

## CONTINUUM FIELD MODEL OF STREET CANYON: THEORETICAL DESCRIPTION. PART I

M. M. D U R A S

**INSTITUTE OF PHYSICS, CRACOW UNIVERSITY OF TECHNOLOGY**

ul. Podchorążych 1, 30–084 Cracow, Poland

Email: mduras@riad.usk.pk.edu.pl

A general proecological urban road traffic control idea for the street canyon is proposed with emphasis placed on development of advanced continuum field gasdynamical (hydrodynamical) control model of the street canyon. The continuum field model of optimal control of street canyon is studied. The mathematical physics' approach (Eulerian approach) to vehicular movement, to pollutants' emission, and to pollutants' dynamics is used. The rigorous mathematical model is presented, using gasdynamical (hydrodynamical) theory for both air constituents and vehicles, including many types of vehicles and many types of pollutant (exhaust gases) emitted from vehicles. The six optimal control problems are formulated.

### 1. DESCRIPTION OF THE MODEL

In the present article we develop a continuum field model of the street canyon. In the next article we will deal with numerical examples [1]. The vehicular flow in the canyon is multilane bidirectional one-level rectilinear, and it is considered with two coordinated signalized junctions [2, 3, 4]. The vehicles belong to different vehicular classes: passenger cars, and trucks. Emissions from the vehicles are based on technical measurements and many types of pollutants are considered (carbon monoxide CO, hydrocarbons HC, nitrogen oxides NO<sub>x</sub>). The vehicular dynamics is based on a hydrodynamical approach [5]. The governing equations are the continuity equation for the number of vehicles, and Greenshields' equilibrium speed-density  $u-k$  model [6].

The model of dynamics of pollutants is also hydrodynamical. The model consists of a set of mutually interconnected nonlinear, three-dimensional, time-

dependent, partial differential equations with nonzero right-hand sides (sources), and of boundary and of initial problem. The pollutants, oxygen, and the remaining gaseous constituents of air, are treated as mixture of noninteracting, Newtonian, viscous fluid (perfect or ideal gases). The complete model incorporates as variables the following fields: density of the mixture, mass concentrations of constituents of the mixture, velocity of mixture, temperature of mixture, pressure of mixture, intrinsic (internal) energy of mixture, densities of vehicles, and velocities of vehicles. The model is based on the assumption of local laws of balance (conservation) of: mass of the mixture, masses of its constituents, momentum and energy of the mixture, the numbers of the vehicles, as well as of the state equations (Clapeyron's law and Greenshields' model). The equations of dynamics are solved by the finite difference scheme.

The six separate monocriterial optimization problems are formulated by defining the functionals of total travel time, of global emissions of pollutants, and of global concentrations of pollutants, both in the studied street canyon and in its two nearest neighbour substitute canyons. The vector of control is a five-tuple composed of two cycle times, two green times, and one offset time between the traffic lights. The optimal control problem consists of minimization of the six functionals over the admissible control domain.

## 2. ASSUMPTIONS

The **geometrical assumptions** of the model are as follows:

**G1.** The street canyon is represented by a cuboid:

$$(2.1) \quad \Omega = [0, a] \times [0, b] \times [0, c],$$

with the boundary  $\partial\Omega$  composed of six walls:

$$(2.2) \quad \partial\Omega = \bigcup_{J=I}^{VI} \partial\Omega_J.$$

The structure of the canyon is simplified by the assumption that the walls of the buildings and the road surface are rectangles. If we put the origin of the Cartesian coordinate system  $(x, y, z)$  in cuboid's corner, then we have two canyon walls  $\Omega_{IV}, \Omega_V$ , at  $y = 0, y = b$ , and road's surface  $\Omega_{VI}$ , at  $z = 0$ . The remaining three non-solid open surfaces of air  $\Omega_I, \Omega_{II}, \Omega_{III}$ , have the coordinates  $x = 0, x = a, z = c$ , respectively. We assume that always:

$$(2.3) \quad (x, y, z) \in \Omega,$$

and

$$\begin{aligned}
 \partial\Omega_{\text{I}} &= \{(x, y, z) \in \Omega : x = 0\}, \\
 \partial\Omega_{\text{II}} &= \{(x, y, z) \in \Omega : x = a\}, \\
 \partial\Omega_{\text{III}} &= \{(x, y, z) \in \Omega : z = c\}, \\
 \partial\Omega_{\text{IV}} &= \{(x, y, z) \in \Omega : y = 0\}, \\
 \partial\Omega_{\text{V}} &= \{(x, y, z) \in \Omega : y = b\}, \\
 \partial\Omega_{\text{VI}} &= \{(x, y, z) \in \Omega : z = 0\}.
 \end{aligned}
 \tag{2.4}$$

**G2.** There are neither holes in the walls nor vegetation alongside the road. The remaining three surfaces of the cuboid also do not have holes since they simulate non-solid open rectangles of air.

**G3.** The road sections which constitute the bottom of the street canyon are rectilinear.

**G4.** At each end of the street canyon there are entrance and exit junctions ( $M = 2$ ) with traffic signals (their coordinates are  $x = 0$ ,  $x = a$ ).

**G5.** The vehicles of  $VT$  distinguishable emission types are material points. The vehicles are treated as hydrodynamical fluid. There are  $n_L = n_1$  left lanes and  $n_2 = n_R$  right ones (the traffic is bidirectional).

The **physical assumptions** of this model are as follows (cf. papers [7 – 50]):

**P1.** The considered mixture of gases consists of  $N = N_E - 1 + N_A$  gases. The first  $N_E - 1 = 3$  gases are the exhaust gases emitted by vehicle engines during combustion (CO, CH, NO<sub>x</sub>, we neglect the presence of SO<sub>2</sub>). The remaining  $N_A = 9$  gases are the constituents of air: O<sub>2</sub>, N<sub>2</sub>, Ar, CO<sub>2</sub>, Ne, He, Kr, Xe, H<sub>2</sub> (we neglect the presence of H<sub>2</sub>O, O<sub>3</sub>).

**P2.** The walls of the canyon and the surface of the road are impervious for all gases of the mixture. The remaining three surfaces of the cuboid are pervious for external fluxes of exhaust gases and air constituents.

**P3.** The internal sources of air constituents are not present with the exception of oxygen, i.e.,  $N_E$  constituent of the gaseous mixture. There are internal mobile sources of exhaust gases (passenger cars and trucks, with many types of engines: diesel or petrol, and with different ages of engines). During combustion, the engine consumes oxygen, therefore with each internal mobile source of exhaust gases, a negative source of oxygen (sink) is connected.

**P4.** The gaseous mixture is treated as a compressible, Newtonian, and viscous fluid. We assume that also the constituents of the mixture are compressible, Newtonian, and viscous fluids. The constituents do not interact with each other. The  $i$ -th constituent possesses individual velocity  $\mathbf{v}_i(x, y, z, t)$ , density

$\rho_i(x, y, z, t)$ , and pressure  $p_i(x, y, z, t)$ , whereas the mixture possesses total velocity  $\mathbf{v}(x, y, z, t)$ , density  $\rho(x, y, z, t)$ , and pressure  $p(x, y, z, t)$ . We assume that

$$(2.5) \quad \mathbf{v}(x, y, z, t) = \sum_{i=1}^N \frac{\rho_i(x, y, z, t)}{\rho(x, y, z, t)} \cdot \mathbf{v}_i(x, y, z, t),$$

$$(2.6) \quad \rho(x, y, z, t) = \sum_{i=1}^N \rho_i(x, y, z, t),$$

$$(2.7) \quad p(x, y, z, t) = \sum_{i=1}^N p_i(x, y, z, t).$$

In order to simplify the set of equations governing the dynamics of mixture, we assume that the total velocity is equal to the velocities of the constituents

$$(2.8) \quad \mathbf{v}(x, y, z, t) = \mathbf{v}_i(x, y, z, t), \quad i = 1, \dots, N.$$

Hence, we can restrict our attention to the equations of balance of total momentum of mixture **E1**, of the total mass of mixture **E2**, of masses of constituents **E3**, and of the energy of mixture **E4**. We assume that the equation of state for mixture **E5** is averaged over the constituents. We also consider equations of balances of the numbers of vehicles **E6**, as well as equations of state for vehicles **E7**.

### 3. VARIABLES

The following set of descriptive dynamic model variables **A0–A10** together with their boundary **B0–B8** (for  $t \geq 0$ ), and initial conditions **C0–C7** (for  $t = 0$ ), and with the set of equations **E1–E8** that governs their dynamics, is assumed [2]. We always consider

$$(3.1) \quad (x, y, z, t) \in \Sigma, \Sigma = \Omega \times [0, T_S],$$

where  $\Sigma$  is manifold of domains of the fields,  $T_S > 0$  is time of simulation. The border  $\partial\Sigma$  of  $\Sigma$  is composed of six subsets  $\partial\Sigma_J$ :

$$(3.2) \quad \begin{aligned} \partial\Sigma &= \bigcup_{J=I}^{VI} \partial\Sigma_J, \\ \partial\Sigma_J &= \partial\Omega_J \times [0, T_S], \quad J = I, \dots, VI. \end{aligned}$$

**A0.**  $T(x, y, z, t)$ , temperature of the gaseous mixture.

**A1.**  $\mathbf{v}(x, y, z, t)$ , total velocity of the gaseous mixture (compare Eq. (2.5)).

**A2.**  $\rho(x, y, z, t)$ , total density of the gaseous mixture (compare Eq. (2.6)).

**A3.**  $c_i(x, y, z, t)$ , mass concentration of the  $i$ -th constituent of gaseous mixture,

$$(3.3) \quad c_i(x, y, z, t) = \frac{\rho_i(x, y, z, t)}{\rho(x, y, z, t)}, \quad i = 1, \dots, N$$

(let us note that due to the condition

$$(3.4) \quad \sum_{i=1}^N c_i(x, y, z, t) = 1,$$

one concentration of the constituent is a dependent variable).

**A4a.**  $p(x, y, z, t)$ , pressure of gaseous mixture (compare Eqs. (2.7), (7.16)).

**A4b.**  $p_i(x, y, z, t)$ ,  $i = 1, \dots, N$ , partial pressure of the  $i$ -th constituent of gaseous mixture (compare Eqs. 2.7), (7.16), (7.17)).

**A5.**  $k_{l,vt}^s(x, t)$ , density of vehicles of type  $vt$  on the  $l$ -th lane measured in  $[\text{veh} \cdot \text{m}^{-1}]$ , where for  $n_1 = n_L$  lanes  $s = 1$  and  $l$  is the left lane's number,  $l = 1, \dots, n_L$ , whereas for  $n_2 = n_R$  lanes  $s = 2$  and  $l$  is the right lane's number,  $l = 1, \dots, n_R$ ,  $vt$  is the vehicular type number,  $vt = 1, \dots, VT$ .

**A6.**  $\mathbf{w}_{l,vt}^s(x, t)$ , velocity of vehicles of type  $vt$  on the  $l$ -th lane.

**A7.**  $e_{l,ct,vt}^s(x, t)$ , emissivity of  $ct$ th constituent of exhaust gases from vehicles of type  $vt$  on  $l$ th lane measured in  $[\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}]$ ,  $ct$  is number of constituent,  $ct = 1, \dots, CT$ .

**A8.**  $\mathbf{u}_m = (g_m, C_m, F)$ , vector of control on the  $m$ -th junction,  $m = 1, \dots, M$  ( $M = 2$ ), which contains traffic signals green times  $g_m$ , and cycle times  $C_m$ , and offset time  $F$  between the traffic signals. The vector of control  $\mathbf{u}$  reads:

$$(3.5) \quad \mathbf{u} = (g_1, C_1, g_2, C_2, F).$$

The admissible control domain set  $U^{\text{adm}}$  for this vector in the simulation time period  $T_S$  reads:

$$(3.6) \quad U^{\text{adm}} = \{(g_1, C_1, g_2, C_2, F) : \\ g_m \in (g_{m,\min}, g_{m,\max}), \quad C_m \in (C_{m,\min}, C_{m,\max}), \quad F \in (F_{\min}, F_{\max}), \\ m = 1, \dots, M\},$$

whereas  $g_{m,\max} = C_m - g_{m,\text{orth}}$ , where  $g_{m,\text{orth}}$  are green times on orthogonal direction of the junctions (on the canyons orthogonal to the one studied),  $F_{\max} = C_2 - \delta_F$ , where  $\delta_F$  is unit step in direction of  $F$  in parameter space (compare **F0**).

**A9.**  $G_{\text{out}}^m(g_m, C_m, F, t)$ , traffic signal on  $m$ th junction at  $x = x_m$ . For  $x_1 = a$  the traffic signal  $G_{\text{out}}^1$  governs all left lanes (outgoing vehicles) and all right lanes (incoming vehicles), whereas for  $x_2 = 0$  the traffic signal  $G_{\text{out}}^2$  governs all right lanes (outgoing vehicles) and all left lanes (incoming vehicles). For the signals we assume the Boolean output values: GREEN and RED.

**A10.**  $\sigma_{i,vt}^s(x, t)$ , rate of change of linear density of energy connected to heat produced by engines of vehicles of type  $vt$  on  $l$ -th lane measured in  $[\text{J} \cdot \text{m}^{-1} \cdot \text{s}^{-1}]$ .

#### 4. BOUNDARY CONDITIONS

We assume the following boundary conditions [2]:

$$\mathbf{B0a-B0c.} \quad T|_{\partial\Sigma_K} = T_K|_{\Sigma'_K}.$$

$$\mathbf{B0d-B0f.} \quad \nabla_{\mathbf{n}} T|_{\partial\Sigma_L} = \vec{0}.$$

$$\mathbf{B1a-B1c.} \quad \mathbf{v}|_{\partial\Sigma_K} = \mathbf{v}_K|_{\Sigma'_K}.$$

$$\mathbf{B1d-B1f.} \quad \nabla_{\mathbf{n}} \mathbf{v}|_{\partial\Sigma_L} = \mathbf{O}.$$

$$\mathbf{B2a-B2c.} \quad \rho|_{\partial\Sigma_K} = \rho_K|_{\Sigma'_K}.$$

$$\mathbf{B2d-B2f.} \quad \nabla_{\mathbf{n}} \rho|_{\partial\Sigma_L} = \vec{0}.$$

$$\mathbf{B3a-B3c.} \quad c_i|_{\partial\Sigma_K} = c_{i,K}|_{\Sigma'_K}.$$

$$\mathbf{B3d-B3f.} \quad \nabla_{\mathbf{n}} c_i|_{\partial\Sigma_L} = \vec{0}.$$

$$\mathbf{B4a-B4c.} \quad p|_{\partial\Sigma_K} = p_K|_{\Sigma'_K}.$$

$$\mathbf{B4d-B4f.} \quad \nabla_{\mathbf{n}} p|_{\partial\Sigma_L} = \vec{0}.$$

$$\mathbf{B5a-B5d.} \quad k_{i,vt}^s(x_m, t) = k_{i,vt,P}^s(t).$$

$$\mathbf{B6a-B6d.} \quad \mathbf{w}_{i,vt}^s(x_m, t) = \mathbf{w}_{i,vt,P}^s(t).$$

$$\mathbf{B7a-B7d.} \quad e_{i,ct,vt}^s(x_m, t) = e_{i,ct,vt,P}^s(t).$$

$$\mathbf{B8a-B8d.} \quad \sigma_{i,vt}^s(x_m, t) = \sigma_{i,vt,P}^s(t).$$

We define additional sets:

$$(4.1) \quad \begin{aligned} \Sigma'_I &= \{(y, z, t) : (x, y, z, t) \in \Sigma\} = \Sigma'_{II}, \\ \Sigma'_{III} &= \{(x, y, t) : (x, y, z, t) \in \Sigma\}, \end{aligned}$$

and we assume that  $K = \text{I, II, III}$ ,  $L = \text{IV, V, VI}$ . The gradient operator  $\nabla_{\mathbf{n}}$  works in direction of unit normal outward vector  $\mathbf{n}$  to border  $\partial\Sigma_L$ .  $\nabla_{\mathbf{n}} \mathbf{v}$  is gradient of vector (so it is a tensor of rank 2),  $\mathbf{O}$  is a zero tensor.  $P = \text{in, out}$ , is the input and output index, and we have the following combinations of triads of

indices:  $(s, P, m) = (1, \text{in}, 2), (2, \text{in}, 1), (1, \text{out}, 1), (2, \text{out}, 2)$ , respectively for **B5-B8**. Conditions **B1d-B1f** result from viscosity of the gaseous mixture since the velocity of viscous fluid on immobile and impervious surface is zero. Similarly, conditions **B0d-B0f**, **B2d-B2f**, **B3d-B3f**, **B4d-B4f** result from the fact that the walls and the surface of the road are impervious solid bodies. According to [5], we assume the boundary conditions **B5a-B5d** in the form:

**B5aS-B5dS.**

$k_{l,vt,P}^s(t) = k_{l,vt,arrival}^s$ , if  $G_{out}^m(C_m, g_m, F, t) = \text{GREEN}$ , and  $\text{QUEUE}(x_m) = \text{FALSE}$ ,

$k_{l,vt,P}^s(t) = k_{l,vt,sat}^s$ , if  $G_{out}^m(C_m, g_m, F, t) = \text{GREEN}$ , and  $\text{QUEUE}(x_m) = \text{TRUE}$ ,

$k_{l,vt,P}^s(t) = k_{l,vt,jam}^s$ , if  $G_{out}^m(C_m, g_m, F, t) = \text{RED}$ ,

where  $\text{QUEUE}(x_m) = \text{TRUE}/\text{FALSE}$  means that there exist/does not exist a queue at  $x = x_m$ , and  $k_{l,vt,arrival}^s$ ,  $k_{l,vt,sat}^s$ , and  $k_{l,vt,jam}^s$ , are arrival, saturation, and jam vehicular densities, respectively (compare Tables 3, and 8 of [1]).

The existence of the queues at the entrances to the canyon (at  $x_2 = 0$  for the left lanes, and at  $x_1 = a$  for the right lanes) is determined by the values of the vehicular densities changing in the following way:

**B5aSS-B5dSS.**

$$(4.2) \quad k_{l,vt}^s(\xi_s, t) = k_{l,vt,Q}^s, \quad \text{for } t \in A_Q^s,$$

$$(4.3) \quad A_{\text{GREEN}}^s = [0, T_S] \cap \bigcup_{n=-\infty}^{+\infty} [t_s + nC_1, t_s + nC_1 + g_1),$$

$$(4.4) \quad A_{\text{RED}}^s = [0, T_S] - A_{\text{GREEN}}^s, \quad Q = \text{GREEN}, \text{RED},$$

$$(4.5) \quad \xi_1 = -\delta_x, \quad \xi_2 = a + \delta_x, \quad t_1 = 0, \quad t_2 = F,$$

where  $k_{l,vt, \text{GREEN}}^s$ ,  $k_{l,vt, \text{RED}}^s$ , are green and red vehicular densities, respectively [1], and  $\delta_x$  is the unit step in x-direction in the domain space  $\Sigma$ .

REMARK

The functions:  $T_J$ ,  $\mathbf{v}_J$ ,  $\rho_J$ ,  $c_{i,J}$ ,  $p_J$ ,  $k_{l,vt,P}^s$ ,  $\mathbf{w}_{l,vt,P}^s$ ,  $e_{l,ct,vt,P}^s$ ,  $\sigma_{l,vt,P}^s$ , are given and they fulfill the natural constraints:

$$(4.6) \quad \sum_{i=1}^N c_{i,J}(x, y, z, t) = 1.$$

## 5. INITIAL CONDITIONS

The initial conditions are as follows [2]:

$$\mathbf{C0.} \quad T|_{\Sigma_0} = T_0|_{\Omega}.$$

$$\mathbf{C1.} \quad \mathbf{v}|_{\Sigma_0} = \mathbf{v}_0|_{\Omega}.$$

$$\mathbf{C2.} \quad \rho|_{\Sigma_0} = \rho_0|_{\Omega}.$$

$$\mathbf{C3.} \quad c_i|_{\Sigma_0} = c_{i,0}|_{\Omega}.$$

$$\mathbf{C4.} \quad p|_{\Sigma_0} = p_0|_{\Omega}.$$

$$\mathbf{C5a-C5b.} \quad k_{i,vt}^s(x, 0) = k_{i,vt,0}^s(x).$$

$$\mathbf{C6a-C6b.} \quad \mathbf{w}_{i,vt}^s(x, 0) = \mathbf{w}_{i,vt,0}^s(x).$$

$$\mathbf{C7a-C7b.} \quad e_{i,ct,vt}^s(x, 0) = e_{i,ct,vt,0}^s(x).$$

$$\mathbf{C8a-C8b.} \quad \sigma_{i,vt}^s(x, 0) = \sigma_{i,vt,0}^s(x).$$

## REMARK

The functions:  $T_0$ ,  $\mathbf{v}_0$ ,  $\rho_0$ ,  $c_{i,0}$ ,  $p_0$ ,  $k_{i,vt,0}^s$ ,  $\mathbf{w}_{i,vt,0}^s$ ,  $e_{i,ct,vt,0}^s$ ,  $\sigma_{i,vt,0}^s$ , are given and they fulfill the natural constraint:

$$(5.1) \quad \sum_{i=1}^N c_{i,0}(x, y, z, t) = 1.$$

## 6. SOURCES

In order to represent the emission process, we assume the following internal sources [2]:

**D0.**  $\sigma(x, y, z, t)$ , the rate of change of the volume density of internal sources of energy connected with the production of heat by vehicular engines, measured in  $[\text{J} \cdot \text{m}^{-1} \cdot \text{s}^{-1}]$ . We assume that the sources of energy are situated in  $n_s$  left and right lanes at  $y = y_i^s$ , at the level of the road  $z = 0$ :

$$(6.1) \quad \sigma(x, y, z, t) = \frac{1}{b \cdot c} \cdot \sum_{s=1}^2 \sum_{l=1}^{n_s} \sum_{vt=1}^{VT} \sigma_{i,vt}^s(x, t) \chi_{D_l^s}(x, y, z),$$

where

$$(6.2) \quad D_l^s = \{(x, y, 0) : (x, y, 0) \in \Omega, y = y_i^s\},$$

are the vehicular lanes, and

$$(6.3) \quad \chi_D(x, y, z) = \begin{cases} 1 & \text{for } (x, y, z) \in D \\ 0 & \text{for } (x, y, z) \notin D \end{cases},$$

is the characteristic function of set  $D$ .



**D1.**  $S(x, y, z, t)$ , the rate of change of the volume density of internal sources of gaseous mixture consisting of exhaust gases and oxygen, measured in  $[\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}]$ .

**D2.**  $S_{ct}^E(x, y, z, t)$ , the rate of change of the volume density of internal sources (the emission rate) of the  $ct$ -th constituent of exhaust gases emitted by all vehicles in the canyon, measured in  $[\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}]$ . We assume that the sources of exhaust gases are situated in  $n_s$  left and right lanes  $y = y_i^s$ , at the level of the road  $z = 0$ :

$$(6.4) \quad S_{ct}^E(x, y, z, t) = \frac{1}{b \cdot c} \cdot \sum_{s=1}^2 \sum_{l=1}^{n_s} \sum_{vt=1}^{VT} e_{l,ct,vt}^s(x, t) \chi_{D_l^s}(x, y, z).$$

$S_{N_E}^E(x, y, z, t)$ , the volume density of negative internal sources (the emission rate) of oxygen absorbed by all vehicles in the canyon, measured in  $[\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}]$ . We assume that

$$S_{N_E}^E(x, y, z, t) = \text{ONOX} \cdot S_{N_E-1}^E(x, y, z, t),$$

where  $\text{ONOX} = -0.5308$ . The following relation holds:

$$(6.5) \quad S(x, y, z, t) = \sum_{ne=1}^{N_E} S_{ne}^E(x, y, z, t).$$

### 7. EQUATIONS OF DYNAMICS

Under the above model specifications, the complete set of equations of dynamics of the model is formulated as follows (we follow the general idea presented in [7, 8]):

**E1. Balance of momentum of mixture - Navier Stokes equation.**

$$(7.1) \quad \rho \cdot \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \circ \nabla) \mathbf{v} \right) + S \cdot \mathbf{v} = -\nabla p + \eta \cdot \Delta \mathbf{v} + \left( \xi + \frac{\eta}{3} \right) \cdot \nabla(\text{div} \mathbf{v}) + \mathbf{F},$$

where  $\eta$  is the first viscosity coefficient ( $\eta = 18.1 \cdot 10^{-6} [\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}]$  for air at temperature  $T = 293.16 [\text{K}]$ ),  $\xi$  is the second viscosity coefficient ( $\xi = 15.6 \cdot 10^{-6} [\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}]$  for air at temperature  $T = 293.16[\text{K}]$ , compare [51]),  $\mathbf{F} = \rho \mathbf{g}$  is the gravitational body force density,  $\mathbf{g}$  is the gravitational acceleration of Earth ( $\mathbf{g} = (0, 0, -9.81)[\text{m} \cdot \text{s}^{-2}]$ ),  $\nabla \mathbf{v}$  is gradient of the vector (so it is a tensor of rank 2). We assume that the gaseous mixture is a compressible and viscous fluid.

**E2. Balance of mass of mixture – Equation of continuity.**

$$(7.2) \quad \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = S.$$

We have assumed the source **D1**.

**E3. Balances of masses of constituents of mixture – Diffusion equations.**

**E3a.**

$$(7.3) \quad \rho \cdot \left( \frac{\partial c_i}{\partial t} + \mathbf{v} \circ \nabla c_i \right) = S_i^E - c_i S \\ + \sum_{m=1}^{N-1} \left\{ (D_{im} - D_{iN}) \cdot \operatorname{div} \left[ \rho \nabla \left( c_m + \frac{k_{T,m}}{T} \cdot \nabla T \right) \right] \right\}, \quad i = 1, \dots, N_E.$$

**E3b.**

$$(7.4) \quad \rho \cdot \left( \frac{\partial c_i}{\partial t} + \mathbf{v} \circ \nabla c_i \right) = -c_i S \\ + \sum_{m=1}^{N-1} \left\{ (D_{im} - D_{iN}) \cdot \operatorname{div} \left[ \rho \nabla \left( c_m + \frac{k_{T,m}}{T} \cdot \nabla T \right) \right] \right\}, \\ i = (N_E + 1), \dots, N,$$

where  $D_{im} = D_{mi}$  is the mutual diffusivity coefficient from the  $i$ -th constituent to  $m$ -th one, and  $D_{ii}$  is the autodiffusivity coefficient of the  $i$ -th constituent, and  $k_{T,m}$  is the thermodiffusion ratio of the  $m$ -th constituent. The diffusivity coefficients and thermodiffusion ratios are constant and known (compare [52]). In **E3a** we have assumed the sources **D1-D2**. In **E3b** only the source **D1** is taken into account. Since the mixture is in motion, we cannot neglect the convection term:  $\mathbf{v} \circ \nabla c_i$ . We assume that the barodiffusion and gravitodiffusion coefficients are equal to zero.

**E4. Balance of energy of mixture.**

$$(7.5) \quad \rho \cdot \left( \frac{\partial \epsilon}{\partial t} + \mathbf{v} \circ \nabla \epsilon \right) = - \left( -\frac{1}{2} \mathbf{v}^2 + \epsilon \right) \cdot S + \mathbf{t} : \nabla \mathbf{v} + \operatorname{div}(-\mathbf{q}) + \sigma,$$

where  $\epsilon$  is the mass density of intrinsic (internal) energy of the air mixture,  $\mathbf{t}$  is the stress tensor, symbol  $:$  denotes the contraction operation,  $\mathbf{q}$  is the vector of flux of heat. We assume that [2]:

$$(7.6) \quad \epsilon = \sum_{i=1}^N \epsilon_i,$$

$$(7.7) \quad \epsilon_i = \frac{1}{m_i} \left\{ c_i k_B T \exp\left(-\frac{m_i |\mathbf{g}| z}{k_B T}\right) \cdot \left[ \left(-\frac{z}{c}\right) \cdot \left(1 - \exp\left(-\frac{m_i |\mathbf{g}| c}{k_B T}\right)\right) - \exp\left(-\frac{m_i |\mathbf{g}| c}{k_B T}\right) \right] \right\} + \tilde{\mu}_i c_i,$$

$$(7.8) \quad \tilde{\mu}_i = \frac{\mu_i}{m_i},$$

$$(7.9) \quad \mu_i = k_B T \cdot \left\{ \ln \left[ (c_i p) (k_B T)^{-\frac{c_{p,i}}{k_B}} \left(\frac{m_{\text{air}}}{m_i}\right) \left(\frac{2\pi h^2}{m_i}\right)^{\frac{3}{2}} \right] \right\} + m_i |\mathbf{g}| z,$$

$$(7.10) \quad \mathbf{t} : \nabla \mathbf{v} = \sum_{m=1}^3 \sum_{k=1}^3 t_{mk} \cdot \frac{\partial v_m}{\partial x_k},$$

$$(7.11) \quad t_{mk} = -p \delta_{mk} + \eta \cdot \left[ \left( \frac{\partial v_m}{\partial x_k} + \frac{\partial v_k}{\partial x_m} - \frac{2}{3} \delta_{mk} \operatorname{div}(\mathbf{v}) \right) \cdot \frac{\partial v_k}{\partial x_m} \right] + \xi \cdot [(\delta_{mk} \operatorname{div}(\mathbf{v}))^2], \quad m, k = 1, \dots, 3,$$

$$(7.12) \quad m \mathbf{q} = \sum_{i=1}^N \left\{ \left[ \left( \frac{\beta_i T}{\alpha_{ii}} + \tilde{\mu}_i \right) \cdot \mathbf{j}_i \right] + [(-\kappa) \cdot \nabla T] \right\},$$

$$(7.13) \quad \alpha_{ii} = \rho D_{ii} \cdot \left[ \left( \frac{\partial \tilde{\mu}_i}{\partial c_i} \right)_{(c_n)_{n=1, \dots, N, i \neq n, T, p}} \right]^{-1},$$

$$(7.14) \quad \beta_i = [\rho D_{ii}] \cdot \left\{ \frac{k_{T,i}}{T} - \left[ \left( \frac{\partial \tilde{\mu}_i}{\partial T} \right)_{(c_n)_{n=1, \dots, N, p}} \right] \right\},$$

$$(7.15) \quad \mathbf{j}_i = -\rho D_{ii} \cdot \left( c_i + \frac{k_{T,i}}{T} \cdot \nabla T \right), \quad \left[ \left( \frac{\partial \tilde{\mu}_i}{\partial c_i} \right)_{(c_n)_{n=1, \dots, N, i \neq n, T, p}} \right]^{-1} \right\},$$

where  $\epsilon_i$  is the mass density of intrinsic (internal) energy of the  $i$ th constituent of the air mixture,  $m_i$  the molecular mass of the  $i$ th constituent,  $k_B = 1.3807 \cdot 10^{-23} [\text{J} \cdot \text{K}^{-1}]$  is Boltzmann's constant,  $\mu_i$  is the complete partial chemical potential of the  $i$ th constituent of the air mixture (it is complete since it is composed of chemical potential without external force field and of external potential),  $m_{\text{air}} = 28.966 [\text{u}]$  is the molecular mass of air ( $1[\text{u}] = 1.66054 \cdot 10^{-27} [\text{kg}]$ ),

$\delta_{mk}$  is Kronecker's delta,  $c_{p,i}$  is the specific heat at constant pressure of the  $i$ th constituent of air mixture,  $h = 6.62608 \cdot 10^{-34}$  [J · s] is Planck's constant,  $\mathbf{j}_i$  is the vector of flux of mass of the  $i$ th constituent of the air mixture, and  $\kappa$  is the coefficient of thermal conductivity of air. These magnitudes are derived from Grand Canonical ensemble with external gravitational Newtonian field.

**E5. Equation of state of the mixture – Constitutive equation – Clapeyron's equation.**

$$(7.16) \quad \frac{p}{\rho} = \frac{R}{m_{\text{air}}} \cdot T$$

is Clapeyron's equation of state for a gaseous mixture, where  $R = 8.3145$  [J · mole<sup>-1</sup> · K<sup>-1</sup>] is the gas constant.

$$(7.17) \quad p_i = c_i \cdot \frac{m_{\text{air}}}{m_i} \cdot p$$

are partial pressures of constituents according to Dalton's law.

**E6. Balances of numbers of vehicles – Equations of continuity of vehicles.**

$$(7.18) \quad \frac{\partial k_{l,vt}^s}{\partial t} + \text{div}(k_{l,vt}^s \mathbf{w}_{l,vt}^s) = 0.$$

**E7. Equations of state of vehicles – Greenshields model.**

$$(7.19) \quad \mathbf{w}_{l,vt}^s(x, t) = \left( w_{l,vt,f}^s \cdot \left( 1 - \frac{k_{l,vt}^s(x, t)}{k_{l,vt,jam}^s} \right), 0, 0 \right).$$

The Greenshields equilibrium speed-density u-k model is assumed [6]. The values of maximum free flow speed  $w_{l,vt,f}^s$ , and of jam vehicular densities  $k_{l,vt,jam}^s$ , are given in Tables 3 and 8 of [1].

**E8. Technical parameters.**

The dependence of emissivity on the density and velocity of vehicles is assumed in the form [54]:

**E8a.**

$$(7.20) \quad e_{l,ct,vt}^s(x, t) = k_{l,vt}^s(x, t) \cdot \left[ \left( \frac{|\mathbf{w}_{l,vt}^s(x, t)| - \tilde{w}_{ct,vt,i_l}}{\tilde{w}_{ct,vt,i_l+1} - \tilde{w}_{ct,vt,i_l}} \right) \cdot (\tilde{e}_{ct,vt,i_l+1} - \tilde{e}_{ct,vt,i_l}) + \tilde{e}_{ct,vt,i_l} \right],$$

where  $\tilde{w}_{ct,vt,i_l}$  are experimental velocities,  $|\mathbf{w}_{l,vt}^s(x, t)| \in (\tilde{w}_{ct,vt,i_l}, \tilde{w}_{ct,vt,i_l+1})$ ,  $\tilde{e}_{ct,vt,i_l}$ , are experimental emissions of the  $ct$ th exhaust gas from single vehicle of  $vt$ th type at velocities  $\tilde{w}_{ct,vt,i_l}$ , respectively, measured in [kg · veh<sup>-1</sup> · s<sup>-1</sup>],

$i_l = 1, \dots, N_{EM}$ ,  $N_{EM}$  is the number of experimental measurements. Similarly, the dependence of the change of the linear density of energy on the density and velocity of vehicles is taken in the form:

**E8b.**

$$(7.21) \quad \sigma_{l,vt}^s(x, t) = q_{vt} \cdot k_{l,vt}^s(x, t) \cdot \left[ \left( \frac{|\mathbf{w}_{l,vt}^s(x, t)| - \bar{w}_{vt,i_l}}{\bar{w}_{vt,i_l+1} - \bar{w}_{vt,i_l}} \right) \cdot (\sigma_{vt,i_l+1} - \sigma_{vt,i_l}) + \sigma_{vt,i_l} \right],$$

where  $\sigma_{vt,i_l}$ , are experimental values of consumption of gasoline/diesel for a single vehicle of  $vt$ th type at velocities  $\bar{w}_{vt,i_l}$ , respectively, measured in  $[\text{kg} \cdot \text{veh}^{-1} \cdot \text{s}^{-1}]$ ,  $q_{vt}$  is the emitted combustion energy per unit mass of gasoline/diesel  $[\text{J} \cdot \text{kg}^{-1}]$  (compare [53]).

## 8. OPTIMIZATION PROBLEMS

Our control task is the minimization of the measures of the total travel time (TTT) [5], emissions (E), and concentrations (C) of exhaust gases in the street canyon, therefore the appropriate optimization problems may be formulated as follows [2]:

**F0. Vector of control.**

$$(8.1) \quad \mathbf{u} = (g_1, C_1, g_2, C_2, F) \in U^{\text{adm}},$$

where  $\mathbf{u}$  is vector of boundary control,  $g_m$  are green times,  $C_m$  are cycle times,  $F$  is offset time, and  $U^{\text{adm}}$  is a set of admissible control variables (compare **A8**, **A9**, **B5**, **B5S**, **B5SS**).

We define six functionals **F1-F6** of the total travel time, emissions, and concentrations of pollutants in single canyon, and in canyon with the nearest neighbour substitute canyons, respectively.

**F1. Total travel time for a single canyon.**

$$(8.2) \quad J_{\text{TTT}}(\mathbf{u}) = \sum_{s=1}^2 \sum_{l=1}^{n_s} \sum_{vt=1}^{VT} \int_0^a \int_0^{T_s} k_{l,vt}^s(x, t) dx dt.$$

**F2. Global emission for a single canyon.**

$$(8.3) \quad J_E(\mathbf{u}) = \sum_{s=1}^2 \sum_{l=1}^{n_s} \sum_{ct=1}^{CT} \sum_{vt=1}^{VT} \int_0^a \int_0^{T_s} e_{l,ct,vt}^s(x, t) dx dt.$$

**F3. Global pollutants concentration for a single canyon.**

$$(8.4) \quad J_C(\mathbf{u}) = \rho_{\text{STP}} \cdot \sum_{i=1}^{N_E-1} \int_0^a \int_0^b \int_0^c \int_0^{T_S} c_i(x, y, z, t) dx dy dz dt.$$

**F4. Total travel time for the canyon in street subnetwork.**

$$(8.5) \quad J_{\text{TTT,ext}}(\mathbf{u}) = J_{\text{TTT}}(\mathbf{u}) + a \cdot \sum_{s=1}^2 \alpha_{\text{TTT,ext}}^s \sum_{l=1}^{n_s} \sum_{vt=1}^{VT} k_{l,vt,\text{jam}}^s \cdot (C_s - g_s).$$

**F5. Global emission for the canyon in street subnetwork.**

$$(8.6) \quad J_{\text{E,ext}}(\mathbf{u}) = J_{\text{E}}(\mathbf{u}) + a \cdot \sum_{s=1}^2 \alpha_{\text{E,ext}}^s \sum_{l=1}^{n_s} \sum_{ct=1}^{CT} \sum_{vt=1}^{VT} e_{l,ct,vt,\text{jam}}^s \cdot (C_s - g_s).$$

**F6. Global pollutants concentration for the canyon in street subnetwork.**

$$(8.7) \quad J_{\text{C,ext}}(\mathbf{u}) = J_C(\mathbf{u}) + \rho_{\text{STP}} \cdot a \cdot b \cdot c \cdot \sum_{i=1}^{N_E-1} c_{i,\text{STP}} \cdot \sum_{s=1}^2 \alpha_{\text{C,ext}}^s \cdot (C_s - g_s).$$

The integrands  $k_{l,vt}^s$ ,  $e_{l,ct,vt}^s$ ,  $c_i$  in functionals **F1-F6** depend on the control vector  $\mathbf{u}$  **F0** through the boundary conditions **B0-B8**, through the equations of dynamics **E1-E8**, as well as, through the sources **D0-D2**. The value of the vector of control  $u$  directly affects the boundary conditions **B5**, **B5S**, **B5SS**, and then the boundary conditions **B6-B8** for vehicular densities, velocities, and emissivities. It also affects the sources **D0-D2**. Next, it propagates to the equations of dynamics **E1-E8** and then it influences the values of functionals **F1-F6**. We only deal with six monocriterial optimization problems **O1-O6**, and not with one multicriterial problem. We put the scaling parameters equal to unity:  $\alpha_{\text{TTT,ext}}^s = \alpha_{\text{E,ext}}^s = \alpha_{\text{C,ext}}^s = 1.0$ , in functionals **F4-F6**.  $\rho_{\text{STP}}$  is the density of air at standard temperature and pressure STP,  $c_{i,\text{STP}}$  is concentration of the  $i$ th constituent of air at standard temperature and pressure.  $J_{\text{TTT}}$  and  $J_{\text{TTT,ext}}$  are measured in [veh · s],  $J_{\text{E}}$  and  $J_{\text{E,ext}}$  are measured in [kg], and  $J_C$  and  $J_{\text{C,ext}}$  are measured in [kg · s], respectively.

Now we formulate six separate monocriterial optimization problems **O1-O6** that consist in minimization of functionals **F1-F6** with respect to control vector **F0** over admissible domain, while the equations of dynamics **E1-E8** are fulfilled.

**O1. Minimization of total travel time for a single canyon.**

$$(8.8) \quad J_{\text{TTT}}^* = J_{\text{TTT}}(\mathbf{u}_{\text{TTT}}^*) = \min\{\mathbf{u} \in U^{\text{adm}} : J_{\text{TTT}}(\mathbf{u})\};$$

**02. Minimization of global emission for a single canyon.**

$$(8.9) \quad J_E^* = J_E(\mathbf{u}_E^*) = \min\{\mathbf{u} \in U^{\text{adm}} : J_E(\mathbf{u})\};$$

**03. Minimization of global pollutants concentration for a single canyon.**

$$(8.10) \quad J_C^* = J_C(\mathbf{u}_C^*) = \min\{\mathbf{u} \in U^{\text{adm}} : J_C(\mathbf{u})\};$$

**04. Minimization of total travel time for a canyon in street subnetwork.**

$$(8.11) \quad J_{\text{TTT,ext}}^* = J_{\text{TTT,ext}}(\mathbf{u}_{\text{TTT,ext}}^*) = \min\{\mathbf{u} \in U^{\text{adm}} : J_{\text{TTT,ext}}(\mathbf{u})\};$$

**05. Minimization of global emission for a canyon in street subnetwork.**

$$(8.12) \quad J_{E,\text{ext}}^* = J_{E,\text{ext}}(\mathbf{u}_{E,\text{ext}}^*) = \min\{\mathbf{u} \in U^{\text{adm}} : J_{E,\text{ext}}(\mathbf{u})\};$$

**06. Minimization of global pollutants concentration for a canyon in street subnetwork.**

$$(8.13) \quad J_{C,\text{ext}}^* = J_{C,\text{ext}}(\mathbf{u}_{C,\text{ext}}^*) = \min\{\mathbf{u} \in U^{\text{adm}} : J_{C,\text{ext}}(\mathbf{u})\},$$

where  $J_{\text{TTT}}^*$ ,  $J_E^*$ ,  $J_C^*$ ,  $J_{\text{TTT,ext}}^*$ ,  $J_{E,\text{ext}}^*$ ,  $J_{C,\text{ext}}^*$ , are the minimal values of the functionals **F1-F6**, and  $\mathbf{u}_{\text{TTT}}^*$ ,  $\mathbf{u}_E^*$ ,  $\mathbf{u}_C^*$ ,  $\mathbf{u}_{\text{TTT,ext}}^*$ ,  $\mathbf{u}_{E,\text{ext}}^*$ ,  $\mathbf{u}_{C,\text{ext}}^*$ , are control vectors at which the functionals reach the minima, respectively.

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