ENGINEERING TRANSACTION • *Engng. Trans.* • **48**, 4, 357–372, 2000
Polish Academy of Sciences • Institute of Fundamental Technological Research
10.24423/engtrans.580.2000

IDENTIFICATION OF THE MATERIAL PHASES LOCATION FOR THE ONE-DIMENSIONAL UNSTEADY HEAT CONDUCTION PROBLEM

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The present paper is devoted to identification of the material phases location for one-dimensional structure with respect to the first-order sensitivity of the identification functional. A transient heat conduction problem within a thermal anisotropic one-dimensional structure is formulated. The material derivative concept and both the direct and adjoint approaches are used in considering the shape identification of the problem domain. The identification functional is assumed in the form of the "distance" between the temperature of the identified body and the measured temperature of real structure. Stationarity conditions are formulated with respect to the obtained first-order sensitivities. Numerical examples of internal boundary identification are presented.

1. Primary problem formulation

The problem formulation is typical for a class of one-dimensional problems. Parameters describing the locations of material phases are unknown whereas the state variable (for example the temperature) can be measured in fixed points by using the contact thermometer or solid rod thermometer. The measuring points can be located at the end or within the one-dimensional structure.

Some of the recent results concerning the sensitivity analysis and shape identification for steady heat conduction problem are developed in this paper in a more general setting. The first-order sensitivity was analyzed by DEMS [1] for the steady conduction problem and isotropic body material. The same problem for the anisotropic body was discussed by DEMS and KORYCKI [3] and DEMS, KORYCKI and ROUSSELET [4]. DEMS and HAFTKA [2] and DEMS and MRÓZ [5] used the material derivative concept in order to obtain the sensitivity analysis equations. KORYCKI [6] identified the shape of the anisotropic body for the steady conduction problem. Some numerical methods for shape identification were shown by ROCHE and SOKOŁOWSKI [8]. The heat transfer problems and the identification processes were discussed e.g. by KOSTOWSKI [7], SZURGUT [9], TALER [10]. The time-dependent problems given in this case in the form of variational conduction equation were solved by the method discussed by ZIENKIEWICZ [11].

Let us consider a one-dimensional structure made of finite number P of material layers (Fig. 1). Design parameters are in this case the coordinates of the material phases location b. The thermal properties of each material layer are characterized by thermal conductivity λ and material heat capacity c.

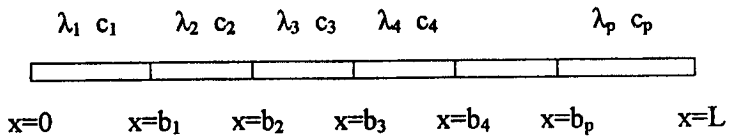

FIG. 1. The one-dimensional primary problem.

The transient heat conduction problem in one-dimensional structure can be formulated as the variational conduction equation, the proper set of boundary (i.e. Dirichlet and Neumann) conditions, and the initial conditions

$$(1.1)\qquad \begin{cases} q_{,x} + f = c\dot{T} \\ \\ q = \lambda T_{,x} + q^* \end{cases} \quad \text{for} \quad 0 \le x \le L,$$

$$T(x,t) = T^0 \text{for} \Gamma_T, \quad q(x,t) = q^0 \quad \text{for} \quad \Gamma_q, \quad T(x,0) = \overline{T}$$

$$\text{for} \quad 0 \le x \le L,$$

where we can introduce the following notations: T and q – temperature field and the heat flux, c – material heat capacity, f – heat generation source within the structure, λ – thermal conductivity, q^* – initial heat flux, $T = dT/dt$ – derivative of the temperature with respect to time t.

We can now assume the continuity of temperature and of the normal heat flux on the internal boundaries

$$(1.2)\qquad \langle T^S(x,t,\mathbf{b})\rangle = 0; \qquad \langle q^S(x,t,\mathbf{b})\rangle = 0.$$

The modification process can be described now for the one-dimensional structure as the translation of the material phases location

$$(1.3)\qquad L \to L^t : x^t = x + \delta\varphi(x,t,\mathbf{b}) = x + v^p(x,t,\mathbf{b}),$$

where $\varphi(x,t,\mathbf{b})$ is a known function and $v^p = \partial\varphi/\partial b_p$ is the transformation velocity field associated with the design parameter b_p; $p = 1..P$.

The material derivative of the length element has the following form:

$$(dL)_p = \frac{D}{Db_p}(dL) = v^p_{,x} dL. \tag{1.4}$$

Let us consider an arbitrary behavioral functional defined over a fixed time period

$$F = \int_0^{t_f} G dt = \int_0^{t_f} \left[\int_L \Psi\left(T; T_{,x}; q; \dot{T}; f\right) dL + \sum_{i=1}^{2} \gamma(T,q) \right] dt, \tag{1.5}$$

where Ψ, γ are continuous and differentiable functions of their arguments. The second term on the right-hand side of (1.5) is a sum of the function γ at both the ends of the structure described by the coordinates $x = 0$ and $x = L$.

The material derivative concept was used in shape identification of the problem domain. The first variation of the above functional has the form

$$\delta F = \frac{DF}{Db_p}\delta b_p = F_p \delta b_p, \tag{1.6}$$

where F_p denotes the first-order sensitivity of the assumed functional F with respect to design parameter b_p, $p = 1..P$.

The first-order sensitivity of functional (1.5) can be expressed with respect to (1.4) as follows:

$$\begin{aligned} F_p = \int_0^{t_f} G_p dt &= \int_0^{t_f} \left\{ \int_L [\Psi_p dL + \Psi (dL)_p] + \sum_{i=1}^{2} \gamma_p \right\} dt \\ &= \int_0^{t_f} \left\{ \int_L \left[\Psi_p dL + \Psi v^p_{,x} dL\right] + \sum_{i=1}^{2} \gamma_p \right\} dt \\ &= \int_0^{t_f} \left[\int_L \left(\Psi_p + \Psi v^p_{,x}\right) dL + \sum_{i=1}^{2} \gamma_p \right] dt \\ &= \int_0^{t_f} \left\{ \int_L \left[\Psi_{,T} T_p + \nabla_{Tx}\Psi (T_{,x})_p + \Psi_{,q} q_p + \Psi_{,\dot{T}} \dot{T}_p + \Psi_{,f} f_p + \Psi v^p_{,x}\right] dL \right. \\ &\qquad \left. + \sum_{i=1}^{2} (\gamma_{,T} T_p + \gamma_{,q} q_p) \right\} dt, \end{aligned} \tag{1.7}$$

where we can denote

$$\Psi_{,T} = \frac{\partial \Psi}{\partial T}; \quad \nabla_{Tx}\Psi = \frac{\partial \Psi}{\partial T_{,x}}; \quad \Psi_{,q} = \frac{\partial \Psi}{\partial q}; \quad \Psi_{,\dot{T}} = \frac{\partial \Psi}{\partial \dot{T}}; \quad \Psi_{,f} = \frac{\partial \Psi}{\partial f}$$

and $\gamma_{,T} = \dfrac{\partial \gamma}{\partial T}; \quad \gamma_{,q} = \dfrac{\partial \gamma}{\partial q}$.

The first-order sensitivity of the functional (1.5) can be next analyzed using both the direct and adjoint approaches. The starting points of the calculations are the primary problem formulation in the form of Eqs. (1.1) and general form of the first-order sensitivity given by Eq. (1.7).

2. Direct approach

Direct approach requires the solution of an additional time-dependent conduction problem associated with variation of each design parameter. The necessary equations are obtained by differentiating the conduction equation, boundary and initial conditions of the primary problem (1.1) with respect to each parameter. They have now for the additional structure the form of conduction equation, the set of boundary conditions and the initial conditions

$$(2.1) \qquad \begin{aligned} &\begin{cases} q^p_{,x} + f^p = c\dot{T}^p & \text{for} \quad 0 \le x \le L, \\ q^p = \lambda T^p_{,x} + q^{*p} \end{cases} \\ &T^p = T^{0p} = T^0_p - T^0_{,x} v^p \quad \text{for} \ \ \Gamma_T, \qquad q^p = q^{0p} = q^0_p - q^0_{,x} v^p \quad \text{for} \ \Gamma_q, \\ &T^p(x,0) = \overline{T}_p - \overline{T}_{,x} v^p \quad \text{for} \quad 0 \le x \le L, \end{aligned}$$

where we can introduce the following notations: $(\bullet)^p = \partial(\bullet)/\partial b_p$ – the local derivative, and $(\bullet)_p = D(\bullet)/Db_p$ – the global derivative of the appropriate quantity with respect to the design parameter.

In the above equations, the thermal conductivity λ and the material heat capacity c are assumed to be independent design parameters. Thus, T^p and q^p are the state fields of the additional problem and they should be now determined in order to solve the direct approach. After some transformations, Eq. (1.7) can be expressed, using the direct approach, in the final form:

$$(2.2) \qquad F_p = \left[\int\limits_L \Psi_{,\dot{T}} T^p dL \right]_0^{t_f} + \int\limits_0^{t_f} \sum_{i=1}^{2} \Psi v^p dt + \int\limits_0^{t_f} \left\{ \int\limits_L \left\{ \left[\Psi_{,T} - \frac{d}{dt}\left(\Psi_{,\dot{T}}\right) \right] T^p \right.\right.$$

(2.2) [cont.]

$$+ \left(\nabla_{Tx}\Psi + \lambda\Psi_{,q}\right) T^{p}_{,x} + \Psi_{,q} q^{*p} \Big\} dL + \sum_{i=1}^{2} \left[\gamma_{,T} T^{0}_{p} + \gamma_{,q}(q^{p} + q_{,x} v^{p})\right]_{\Gamma_T}$$

$$+ \sum_{i=1}^{2} \left[\gamma_{,T}(T^{p} + T_{,x} v^{p}) + \gamma_{,q} q^{0}_{p}\right]_{\Gamma_q} \Bigg\} dt,$$

where we can denote

$$\left[\int_L \Psi_{,\dot{T}} T^{p} dL\right]_0^{t_f} = \left[\int_L \Psi_{,\dot{T}} T^{p} dL\right]_{t=t_f} - \left[\int_L \Psi_{,\dot{T}} T^{p} dL\right]_{t=0} .$$

The first-order sensitivity expressions using the direct approach – see the Eq. (2.2) – are obtained as a sum of a few terms defined over the length of the structure and at its both ends.

Introducing the direct approach, we should solve P additional heat conduction problems for P design parameters and the primary heat conduction problem. In other words, evaluation of the first-order sensitivity vector requires the solution of $(P + 1)$ problems. The conduction equation and the set of boundary – initial conditions (2.1) describe each problem.

The primary and direct solutions are solved at the same time t.

3. Adjoint approach

An alternative method to calculate the first-order sensitivity is the adjoint approach in which only one adjoint heat transfer problem is solved and the adjoint state field is found. The adjoint and primary structures have the same shape and thermal properties (i.e. the thermal conductivity λ and the material heat capacity c). The conduction equation and a set of boundary and initial conditions describes the adjoint structure. The conduction equation for adjoint structure is assumed in the same form as that for the primary structure – see the Eq. (1.1),

$$\text{(3.1)} \qquad \begin{cases} q^{a}_{,x} + f^{a} = c\dot{T}^{a} & \text{for} \quad 0 \le x \le L, \\ q^{a} = \lambda T^{a}_{,x} + q^{*a} \end{cases}$$

where $\dot{T}^{a} = \dfrac{dT^{a}}{d\tau}$ is the derivative of the temperature with respect to time τ. We can next assume that τ is the time determined now for the adjoint structure, and τ can not be equal to t.

Let us multiply the Eq. (3.1) by the test function z and integrate along the structure

$$\int_L z\left(q^a_{,x} + f^a - c\dot{T}^a\right) dL = 0. \tag{3.2}$$

Assume $z = T^p$ (temperature for the additional structure in direct approach) and integrate the Eq. (3.2) with respect to time

$$\int_0^{t_f}\int_L [(T^p q^a)_{,x} - T^p_{,x} q^a + T^p f^a] dL dt - \int_0^{t_f}\int_L cT^p \frac{dT^a}{d\tau} dL dt = 0. \tag{3.3}$$

After some transformations of Eq. (2.1) we have obtained

$$\begin{aligned}\int_0^{t_f}\int_L [(\lambda T^p T^a_{,x} - \lambda T^p_{,x} T^a + T^p q^{*a} - T^a q^{*p})_{,x} \\ + T^p f^a - T^a f^p - T^p_{,x} q^{*a} + q^{*p} T^a_{,x}] dL dt \\ + \left[\int_L cT^a T^p dL\right]_0^{t_f} - \int_0^{t_f}\int_L cT^p \frac{dT^a}{dt} dL dt - \int_0^{t_f}\int_L cT^p \frac{dT^a}{d\tau} dL dt = 0.\end{aligned} \tag{3.4}$$

The solution of above problem can be considerably simplified under vanishing sum of the last two integrals in (3.4). Thus, it is convenient to assume that

$$-\frac{dT^a}{d\tau} = \frac{dT^a}{dt}. \tag{3.5}$$

Under the above assumption, we have the sum of the two last integrals in (3.4)

$$-\int_0^{t_f}\int_L cT^p \frac{dT^a}{dt} dL dt - \int_0^{t_f}\int_L cT^p \frac{dT^a}{d\tau} dL dt = 0. \tag{3.6}$$

From (3.5) it may be concluded that the following transformation between the primary and adjoint time is considered

$$\begin{aligned}\tau = t_f - t; \quad t = t_f \quad &\Rightarrow \quad \tau = 0, \\ t = 0 \quad &\Rightarrow \quad \tau = t_f.\end{aligned} \tag{3.7}$$

Thus, the adjoint problem should be solved backwards in time in relation to the primary and direct solution.

The first term on the left-hand side of Eq. (3.4) can be expressed now in the form

$$
(3.8)\quad \int_0^{t_f}\int_L [(\lambda T^p T^a_{,x} - \lambda T^p_{,x} T^a + T^p q^{*a} - T^a q^{*p})_{,x} + T^p f^a - T^a f^p - T^p_{,x} q^{*a}
$$

$$
+ q^{*p} T^a_{,x}] dL dt = \int_0^{t_f}\int_L (T^p f^a - T^a f^p - T^p_{,x} q^{*a} + q^{*p} T^a_{,x}) dL dt
$$

$$
+ \int_0^{t_f} \sum_{i=1}^{2} (\lambda T^p T^a_{,x} - \lambda T^p_{,x} T^a + T^p q^{*a} - T^a q^{*p}) dt \,|_{\Gamma_T}
$$

$$
+ \int_0^{t_f} \sum_{i=1}^{2} (\lambda T^p T^a_{,x} - \lambda T^p_{,x} T^a + T^p q^{*a} - T^a q^{*p}) dt \,|_{\Gamma_q} .
$$

It is easily seen that after simple transformations we obtain from (3.4) and Eqs. (3.5), (3.6) and (3.8)

$$
(3.9)\quad \left[\int_L c T^a T^p dL\right]_{t=t_f} + \int_0^{t_f}\int_L (T^p f^a - T^p_{,x} q^{*a}) dL dt
$$

$$
+ \int_0^{t_f} \sum_{i=1}^{2} (\lambda T^p T^a_{,x} + T^p q^{*a}) dt \,|_{\Gamma_q} - \int_0^{t_f} \sum_{i=1}^{2} (\lambda T^p_{,x} T^a + T^a q^{*p}) dt \,|_{\Gamma_T}
$$

$$
= \left[\int_L c T^a T^p dL\right]_{t=0} + \int_0^{t_f}\int_L (T^a f^p - q^{*p} T^a_{,x}) dL dt
$$

$$
- \int_0^{t_f} \sum_{i=1}^{2} (\lambda T^p_{,x} T^a + T^a q^{*p}) dt \,|_{\Gamma_q} - \int_0^{t_f} \sum_{i=1}^{2} (\lambda T^p T^a_{,x} + T^p q^{*a}) dt \,|_{\Gamma_T} .
$$

Our next goal is to determine the conditions for the adjoint structure. Let us compare the suitable terms in Eqs. (2.2) and (3.9),

$$
\begin{aligned}
&T^a(x,\tau=0) = \frac{1}{c}\Psi_{,\dot{T}}(x,t=t_f) \quad \text{for} \quad 0 \le x \le L,\\
&f^a = \Psi_{,T} - \frac{d}{dt}\left(\Psi_{,\dot{T}}\right) \quad \text{for} \quad 0 \le x \le L,\\
(3.10)\qquad &q^{*a} = -(\nabla_{Tx}\Psi + \lambda\Psi_{,q}) \quad \text{for} \quad 0 \le x \le L,\\
&q^{0a}(x,\tau) = \gamma_{,T} \quad \text{on} \quad \Gamma_q,\\
&T^{0a}(x,\tau) = -\gamma_{,q} \quad \text{on} \quad \Gamma_T.
\end{aligned}
$$

Introducing now (3.10) and (3.9) into (2.2), the equation of the direct approach of the sensitivity analysis can be transformed to the final form

$$
\begin{aligned}
(3.11)\qquad F_p = {}& \left[\int_L (cT^a - \Psi_{,\dot{T}})(T_p - T_{,x}v^p)dL\right]_{t=0}\\
&+ \int_0^{t_f}\int_L (f^pT^a - q^{*p}T^a_{,x} + \Psi_{,q}q^{*p})dLdt + \int_0^{t_f}\sum_{i=1}^{2}\Psi v^p dt\\
&+ \int_0^{t_f}\left\{\sum_{i=1}^{2}\left[\gamma_{,T}T^0_p + \gamma_{,q}q_{,x}v^p + (\lambda T^p_{,x} + q^{*p})T^a\right]_{\Gamma_T}\right.\\
&\left.+ \sum_{i=1}^{2}\left[\gamma_{,T}T_{,x}v^p + \gamma_{,q}q^0_p - (\lambda T^a_{,x} + q^{*a})T^p\right]_{\Gamma_q}\right\} dt.
\end{aligned}
$$

The first-order sensitivity due to the adjoint approach – see the Eq. (3.11) – is given as a sum of few integrals defined along the structure and at its both ends.

Introducing the adjoint approach we should solve one adjoint heat conduction problem and one primary heat conduction problem. Thus, the evaluation of the first-order sensitivity vector requires the solution of 2 problems. The conduction equation (3.1) and the set of boundary and initial conditions (3.10) describe each problem.

The adjoint solution is solved at the time τ, backward in time t in relation to the primary and direct solution.

Using the equations derived above for direct and adjoint approaches of the sensitivity analysis let us formulate the identification formulation of the problem.

4. Identification of the material phases location

The identification functional can be assumed as the "distance" between the temperature of the identified body T and the measured temperature T_m of the real structure at the end $\Gamma_m \in \Gamma_q$:

$$J = \frac{1}{2} \int_0^{t_f} (T - T_m)^2 dt \Big|_{\Gamma_m} . \tag{4.1}$$

The stationarity conditions of the identification functional have the form

$$J_p = \frac{DJ}{Db_p} = 0, \tag{4.2}$$

where DJ/Db_p are the first-order sensitivity expressions formulated by adaptation of direct and adjoint approaches, respectively. The local derivatives of the integrand of (4.1) with respect to $\gamma = \gamma(T)$ and $\Psi = 0$ are shown in the Table 1.

Table 1. Local derivatives of the integrand for identification functional (4.1).

	$\Gamma_m \in \Gamma_q$	$\Gamma_q - \Gamma_m$	Γ_T
$\gamma_{,q}$	0	0	0
$\gamma_{,T}$	$T - T_m$	0	0

Using the direct approach, the first-order sensitivity of the identification functional can be expressed now with respect to (2.2)

$$J_p = \int_0^{t_f} \left\{ \sum_{i=1}^{2} [(T - T_m)(T^p + T_{,x} v^p)]_{\Gamma_m \in \Gamma_q} \right\} dt. \tag{4.3}$$

The additional heat conduction problem has the form of conduction equation, the set of boundary conditions and the initial conditions (2.1).

Using the adjoint approach, the first-order sensitivity of the identification functional can be expressed (cf. Eq. (3.11))

$$J_p = \left[\int_L cT^a (T_p - T_{,x} v^p) dL \right]_{t=0} + \int_0^{t_f} \int_L (f^p T^a - q^{*p} T^a_{,x}) dL dt \tag{4.4}$$

$$+\int_0^{t_f}\left\{\sum_{i=1}^{2}\left[\left(\lambda T^p_{,x}+q^{*p}\right)T^a\right]_{\Gamma_T}+\sum_{i=1}^{2}\left[(T-T_m)\,T_{,x}v^p - \left(\lambda T^a_{,x}+q^{*a}\right)T^p\right]_{\Gamma_m\in\Gamma_q}\right\}dt. \tag{4.4 [cont.]}$$

The adjoint heat conduction problem has the form of the conduction equation (3.1) and the set of conditions (3.10).

The calculation of temperature requires the solution of the conduction equation for primary and additional or primary and adjoint structures. Using the analytical methods (in fact, explicite methods – e.g. the method of separation of variables), it is difficult to solve the conduction equation in real engineering problems; the temperature should satisfy the boundary and initial conditions (1.1) and the continuity condition on the internal boundary (1.2).

The conduction equation should be integrated in time, i.e. the time derivatives dT/dt, dT^p/dt, dT^a/dt should be calculated. It is convenient to integrate the above equations using the method described by ZIENKIEWICZ [11]. The temperature is interpolated in each Finite Element by equation

$$T = N_i(t)T_i^e, \tag{4.5}$$

where $N_i(t)$ – the continuous shape functions in time interval, T_i^e – the nodal values of the temperature at the specified time t.

For linear interpolation of the problem only the values at the time $t_0 = 0$ and $t_1 = \Delta t$ are considered. The Eq. (4.5) have now the following form:

$$T = |N_0\ N_1|\left\{\begin{array}{c}T_0\\ T_1\end{array}\right\}, \tag{4.6}$$

and the shape functions are expressed as follows:

$$N_0 = (\Delta t - t)/\Delta t; \quad N_1 = t/\Delta t. \tag{4.7}$$

The Eqs. (4.6) and (4.7) are considered in (1.1), (2.1) and (3.1) and the problems can be solved with respect to time. $T_1(t = \Delta t)$ is calculated with respect to $T_0(t = 0)$.

In this paper we have not analyzed the stability and stationarity conditions of the above equations (4.5) and (4.6). The problem will be solved in a separate paper.

5. Numerical examples

Let us assume the primary structure defined by Fig. 2. The one-dimensional structure made of two material layers, is characterized by the material heat capacities c_1, c_2 and the thermal conductivities λ_1, λ_2. At the right-hand end of the structure the heat flux $q(b,t) = q^0 be^{-t}$ is considered. We have also one design parameter, the coordinate of the material phases location b. The identification functional was assumed at the end $\Gamma_m \in \Gamma_q$ in the form (4.1). The shape modification is in this case the translation of the location of the material phases,

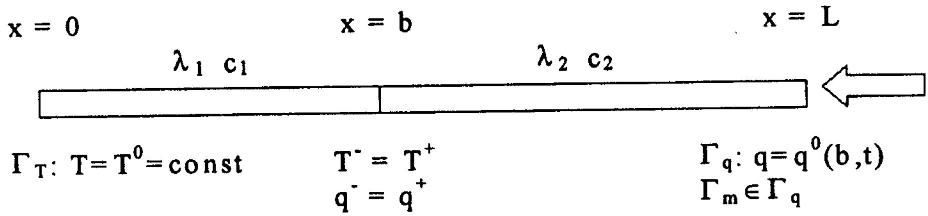

FIG. 2. The primary structure assumed in numerical examples.

$$L \to L^t : x^t = x + \delta b. \tag{5.1}$$

This modification gives us the configuration, which minimizes the identification functional. It is convenient to assume the initial heat flux $q^* = 0$ and the heat source $f = 0$. The characteristic values of transformation velocity are assumed now: $v(x=0) = 0; v(x=L) = 0; v(x=b) = 1$. The velocity can be considered in the form

$$v(x) = \begin{cases} \dfrac{x}{b} & \text{for} \quad 0 \le x \le b, \\ \dfrac{L-x}{L-b} & \text{for} \quad b \le x \le L. \end{cases} \tag{5.2}$$

In this case we assume the linear interpolation of primary and adjoint problems in time.

Under the above assumptions, the primary unsteady conduction problem can be simplified in relation to (1.1) and has now the form

$$\begin{aligned} \lambda T_{,xx} &= c\dot{T} && \text{for} \quad 0 \le x \le L, \\ T(0,t) = T^0 &= \text{const} && \text{on} \quad \Gamma_T, \\ q(L,t) &= q^0 be^{-t} && \text{on} \quad \Gamma_q, \\ T(x,0) &= \overline{T} && \text{for} \quad 0 \le x \le L. \end{aligned} \tag{5.3}$$

The problem was solved using the direct approach. The additional structure has now the form shown in Fig. 3.
The conditions of the additional problem are obtained by applying Eqs. (3.10),

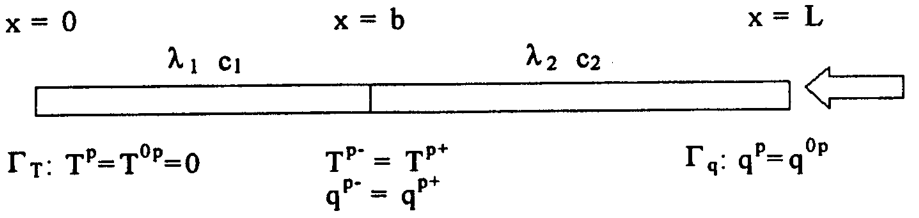

FIG. 3. The additional structure in numerical examples.

$$(5.4)\qquad \begin{aligned} \lambda T^p_{,xx} &= c\dot{T}^p \quad \text{for} \quad 0 \le x \le L, \\ T^p = T^{0p} &= 0 \quad \text{on} \quad \Gamma_T, \\ q^p = q^{0p} = q^0_p &= q^0 e^{-t} \quad \text{on} \quad \Gamma_q, \\ T^p(x,0) &= \overline{T}_p - \overline{T}_{,x} v^p \quad \text{for} \quad 0 \le x \le L. \end{aligned}$$

The first-order sensitivity of the identification functional can be now expressed with the aid of Eqs. (4.3) and (5.2) as follows:

$$(5.5)\qquad J_p = \int_0^{t_f} (T - T_m) T^p \,|_{\Gamma_m \in \Gamma_q} dt.$$

5.1. Example 1

Let us consider a one-dimensional structure, the overall length of this structure is L. The initial location of the material phases is described by the coordinate $b_0 = 0,45L$. The left-hand layer is the thermal isotropic chrominium steel (0.8% Cr, 0.2% C), characterized by $\lambda_1 = 40$ W/mK, $c_1 = 20$ kJ/K. The right-hand layer is the thermal isotropic steel V2A, and thermal parameters equal $\lambda_2 = 15$ W/mK, $c_2 = 10$ kJ/K. At the right-hand end of the structure the heat flux $q(L,t) = q^0 b e^{-t}$ is assumed. At the same end of the structure we have the boundary $\Gamma_m \in \Gamma_q$, the temperature T_m is measured and its values in time are shown in the Fig. 4. At the left-hand end of the structure the value of temperature $T(0,t) = 0$ is known. The finite element net used in the analysis has 25 nodes.

Calculations were performed by using the external penalty function. Figure 5 shows the initial and final material phases location for $q^0 = 100$, whereas Fig. 6 presents the current identification history.

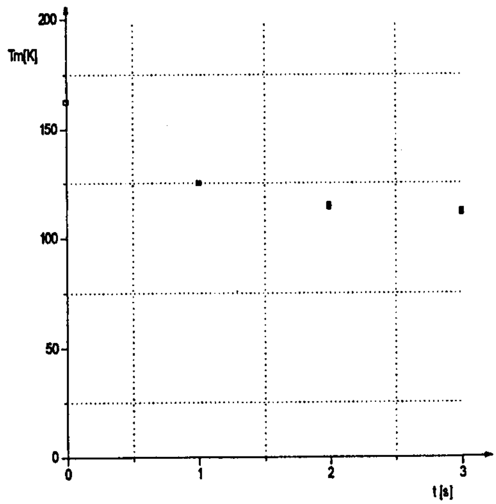

FIG. 4. The measured temperature T_m at the end $\Gamma_m \in \Gamma_q$.

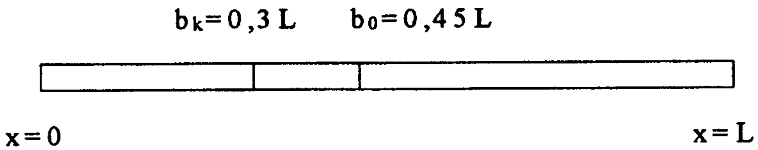

FIG. 5. Initial and final material phases location for $q^0 = 100$.

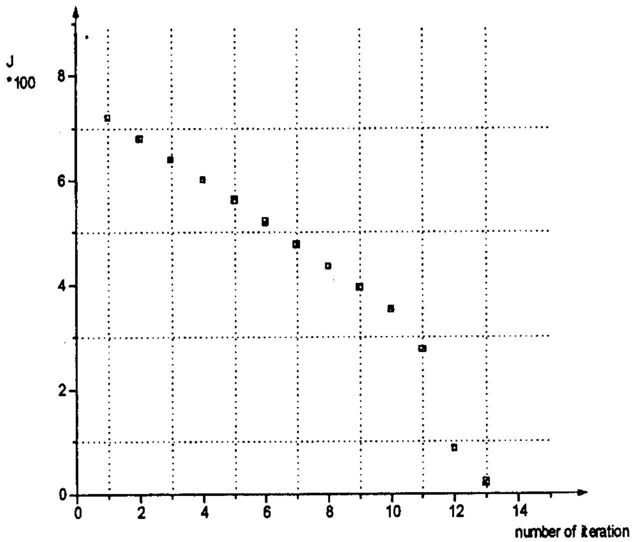

FIG. 6. The identification history.

5.2. Example 2

Let us consider the one-dimensional structure, the overall length of this structure is L. The initial location of the material phases is described by the coordinate $b_0 = 0.8L$. At the right-hand end of the structure the heat flux $q(L,t) = q^0 b e^{-t}$ is assumed. At the right-hand end of the structure we have the boundary $\Gamma_m \in \Gamma_q$: the temperature T_m is measured and its values in time are shown in Table 2 and in Fig. 7.

Table 2. The identified T_i and measured T_m values of the temperature for the Example 2.

Time [s]	1	2	3	4	5	6	7	8
T_m [°C]	53.9773	20.4797	12.0066	9.8633	9.3211	9.1840	9.1493	9.1406
T_i [°C]	53.9773	20.4797	12.0066	9.8633	9.4211	9.1840	9.1493	9.1406

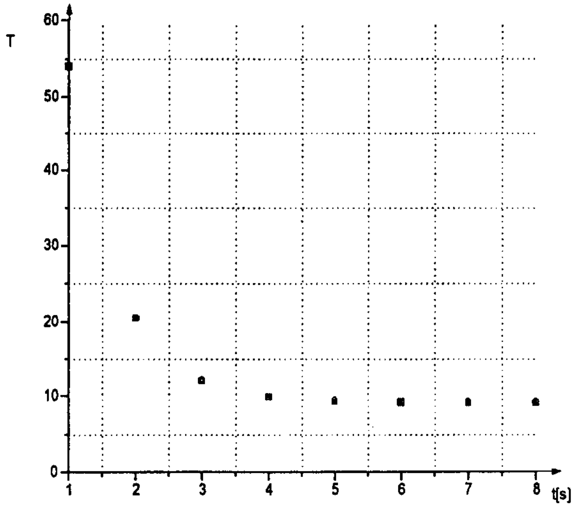

FIG. 7. The measured temperature T_m at the end $\Gamma_m \in \Gamma_q$.

The left-hand layer is in this case the thermal isotropic brass, characterized by thermal coefficient $\lambda_1 = 80$ W/mK, $c_1 = 10$ kJ/K. The right-hand layer is now the thermal isotropic zinc, $\lambda_2 = 110$ W/mK, $c_2 = 20$ kJ/K. At the left-hand end of structure the value of the temperature $T(0,t) = 0$ is known.

The finite element net used in the analysis has 25 nodes. Calculations were performed by using the external penalty function. Fig. 8 shows the identification history and Fig. 9 the initial and final material phases location for $q^0 = 100$.

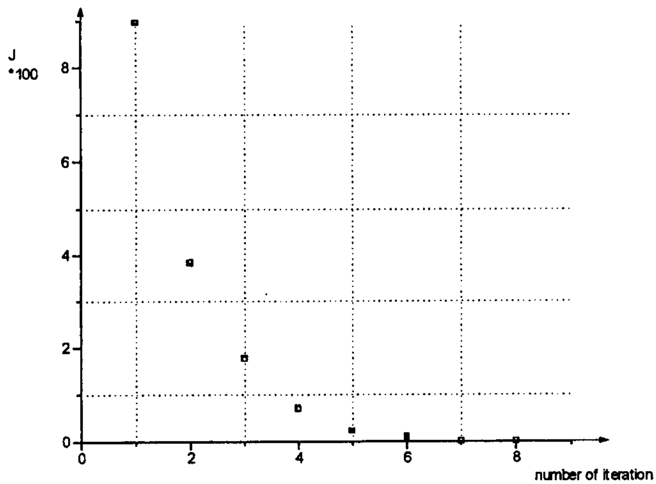

FIG. 8. The identification history.

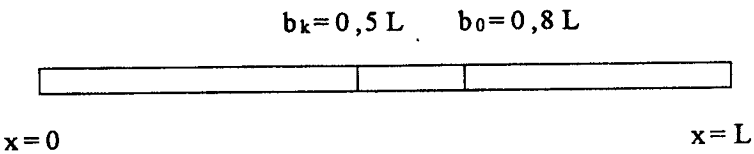

FIG. 9. The initial and final material phases location for $q^0 = 100$.

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Received October 22, 1999; revised version March 29, 2000.