depth of the layer (a), the de-trended values (b) and fitting the experimental correlation function with theoretical model (c) has been presented in Fig. 1.

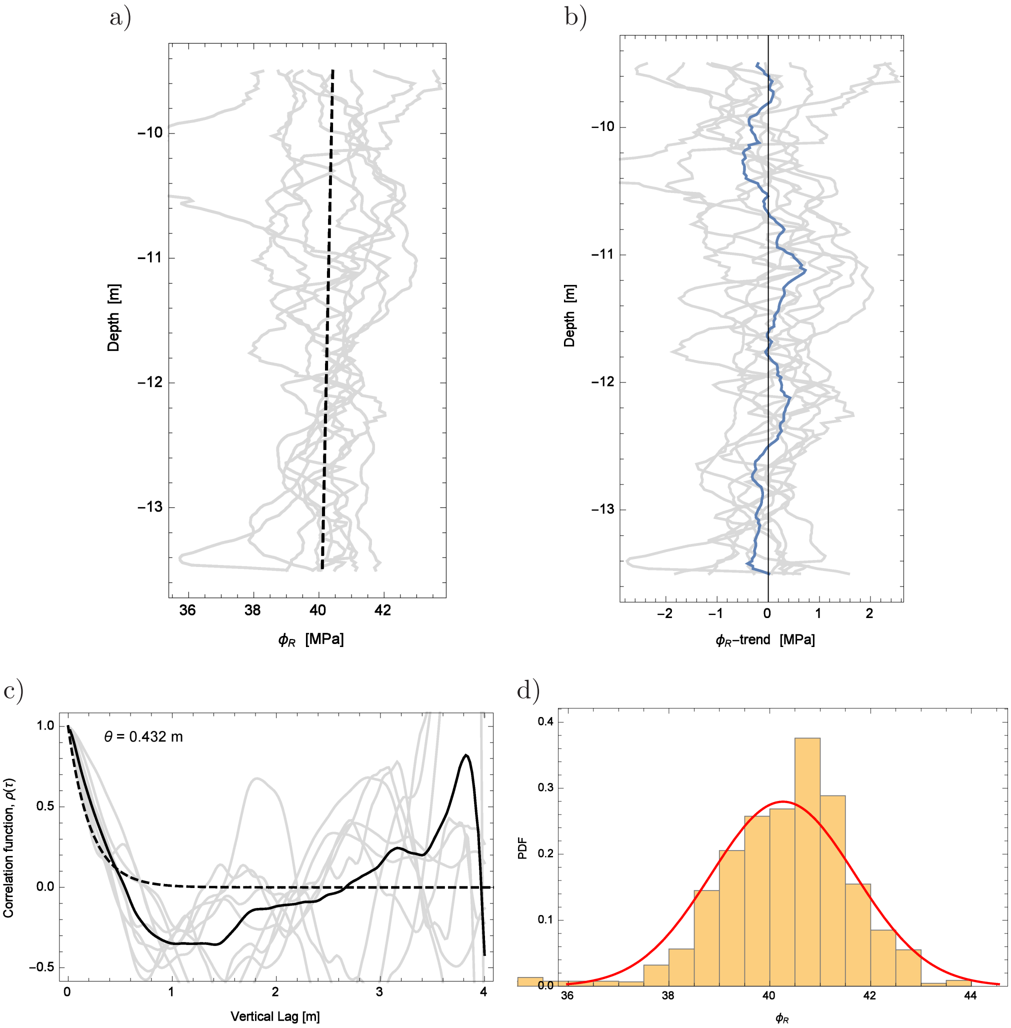

Fig. 1. a) ϕ' values for all test (gray) with mean linear trend line (dashed); b) variability of detrended mean values of ϕ' over depth; c) fitting the experimental correlation function (solid black) with theoretical model (dashed); d) histogram of the values of ϕ' obtained from correlation (3.1).

4. Numerical calculations

The identified measures of random field of friction angle has been used for generation of random field for numerical model. The boundary value problem of 4 m excavation supported with 8 m cantilever sheet pile wall has been analyzed. The individual realizations of Monte Carlo simulation has been solved using FLAC software. The geometry and boundary conditions of the 2-D model used for calculations has been shown in Fig. 2. The moment of inertia of sheet pile wall and the area of its cross section have been assumed to be equal 11.54E–5 m^4/m and 1.0E–3 m^2/m, respectively (GU 67 profile). Upper surface loading has been assumed equal 10 kPa. The soil has been assumed as elastic-perfectly plastic with non-associated flow rule. The values of the friction angle has been described using random field generated separately for each realization with parameters obtained from CPTu (for simplicity $\mu = 40°$, $\sigma = 1.5°$, $\theta = 0.5$ m have been used). Since only the data on vertical fluctuation scale has been available the field has been generated as unidirectional: to account for anisotropy of the field the horizontal scale of fluctuation has been assumed to be infinite. Resulting random field of friction with variability in only one direction has been generated using implementation of LAS algorithm [3]. The mesh density has been assumed such that the model has been vertically divided into 64 layers with 0.25 m of thickness. The values of the cohesion, unit weight and dilations angle has been assumed deterministically as 1 kPa, 20 kN/m^3 and 4°, respectively. The strength parameters of interface layer modeled between sheet pile wall and the soil follow values of the random field reducing the friction angle with the factor of 1/3. The process of excavation has been modeled in four steps – in each step 1 m thick layer of the soil has been removed. The Monte Carlo Simulations consisted of 1000 realizations with the total estimated calculation time on the modern PC equal to 2 days. Both displacement of the top of the wall as well as maximum bending moment has been tested. Due to small values of the obtained moment only the results of displacement have been presented. Apart from simulation with identified fluctuation scale two different simulation has been also performed (each consisted of 1000 realization). In the first case the scale has been assumed as infinity (in single realization value of friction angle stays constant over the domain). In the second each of 0.25 m thick layers has been assumed as independent (for each layer angle has been generated independently) resulting in θ being close to 0.

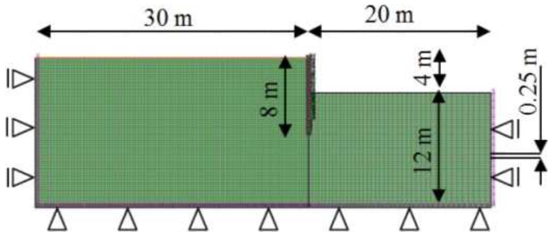

FIG. 2. Geometry and boundary conditions of the finite difference model.

5. Results of Monte Carlo Simulation

The obtained values of displacement of top of the sheet pile wall based on 1000 MCS runs have been presented in Fig. 3 in form of cumulative distribu-

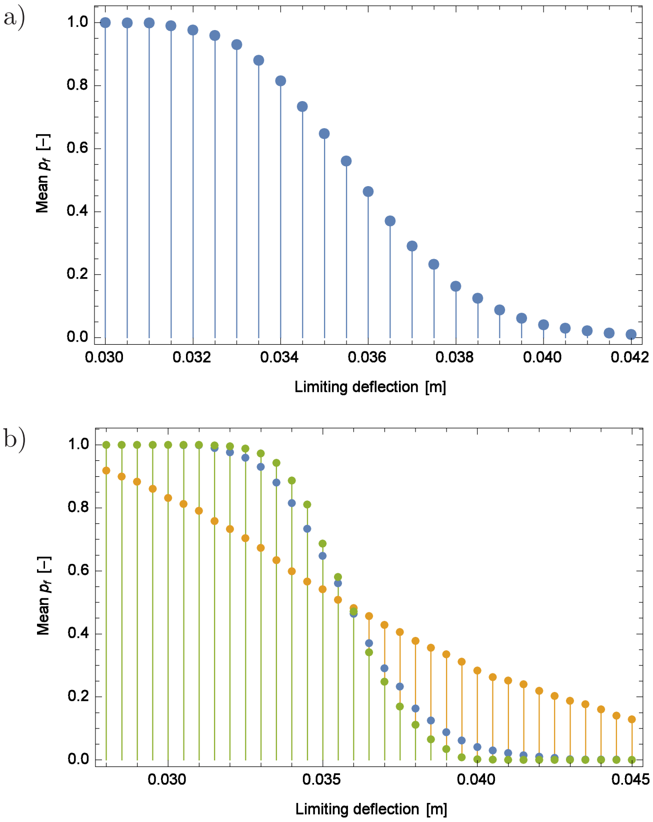

FIG. 3. a) Probability of failure for the assumed displacements top of the wall; b) probability of failure for the assumed displacements top of the wall for different values scale of fluctuation: blue $\theta = 0.50$ m, green $\theta \approx 0.0$ m, yellow $\theta = \infty$.

tion that describes the probability of failure for differently assumed criterion of serviceability limit state. As can be seen in Fig. 3b the case with limited value of fluctuation scale is intermediate case. The case of independent random 0.25 m layers resulting in fluctuation scale being almost 0 makes the curve of the distribution steeper. On the contrary for the infinite fluctuation scale obtained cumulative distribution is almost linear with mild slope.

The values of displacement of top of the sheet pile wall obtained from Monte-Carlo simulations has been summarized in Table 1. Both mean and standard deviation as well as probability of exceeding of two selected values of ultimate displacements a_{ult} has been presented. As can be seen even small change in serviceability condition strongly changes probability of failure. Also it can be noticed that solution obtained for $\theta = \infty$ is very conservative and not economic whereas solution for scale equal to zero is too optimistic in comparison to the real situation.

Table 1. Displacements of the sheet pile wall top and probability of failure.

Scale of fluctuation [m]	Mean displacement μ [cm]	Standard deviation σ [cm]	Probability of failure for $a_{ult} = 3.75$ cm [–]	Probability of failure for $a_{ult} = 4.0$ cm [–]
$\theta = 0.50$	3.59	0.21	0.233	0.004
$\theta \approx 0.00$ (random layers)	3.66	0.67	0.170	0.002
$\theta = \infty$	3.59	0.16	0.406	0.28

6. Concluding remarks

Reliability analysis of cantilever sheet pile wall serviceability limit state has been performed. Random field of soil friction angle used in the analysis has been identified based on CPTu results. It has been shown that spatial variability has a strong impact on the predicted variability of the maximum lateral wall deflection. The presented method requires relatively lot of computing time especially for small values of failure probability.

Based on the results of the presented research, the following conclusions can be drawn:

- CPTu results provide sufficient data series for identification of random field of soil properties,
- apart from point statistic the fluctuation scale of the random field is important parameter in reliability assessment,
- presented results illustrate that neglecting spatial soil variability in the geotechnical design can result in either overestimation or underestimation of the probability of failure.

References

1. Allahverdizadeh P., Griffiths D.V., Fenton G.A., *The Random Finite Element Method (RFEM) in probabilistic slope stability analysis with consideration of spatial variability of soil properties*, [in:] IFCEE 2015, pp. 1946–1955, 2015, doi: 10.1061/9780784479087.178.

2. Fenton G.A., Griffiths D.V., *Risk assessment in geotechnical engineering*, Wiley, 2008.

3. Fenton G.A., Vanmarcke E.H., *Simulation of random fields via local average subdivision*, Journal of Engineering Mechanics, **116**(8): 1733–1749, 1990.

4. International Standard ISO 2394:1998, *General principles of engineering structure reliability*, 1998.

5. Lloret-Cabot M., Fenton G.A., Hicks M.A., *On the estimation of scale of fluctuation in geostatistics*, Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, **8**(2): 129–140, 2014.

6. Phoon K.K., Ching J. [Eds.], *Risk and reliability in geotechnical engineering*, CRC Press., 2014.

7. Robertson P.K., Campanella R.G., *Interpretation of cone penetration tests. Part I: Sand*, Canadian Geotechnical Journal, **20**(4): 718–733, 1983.

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