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Efficient Generator of Structural Topologies based on Irregular Cellular Automata

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Recent development of Cellular Automata implementation into optimal design problems has shown that the automaton can be an effective tool for generation of optimal topologies in engineering applications. Nevertheless, the vast majority of results have been obtained to date for regular lattices of cells. The aim of the present paper is therefore to extend the concept of Cellular Automata towards irregular grid of cells related to non-regular mesh of finite elements. Introducing irregular lattice of cells allows to reduce number of design variables without losing accuracy of results and without excessive increase of number of elements caused by using fine mesh for a whole structure. This paper proposes a novel Irregular Cellular Automata formulation that can be adapted to topology optimization of real structural elements. The effectiveness of proposed local update rule is illustrated by results of numerical generation of optimal topologies for selected spatial engineering structures.

Key words: topology optimization, irregular cellular automata, local update rules.

1. Motivation behind present study

Appearance of efficient and versatile optimization algorithms stimulates fast development within structural topology optimization research area. Utilizing this progress in recent years increasing range of implementation of structural topology optimization, especially to practical engineering problems, has been observed. One of the most important problems to cope with is to adjust optimization algorithms abilities to high requirements imposed on effectiveness and reliability of structural analysis tools. It is well known that for real structural elements implementation of regular finite element meshes is in many cases inadequate. For example complicated shapes, holes and sharp edges indicate stress concentration, and in order to obtain reliable stress distribution the regions of such intensity should be covered with a more fine mesh. On the other hand to avoid an increase of computational cost one wish to use rough mesh for regions, where element concentration is not necessary. As the result, a non-uniform density of elements represented by irregular meshes should be used in order to achieve an accurate solution without excessive increase of number of elements. Since structural analysis is often a part of optimization problem therefore irregular mesh problem arises also for performing design process. Although irregular meshes have been frequently used in structural finite element analysis, implementation of unstructured design elements distribution in topology optimization tasks is not in common use.

2. Topology optimization for minimal compliance

Topology optimization started in late eighties of 20th century [1] and since then has become one of the most important branches of structural design. A broad discussion on topology optimization concepts is provided by many survey papers e.g. [5, 7, 8]. Methods adapted to topology optimization represent wide range of approaches from classical gradient based algorithms to heuristic techniques. Among the latter ones Cellular Automata can be found, efficient and simple approach which has been chosen in this paper as an optimization tool. Cellular Automata technique requires local formulation of optimization problem. It means, that considered problem has to be stated as the set of local minimizations posted for each cell.

Many topology optimization problems regard minimization of structure compliance under applied loads and supports. The formulation of such problem within the frame of this paper is as follows:

$$\begin{aligned} &\text{minimize} \quad && U(d_i) = d_i^p \boldsymbol{u}_i^T \boldsymbol{k}_i \boldsymbol{u}_i, \\ &\text{subject to} \quad && 0 \leqslant d_{\min} \leqslant d_i \leqslant 1, \end{aligned} \tag{2.1}$$

where $\boldsymbol{u}_i$ and $\boldsymbol{k}_i$ are the element displacement vector and stiffness matrix, respectively. In this study well known SIMP approach is used [2], where elastic modulus E_i of each element is represented as a function of design variables being relative densities d_i of material $E_i = d_i^p E_0$. In this formula p is a penalization power usually equal 3 and E_0 is the elastic modulus of a solid material. The total volume constraint $V = \kappa V_0$, if present, is set globally and imposed after each iteration. The quantity κ stands for a prescribed volume fraction and V_0 is a design domain volume.

3. Irregular cellular automata

The basic idea and first application of Cellular Automata to structural optimization has been proposed and described in [6]. Classical formulation of Cellular Automata approach requires decomposition of considered domain into a uniform lattice of cells, which in the most common approach, coincide with finite element mesh. During optimization process cells interact only with their neighbors according to specified local update rules, which are responsible for the evolution of cell states, e.g. [3, 4, 9]. The implementation of non-uniform lattice of cells of Cellular Automaton requires a reformulation of local rules provided for regular lattices. In what follows, new local update rule dedicated to irregular lattices of cells is proposed. The rule incorporates influence of cell sizes on design variables updating process. Assuming that a spatial structure is considered and quantities V_i and V_{ik} stand for volumes of central and neighboring cells, respectively, the proposed update rule takes the following form:

$$\delta d_i = \left[\frac{V_i}{V_{nb}} (-1)^{\alpha_0} + \sum_{k=1}^{N} \frac{V_{ik}}{V_{nb}} (-1)^{\alpha_k} \right] m = \tilde{\alpha} m, \tag{3.1}$$

where $V_{nb} = V_i + \sum_{k=1}^{N} V_{ik}$ is a neighborhood volume and m stands for a design variable admissible change. The specified values of power α_0 and α_k are transferred to the update rule (3.1) according to the following relations:

$$\alpha_0 = \begin{cases} 1 & \text{if} \quad U_i^{(t)} \leqslant U^*, \\ 2 & \text{if} \quad U_i^{(t)} > U^*, \end{cases} \qquad \alpha_k = \begin{cases} 1 & \text{if} \quad U_{ik}^{(t)} \leqslant U^*, \\ 2 & \text{if} \quad U_{ik}^{(t)} > U^*, \end{cases} \tag{3.2}$$

in which $U_i^{(t)}$ and $U_{ik}^{(t)}$ represent values of compliance of considered cell and its neighbors (at iteration t) and U^* is a specified compliance threshold value. The form of rule (3.1) together with relations (3.2) guarantee that $-1 \leqslant \tilde{\alpha} \leqslant 1$.

The above new proposal can be treated as generalization of the original rule [3] extended here towards irregular three dimensional lattice of cells. It is worth underlining that the above rule can be easily adapted also to plain structures. The only modification to make is to replace spatial cells volumes by plane cells areas.

4. Numerical examples

As the first example of Irregular Cellular Automata implementation to generation of minimal compliance topology a mechanical part (bell crank) presented in Fig. 1 has been chosen. Loads are distributed along lines of nodes inside both bottom cylindrical holes, while the inner right area of upper cylinder is supported. Cylindrical parts (blue colored regions) are treated as non-optimized. Due to symmetry only half of the structure has been considered.

The regular (321 110 elements) and irregular (117 591 elements) meshes that consist of tetrahedral elements/cells have been applied. Mesh refinement, in the

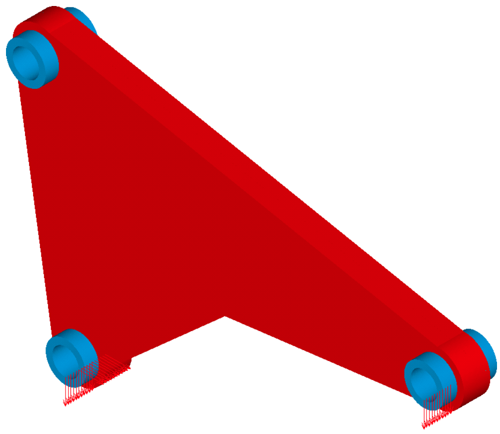

FIG. 1. Mechanical part (bell crank). Initial structure.

latter case, is applied only in the region surrounding structure support, what was indicated by stress concentration area. The numbers of elements for both lattices are selected so as to obtain the same level of maximal equivalent stresses, maximal displacements and values of compliance for initial structure. In what follows resulting values are: $48.3 \cdot 10^6$ Pa, $0.823 \cdot 10^{-4}$ m and $4.39 \cdot 10^{-3}$ N · m for regular lattice and $48.2 \cdot 10^6$ Pa, $0.824 \cdot 10^{-4}$ m and $4.39 \cdot 10^{-3}$ N · m for irregular one, respectively.

For both cases the minimal compliance topologies have been generated and the final results are presented in Fig. 2. The values of maximal equivalent stress, maximal displacement and value of compliance calculated for final topologies are: $44.3 \cdot 10^6$ Pa, $0.575 \cdot 10^{-4}$ m and $3.22 \cdot 10^{-3}$ N · m for regular and $40.4 \cdot 10^6$ Pa, $0.607 \cdot 10^{-4}$ m and $3.32 \cdot 10^{-3}$ N · m for irregular lattice.

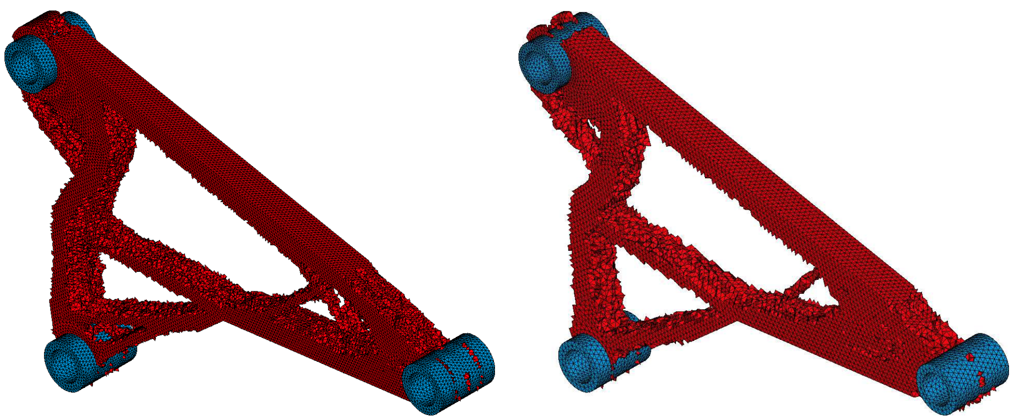

FIG. 2. Final topology for regular (left) and irregular (right) lattice.

One can notice, that in order to obtain the same level of representative data, the number of elements/design variables for regular lattice should increase almost three times in comparison with irregular lattice.

As the second example a truncated cone under torsion has been selected. Distributed loads are applied across the upper edge section, which is supported as shown in Fig. 3. All nodes of bottom edge of cylinder are fixed. The thickness of cone wall is constant. The irregular (55192 elements) mesh has been applied, where the regions surrounding acting loads are covered with finer mesh. The results of structural analysis are: maximal equivalent stresses $0.85 \cdot 10^6$ Pa, maximal displacement $0.479 \cdot 10^{-6}$ m and compliance $3.58 \cdot 10^{-3}$ N · m. The regular mesh which consists of 125 056 elements has been applied next. That many elements are required to get nearly the same results of analysis as for irregular lattice, namely: maximal equivalent stresses $0.85 \cdot 10^6$ Pa, maximal displacement $0.483 \cdot 10^{-6}$ m and compliance $3.60 \cdot 10^{-3}$ N · m.

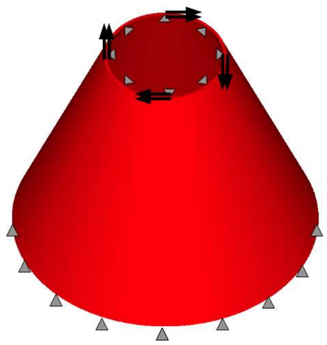

Fig. 3. Truncated cone. Initial structure.

The optimization has been performed and the generated final topologies are presented in Fig. 4. The resulting data are: maximal equivalent stress $0.86 \cdot 10^6$ Pa and $0.85 \cdot 10^6$ Pa, maximal displacement $0.342 \cdot 10^{-6}$ m and $0.330 \cdot 10^{-6}$ m, com-

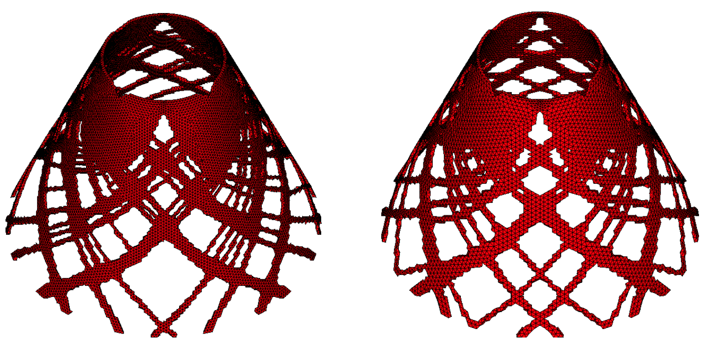

Fig. 4. Final topology for regular (left) and irregular (right) lattice.

pliance $2.59 \cdot 10^{-3}$ N · m and $2.55 \cdot 10^{-3}$ N · m for regular and irregular lattice, respectively.

One can observe, that implementation of irregular mesh allows for a significant reduction of a number of elements/cells, what is equivalent to a reduction of number of design variables. That indicates smaller computing cost, since the implementation of irregular lattice does not affect the value of compliance of optimized structure.

5. Concluding remarks

The Irregular Cellular Automata concept presented in this paper can be efficiently adapted to topology optimization of engineering structures. It demonstrates a significant potential of application especially to problems which cannot be adequately represented by regular grids. While generating structural topologies it is not necessary to use a very fine mesh for whole structure therefore number of elements and design variables can be substantially reduced. Although number of cells is limited, because of only local mesh refinement, information about stresses and displacements can still be correct. The use of irregular meshes can be helpful while modeling a complicated domain geometry, accurately specify design loads or supports and finally compute structure response. What is also worth underlining the Irregular Cellular Automata approach offers relatively easy implementation into professional finite element codes.

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