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# CONSTITUTIVE LAWS OF VISCOPLASTICITY IN DYNAMIC RESPONSE OF STRUCTURES

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The aim of the paper is to discuss the differences which appear in elasto-viscoplastic dynamic analysis of structures using different types of constitutive equations. Three types of constitutive laws are examined: Chaboche, Perzyna and Bodner–Partom models. Material data were taken for calculations from literature. It is shown that for a wide range of strain rates, these different sets of laws produce different functions of yield limit, even for the same material and the same constitutive formulation. Their choice has a large influence on the calculus of dynamic behaviour of structures, where during the deformation process, the strain rates change their values very much. On the other hand, different considerations of hardening in constitutive rules induce also significant differences in dynamic response of viscoplastic structures. We give examples where the published results of experimental investigations in different laboratories lead even to contradictory effects. The authors propose some sets of material parameters which improve the dynamic results and allow to compare the calculated vibrations with different constitutive laws.

## 1. Introduction

The knowledge of the viscoplastic behaviour of structures has an indisputable technical importance, and as such, it has been the subject of scientific research for many years. The first works on dynamic response of viscoplastic structures were published already in the sixties [1–6]. Among many models of elasto-viscoplastic constitutive equations [7], the majority of authors, for their geometrically linear or nonlinear investigations of dynamics, use the constitutive viscoplastic law of Perzyna [8]. Lack of experimental results limits the number of works where numerical comparison of two or more different types of constitutive laws is published. Moreover, the experimental results are rarely published [6, 9, 10].

For static problems, the numerical comparison of the Chaboche [11] and Bodner–Partom [12] set of constitutive equations can be often found. Usually the experiments are performed for uni-dimensional specimens [13–15]. In [13], the numerical analysis of viscoplastic behaviour of INCO718 specimens at 650°C is presented. Different types of static tests to compare the Chaboche and Bodner - Partom theories have been described. Some of them are compared with the experimental results and in some cases, a good agreement between both types of constitutive laws has been obtained. The comparison of the same types of laws for INCO738 at 650°C can be found in [14, 15], where static tests have also been presented. It has been observed that both constitutive models adjusted to the material response on a uniaxial data base, are capable of describing the multiaxial out-of-phase deformation behaviour accurately.

Some results from the elasto-plastic dynamic analysis of structures [16] can be used for limited elasto-viscoplastic approach, but in this case some additional approximations are necessary. So when differences between two different constitutive laws are observed, it is difficult to justify which model produces better results.

In papers [17, 18], the present authors considered the problems of geometrically nonlinear vibrations of elastic-viscoplastic plates and shells. It has been shown that the Chaboche and Bodner-Partom types of constitutive equations lead to significant differences in the dynamic approach. To the authors knowledge, it is not possible to find in the literature the papers on dynamic viscoplastic behaviour of structures based on different types of constitutive laws where the results would be in good agreement.

The aim of the paper is to present the reasons of these differences and to propose the methods of minimising them. The problem has been investigated in two stages. At first, uniaxial numerical calculations necessary for obtaining a good agreement of both theories in dynamically important tests were made. Then the results of material parameters identification were verified by the finite element analysis of a circular plate under impact loading.

## 2. General assumptions

We considered the homogeneous and isotropic material with elasto-viscoplastic properties. It is assumed that strains are small enough, so that the additive decomposition of the Green-Lagrange strain rate tensor $\dot{\mathbf{E}}$ into a purely elastic part $\dot{\mathbf{E}}^E$ and an inelastic part $\dot{\mathbf{E}}^I$ is justified:

$$\dot{\mathbf{E}} = \dot{\mathbf{E}}^E + \dot{\mathbf{E}}^I.$$

The rates of the second Piola-Kirchhoff stress tensor $\dot{\mathbf{s}}$ can be calculated using the generalised tensor of elastic coefficients $\mathbf{D}$:

$$\dot{\mathbf{s}} = \mathbf{D} : \dot{\mathbf{E}}^E = \mathbf{D} : \left(\dot{\mathbf{E}} - \dot{\mathbf{E}}^I\right). \tag{2.1}$$

The expressions for the inelastic strain rates $\dot{\mathbf{E}}^I$ for different types of the constitutive laws used in the paper are given in Table 1. Three types of viscoplastic laws

**Table 1. Constitutive equations.**

| Model | Perzyna | Chaboche | Bodner–Partom |
|---|---|---|---|
| inelastic strain rate | $\dot{\mathbf{E}}^I = \frac{3}{2}\dot{p}\frac{\mathbf{s}'}{J(\mathbf{s}')}$ | $\dot{\mathbf{E}}^I = \frac{3}{2}\dot{p}\frac{\mathbf{s}' - \mathbf{X}'}{J(\mathbf{s}' - \mathbf{X}')}$ | $\dot{\mathbf{E}}^I = \frac{3}{2}\dot{p}\frac{\mathbf{s}'}{J(\mathbf{s}')}$ |
| cumulated inelastic strain rate | $\dot{p} = \gamma\left\langle\frac{J(\mathbf{s}') - k}{k}\right\rangle^n$ | $\dot{p} = \gamma\left\langle\frac{J(\mathbf{s}' - \mathbf{X}') - R - k}{K}\right\rangle^n$ | equation A<br>$\dot{p} = \frac{2D_0}{\sqrt{3}}\exp\left[-\left(\frac{Z}{J(\mathbf{s}')}\right)^{2n}\frac{n+1}{2n}\right]$<br>equation B<br>$\dot{p} = \frac{2D_0}{\sqrt{3}}\exp\left[-\left(\frac{Z}{J(\mathbf{s}')}\right)^{2n}\right]$ |
| | $J(\mathbf{a} = \left(\frac{3}{2}\mathbf{a} : \mathbf{a}\right)^{1/2} = \left(\frac{3}{2}a^{ij}a_{ij}\right)^{1/2}$<br>$\langle x\rangle = \frac{1}{2}(x + \lvert x\rvert)$ | | $Z = R + \mathbf{X} : \frac{\mathbf{s}}{J(\mathbf{s})}$<br>$\dot{W}^I = \mathbf{s} : \dot{\mathbf{E}}^I$ |
| isotropic hardening | – | $\dot{R} = b(R_1 - R)\dot{p}$<br>$R(t = 0) = 0$ | $\dot{R} = m_1(R_1 - R)\dot{W}^I$<br>$R(t = 0) = R_0$ |
| kinematic hardening | – | $\dot{\mathbf{X}} = \frac{2}{3}a\dot{\mathbf{E}}^I - cf(p)\mathbf{X}\dot{p}$<br>$f(p) = l + (1 - l)e^{-\beta p}$ | $\dot{\mathbf{X}} = m_2\left(\frac{3}{2}D_1\frac{\mathbf{s}}{J(\mathbf{s})} - \mathbf{X}\right)\dot{W}^I$ |
| material parameters | $\gamma,\ k,\ n$ | $k, K, n, a, c, b, R_1, \gamma, l, \beta$ | $n, D_0, D_1, R_0, R_1, m_1, m_2$ |

are considered: the PERZYNA [8], CHABOCHE [11], and BODNER–PARTOM [12] models. They express the inelastic strain rate as a function of the stress deviator $\mathbf{s}'$ and, except for the Perzyna law, as a set of functions of internal variables $R$ and $\mathbf{X}$, linked respectively with isotropic and kinematic hardening. Additionally the Bodner - Partom approach, contrary to the Chaboche and Perzyna models, does not require any explicit yield function to govern the inelastic response of a material. In the Bodner - Partom model, the viscoplastic effects appear from the very beginning of the deformation process. Although the general mathematical framework of this model is similar, it significantly differs from the previously discussed models. For example, in this model, as in the Perzyna model, the inelastic strain rate is expressed as a function of stress but not as a function of the backstress $\mathbf{X}$ (the case of the Chaboche model). Also, the measure of inelastic hardening in this model is assumed to be a function of the inelastic work and not the inelastic strain, as it is done in the Chaboche model. It should be noticed that in that particular case, when $\mathbf{X} = R = 0$, $K = k$, the Chaboche model transforms into the Perzyna model.

To perform the calculations in each theory, the constitutive equations must be integrated. From many numerical routines, we have chosen the trapezoidal integration scheme as the simplest one. The algorithm of the integration is presented in [17].

The dynamic analysis of structures has been restricted to plates and shells. For this approach the first order shear deformation and the moderate rotation theory [19, 20] was applied. For the numerical solution of the problem, we used the finite element method. The structure was discretized by the nine-node isoparametric shell elements. To integrate the constitutive equations it was necessary to divide the volume of the structure into layers parallel to the middle surface. It was assumed that in each layer stresses are constant across the layer thickness. To integrate the equations of motion, the central difference method in the form described in [21] was used. In this case the stiffness matrix may be not used. All nonlinear effects are included in the vector of balanced forces. This type of integration needs a small time step to be applied to fulfil the stability conditions. Since for the integration of constitutive equations, a relatively small time increment is also required, we applied the same time step for both types of integration.

## 3. PROBLEM OF CHOICE OF MATERIAL PARAMETERS

In papers [17, 18] significant differences between the numerical results produced by the Chaboche and Bodner - Partom laws in some dynamic problems of plates and shells have appeared.

The questions is: do these differences come from the applied constitutive models, or rather from the published experimental data, which could be obtained in different conditions. It is necessary to remind that, according to [11], the Chaboche constitutive equations are usually applied for quasi-static cases, contrary to the Bodner – Partom and Perzyna laws which are also used for faster processes.

One way to compare these laws is to examine sensibility of the initial yield limit to the strain rates. From (2.1), for a uniaxial case we can write

$$\frac{d\sigma}{d\varepsilon} = E\left(1 - \frac{\dot{\varepsilon}^i}{\dot{\varepsilon}}\right), \tag{3.1}$$

where $\sigma$ and $\varepsilon$ are the uniaxial stress and strain respectively; dot stands for time derivative.

We will assume that the specimen is reaching the yield limit for elastic perfectly viscoplastic material when

$$\dot{\varepsilon}^i \to \dot{\varepsilon} \;\Rightarrow\; \frac{d\sigma}{d\varepsilon} = 0.$$

The value of yield stress $\sigma_0$ can be determined from equations for the accumulated inelastic strain rate given in Table 1 as follows:

- for the Perzyna model:

$$\sigma_0 = k\left(\frac{\dot{\varepsilon}\cdot\operatorname{sign}(\sigma)}{\gamma}\right)^{1/n} + k; \tag{3.2}$$

- for the Chaboche model:

$$\sigma_0 = K\left(\dot{\varepsilon}\cdot\operatorname{sign}(\sigma)\right)^{1/n} + k; \tag{3.3}$$

- for the Bodner – Partom model (Equation A):

$$\sigma_0 = \frac{R_0}{\left[\dfrac{2n}{n+1}\cdot\ln\left(\dfrac{2D_0}{\sqrt{3}\cdot\dot{\varepsilon}\cdot\operatorname{sign}(\sigma)}\right)\right]^{1/2n}}. \tag{3.4}$$

For $\dot{\varepsilon} \to 0$ in the Perzyna and Chaboche approach we have $\sigma_0 \to k$, and for the Bodner – Partom equations $\sigma_0 \to 0$, what is in agreement with the assumptions of these formulations. In the Bodner – Partom case, we can consider for the "static" yield limit the value of $\sigma_0$ for very low strain rate, for example less than $10^{-4}\text{s}^{-1}$.

In Fig. 1, we present the initial yield limit as a function of strain rates for INCO718 at 650° C calculated with parameters taken from the literature. The

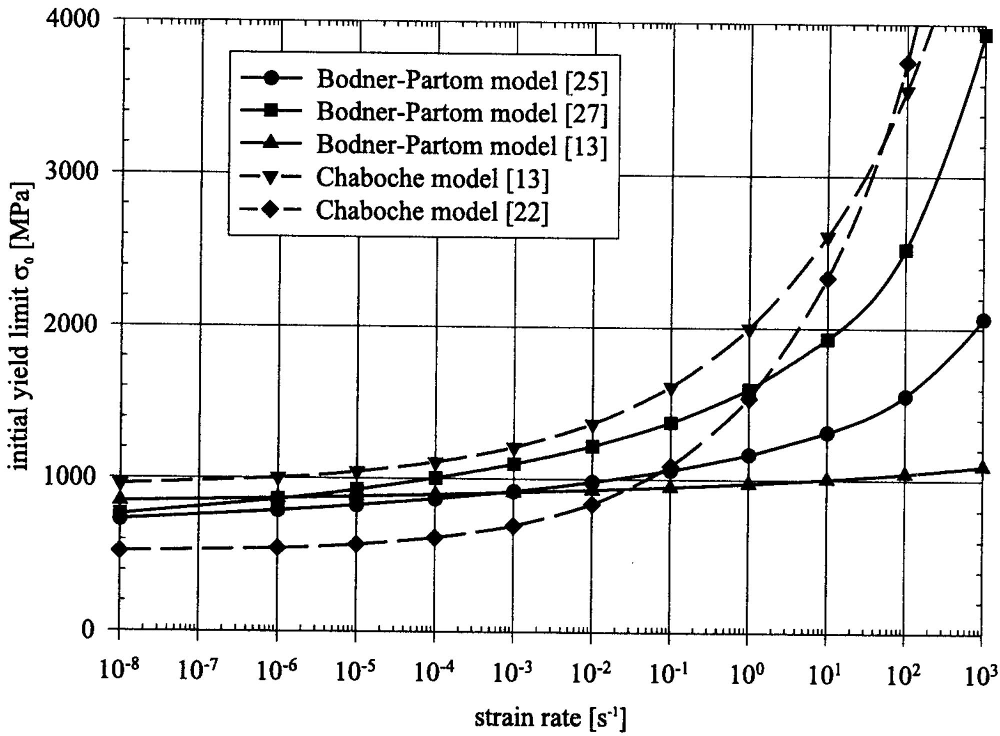


FIG. 1. Initial yield limit as the function of strain rate for INCO718 at 650° C.

values of the material parameters according to the set of constitutive equations which are used are given in Tables 2–4. But we remark that those parameters coming from different laboratories (we hope that under the same conditions) for the same type of constitutive laws are different. Each work gives different values of elastic modulus. With the Chaboche model, the obtained material parameters describe isotropic softening, whereas the same parameters in the Bodner - Partom law describe isotropic hardening. The parameters for the Bodner - Partom law disregard the kinematic hardening. The curves of the initial yield limit presented in Fig. 1 differ significantly depending on which laboratory they are coming from. Even for very small strain rates, the "static" yield limit determined with these

**Table 2. INCO718 at 650° C Chaboche model.**

| | $E$ GPa | $\gamma$ 1/s | $n$ – | $k$ MPa | $K$ MPa | $c$ – | $a$ GPa | $b$ – | $R_1$ MPa | $l$ – | $\beta$ – |
|---|---|---|---|---|---|---|---|---|---|---|---|
| [13] | 170.22 | 1 | 5.1 | 932.2 | 1066.86 | 350 | 972.27 | 60 | −587.44 | 1 | 0 |
| [22] | 159.0 | 1 | 4 | 514.21 | 1023.5 | 500 | 170.0 | 60 | −194.4 | 1 | 0 |
| modified | 169 | 1 | 6 | 780 | 400 | 0 | 0 | 30 | 360 | 1 | 0 |

parameters is quite different. Even within the same set of constitutive equations, differences are remarkable. We have seen that the function of the yield limit calculated using the Chaboche model is more sensitive to changes of strain rates than the same function obtained by means of the Bodner - Partom formulation. In Table 4, the parameters for the Perzyna law have been recalculated from parameters for the Chaboche law given in [22] where the hardening effects had to be neglected.

**Table 3. INCO718 at 650° C Bodner – Partom model (Equation A).**

| | $E$ MPa | $D_0$ 1/s | $n$ – | $R_0$ MPa | $R_1$ MPa | $R_2$ MPa | $D_1$ – | $m_1$ MPa$^{-1}$ | $m_2$ MPa$^{-1}$ |
|---|---|---|---|---|---|---|---|---|---|
| [13] | 162.4 | $10^6$ | 3 | 1621.5 | 1794.7 | 718 | 0 | 0.42 | 0 |
| [25] | 169 | $10^4$ | 1.17 | 3130 | 4140 | 2760 | 0 | 0.024 | 0 |
| [27] | 172 | $1.03\cdot10^4$ | 0.7374 | 6520 | 7030 | 3690 | 0 | 0.686 | 0 |

**Table 4. INCO718 at 650° C Perzyna model.**

| | $E$ GPa | $n$ – | $k$ MPa | $\gamma$ 1/s |
|---|---|---|---|---|
| calculated from [22] | 159 | 4 | 514.2 | 0.0632 |
| modified | 159 | 6 | 780 | 54.98 |

On the contrary, a good agreement exists for INCO738 at 850° C (Fig. 2) calculated using the Chaboche and the Bodner - Partom models with the values of material parameters given in [14, 15] (Tables 5 – 6). It should be stressed that the results for both curves were obtained in the same laboratory. The curves for titanium (Fig. 3) according to results published in three different experimental centres (Tables 7 – 8) are quite similar.

**Table 5. INCO738 at 850° C Chaboche model.**

| | $E$ GPa | $\gamma$ 1/s | $n$ – | $k$ MPa | $K$ MPa | $c$ – | $a$ GPa | $b$ – | $R_1$ MPa | $l$ – | $\beta$ – |
|---|---|---|---|---|---|---|---|---|---|---|---|
| [14, 15] | 149.65 | 1 | 7.7 | 153 | 1050 | 201 | 62.51 | 317 | 0 | 1.1 | 0.04 |

**Table 6. INCO738 at 850° C Bodner–Partom model (Equation B).**

| | $E$ MPa | $D_0$ 1/s | $n$ – | $R_0$ MPa | $R_1$ MPa | $R_2$ MPa | $D_1$ – | $m_1$ MPa$^{-1}$ | $m_2$ MPa$^{-1}$ |
|---|---|---|---|---|---|---|---|---|---|
| [14, 15] | 149.65 | $2.45\cdot10^6$ | 0.289 | $4.18\cdot10^5$ | $3.76\cdot10^5$ | $3.07\cdot10^5$ | $1.54\cdot10^5$ | 0.581 | 0.344 |

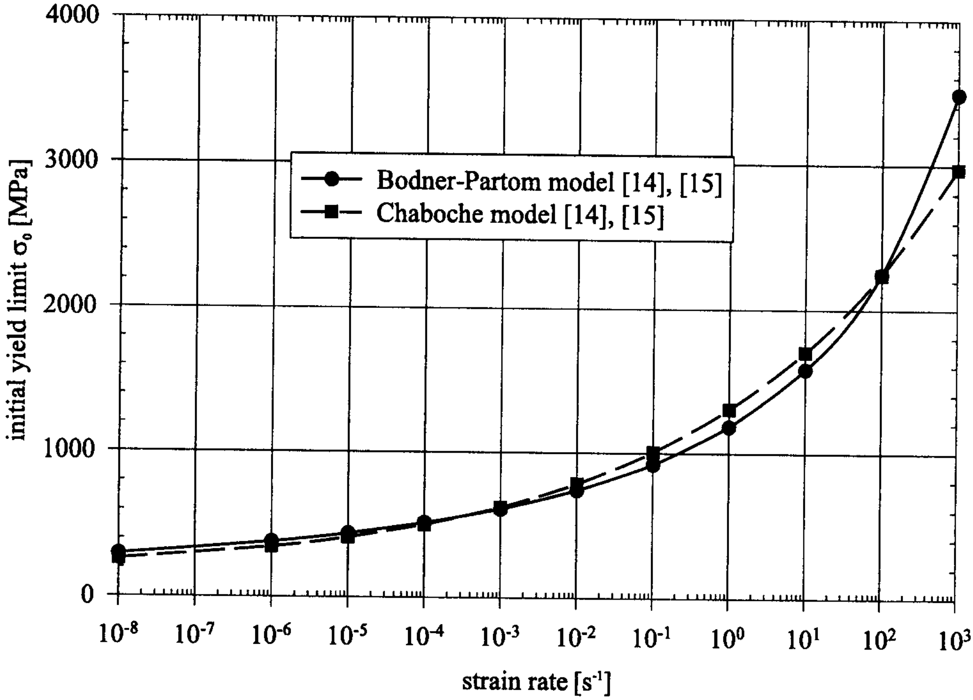


FIG. 2. Initial yield limit as the function of strain rate for INCO738 at 850° C.

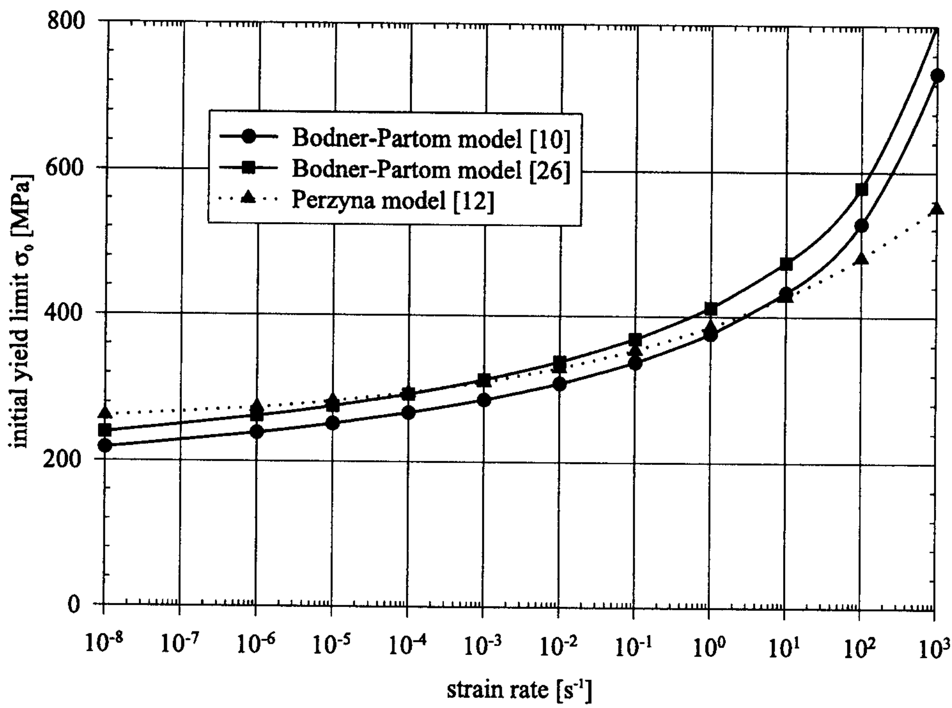


FIG. 3. Initial yield limit as the function of strain rate for titanium at 20° C.

**Table 7. Titanium at 20° C Perzyna model.**

| | $E$ GPa | $n$ – | $k$ MPa | $\gamma$ 1/s |
|---|---|---|---|---|
| [12] | – | 9 | 243 | 120 |
| modified | 118 | 7 | 230 | 5 |

**Table 8. Titanium at 20° C Bodner–Partom model (Equation A).**

| | $E$ MPa | $D_0$ 1/s | $n$ – | $R_0$ MPa | $R_1$ MPa | $R_2$ MPa | $D_1$ – | $m_1$ $\mathrm{MPa}^{-1}$ | $m_2$ $\mathrm{MPa}^{-1}$ |
|---|---|---|---|---|---|---|---|---|---|
| [10] | 118 | $10^4$ | 1 | 1150 | 1400 | 0 | 0 | 0.087 | 0 |
| [26] | – | $10^4$ | 1 | 1260 | 1950 | 0 | 0 | 0.01 | 0 |

## 4. Material parameters fitting for dynamic analysis

Application of the parameters discussed above to a quasi-static case gives a good correlation between the results of calculations of structures based on different models [13–15]. Comparisons were made for rather small and constant strain rates. In dynamics this correlation is much worse. The strain rate in constructions submitted to dynamic loads can change within a wide range. Therefore, in order to be able to compare the dynamic responses, it is necessary to ensure a good agreement of the initial yield limit for the large range of strain rates. In the same way, it is necessary to match the hardening modelling in the different constitutive laws.

For example, in the case of INCO718 it is possible to minimise the differences between the Chaboche and Bodner - Partom model curves giving in the Chaboche model new values of $k$, $K$, $n$ parameters (see last row of Table 2) calculated by the formula (3.3). This change of the Chaboche's parameters can be justified, because this model is used for quasi-static case and low strain rates, contrary to the Bodner - Partom formulation. Effects of these changes are presented in Fig. 4. These new parameter values allowed for a good correlation between the yield limit curves of both models (Fig. 4 A). The curve obtained from the Perzyna law is identical to the modified Chaboche curve. Figure 4 B presents the comparison of cyclic load test (without inertial effects) with the strain rate $\dot{\varepsilon} = 10\,\mathrm{s}^{-1}$ for Chaboche and Bodner - Partom approaches with original parameters. We can observe significant differences. The different approaches used by these two models for the problem of hardening for INCO718 is also easy to see. The im-

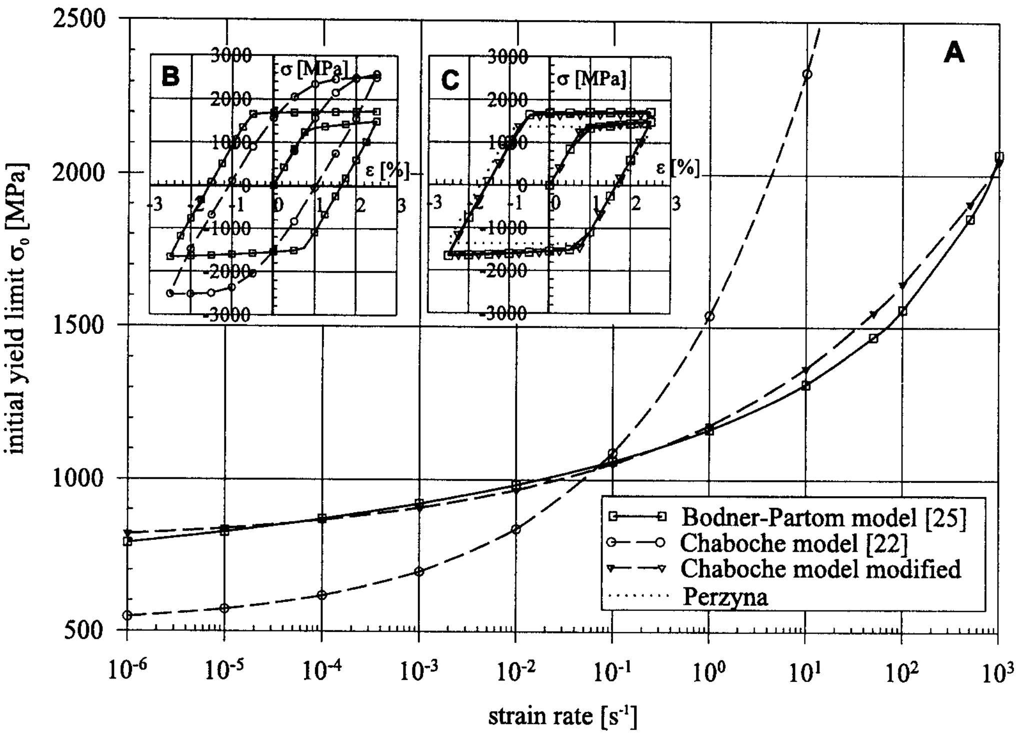


FIG. 4. A) Initial yield limit as the function of strain rate INCO718 at 650° C after modifications, B) Cyclic loading test – original data, C) Cyclic loading test – with modified parameters.

provement of this correlation after modification of the Chaboche law parameters is shown in Fig. 4 C. But the differences due to hardening between the Perzyna and both Bodner-Partom and modified Chaboche approach still exist. Values of material parameters after modifications are given in the last rows of Table 2 and 4.

Similar changes can be made for titanium. We adjust the Perzyna law parameters in such a way that the curve of initial yield limit is superimposed on the curve obtained according to the Bodner-Partom formulation (Fig. 5). In this case, the proposed values of the parameters $k$, and $\gamma$, calculated from Eq. (3.2) are given in the last row of Table 7. But even that change does not allow us to arrive at the same dynamic response with both data sets. In order to compare both models, besides the changes of the parameters for the Perzyna formulation to adopt the initial yield function, it was necessary to cancel both types of hardening of the Bodner-Partom model.

For INCO738 no adjusting of material parameters has been done due to a good agreement in the static behaviour of the two laws presented in [14, 15].

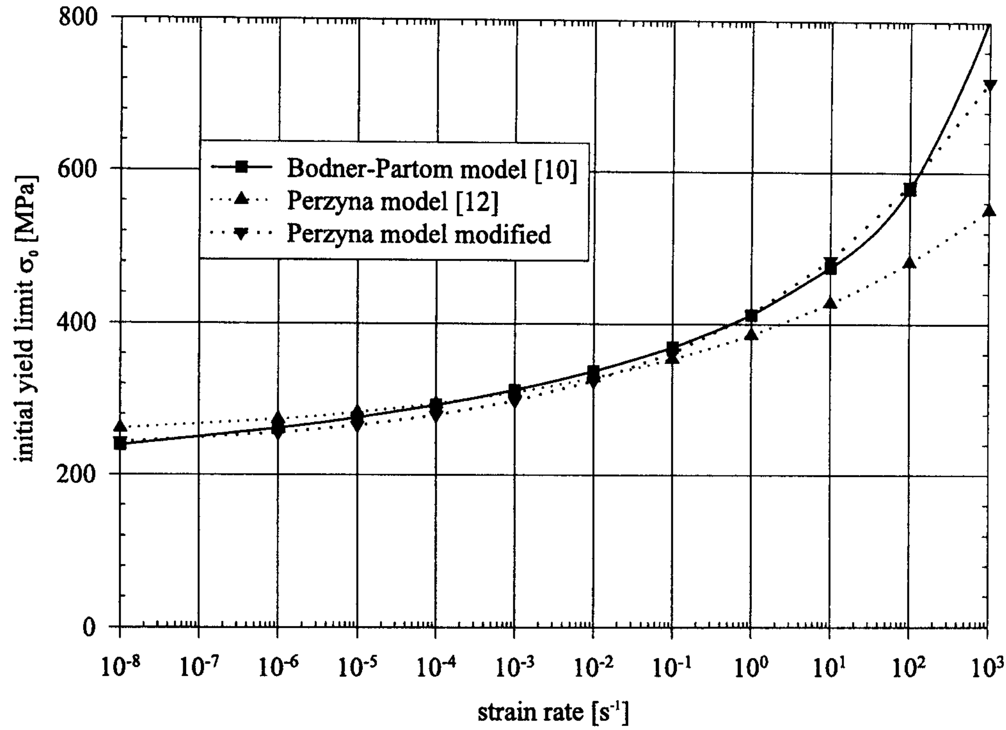


FIG. 5. Initial yield limit as the function of strain rate for titanium at 20° C after modifications.

## 5. Transient analysis of circular plate

To examine the dynamic behaviour of elasto-viscoplastic structures, the set of results obtained for the circular plate shown in Fig. 6 is presented. The plate was subjected to load impulse being a function of time $p(x, y, t) = p(t)$ corresponding to explosion [23, 24], the diagram of which is also given in the same picture. Taking advantage of the symmetry of the problem, a quarter of the plate was divided into eight nine-node isoparametric finite elements. To integrate the viscoplastic laws, the thickness was divided into four equal layers (a larger number of layers did not change the results, see [17]). The plate was investigated for all three materials studied in previous sections. The centre deflection functions for INCO718 plate are shown in Fig. 7. Application of the material data given in [22] for the Chaboche law leads to significant difference between the graph obtained for the Bodner-Partom equations and the data taken from [25]. The vibrations according to [22] have a larger amplitude and a shorter "period" of vibrations. Even the average level of deflection is much higher.

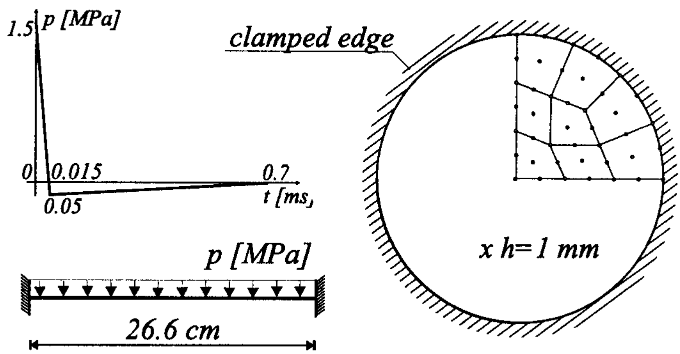


FIG. 6. Geometry, finite element mesh of circular plate and loading function.

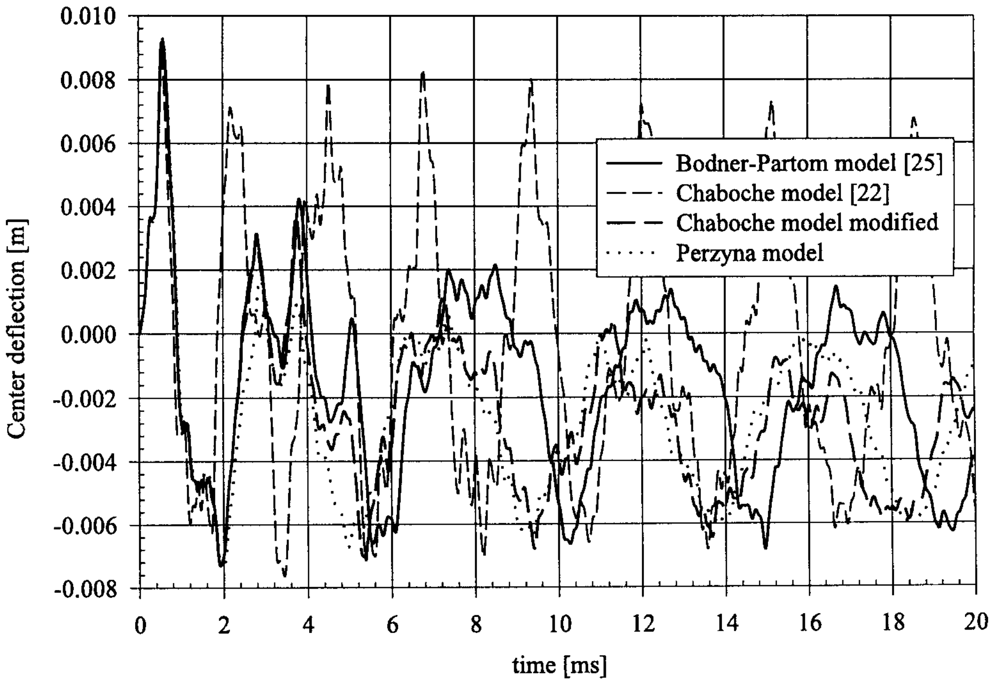


FIG. 7. Dynamic response of INCO718 plate at 650° C to explosive type of loading.

A relatively good correlation between graphs for the Bodner-Partom and Perzyna sets of data recalculated from the data published in [22] can be observed. Lack of hardening functions in the Perzyna formulation can explain some differences. Change of material parameters in the Chaboche law according to the fitting of the yield limit for different strain rates and hardening evolution improves the results. During 4 ms, the two vibration curves of the middle point of the plate are almost identical. Above 4 ms, the amplitude of vibrations in the Chaboche model is smaller.

In spite of a good agreement of yield limit for INCO738 as a function of strain rates and the same static behaviour, in dynamic response the graphs of the middle point deflections differ very much (Fig. 8). Here again, vibrations are similar only up to 4 ms. Afterwards, a rapid decrease of amplitude for the Bodner - Partom model can be observed. Vibrations for the Chaboche model are qualitatively similar to those for INCO718.

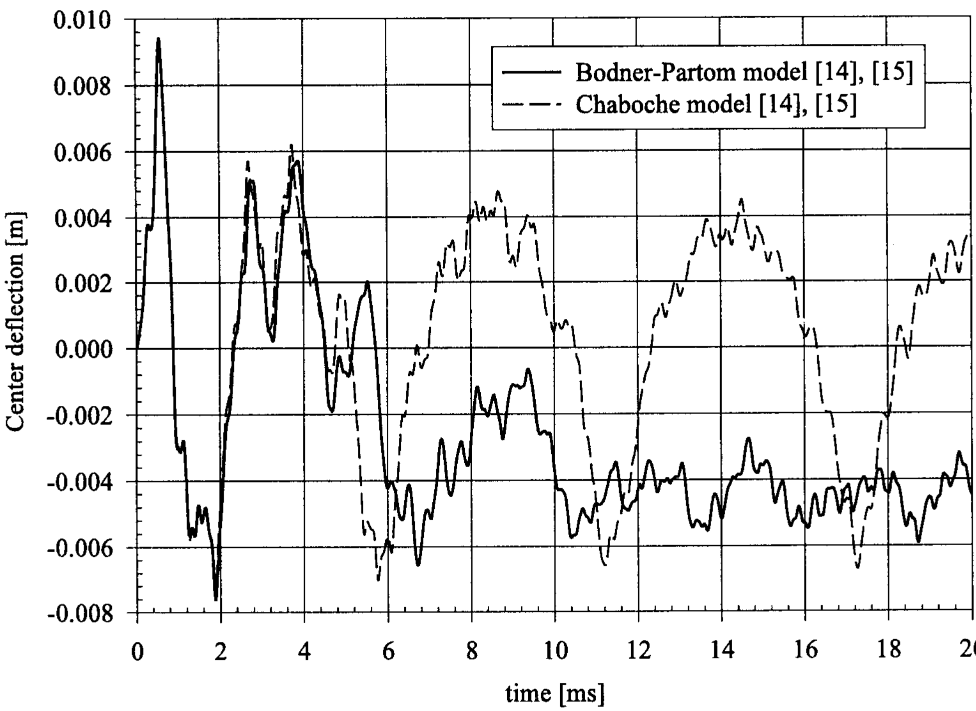


FIG. 8. Dynamic response of INCO738 plate at 850° C to explosive type of loading.

In Fig. 9 the time functions of the centre deflection for the titanium plate are presented. Also here a significant difference between the solution obtained using the Bodner - Partom material parameters proposed in [26] and solution for the Perzyna-type constitutive equations with parameters given in [12] can be seen. Because the Perzyna law can not express the evolution of the hardening, therefore calculations using the elastic perfectly viscoplastic model of Bodner - Partom was performed. The obtained curves of the centre deflection are still not in good agreement with the Perzyna curves. This can be explained by differences which appeared for high strain rate in uniaxial analysis. If the proposed modification of the physical parameters is made, then the agreement between both models

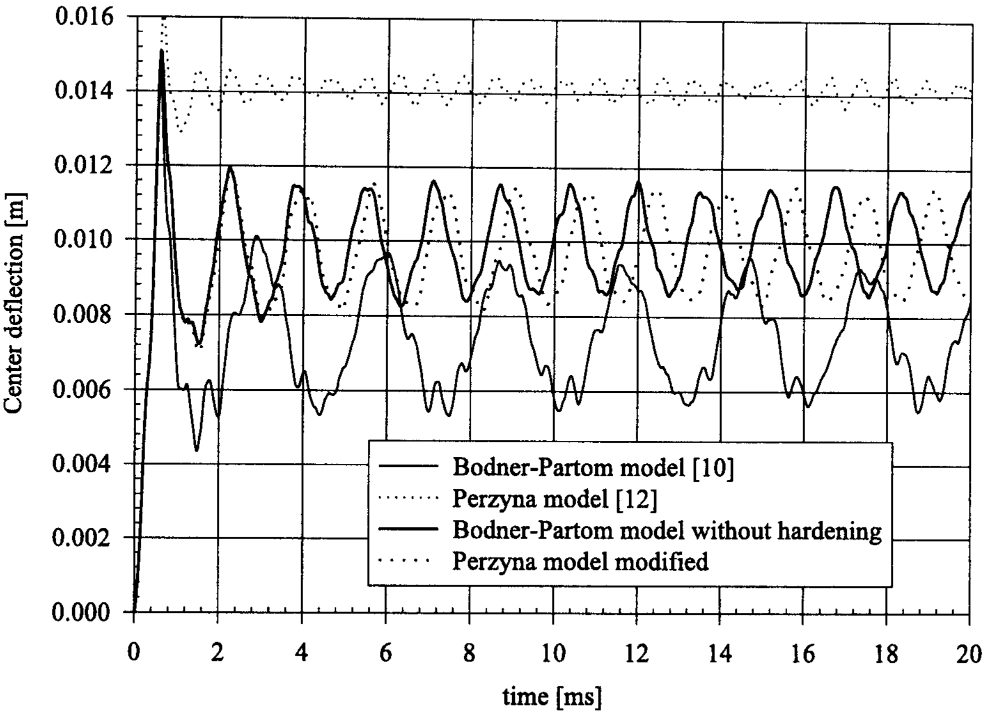


FIG. 9. Dynamic response of titanium plate at 20° C to explosive type of loading.

becomes satisfactory. The amplitude of vibrations and average deflections are the same. Only some differences in the period of vibrations can be observed.

## 6. Conclusions

The analysis presented above shows that the results of calculations of the dynamic behaviour of a construction strongly depend on the constitutive law which is applied. The values of material parameters available in the literature (the conditions of evaluation of which are generally unknown) can lead for certain materials, giving a good agreement in statics, to significant differences in dynamic approach. In this case the strain rates change their values very much and the comparison of numerical results for different types of constitutive equations is possible if the initial yield limit (as a function of strain rates) is the same for each law. Evidently, the same description of the hardening should be used too. This paper proposes some modifications of material parameters which improve the dynamic answer of the structures. Nevertheless, modifications in this paper were mainly proposed for the Chaboche and the Perzyna law, but it can also happen that the parameters of the Bodner - Partom formulation should also be changed. To decide which laws should be modified, it is necessary to compare the numerical results with laboratory experiments, which unfortunately are not available. The authors wanted only to show the origin of the problem and suggest the ways of its solution, but not to decide which set of parameters should be adjusted or which type of constitutive equations is better.

There is also the question which should be answered. Why under a similar yield condition and hardening description, different laws lead, after a short time period, to considerable differences in construction behaviour? The research is continued in this direction.

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