A SIMPLE SINGLE SPECIMEN METHOD FOR MEASURING
THE FRACTURE SURFACE ENERGY OF RUBBER MATERIALS

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In this investigation, particular attention is paid to the generalized theory of fracture mechanics developed by Andrews and based on the spatial strain energy density distribution W in the case of an infinite lamina. A numerical analysis using a finite element method brings out some serious shortcomings in the fundamental equation of this theory when dealing with laboratory specimens of finite dimensions. It is proven that the J integral derived from this equation has no physical meaning. We have proposed an alternative expression of W which leads to a simplified J integral requiring measurements on one specimen only.

Key words: fracture, rubber, J integral, nonlinearity, finite element.

1. Introduction

Several methods based on energy considerations exist for characterization of the fracture of materials. One of the most popular methods is the J-integral of RICE [1] defined as:

$$(1.1) \qquad J = \int_{\Gamma} \left(W \, dY - \mathbf{T}_i \cdot \frac{\partial \mathbf{u}_i}{\partial X} dS \right),$$

where W is the strain energy density, $\mathbf{T}_i$ and $\mathbf{u}_i$ denote the traction and displace-ment vectors respectively, and dS is a small element on a curve Γ surrounding the notch tip. The integral assumes the same value for all paths Γ, both in the cases of linear or nonlinear reversible elasticity.

The J integral is commonly employed to study the fracture of metallic mate-rials and extended by several researchers to investigate the onset of crack growth in rubber-like materials [2, 3]. The fracture event is characterized by the critical value J_c which is an energy necessary to create new surfaces.

In 1974, ANDREWS [4] developed his theory which gives fracture criteria for solids in general, without limitations as to their linearity, elastic behaviour or infinitesimal strain. He considered an infinite sheet of an elastic material with a crack of length 'a' and loaded at infinity by a uniform stress, σ_0, acting normally to the crack axis (Fig. 1). X_0 and Y_0 are Cartesian co-ordinates of a point P of the specimen, referred to the origin fixed at the centre of the crack and to the undeformed state.
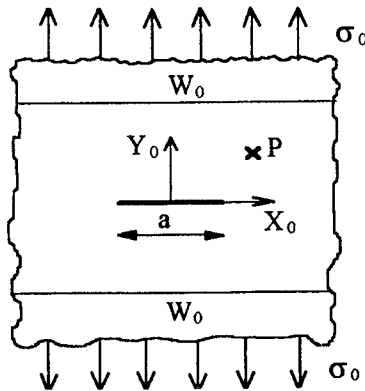

FIG. 1. Infinite lamina containing a crack.

The component of the strain energy density, $W(P)$, at any point P, is ex-pressed as follows:

(1.2) $$W(P) = W_0 f(x, y, \varepsilon_0),$$

where ε_0 is the strain and W_0 is the corresponding energy density at infinity, $x = X_0/a$, and $y = Y_0/a$ are the reduced variables, f is a function.

According to the Eq. (1.2), Andrews has shown that the net energy available for crack propagation can be written as:

(1.3) $$J = - \left(\frac{dU}{B\,da} \right)\bigg|_u = 2k(W_0)W_0 a,$$

where U is the elastic energy stored in the body, B is the undeformed thickness of the sheet, $k(W_0)$ is a dimensionless function depending of the material and varying with the strain level far from the crack.

In analogy with Linear Elastic Fracture Mechanics, a corrective factor $F(a/w, a/h, ...)$, taking into account the dimension effects (width w, height h, ...), has been introduced for the laboratory specimen [5, 6], and J has been written as:

(1.4)
$$J = 2k(W_0)W_0 a F(a/w, a/h, ...).$$

In our recent investigation [6, 7], the experimental terms of Eq. (1.4) have been determined using an original technique derived from the load separation criterion [8]. This procedure needs several specimens with different crack lengths.

The aim of this paper is evaluation of the energy parameter J from the spatial distribution of the numerical strain energy density and comparison of the results obtained in this way to the J integral of Rice. This approach brings us some indications about the truth of the relation (1.2) of Andrews in the case of the laboratory specimen. We finally suggest an expression of the strain energy density leading to a simple method which requires to perform the measurements on only one specimen to determine the fracture surface energy.

2. EXPERIMENTAL

2.1. Materials, specimens, tests

The materials selected in this investigation are:

• An ethylene-propylene-diene rubber (E.P.D.M), reticulated with sulphur and filled with carbon black. The volume fraction of carbon black is about 30 percent and the average size of the particles is 5 μm. This material displays deformation up to 15% and exhibits a non-linear but dissipative behaviour involved by the detachment of the particles from the polymeric matrix when loading the specimen.

• A polybutadiene (P.B.) which is a synthetic elastomer used as a binding agent in the preparation of solid propellants. This material displays large deformation (up to 130%) and its mechanical behaviour, which is nonlinear elastic, can be described using the hyperelastic formalism.

The fracture tests (Mode I) have been performed using the Single Edge Notch in Tension (S.E.N.T.) and the Double Edge Notch in Tension (D.E.N.T.) specimens (Figs. 2 and 3), on a conventional Instron tensile machine, at room temperature and under a constant crosshead speed of 10 mm/mn. The specimen, the

dimensions of which are summarised in Table 1, were cut from the same material plate. The cracks were introduced using a razor blade.
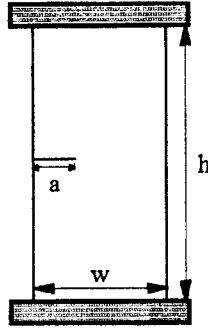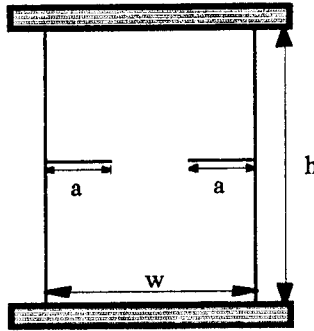

FIG. 2. S.E.N.T. specimen.

FIG. 3. D.E.N.T. specimen.

Table 1. Dimensions of specimens.

Material	specimen	Height h (mm)	width w (mm)	thickness B (mm)	crack length/width (a/w)
E.P.D.M.	S.E.N.T.	200	40	3.4	0.3, 0.4, 0.5, 0.6, 0.7
polybutadiene	S.E.N.T.	200	40	4.5	0.3, 0.4, 0.5, 0.7
	D.E.N.T.	200	80	4.5	0.4, 0.5, 0.6, 0.7

A self-gripping fixture screwed to the tensile machine theoretically ensures the specimen to be loaded under displacement control. Only one test is conducted for a given crack length. The obtained load-displacement curves, recorded until total breaking, are shown in Fig. 4. The circles denote the crack initiation in the case of the SENT specimens, and the locus of these points is represented by a dotted line.
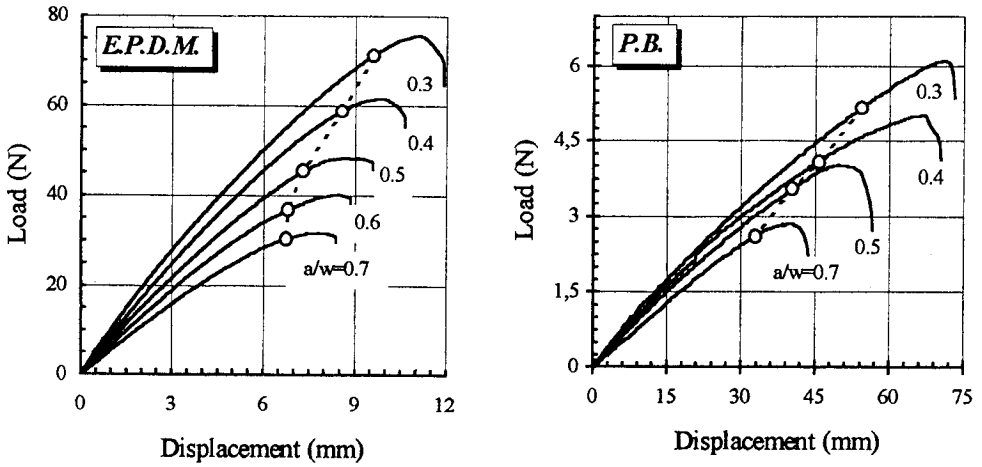
FIG. 4. Load-displacement curves of S.E.N.T. specimens. Circles denote the crack initiation points.

3. FINITE ELEMENT PROCEDURE

The numerical study was carried out by means of the ANSYS finite element program. Although this program contains hyperelastic elements, these one are developed only for the plane-strain and axisymmetric problems. So, since the sheets we used were thin, we tacked up another element which provides a capability to model plane-stress problems with large strains, including geometrical non-linearity. Unlike the case of plasticity, no permanent inelastic strains are induced and no energy is lost (the process is conservative).

The experimental identification of the constitutive law of our materials has been clearly described in our previous work [6]. Figure 5 shows the obtained results in terms of the engineering stress as a function of the extension ratio λ (final length/original length).

The discrete values (about one hundred) of true stress:

$$(3.1) \qquad \sigma = \frac{F}{S_0}\lambda,$$

as functions of logarithmic strains:

$$(3.2) \qquad \varepsilon = \ln(\lambda)$$

introduced in the numerical model, are extracted from an average curve schematised by the solid line in Fig. 5. In Eq. (3.1), S_0 represents the undeformed

section of the specimen, and F is the applied load. The materials are assumed to be elastic and virtually incompressible (Poisson's ratio $= 0.499$). Regarding the symmetries, only a half and a quarter of the plate were analysed for the S.E.N.T and the D.E.N.T. specimens, respectively.
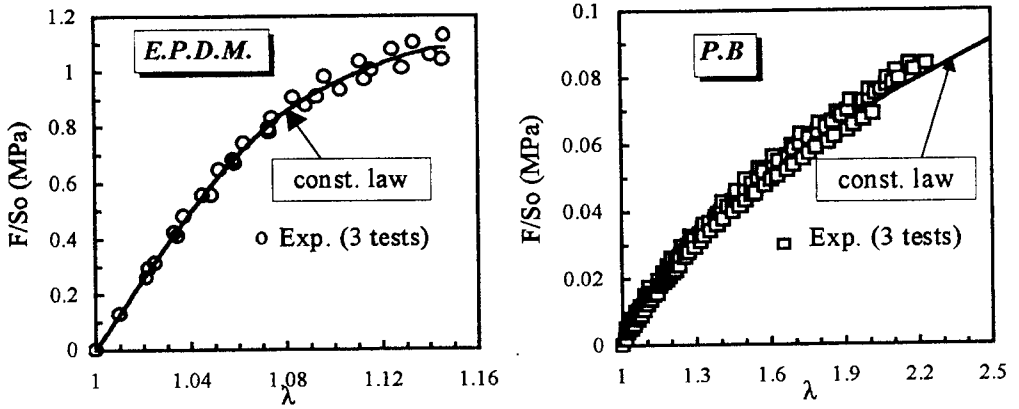

FIG. 5. Mechanical behaviour of the materials.

Discretization consists of approximately:

• 1440 isoparametric quadrilaterals with eight nodes (Fig. 6) for the calculation based upon the Andrews approach (Sec. 5.1);
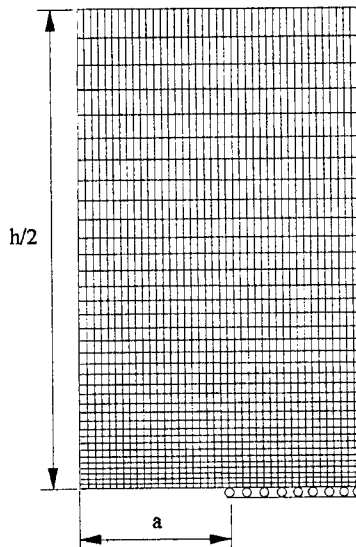

FIG. 6. Finite element meshing used for the calculation of J based upon the Andrews approach, Eq. (5.5).

• 800 elements for the evaluation of the contour J integral (Eqs. (1.1) and (5.18)). This meshing include triangles with six nodes in the vicinity of the crack tip and quadrilaterals with eight nodes far from the crack (Fig. 7).
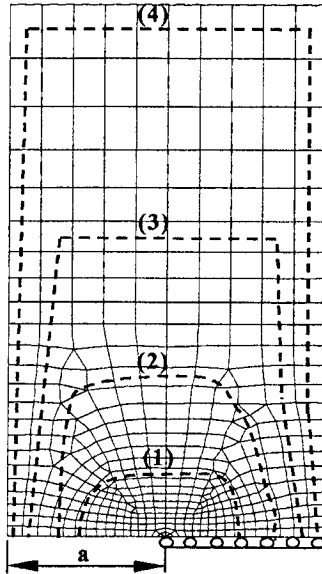

FIG. 7. Gridwork used for the calculation of the contour J integral, Eqs. (1.1), (5.18). Dotted lines represent the contours.

The loading is simulated by gradual increase of the displacement of the nodes situated on the side opposite to the crack surface, with equilibrium iteration at each step (Newton-Raphson method). The numerical analysis gives the components of the strain and the stress tensors. Then, we evaluate in the postprocessor the strain energy density using the von Mises equivalent strain (ε_{eq}) and equivalent stress (σ_{eq}):

$$(3.3) \qquad W = \int_0^{\varepsilon_{eq}} \sigma_{eq} d\varepsilon_{eq}.$$

4. COMPARISON OF CALCULATED RESULTS WITH EXPERIMENT

Our previous papers [6, 7] dealt with the experimental and the numerical evaluation of the J integral (Eq. (1.1)) on the same rubber-like materials studied here. A positive correlation has been observed between the experimental and the numerical results which, in particular, confirm the validity of the numerical model. Moreover, it has been clearly shown that crack initiation in such materials

is governed by the critical value of the J integral of Rice, allowing us to consider the values of this integral as a reference in the present study.

5. Determination of the Energy Parameter Using the Strain Energy Density Distribution

5.1. Calculation based upon the Andrews approach

According to the Eq. (1.2), an increment of crack growth, at constant W_0 (or ε_0), changes at point P the energy density:

(5.1) $$\left. \frac{dW(P)}{da} \right|_{W_0} = \frac{\partial W}{\partial x}\frac{dx}{da} + \frac{\partial W}{\partial y}\frac{dy}{da}$$

$$= a \left[\frac{\partial W}{\partial X_0}\frac{d(X_0/a)}{da} + \frac{\partial W}{\partial Y_0}\frac{d(Y_0/a)}{da} \right] dX_0 dY_0.$$

The energy parameter J derived from the relation (5.1) is expressed as:

(5.2) $$J = -\left. \frac{dU}{Bda} \right|_u = -\int_X \int_Y a \left[\frac{\partial W}{\partial X_0}\frac{d(X_0/a)}{da} + \frac{\partial W}{\partial Y_0}\frac{d(Y_0/a)}{da} \right] dX_0 dY_0.$$

where the subscript u indicates that the differentiation is carried out at constant displacement of the external clamped boundaries.

Let $R_0(O_0, X_0, Y_0)$ be any fixed orthogonal co-ordinate system, and $R(O, X, Y)$ be the one located at the crack tip, so that its origin moves with the advancing crack. The axes OX_0 and OX are parallel to the crack surface. In this case, assuming that OX is the crack growth direction $(dX = -da)$, we obtain:

$$\frac{d(X_0/a)}{da} = -\frac{X_0}{a^2}; \qquad \frac{d(Y_0/a)}{da} = -\frac{Y_0}{a^2}; \qquad \frac{d(X/a)}{da} = \frac{1}{a}\frac{dX}{da} - \frac{X}{a^2} = -\frac{1}{a} - \frac{X}{a^2}$$

$$\text{and} \quad \frac{d(Y/a)}{da} = -\frac{Y}{a^2}.$$

Thus, the Eq. (5.2) becomes:

(5.3) $$J = \frac{1}{a} \int_{X_0} \int_{Y_0} \left[\frac{\partial W}{\partial X_0}X_0 + \frac{\partial W}{\partial Y_0} \right] dX_0 dY_0,$$

when a fixed co-ordinate system $R_0(O_0, X_0, Y_0)$ is considered, and:

(5.4) $$J = \int_X \int_Y \left[\frac{\partial W}{\partial X} + \frac{1}{a}\left(\frac{\partial W}{\partial X}X + \frac{\partial W}{\partial Y}Y \right) \right] dX dY,$$

when considering the co-ordinate system $R(O, X, Y)$ which moves with the advancing crack.

The relation (5.4) can be rewritten as:

$$(5.5) \quad J = \frac{1}{a} \left[\left(\int_X \int_Y X \frac{\partial W}{\partial X} dX dY \right) + \left(\int_X \int_Y Y \frac{\partial W}{\partial Y} dY dX \right) \right]$$

$$+ \int_X \int_Y \frac{\partial W}{\partial X} dX dY = \frac{1}{a} \left[\int_Y R_{1X} dY + \int_X R_Y dX \right] + \int_Y R_{2X} dY,$$

where:

$$(5.6) \qquad R_{1X} = \int_X \frac{\partial W}{\partial X} dX = \int_P X dW,$$

$$(5.7) \qquad R_Y = \int_Y \frac{\partial W}{\partial Y} dY = \int_P Y dW,$$

$$(5.8) \qquad R_{2X} = \int_X \frac{\partial W}{\partial X} dX = W(w - a, Y) - W(-a, Y).$$

$W(w - a, Y)$ and $W(-a, Y)$ are the strain energy density at the right-hand side and on the left-hand side of the specimen over the range Y, respectively. The equation (5.3) referred to a fixed co-ordinate system can be rewritten as the expression (5.5) without the term R_{2X}.

To evaluate R_{1X}, plots are made of the values of energy density W (at the centre of elements) as a function of the abscissa X for various values of co-ordinates Y (Fig. 8). The co-ordinates (X, Y) are always referred to the unstrained specimen. R_{1X} is then evaluated by graphical integration of X with respect to W, and the values obtained are shown versus Y in the Fig. 9. In a similar manner, R_Y is obtained from the graphs of W versus Y (Fig. 10) and takes the form shown in Fig. 11 when plotted against X. The term R_{2X} evaluated analytically is plotted versus Y in Fig. 12.

Graphical integration finally gives the terms $\int_Y R_{1X} dY$, $\int_X R_Y dX$ and $\int_Y R_{2X} dY$. According to the Eq.(5.5), the summation of these terms gives the energy parameter J. The results obtained, referred to three Cartesian co-ordinate systems, are plotted in Fig. 13 against the strain energy density W_0 for both the two materials studied. It is clearly shown that the values of J depend upon the co-ordinate system position, and they are negative in the case of the polybutadiene, thus leading to the conclusion that the parameter J of Eq. (5.5) has no physical meaning.
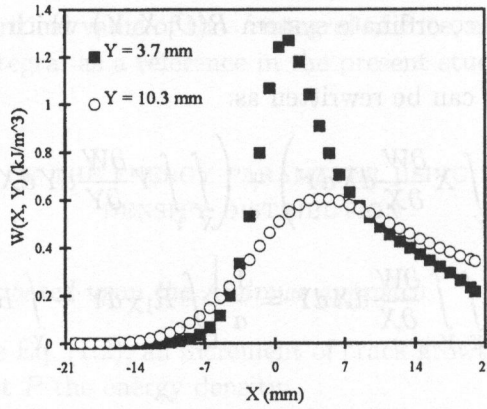
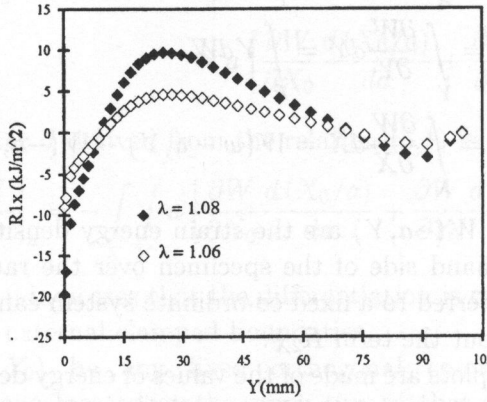FIG. 8. Plots of input energy density versus X for P.B.
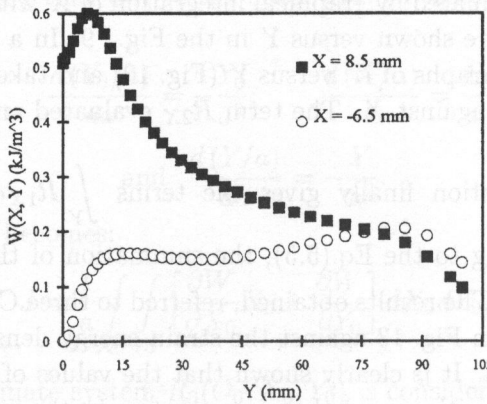

FIG. 9. Plots of R_{1X} versus Y for P.B.

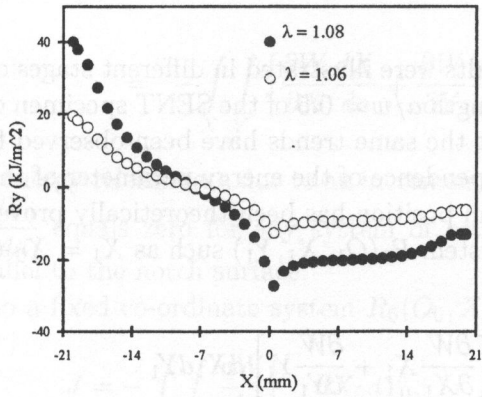FIG. 10. Plots of input energy density versus Y for P.B.

FIG. 11. Plots of R_Y versus X for P.B.
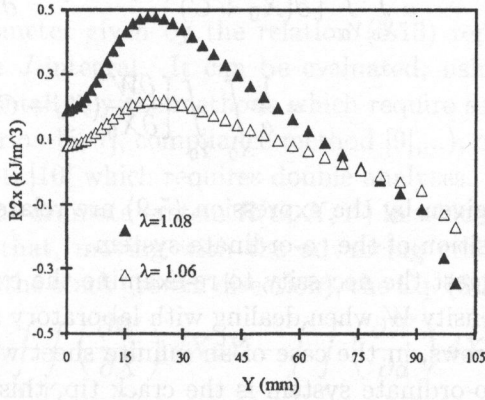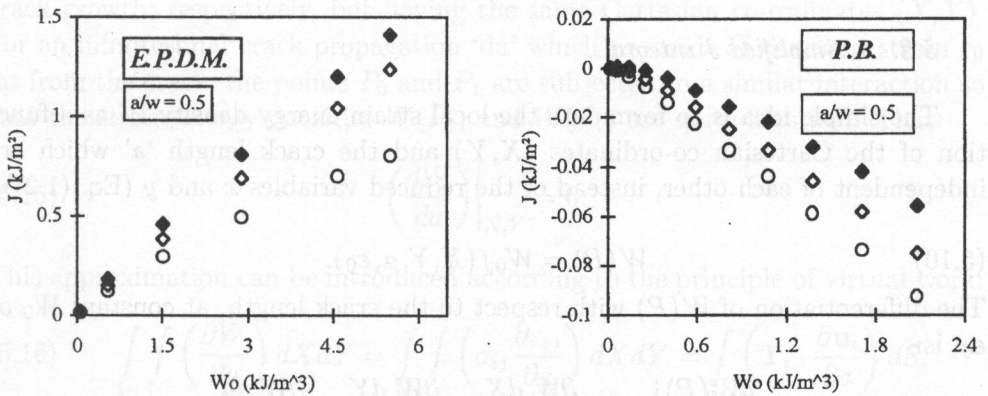

FIG. 12. Plots of R_{2X} versus Y for P.B.

FIG. 13. Energy parameter J (Eq. (5.5)) versus the uniform strain energy density W_0 (SENT specimen). ($\diamondsuit$) – co-ordinate system $R(O, X, Y)$ located at the crack tip; ($\diamond$) – co-ordinate system $R_1(O_1, X_1, Y)$ such as $X_1 = X + 21$; ($\circ$) – co-ordinate system $R_2(O_2, X_2, Y)$ such as

$$X_2 = X + 23.$$

[39]

Although the results were illustrated in different stages of the previous calculation for the crack length $a/w = 0.5$ of the SENT specimen of the polybutadiene, it must be noted that the same trends have been observed for all other cases.

The presented dependence of the energy parameter of the relation (5.5) upon the co-ordinate system position has been theoretically proved by expressing J in a fixed co-ordinate system $R_1(O_1, X_1, Y_1)$ such as $X_1 = X_0 + C$ (C is a constant) and $Y_1 = Y_0$:

$$(5.9) \quad J = \frac{1}{a} \int\limits_{X_1} \int\limits_{Y_1} \left[\frac{\partial W}{\partial X_1} X_1 + \frac{\partial W}{\partial Y_1} Y_1 \right] dX_1 dY_1$$

$$= \frac{1}{a} \int\limits_{X_0} \int\limits_{Y_0} \left[\frac{\partial W}{\partial (X_0 + C)}(X_0 + C) + \frac{\partial W}{\partial Y_0} Y_0 \right] d(X_0 + C) dY_0$$

$$= \frac{1}{a} \int\limits_{X_0} \int\limits_{Y_0} \left[\frac{\partial W}{\partial X_0}(X_0 + C) + \frac{\partial W}{\partial Y_0} Y_0 \right] dX_0 dY_0.$$

The values of J given by the expression (5.9) are related to the constant C which defines the position of the co-ordinate system.

These results suggest the necessity to re-examine the expression (1.2) of the local strain energy density W when dealing with laboratory specimens. However, as suggested by Andrews, in the case of an infinite sheet where the only identifiable origin of the co-ordinate system is the crack tip, this equation is perhaps correct.

5.2. A simplified J integral

The simple idea is to formulate the local strain energy density W as a function of the Cartesian co-ordinates (X, Y) and the crack length 'a' which are independent of each other, instead of the reduced variables x and y (Eq. (1.2)):

$$(5.10) \qquad\qquad W(P) = W_0 f(X, Y, a, \varepsilon_0).$$

The differentiation of $W(P)$ with respect to the crack length, at constant W_0 or ε_0, is:

$$(5.11) \qquad \left. \frac{dW(P)}{da} \right|_{W_0} = \frac{\partial W}{\partial X} \frac{dX}{da} + \frac{\partial W}{\partial Y} \frac{dY}{da} + \frac{\partial W}{\partial a} \frac{da}{da}.$$

The energy parameter J is then expressed as:

$$(5.12) \qquad J = -\sum_P \left. \frac{dW(P)}{B da} \right|_{W_0} dX dY$$

(5.12)
[cont.]
$$= -\int\limits_X \int\limits_Y \left[\frac{\partial W}{\partial X}\frac{dX}{da} + \frac{\partial W}{\partial Y}\frac{dY}{da} + \frac{\partial W}{\partial a}\frac{da}{da}\right] dX\,dY,$$

which represents the energy release rate due to an increment of crack growth 'da'.

The derivative $\dfrac{dY_0}{da}$ equals zero for any system of orthogonal co-ordinates whose X-axis is parallel to the notch surface.

When referring to a fixed co-ordinate system $R_0(O_0, X_0, Y_0)$, the Eq. (5.12) becomes:

(5.13)
$$J = -\int\limits_{X_0} \int\limits_{Y_0} \frac{dW}{da}\bigg|_{W_0} dX_0 dY_0,$$

since $dX_0/da = 0$.

The energy parameter given by the relation (5.13) represents an energetic interpretation of the J-integral. It can be evaluated, using a total strain energy, either experimentally by the methods which require several specimens (energy separation criterion [5, 7], compliance method [9],...), or numerically by the method of WATWOOD [10] which requires double analyses.

Considering the co-ordinate system $R(O, X, Y)$ in which the origin is located at the crack tip so that, moving with the advancing crack (i.e. $dX = -da$, assuming that OX is the crack growth direction), the Eq. (5.12) can be written as:

(5.14)
$$J = \int\limits_X \int\limits_Y \left(\frac{\partial W}{\partial X}\right) dX\,dY - \int\limits_X \int\limits_Y \left(\frac{\partial W}{\partial a}\right) dX\,dY.$$

Let P_0 and P_1 be the two points of the sheet defined before and after the crack growth, respectively, but having the same Cartesian co-ordinates (X, Y). For an infinitesimal crack propagation 'da' which proceeds at constant strain ε_0 far from the crack, the points P_0 and P_1 are subjected to a similar interaction so that the strain energy densities $W(P_0)$ and $W(P_1)$ are quite identical. Thus:

(5.15)
$$\left(\frac{dW}{da}\right)\bigg|_{X,Y} = 0.$$

This approximation can be introduced according to the principle of virtual work:

(5.16)
$$\int\limits_X \int\limits_Y \left(\frac{\partial W}{\partial a}\right) dX\,dY = \int\limits_X \int\limits_Y \left(\sigma_{ij}\frac{\partial \varepsilon_{ij}}{\partial a}\right) dX\,dY = \int\limits_{\Gamma_0} \left(\mathbf{T}_i \cdot \frac{\partial \mathbf{u}_i}{\partial a}\right) dS,$$

where Γ_0 is the curve bounding the total area of the body, dS is a small element on Γ_0, ε_{ij} is the strain tensor, $\mathbf{T}_i$ is the traction vector related to the stress tensor by $\mathbf{T}_i = \sigma_{ij}n_j$, $\mathbf{u}_i$ is the displacement vector, and n_j is the outward normal on the border of the specimen.

Assuming that the crack grows at constant displacement of the clamped boundary, the right-hand side of Eq. (5.16) vanishes. Then, the energy parameter J given by the relation (5.14) is reduced to the following form:

$$(5.17) \qquad J = \int\limits_X \int\limits_Y \left(\frac{\partial W}{\partial X}\right) dX\, dY.$$

According to the theorem of Gauss which allows to transform an integration over a surface into an integration along the curve bounding the total area of this surface, the energy parameter of the relation (5.17) becomes:

$$(5.18) \qquad J_1 = \int\limits_X \int\limits_Y \left(\frac{\partial W}{\partial X}\right) dX\, dY = \int_{\Gamma_0} W n_x dS = \int_{\Gamma_0} W\, dY$$

as suggested by OH [11], who pointed out that, taking a path along a stress-free boundary $(\mathbf{T}_i = \mathbf{0})$ or along a clamped boundary parallel to the X-axis $(\partial \mathbf{u}_i / \partial X = \mathbf{0})$, reduces the second term $\left(J_2 \int_{\Gamma_0} \mathbf{T}_i \cdot \frac{\partial \mathbf{u}_i}{\partial X} dS\right)$ of the J integral of Rice to zero.

We have evaluated the term J_2 for one crack length $(a/w = 0.5)$ of the S.E.N.T. specimen of the polybutadiene by considering four contours at increasing distances from the crack tip (Fig. 7). The results obtained are compared to the corresponding values of the J integral for different levels of the extension ratio λ (Table 2). It is clearly seen that the term J_2 decreases as the integration curve moves away from the notch. It represents 65 to 70 percent of the J integral when it is evaluated along a contour chosen in the vicinity of the crack tip (contour 1), and it becomes insignificant when the integration path is sufficiently close to the boundary of the specimen (contour 4).

Table 2. Comparison between the term J_2 and the J integral for four contours (Fig. 7) at increasing distances from the vicinity of the crack tip.

J_2/J (Rice integral)				
λ	Contour 1 %	Contour 2 %	Contour 3 %	Contour 4 %
1.04	70	40	18	0.3
1.08	68	39	17	0.1
1.12	67	37	17	0.2
1.16	66	36	17	0.8
1.20	65	35	17	1.5

The term J_1 of Eq. (5.18) is then calculated, for all the studied configurations, through the contour (4) which bounds the total area of the body (Fig. 7). For

the SENT specimens, the values of this parameter are close to those of the J integral with a difference less than 4 percent (Fig. 14). Concerning the DENT specimens, the deviation reaches 10 percent for small values of the uniform strain energy density W_0 and becomes smaller for higher values of W_0 (Fig. 15).
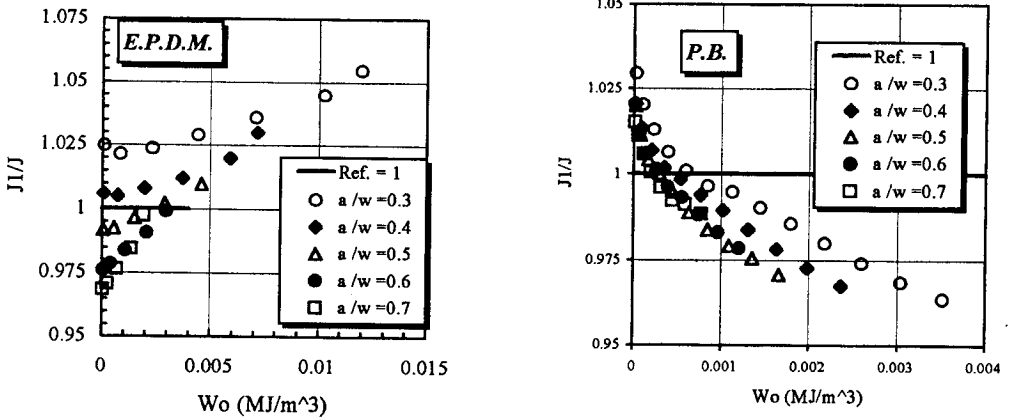

FIG. 14. Ratio J_1/J (Rice integral) as a function of the uniform strain energy density W_0 (SENT specimen).
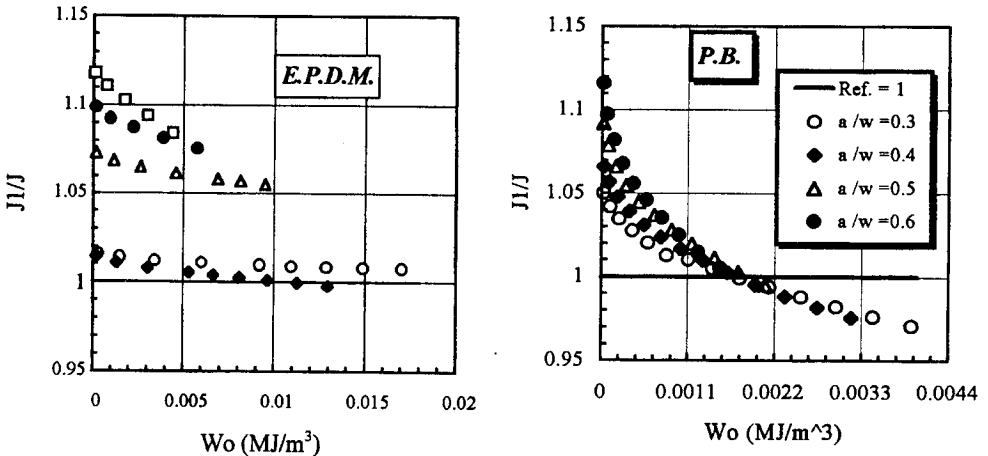

FIG. 15. Ratio J_1/J as function of the uniform strain energy density W_0 (DENT specimen).

Figure 16 shows the numerical values J_c of J_1, corresponding to the onset of crack growth (Fig. 4) and computed using the experimental critical displacements. The horizontal solid lines indicate the average critical J_1 value of 0.312 kJ/m^2 for the P.B. and of 1.88 kJ/m^2 for the E.P.D.M. The variation of

J_c evaluated for different crack lengths is about ±1 percent and ±2 percent for the P.B. and the E.P.D.M., respectively. These results lead us to the conclusion that the crack initiation is governed by a critical value of J_1 in such materials. Moreover, a direct experimental determination of the fracture toughness J_c using a single specimen is therefore possible provided that the strain distribution along the border of the sheet is measured. In this case, by symmetry, only one half of the body needs to be considered.
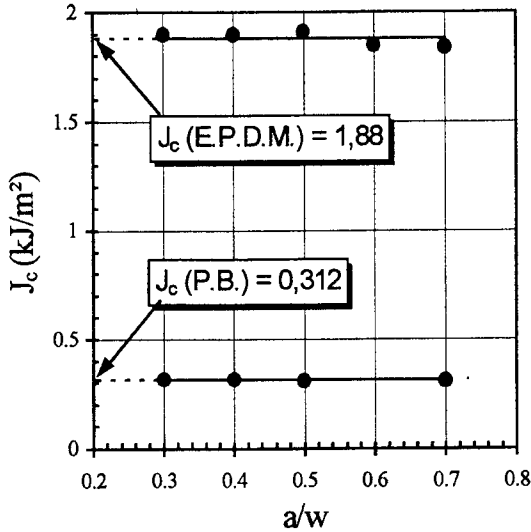

J_c (E.P.D.M.) = 1,88

J_c (P.B.) = 0,312

FIG. 16. Numerical fracture energy J_c evaluated from the relation (5.18) as a function of the crack length (S.E.N.T. specimen).

6. CONCLUSION

The values of Rice integral J have been considered as a reference according to the results obtained in our previous works [6, 7].

Some serious shortcomings have been underlined in the equation of the strain energy density distribution proposed by Andrews, when dealing with the laboratory specimens. The energy parameter J derived from this equation has no physical meaning since its values depend on the position of the co-ordinate system.

The expression of the strain energy density we suggested leads to a simplified J integral evaluated along the borderlines of the specimen. In this case, J depends only on the strain energy density, so, provided the strain distribution along this path is experimentally measured, this parameter and the fracture surface energy can be determined by testing only one specimen.

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