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EVOLUTIONARY PROGRAMS IN THE OPTIMIZATION OF CEMENTLESS HIP PROSTHESIS

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The optimal design of cementless hip prosthesis is investigated in the paper. Design variables are materials (represented by their Young's moduli) of the non-homogenous stem prosthesis, disposed in vertical layers. The minimisation of the interface stress function with constraints on the resorbed bone mass fraction is presented. A simplified two-dimensional FEM model of a stem-bone configuration is considered, enabling however to obtain essential characteristics of the stem-bone load-transfer mechanism. Evolutionary algorithm approach is applied to find the optimal solution.

1. Introduction

First papers devoted to the artificial joint design had a form either based on the creativity of the analyst, guided by FEM results, or based on systematic parametric variations in subsequent FEM calculations. Further papers were based on the numerical optimization methods. YANG *et al.* [1] introduced a design sensitivity analysis in combination with the FEM, analysing the dependence of cement strain-energy density levels on cement and stem elastic moduli in the femoral hip prosthesis.

Mathematical shape optimization of femoral hip stem was considered by HUISKES and BOEKLAGEN [2]. Design objective was to minimize stress peeks in the cement and at the cement/bone interfaces, which one believed to be the prime causes for fixation failure. Two mechanical models were applied. The first one, very simple, using the beam on elastic foundation theory and the second one as a two-dimensional FEM model.

It was concluded that concerning "traditional" stem shapes, and depending on the loads and properties, the optimal stem produce cement and cement/interface stress reduction in the range of 30 – 70%. The parametric analysis showed that to obtain maximal stress reductions, the stem should not be extremely short, and preferably made of a Co Cr-alloy, rather than titanium.

Evidently, the shape of prosthesis will not be dictated by the outcome of an optimization process, because other considerations (e.g. surgical technique, possibilities of removal, manufacturing techniques) will have to play a role as well. The mechanically optimized shape can only serve as a guideline in a design process. KOWALCZYK [3], using three-dimensional model of femur with a cementless implant and design sensitivity analysis, considered the stress concentrations and their sensitivity to various geometric parameters of the implant. Results provide a good qualitative information on the influence of geometric parameters of the implant stem on the stress distribution in the bone tissue.

The first very simple model mentioned above – beams on elastic foundation – in combination with the predictor based on a statistical model (CHANG *et al.* [4]) gives results which are in agreement with three-dimensional finite element computer simulations, and experimental and clinical results.

The designer of a cementless hip stem in total hip replacement is faced with two conflicting demands. On the one hand, a stiff stem shields the surrounding bone from mechanical loading (stress shielding). It may cause massive bone resorption mainly around the proximal part of the stem. Reduction of the stem stiffness decreases the amount of stress shielding but it promotes higher proximal interface stresses and increases the risk of proximal interface failure.

These two objectives (less stress shielding and more uniform load transfer) lead to a design conflict. It is known that more uniform load transfer requires a nonhomogeneous stem. The simplest example of a nonhomogeneous implant is the one consisting of two separate homogeneous materials, such as a cemented stem. The stiffness of the implant (stem and cement) can be controlled by changing the parameters describing the stem diameter and cement mantle thickness. Optimal distribution of elastic properties in a stem has been presented by KUIPER and HUISKES [5]. For a simplified two-dimensional FEM-model of a stem-bone configuration, the minimization problem of the interface stress function with constraints on the resorbed bone mass fraction has been solved, with Young's modulus in particular points of the stem as design variables. Some numerical results for different meshes of finite elements and various values of upper bounds for the prosthetic Youngs modulus, have been presented. In the present paper, the minimization problem of the interface stress function with constraints on the resorbed bone mass function for a similar two-dimensional FEM-model has been solved. We assume a layered medium. In each layer we assume a constant value of the Young modulus. Design variables are Young's moduli in each layer (Fig. 1b). The number of layers is given.

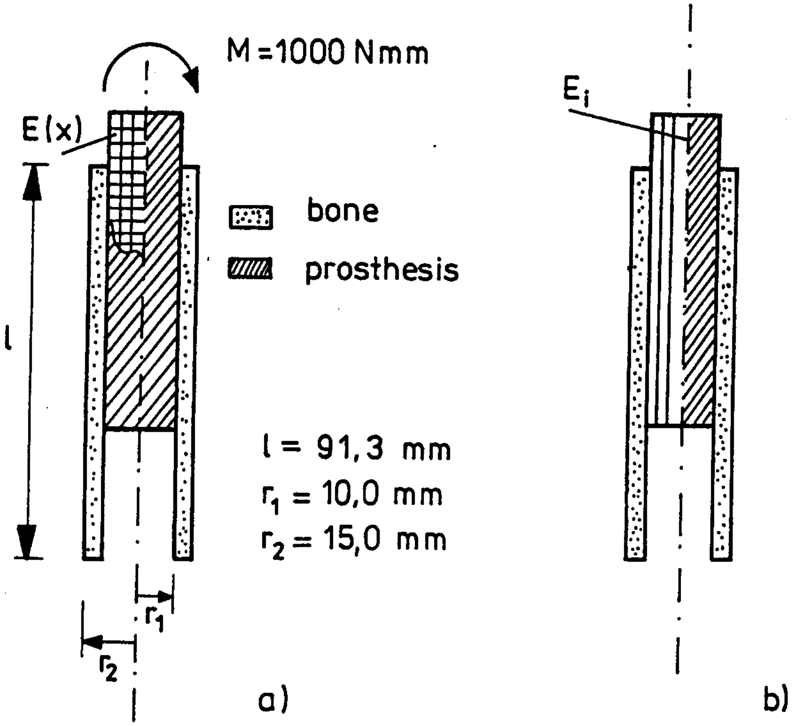

FIG. 1.

2. Resorbed bone mass fraction

Underloading of a bone may lead to bone loss. Bone can be considered as locally underloaded when its local strain energy per unit of the bone mass (S), averaged over n loading cases ($S = \sum_{i=1}^{n} U_i/g$), is beneath the local reference value S_{ref}, which is the value of S when no prosthesis is present [5, 6]. It has been observed that not every underloading leads to resorption: a certain function of underloading (the threshold level or dead zone "s") is tolerated. Hence, bone is considered to be underloaded when the local value of S is beneath the local value of $(1-s)S_{\text{ref}}$. Using this definition, the resorbed bone mass fraction m_r can be expressed in the following way:

$$m_r = \frac{1}{M}\int\limits_{\Omega} r(S(\mathbf{b};\mathbf{x}))g(\mathbf{x})d\mathbf{x} \tag{2.1}$$

where: m_r – resorbed bone mass fraction, $\mathbf{b}$ – vector of design variables, M – original bone mass, Ω – original bone volume, $\mathbf{x}$ – volume coordinates, $g(\mathbf{x})$ – local bone density, $r(S(\mathbf{b};\mathbf{x}))$ – resorptive function,

$$r(S(\mathbf{b},\mathbf{x})) = \begin{array}{ll} 1 & \text{if} \quad S < (1-s)S_{\text{ref}}, \\ 0 & \text{if} \quad S > (1-s)S_{\text{ref}}. \end{array}$$

The function $r(S)$ is a kind of "influence function". In the paper [6] by KUIPER and HUISKES, two influence function were tested. The first function is the normal or Gaussian cumulative distribution function (curve A, Fig. 2). The second one is a step function (curve B, Fig. 2). In the present paper the step function (curve B) is applied.

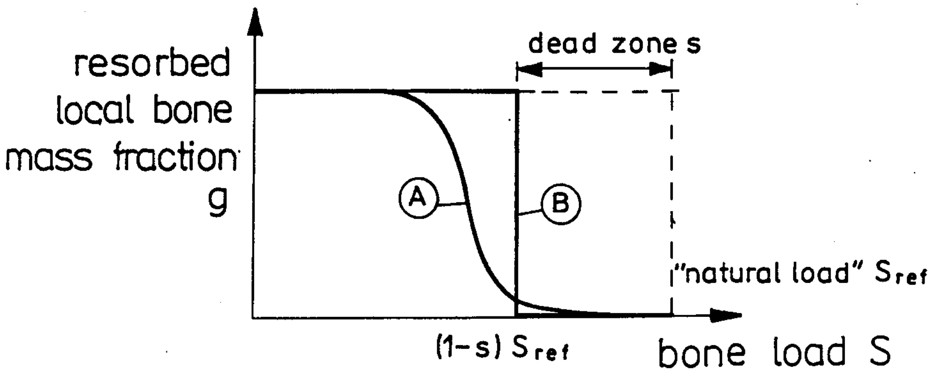

FIG. 2.

3. PROBLEM FORMULATION

The present study investigates the stem-bone load-transfer mechanism in order to minimize the shear stress function and satisfy the resorption limits. Homogenous stem solution is compared to the stem composed of different materials, distributed symmetrically in vertical layers. For the two-dimensional plane stress FEM model of a stem-bone configuration (Fig. 1b), the three following optimization problems have been solved:

• the minimization of the resorption coefficient

$$m_r \to \min; \tag{3.1}$$

• the minimization of the shear stress in the bone interface zone

$$\frac{1}{\Pi}\int_{\Pi}(\sigma_{xy}/\tau_{\text{ref}})^2 d\Pi \to \min; \tag{3.2}$$

• the minimization of the shear stress function in the interface area subjected to the constraints on the resorption coefficient

$$\min\left[\frac{1}{\Pi}\int_{\Pi}(\sigma_{xy}/\tau_{\text{ref}})^2 d\Pi\right] \tag{3.3}$$

$$\text{subjected to} \quad m_r \le m_0, \tag{3.3a}$$

where: Π – interface area, σ_{xy} – shear stresses in bone elements being in contact with the prosthesis, τ_{ref} – reference stress, m_0 – upper bound of the resorbed mass fraction m_r.

Discrete design variables of nonhomogenous stem optimization problem are material characteristics of the prosthesis vertical layers (Fig. 1b). Young's moduli E_i are chosen from a finite set of q available values. The discretteness constraints used in the three optimization problems can be formulated as follows:

$$E_i \in [E^1, E^2, E3, ..., E^q]. \tag{3.4}$$

4. Presentation of Evolutionary Algorithms

Evolutionary Algorithms (EA) are a class of stochastic search methods inspired by natural phenomena of evolution, genetic inheritance and fight for survival [7]. They attempt to emulate the biological evolutionary theories to solve the optimization problems and use a vocabulary borrowed from natural genetics. The EAs can be considered as extension of the Genetic Algorithms (which are based mainly on fixed-length binary string encoding) and allow any data structure representation suitable for a problem, together with any set of adapted operators [7].

The EAs process at every iteration a fixed number of individuals, called population. Each individual of the population represents a potential solution of the problem, and is characterized by its fitness, i.e. a measure of its performance evaluated with respect to the optimization criteria. The process of simulated evolution uses biologically inspired operators of selection and recombination (reproduction and mutation) and corresponds to a search through a space of potential solutions, coupling the elements of the exploration of the search space and the exploitation of the most promising individuals. During the iterative process, a constant population of potential solutions evolve, the best individuals are selected and its features are recombined to create new propositions of solutions. Probabilistic and random functions are applied to search for the best individual.

```
t = 0
initialize randomly Population (t)
evaluate Population (t)
while (termination condition not satisfied) do
    t = t + 1
    select Population (t) from Population (t − 1)
    recombine Population (t)
    evaluate Population (t)
```

FIG. 3. Flow-chart of a classical evolutionary algorithm (Population $(t) = \{x_1, x_2, ..., x_n\}$ denotes a population of n individuals at iteration t).

New designs are generated by recombining the information contained in the existing "parent" individuals. The crossover operator (applied with the crossover rate probability p_c) forms an offspring by swapping the corresponding segments of parents' features, and exchanges the information between different potential solutions. The mutation operator (used with the mutation rate p_m probability) changes randomly some characteristics of a selected individual and introduces in this manner some extra variability into the population. Selection is the process of creating a new generation, accomplished by copying individuals from the last generation, based upon the evaluation of fitnesses of individuals. According to the evolutionary theories, only the most suited elements are likely to survive. The individuals with higher fitness values will be selected to form the next generation, i.e. the relatively good solutions are reproduced while the relatively bad solutions die. The initial population can be created randomly. New designs are usually better and they replace the members of old generations. This evolutionary process converges after several populations and the best individual represents the solution. It cannot be shown mathematically that the EAs approach always converges to the global optimum but it is possible to find near-optimal solutions for difficult problems, where standard optimization procedures cannot be applied. These robust near-optimum propositions often introduce more realism into the optimization practice and can be acceptable for engineering design.

5. Optimization of hip prosthesis

A simplified FEM model of 2D plane stress is used for static analysis of the hip prosthesis behaviour. The stem-bone structure is meshed with 612 square isoparametric elements Q4, the material characteristics of the stem part are allowed to vary in the optimization procedure. Young's modulus $E_B = 20$ GPa and Poisson's ration $\nu_B = 0.3$ have been assumed for bone material. The stem is loaded with a bending moment of 1000 Nmm (Fig. 1). In the present approach, shear stresses σ_{xy} are calculated in the middle of each bone element staying in contact with the stem. The local strain energy density u evaluated for the bone-stem configuration and necessary to determine the resorbed bone mass fraction m_r ,is obtained using the expression

$$u = \frac{1}{2E}(\sigma_{xx}^2 + \sigma_{yy}^2) - \frac{\nu}{E}\sigma_{xx}\sigma_{yy} + \frac{1+\nu}{E}\sigma_{xy}^2. \tag{5.1}$$

This value is compared with the corresponding energy densities u_B calculated for the bone only structure in a similar manner.

The EA optimization procedure is coupled to FEM 2D plane stress analysis module. It uses real encoding of discrete design parameters, adapted genetic operators and the following exterior penalty method to formulate the fitness function f (which includes the optimization criteria and the normalised constraint violation terms, weighted by penalty coefficients)

$$f = \frac{c_1}{\sum_{nb}(\sigma_{xy}/\tau_{\text{ref}})^2 + 1} + c_2(m_0 - m_r). \tag{5.2}$$

The fitness is maximized in the optimization process. The square of normalized shear stresses is summed up over all nb elements staying in contact with the stem, m_r, m_0 are respectively the resorbed mass fraction and its upper limit, and c_1, c_2 are constant values. For the 612 element mesh model $c_1 = 2$, $c_2 = 5$ have been taken.

The stem is composed of eight vertical layers corresponding to symmetric material distribution. Design variables are Young's modulus of 4 stem layers, chosen from a list of available values. The EP problem encoding uses the prosthesis layers mapping to the material numbers and integer string problem representation.

The numerical results, presented in the next section, have been obtained for a population of 60 individuals. The single arithmetical crossover operator (applied with the probability $p_c = 0.65$) and the non-uniform mutation (the probability $p_m = 0.15$) have been used [7]. The selection procedure applies the tournament ranking using random pairs. Moreover, this approach replaces the worst individual in the new generation by the best individual found in the previous generation.

6. Numerical results

According to [5], the resorbed bone mass fraction has been limited to $m_0 = 0.25$ and the dead zone coefficient s (see Fig. 2) was set equal to 0.5. The results obtained for homogeneous stems clearly illustrate the design conflict. The stem composed of bone material ($E = 20$ GPa) gives the high interface shear stresses $\sigma_{xy,\max} = 3,32$ MPa, whereas stiff stem (titanium $E = 100$ GPa) generates lower shear stresses ($\sigma_{xy,\max} = 1.39$ MPa) but causes much bone resorption ($m_r = 0.830$).

Homogeneous stem with Youngs modulus $E = 47.8$ GPa corresponds to the limit value of bone resorption $m_r = 0.25$.

The optimization results obtained for two different catalogues of Young's modulus of the layered prosthesis are presented:
I : 9 elements catalogue [20.0; 50.0; 55.0; 60.0; 65.0; 70.0; 75.0; 80.0; 100.0][GPa],
II : 14 elements catalogue [2.0; 5.0; 7.0; 10.0; 15.0; 20.0; 30.0; 40.0; 50.0; 60.0; 70.0; 80.0; 90.0; 100.0][GPa].

The distribution of shear stresses in the interface part of the bone obtained for the first Catalogue I are presented in Fig. 4. The optimal material distribution in the prosthesis for the minimisation of m_r coefficient is shown in Fig. 5. The outer layers are much weaker than the inner layers and Young's modulus of the outer part is the same as the bone modulus.

Results for the minimization of shear stresses σ_{xy} in the bone are shown in Fig. 6. The opposite tendency of material stiffness distribution in comparison with the previous test can be noticed -outer layers are much stronger than the inner layers.

Finally, considering a minimization of σ_{xy} with the constraint on the upper limit of m_r coefficient, we obtain a trade-off material distribution in the prosthesis which is a resultant of the two previous solutions (see Fig. 7).

The solutions obtained for the Catalogue II (14 elements) are qualitatively similar to the results in the Case I. The distribution of shear stresses in the bone is shown in Fig. 8 and material distribution in the prosthesis for three minimization problems are given in Figs. 9, 10 and 11, respectively. It is worth to note the difference between the optimal solutions in Figs. 7 and 11, which is caused by various material characteristics available for the two studies.

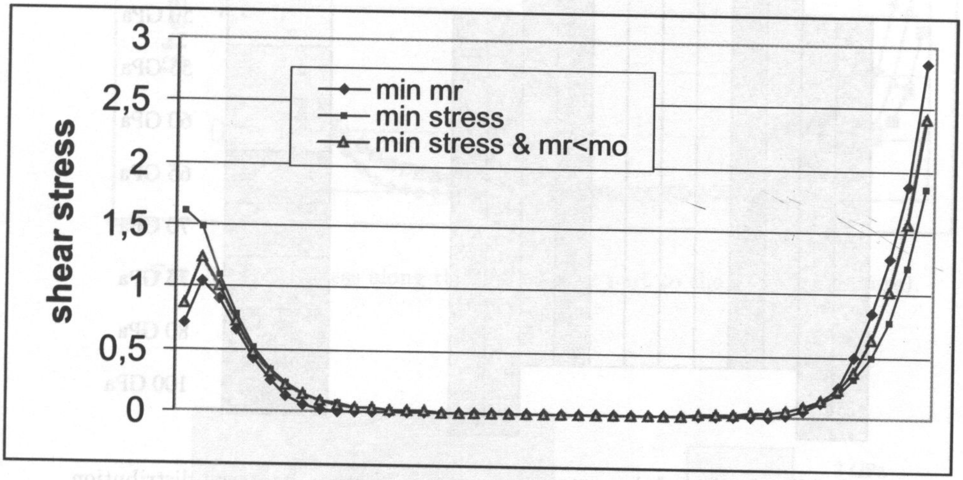

FIG. 4. Shear stress along the bone (lower part to the left of the figure).

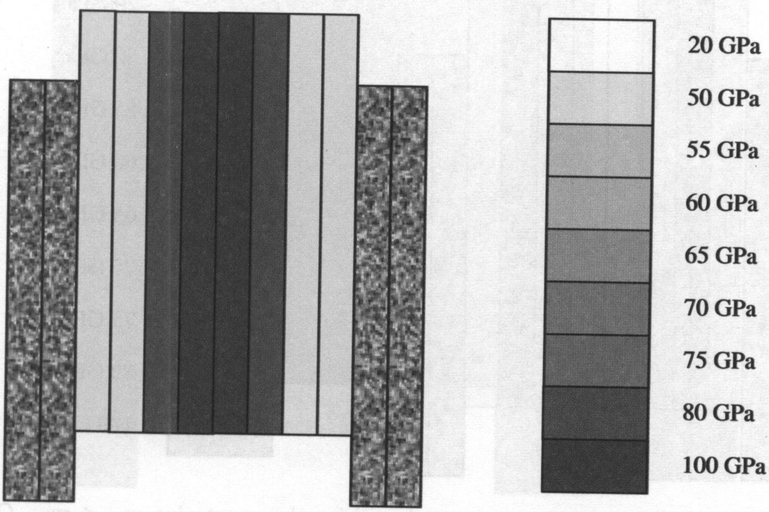

FIG. 5. Minimization of the resorption coefficient $m_r \to \min$. Optimal material distribution.

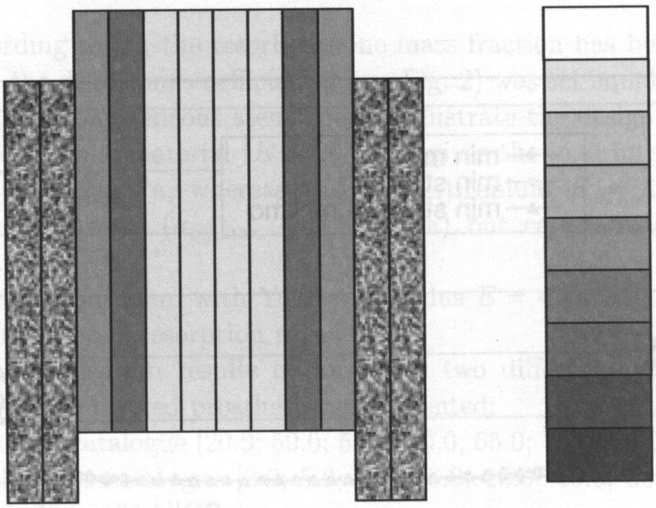

FIG. 6. Minmization of shear stress $\sigma_{xy} \to$ min. Optimal material distribution.

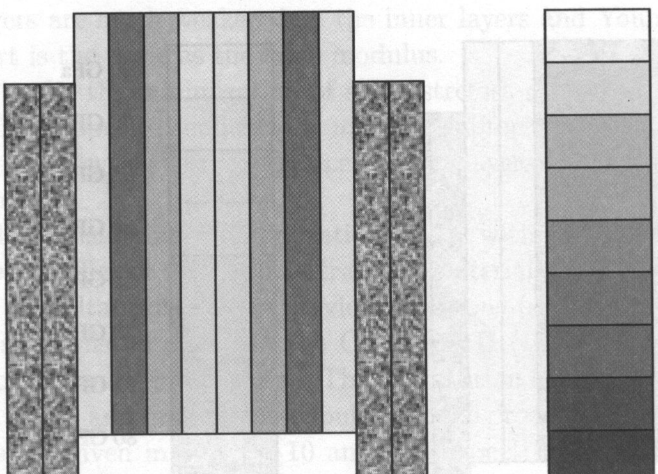

FIG. 7. Minimization of shear stress $\sigma_{xy} \to$ min subjec the constraint $m_r < m_0$. Optimal material distribution.

Case II

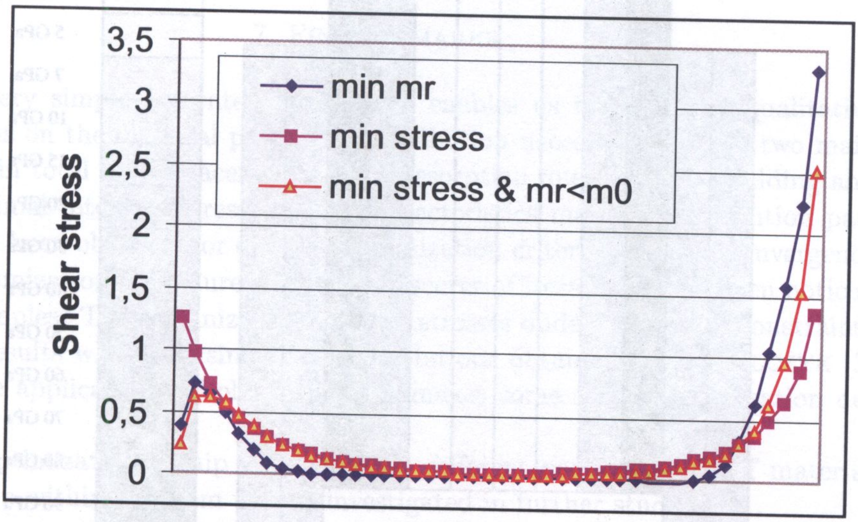

FIG. 8. Shear stress along the bone (lower part to the left of the figure).

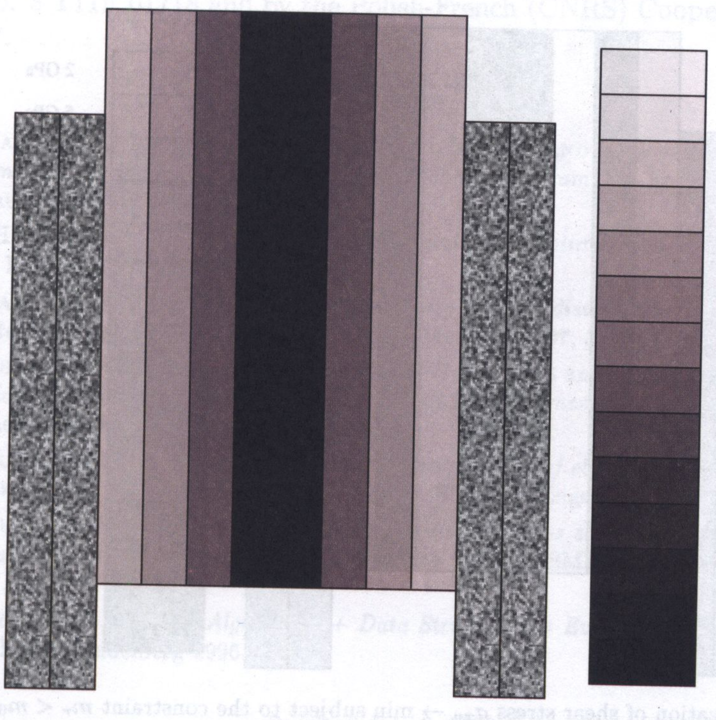

FIG. 9. Minimization of the resorption coefficient $m_r \to \min$. Optimal material distribution.

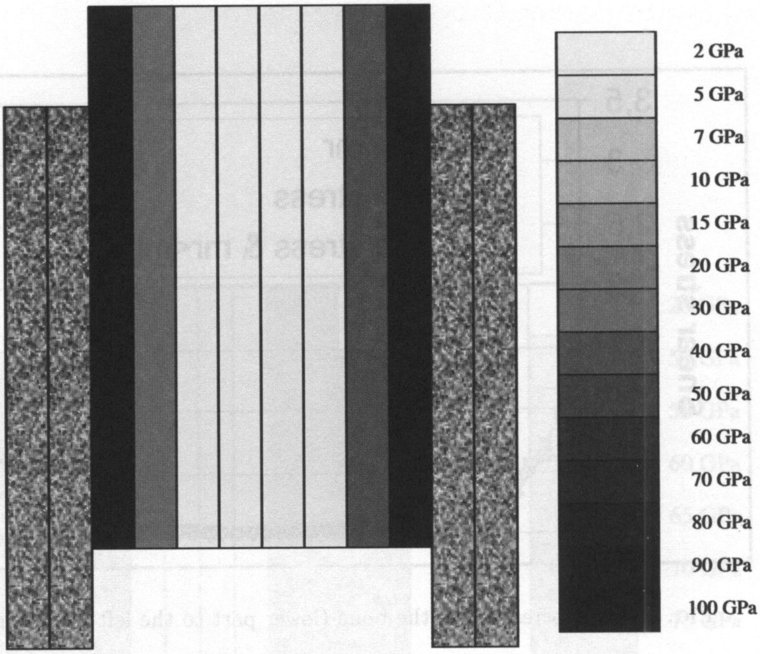

FIG. 10. Minimization of shear stress $\sigma_{xy} \to \min$. Optimal material distribution.

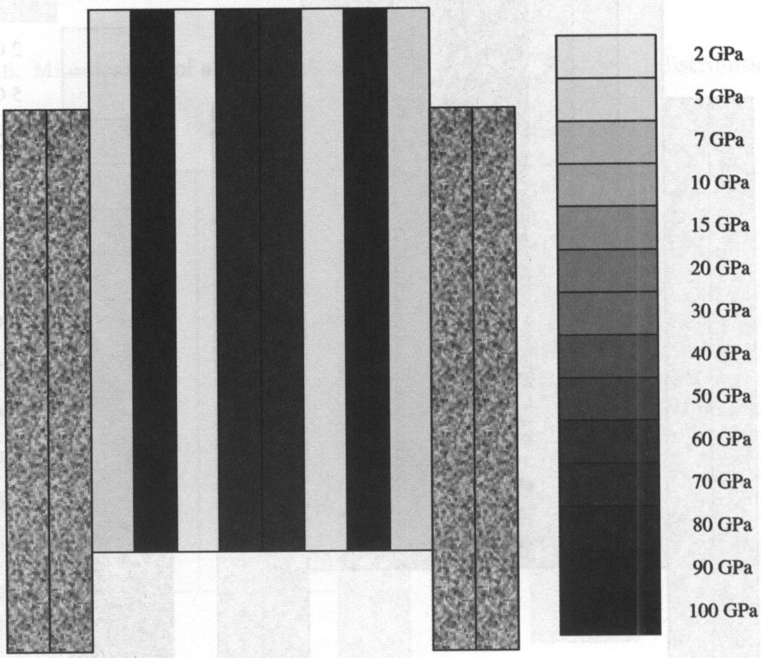

FIG. 11. Minimization of shear stress $\sigma_{xy} \to \min$ subject to the constraint $m_r < m_0$. Optimal material distribution.

7. Final remarks

The very simple presented here model enables us to formulate qualitative conclusions on the material properties distribution necessary to avoid two main problems in total hip replacements: bone resorption due to stress shielding and high proximal interface stresses. The characteristics material distribution patterns have been obtained for different optimization criteria. A rapid convergence of the optimization procedure within some scores of iterations have been noticed in all examples. The minimization of shear stresses under resorption constraints leads to results which are similar to the solutions obtained by Kowalczyk [3] ,where the application of hollow implants makes some stress concentration decrease.

The optimization of hip prosthesis for different configurations of material distribution within the stem will be investigated in further studies.

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