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Influence of Polymer Flexible Joint on Concrete Beams in Three-Point Bending Test

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Problem of three-point bending test for concrete beam with polymer flexible joint is described in this paper. The aim of this work is to present selected potential applications of flexible joint for concrete structures. The results of experimental researches for concrete beams before failure and after repairing with *flexible joint method* are presented. Behaviour of members repaired (bonded) with flexible joint is analysed in comparison to the test results of concrete beam before failure. Attention is paid to beneficial, due to a higher load capacity, stress redistribution in repaired element. Numerical finite element (FE) analysis of influence of polymer flexible joint on the behaviour of elements is conducted. Stresses concentration with their redistribution and displacement pattern are presented. In conclusion, beneficial influence of application of polymer flexible joints as a method of repairing concrete elements is highlighted. Furthermore, their potential applications in concrete structures are specified.

Key words: polymer flexible joint, stress redistribution, concrete strengthening, concrete repair.

1. Introduction

The problem of structures strengthening and repairing is currently intensively studied by numerous research and industrial centres all over the world. In general, the most common way of repairing structures is to restore the original strength parameters of load-bearing elements, including ensuring original stiffness properties and even stiffening the elements more. This paper is aimed at presenting an approach different than common repairing of structures, i.e., application of "softening" for repairing. In this method, called *flexible joint method* [6], the main criterion of material selection and strengthening methodology (which is understood here as restoring the reliability and structure's function to at least original state) is a strain criterion more than a load-capacity, which is the opposite to typical applications of strengthening.

A concept of *polymer flexible joint* application was first applied during the renovation works of cracked bricks vaults conducted by R. Ciesielski and A. Kwiecień in 2003 [1]. The aim of this concept was to solve the problem of new cracks appearing close to strengthening area (Fig. 1). Cracks repaired with too stiff materials (special mortars and epoxides) increased the risk of new cracks due to weakening area (with microcracks) around the strengthening [6]. The concept of polymer flexible joint solved this problem satisfactorily.

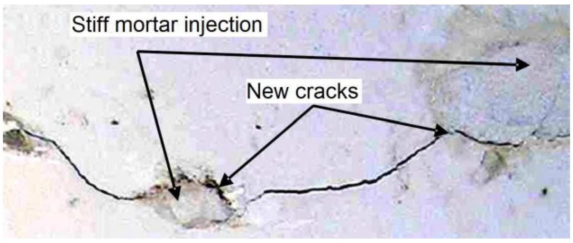

Fig. 1. New cracks of masonry in the area of too stiff strengthening in relation to original strength [6].

2. Teoretical approach

2.1. Flexible joint material description

Polymer, applied to flexible joint, is characterized by high deformation ability and controlled strength parameters (including stiffness). It reveals nonlinear behaviour under loading which can be described as an elasto-viscoplastic material [2]. Its deformability (even up to 30%) under loads leads to energy dissipation of joint (i.e., stress redistribution) without any additional stresses in structure [6]. Properly designed polymer joint restores the strength parameters of cracked structures close to the original state and, what is more, with higher flexibility.

The behaviour of flexible joints depends on many factors such as: temperature, velocity of deformation, character, magnitude and duration of loads, method of mixing polymer (manual, automatic), etc. The polymer, as a part of flexible joint, is similar to rubber and it is treated as an incompressible material [4]. Models which describe the polymer behaviour as a hyper-elastic material are based on continuum mechanics. To describe polymer and flexible joint behaviour a hyperelastic material in logarithmic strain is applied (more details in [6]). The analysis here is limited to flexible joint subjected only to bending moment.

Due to nonlinear behaviour of polymer its stiffness is changing in dependence of deformation. Hence, to obtain one-parametric model (applied in engineering practice) description of material deformation in logarithmic strain is introduced. The initial (calibrating) model is a flexible joint in four-point bending test (Fig. 2). Nonlinear variation of joint stiffness is represented here by the variable modulus of elasticity E_z (moment of inertia J_Z remains constant) which depends, among others, on current load and geometry of element. To define E_z the following transformations of equation are conducted:

$$\frac{F\cdot L_F}{2E_Z J_Z}=\frac{M_z}{E_Z J_Z}=\frac{1}{r}=\frac{y}{L_F\left(\dfrac{d}{2}+\dfrac{\Delta d}{2}\right)}, \tag{2.1}$$

$$u=\frac{y}{L_F}\left(\frac{L}{2}-\frac{d}{2}\right),\qquad \frac{y}{L_F}=\frac{\Delta d}{2\cdot u}, \tag{2.2}$$

$$\varepsilon=\frac{\Delta d}{d}=\frac{2\cdot y\cdot u}{d\cdot L_F}=\frac{2}{d}\cdot\frac{y^2}{L_F^2}\left(\frac{L}{2}-\frac{d}{2}\right), \tag{2.3}$$

$$E_Z=\frac{F\cdot L_F\left[\dfrac{d}{2}+\dfrac{y^2}{L_F^2}\left(\dfrac{L}{2}-\dfrac{d}{2}\right)\right]}{y\cdot J_Z}, \tag{2.4}$$

where M_z – bending moment in joint caused by force F, F, L_F, d – values according to Fig. 2, E_z – longitudinal modulus of elasticity of joint caused by bending, ε – strains at bottom surface of joint, y – vertical displacement of the point under applied force $F/2$, u – deflection of the beam in mid-span (bottom surface of polymer), J_z – second moment of joint area.

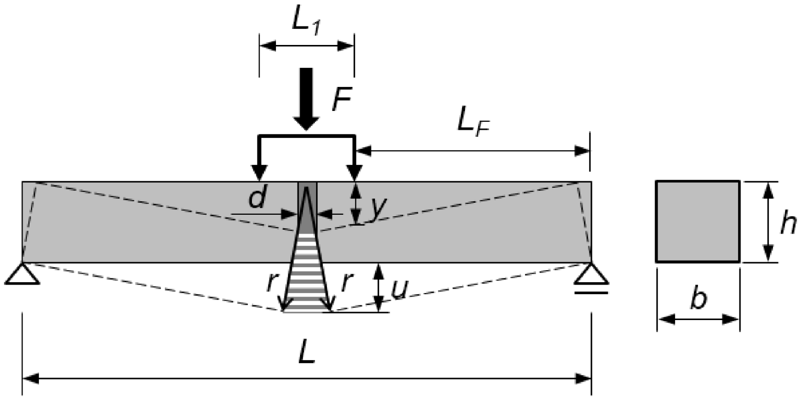

Fig. 2. Geometry and boundary condition of the tested beam (initial model) [6].

The experimental test results [6] allow to define the relation between stresses σ and equivalent strains ε'' according to Eq. (2.5)

$$\sigma = \frac{M_z}{W} = \frac{3 \cdot F \cdot L_F}{b \cdot h^2} = T_{11}^{(0)} = E_z \cdot \varepsilon'' = E_z \cdot \ln \lambda_1, \tag{2.5}$$

where σ – Cauchy stresses, W – bottom section modulus between concrete and polymer, F, L_F, b, h – values according to Fig. 2, $T_{11}^{(0)}$ – Hencky stresses, ε'' – equivalent strains of joint, λ_1 – stretch of eigenvector along 1-1 direction.

Model, formed in this way, is one-parametric and allows for easier definition of joint stress-stiffness relation. It is necessary to emphasize that stiffness of the flexible joint E_z is higher than the corresponding stiffness of the same polymer in uniaxial tension test [6].

2.2. Effort measure of the polymer flexible joint

The polymer flexible joints consist of concrete and polymer materials. All of them reveal brittle character – i.e., they bear compressive stresses better than tensile stresses. Hence, the Burzynski criterion of effort is proposed to describe effort measure of flexible joint energy (more details in [10]). Briefly, it says that material, which reveals permanent strains or loss of integrity (cohesion) after exceeding a limit value of shear-volume energy, will be damaged. For asymmetric distribution of strength parameters ($k_R < k_C$) it can be reduced to plane stress state ($\sigma_2 = 0$) shown in formula (2.6)

$$\sigma_1^2 + \sigma_3^2 - 2 \cdot \left(\frac{k_R k_C}{2k_S} - 1 \right) \cdot \sigma_1 \sigma_3 + (k_C - k_R) \cdot (\sigma_1 + \sigma_3) \leq k_R k_C, \tag{2.6}$$

where σ_1, σ_3 – principal stresses assuming that $\sigma_1 \geq \sigma_3$, k_R, k_C, k_S – yield strength for tension, compression, shear in uniaxial test of tension, compression and shear, respectively.

Due to the above relations, the notion of reduced stress σ_{red} (2.7) can be derived, which is an effort measure of flexible joint in complex stress state [11].

$$\sigma_{\mathrm{red}} = \frac{1}{2\dfrac{k_C}{k_R}} \left[3 \cdot \left(\frac{k_C}{k_R} - 1 \right) \cdot \sigma_m + \sqrt{9 \cdot \left(\frac{k_C}{k_R} - 1 \right)^2 \cdot \sigma_m^2 + 4 \frac{k_C}{k_R} \sigma_e^2} \right], \tag{2.7}$$

where

$$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3},$$

$$\sigma_e = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}.$$

The model built using the above relations can effectively describe behaviour of the polymer flexible joint, what is confirmed by the researches [7, 8].

3. Experimental researches

Concrete beams B2 and B10 were tested in three-point bending test [6, 8]. These elements with cross-sectional dimension of 100×100 [mm] and 500 mm length were simply supported in distance equal to 300 mm and loaded in the centre of symmetry by concentrated force. To avoid random damage localization along the elements, the notch (groove) in the middle of span with the depth of ca. 15% of the cross section height was made. Geometry of tested elements is shown in Fig. 3. The beams were made of concrete without any reinforcement, cf. material properties in Table 1. Bending tests were conducted in testing machine, type Z100 Zwick/Roell with control of velocity of vertical displacement. During a testing, under applied load, a peak of stresses took place in area of notch (zone of tensile stresses). While the stresses reached the tensile strength, the numerous microcracks in overloaded area occurred. The brittle material in this place revealed a nonlinear characteristic, what finally led to rapid connecting of microcracks and formed the principal (main) crack [5, 7]. After failure, all tested beams were bonded with adhesive-bonded joints: B2 beams were bonded with polymer type PT (hard) and B10 beams with polymer type PM (soft). Figure 4 shows the original beams (before failure) and after bonding with polymer.

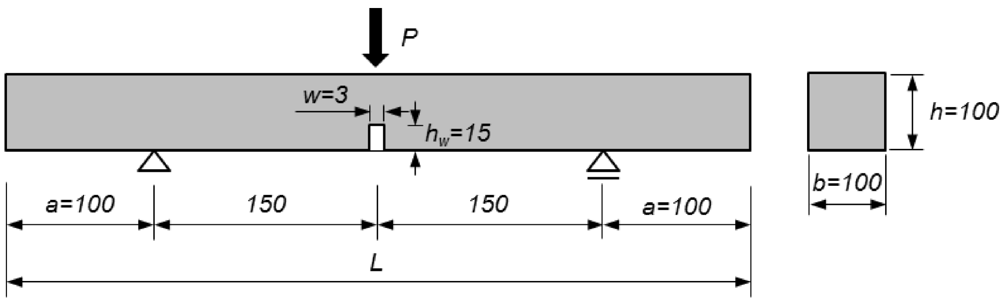

Fig. 3. Geometry and load condition of tested beam.

Table 1. Material properties of tested beam [5, 7].

Material	E [MPa]	ν [–]	γ [kN/m^3]	f_t [MPa]
Concrete	39 000	0.170*	23.14	3.5
Polymer PT	800	0.495	8.83	18
Polymer PM	4.4	0.495	8.83	1.4

* for $G = 33.1$ N/m

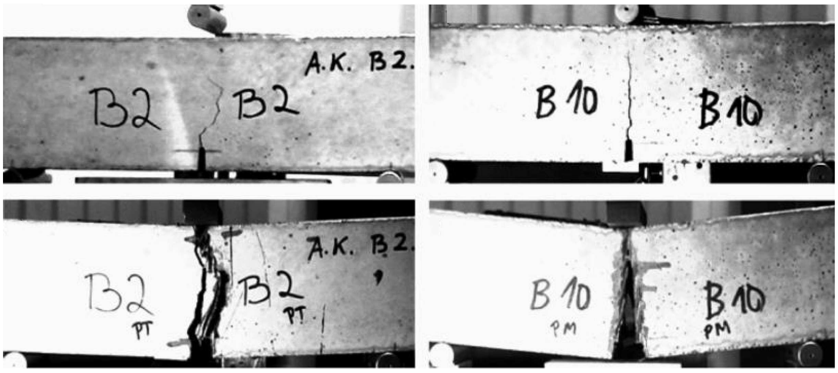

FIG. 4. Illustration of the damage in beams B2 and B10 (top: original beam, bottom: beam connected with polymer flexible joint) [6].

Development of displacement under applied loads is shown in Fig. 5.

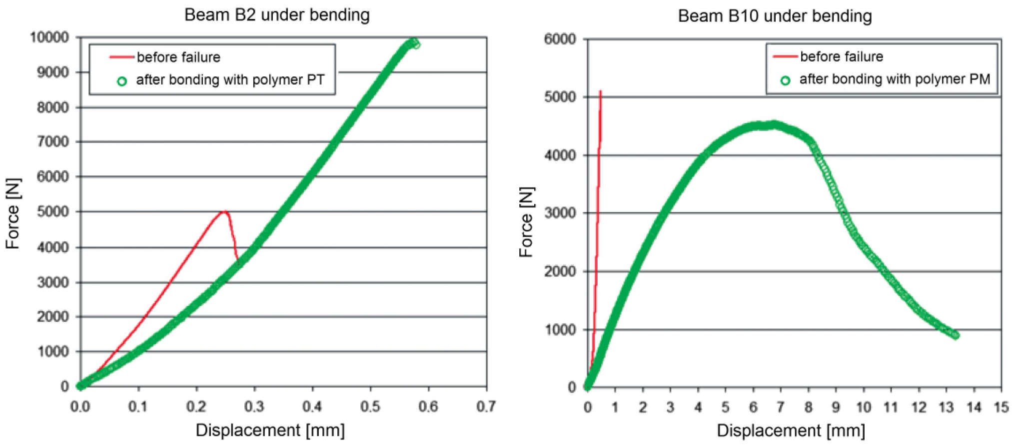

FIG. 5. Force-displacement relation of B2 and B10 beams before failure and after bonding with polymers PT and PM [7].

The original beams (i.e., before bonding with polymers), during an increase of the load, keep linear-elastic character until a tensile strength is reached [9]. After concrete reaches tensile strength (due to force $P = 5000$ N) brittle failure occurs (i.e., rapid connecting of microcracks in place of maximal tensile stresses takes place). The approximate tensile stress in concrete of all analysed beams, after the occurrence of cracks, is defined by Eq. (3.1).

$$\sigma_{t,\max} = 1.5\frac{P\,L}{b\,(h-h_w)^2} = 1.5\frac{5000\cdot 300}{100\cdot(100-15)^2} = 3.11 \quad [\mathrm{MPa}], \tag{3.1}$$

where the $\sigma_{t,\max}$ value is approximately equal to the tensile strength of concrete f_t according to Table 1 [5]. The difference between $\sigma_{t,\max}$ and f_t ($\sigma_{t,\max}/f_t$ = 88.9%) can be explained by difficulty in estimation of this parameter which is typical for concrete [3].

The beams bonded with polymers PT and PM are capable to bear the load almost equal to original load for PM and even higher value for PT. The polymer flexible joint decreases effort of the beams (lower stiffness of a member). During a failure the elements bonded with polymers should do a work of much higher value (cf. area under a curve of force-displacement in Fig. 5). For PT polymer (hard) the damage has a cohesive character, fracture occurs in concrete near bonding area (Fig. 4c); however, in application of soft polymer (PM) the failure has cohesive-adhesive character, both the polymer and the joint between concrete and polymer are damaged (Fig. 4d). Both types of joints reveal far-reaching stress redistribution in notch area, what results in higher capacity of these joints.

4. Numerical analysis in engineering application

The analysis in this section is focused only on qualitative presentation of stress redistribution in flexible joint. For this case, the beams described above were modelled in plane stress state (2D) using finite element method (ABC Tarcza v.6.14). Geometry of the model is shown in Fig. 6 (cf. Fig. 3). Material properties are shown in Table 1. Three models are tested: (1) concrete beam (original), (2) concrete beam bonded with PT joint and (3) concrete beam bonded with PM joint.

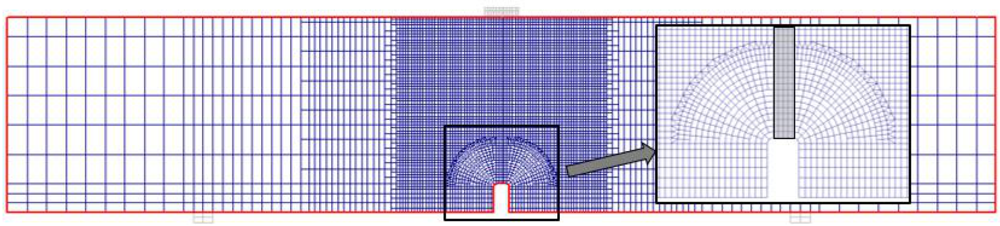

Fig. 6. An FE model of the analysed beam (5614 finite elements, 5804 nodes), compare with Fig. 3; grey area indicates polymer material.

All beams are described by elastic damage model (more on this topic in [9]). This model describes brittle material as a linear-elastic before failure, until it reaches the damage threshold. After that a rapid connecting of previous microcracks occurs and the principal crack is formed [5]. As a result of material damage its parameters obtain different values in cracking area. This is caused by weakening zone formed previously (the rest of microcracks in this area). As a result, a lower value of concrete stiffness is reached ($E_{c.cr} = 2180$ MPa) [5] (this parameter is assumed arbitrarily as ca. 10 widths of the notch for each side; ν ratio remains the same – cf. Table 1). The thickness of adhesive-bonded joint for polymers is 5 mm. Size effect, development of strains after damage and self-weight (as an insignificant) are omitted in this analysis. The connection between concrete and polymer is modelled as rigid (lack of interface layer). According to [5] load is set as a $P = 1500$ N, which is within elastic behaviour range. The results are presented according to the Burzynski stresses, with k_C/k_R ratio $= 1.333$ [6].

Deflection under force P is shown in Fig. 7, the reduced stresses according to Burzynski, for models: (1) concrete beam (original), (2) bonded with PT polymer and (3) bonded with polymer PM are shown in Fig. 8.

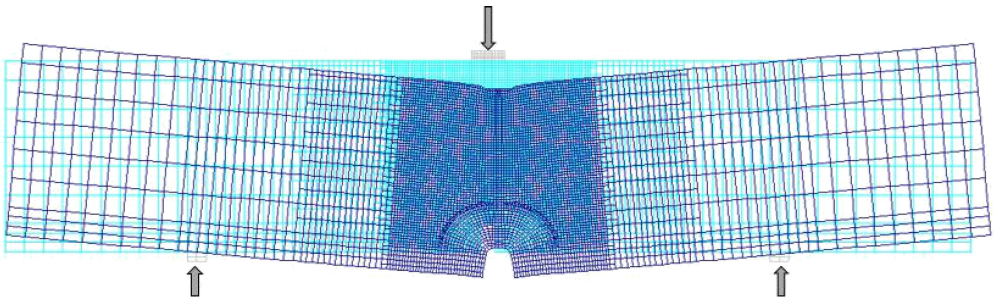

FIG. 7. Deflection of the beam repaired with polymer flexible joint (max. value: original member 0.005 mm, PT 0.055 mm and PM 1.837 mm in mid-span) under force $P = 1500$ N.

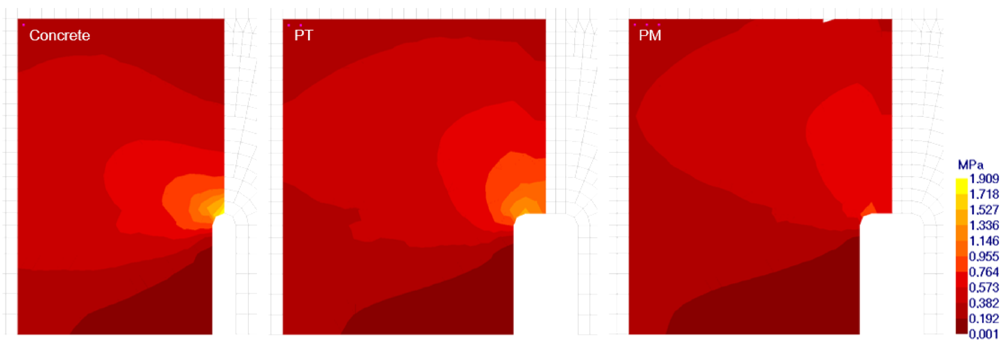

FIG. 8. Distribution of material effort according to the Burzynski theory in notch area of concrete beam (left) and flexible joint PT (middle) and PM (right) at load $P = 1500$ N.

The stress maps (Fig. 8) show the differences in stress distribution between particular models. The concrete beam model (1) reveals the peak of stress in notch area $\sigma_{\text{red}} = 1.909$ MPa; however, for other cases the stresses in notch area are much lower (PT: $\sigma_{\text{red}} = 1.423$ MPa and PM: $\sigma_{\text{red}} = 0.839$ MPa).

The flexible joint models (2) and (3) present significant redistribution along the whole length of joint, and the stress concentration around the notch area is less intensive and more distributed along the joint. This causes that extreme stresses are respectively lower than stresses in model (1) – 75.5% for PT and 43.9% for PM. The redistribution of stresses is stronger for polymer type PM than for type PT ($E_{PM} < E_{PT}$).

5. Summary

The nonlinear character of polymer flexible joints requires an adequate description of behavioural model, what is briefly presented above. A good way to properly describe polymer joint behaviour is to apply the model in logarithmic strain of concrete member with polymer flexible joint. However, in the authors' opinion the linear-elastic model for polymer flexible joints cannot be considered as appropriate for such material. The Burzynski stresses for brittle materials can be used as an effort measure of flexible joint. Problems to solve which still remain are: the quantitative relation between strength parameters of flexible joint and a proper definition of the measure effort for joint.

The flexible joint is able to bear high loads (not less than structural concrete) and thus to increase, in this way, load capacity of structural element through beneficial stress redistribution in the element (cf. analysis above for bending element). The polymer flexible joint can be an effective way of repairing structural elements.

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