ENGINEERING TRANSACTIONS • *Engng. Trans.* • **49**, 4, 529–535, 2001
Polish Academy of Sciences • Institute of Fundamental Technological Research
10.24423/engtrans.541.2001

YIELD CRITERION OF THE SOFTWOOD UNDER CONDITIONS OF VARIABLE HUMIDITY

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Wood is a porous material. In a dry state the unit volume of wood consists of water-free volume of the wood substance, the so-called wood skeleton, and of the air-filled pores. The variable humidity produces the internal forces within the loaded wood bulk loading to destruction in the form of loosening wood fibers, cracks and twists. In the paper the yield criterion or the transition of the softwood into the plastic state is discussed. The outset of yielding is examined in accordance with the Hill criterion for anisotropy with various ratios of humidity. The case of plane stress with the orthotropic structure is used as an example for which the material constants are identified, resulting in a special form of the yield criterion.

Key words: yield criterion, softwood structure, humidity, moisture content.

1. Introduction

The loss of mechanical parameters of wood observed at the increase of wood moisture is interpreted as the effect of the plastifying action of water which decreases the medial energy of high-molecular components of wood substance. The plastifying action of wood occurs when there is more than one molecule of bound moisture to one cell of cellulose (MOLIŃSKI [6]).

The injury in wood results from the acting mechanical stress. When dry wood is loaded, we deal with diversified stress pattern in test samples. The high rigidity of particular anatomical elements due to low humidity of wood adds to heterogeneous distribution of internal stress. Relatively small external loads may induce in the weakest points of wood structure the stresses that locally exceed its strength. A local destruction of xylem may cause levelling of the distribution of internal stresses (RACZKOWSKI [7]) or stress concentration in other areas of the wood structure. The internal stresses may occur already in the early stages of the loading, especially in the zone of tensile stresses (ANSELL [1]). In the case of wet wood loading, because of its considerable susceptibility to strain, the distribution of internal stress is more homogenous because the microfibrils of cell walls are open to a maximum.

The aim of the paper is to present a model the describing criterion stresses in sapwood pine of variable humidity in different states of stress.

2. Theory of yield criterion and preliminaries to the experiments

The considered porous material (wood) is supposed to become plastic when the physical stress components in a given state of deformation satisfy a yield condition. We shall restrict our attention to the motion preserving coincidence of the principal directions of the tensor involved with the co-ordinates axes. For the considered motions, the yield condition has a general form as a scalar function of two quantities:

$$F(\sigma, W) = 0,$$

where σ denotes the current stress state for anisotropy and W is the humidity ratio.

The criterion approximately describing the yielding process of isotropic material is that of Huber–Mises–Hencky, but for anisotropy the generalisation of this criterion, the so-called Mises criterion (Mises [5], Hill [3], Bednarski [2]), may be taken into account. We have:

$$A_{ijkl}\, \sigma_{ij}\, \sigma_{kl} = 1, \tag{2.1}$$

where A_{ijkl} is the matrix of plastic moduli.

For simplicity we shall only consider the states of anisotropy that possess three mutual orthogonal planes of symmetry at every point. The intersections of these planes are known as the principal axes of orthotropy (Fig. 1).

If then the yield criterion (2.1) is assumed to be a quadratic function in the stress components, it has the form:

$$F(\sigma_{ij}, W) = L(\sigma_{yy} - \sigma_{zz})^2 + M(\sigma_{zz} - \sigma_{xx})^2 + N(\sigma_{xx} - \sigma_{yy})^2 + 2P\sigma_{yz}^2 + 2Q\sigma_{zx}^2 + 2R\sigma_{xy}^2 = 1 \tag{2.2}$$

where: L, M, N, P, Q, R are functions of the yield stresses and humidity in the current state of anisotropy (Hill [3]). This criterion reduces to the Huber–Mises law when the anisotropy is negligible.

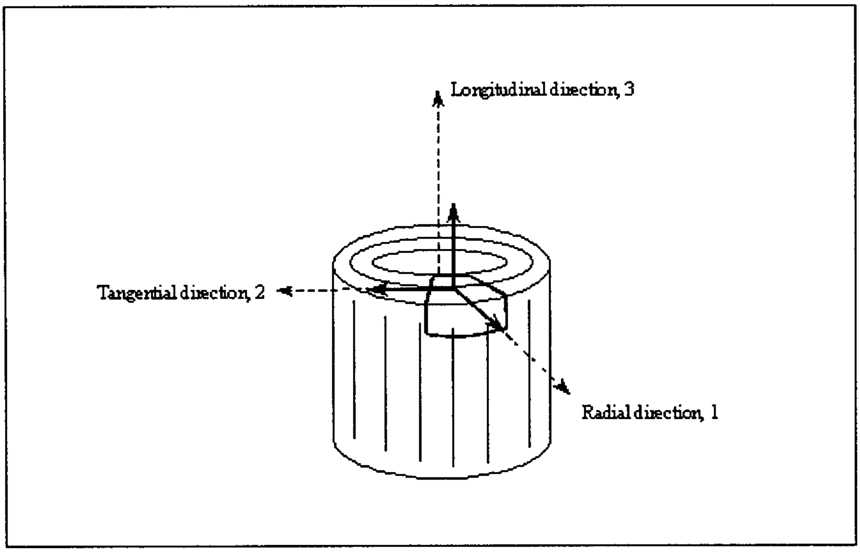

FIG. 1. Local orthogonal co-ordinates for the wood structure.

If Y_1, Y_2, Y_3 are the tensile yield stresses in the principal directions of anisotropy and k_{12}, k_{23}, k_{13} are the yield stresses in shear with respect to these axes, it can be shown that:

$$(2.3)\quad \begin{gathered} 2L = \frac{1}{Y_2^2} + \frac{1}{Y_3^2} - \frac{1}{Y_1^2}, \quad 2M = \frac{1}{Y_3^2} + \frac{1}{Y_1^2} - \frac{1}{Y_2^2}, \\ 2N = \frac{1}{Y_1^2} + \frac{1}{Y_2^2} - \frac{1}{Y_3^2}, \quad 2P = \frac{1}{k_{12}^2}, \quad 2Q = \frac{1}{k_{23}^2}, \quad 2R = \frac{1}{k_{13}^2}. \end{gathered}$$

Now (2.2) can be rewritten in the form:

$$(2.4)\quad [(M+N)\sigma_1^2 - 2N\sigma_1\sigma_2 + (L+N)\sigma_2^2 + 2R\sigma_{12}^2] \\ - 2(M\sigma_1 + L\sigma_2)\sigma_3 + 2(P\sigma_{23}^2 + Q\sigma_{31}^2) + (L+M)\sigma_3^2 = 1.$$

To describe fully the state of anisotropy in an element, we need to know the orientation of the principal axes and the values of the six independent yield stresses $Y_1, Y_2, Y_3, k_{12}, k_{23}, k_{31}$ and their dependences on humidity.

We consider now the plane state of stresses. The sheet of wood has two principal axes in the plane – x_1 (radial) and transversal x_2 (tangential). The yield criterion is:

$$(M+N)\sigma_1^2 - 2N\sigma_1\sigma_2 + (N+L)\sigma_2^2 + 2R\sigma_{12}^2 = 1 \tag{2.5}$$

or

$$\frac{\sigma_1^2}{Y_1^2} - \left(\frac{1}{Y_1^2} + \frac{1}{Y_2^2} - \frac{1}{Y_3^2}\right)\sigma_1\sigma_2 + \frac{\sigma_2^2}{Y_2^2} + \frac{\sigma_{12}^2}{k_{12}^2} = 1. \tag{2.5'}$$

It has been assumed that the influence of the stresses in the radial and tangential directions on the longitudinal strain is very small. The problem of two-dimensional moistening at the plane state of stress induced in radial and tangential directions in which the material constants are independent from temperature has been analyzed.

Initially in the moistened wood only the external layers absorb water. The resulting stresses may cause opening of xylem, clefts and twists. The research was conducted on the sapwood pine samples of the size 30x5x102mm. The following states of stress (0.50MPa) were induced:

- tension in radial direction,
- compression in tangential direction,
- both of the above mentioned states of stress acting together,
- tension in tangential direction,
- compression in radial direction,
- both of the above states of stress acting together.

The loaded samples of wood were changing its moisture content in the following intervals: 15, 21, 28%.

3. Results and their analysis

From the previous experiments we know the relation between moduli E_i and the moisture W, and we know also how to relate the yield stresses to the moisture content (Kowal and Kowalski [4]). For small variation of moisture content, this relation may be presented in the form:

$$Y_i = \frac{E_i^o A(X - X_o)}{1 + a_i E_i^o Y_i^o (X - X_o)}, \qquad i = 1, 2, 3, \tag{3.1}$$

where

$$X = \frac{W - W_o}{W_n - W_o}, \qquad X_o \approx W_o \tag{3.2}$$

W_o – initial moisture content,

W_n – moisture content at the fibre saturation point.

It results from the experiments that the following relations between the yield stresses are fulfilled

$Y_2 = \alpha Y_1$, $Y_3 = \beta Y_1$, $k_{12} = \gamma Y_1$, and the factors α, β and γ for the considered cases are approximately constant (invariable).

Therefore the yield criterion (2.5) becomes:

$$\left[\frac{1+a_1E_1^oY_1^o(X-X_o)}{E_1^oA(X-X_o)}\right]^2\sigma_1^2-\left\{\left[\frac{1+a_1E_1^oY_1^o(X-X_o)}{E_1^oA(X-X_o)}\right]^2\right.$$
$$\left.+\left[\frac{1+\alpha a_1E_1^oY_1^o(X-X_o)}{\alpha E_1^oA(X-X_o)}\right]^2-\left[\frac{1+\beta a_1E_1^0Y_1^0(X-X_0)}{\beta E_1^0A(X-X_0)}\right]^2\right\}\sigma_1\sigma_2$$
$$+\left[\frac{1+\alpha a_1E_1^oY_1^o(X-X_o)}{\alpha E_1^oA(x-X_o)}\right]^2\sigma_2^2+\left[\frac{1+\gamma a_1E_1^oY_1^o(X-X_o)}{\gamma E_1^oA(X-X_o)}\right]^2\sigma_{12}^2=1. \tag{3.3}$$

The values of some material constants and other coefficients occurring in the yield criterion (3.3) are presented in Table 1.

The graphical form of the yield criterion (3.3) for wood for various values of its initial humidity is presented in Fig. 2.

The obtained limiting curves are presented in plane system of relative stresses. The shape of conical curves and their rotation in relation to the assumed system of axes is the function of variations of the moisture content and anisotropic properties of wood.

Table 1. Some particular values of the material constants and other characteristic coefficients

Coefficient	Values
Modulus of elasticity E_1^o	715 MPa
Coefficient of the mechano-sorptive effect a_1	0.008 MPa^{-1}
Plastic strain ratio A	0.003
Proportionality factors α	5.5
β	over 30
γ	4.5

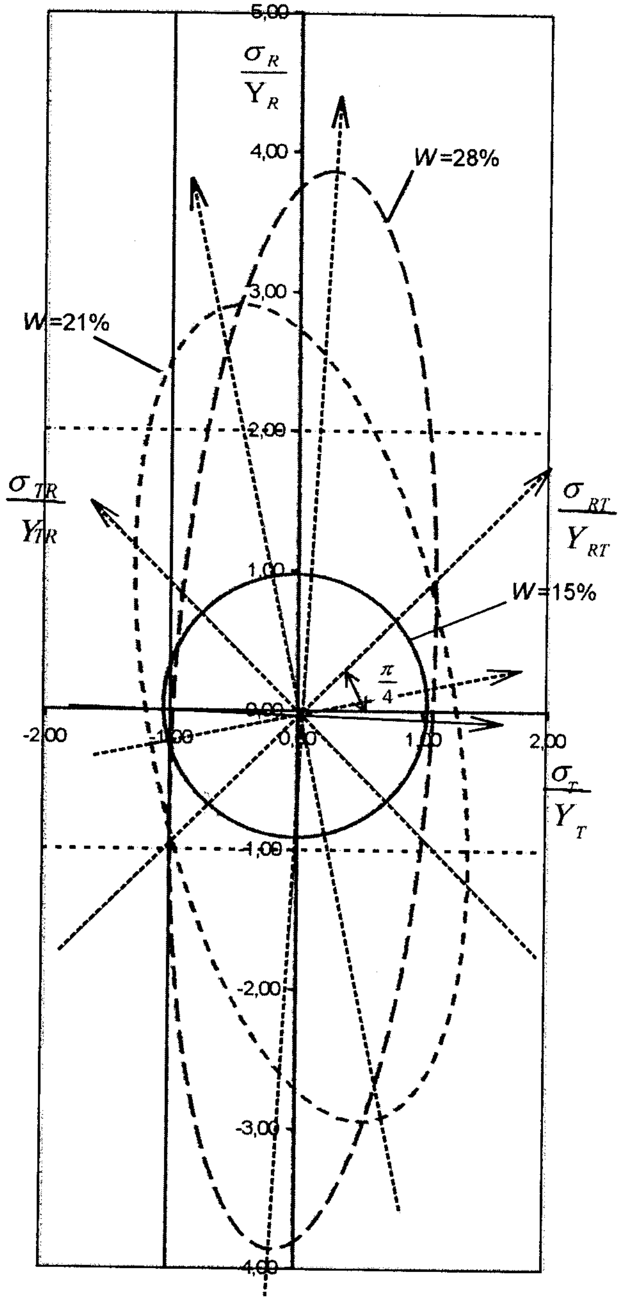

FIG. 2. Curves of the yield criterion

4. Final remarks

The paper presents a yield criterion for wood changing its moisture content and loading in various directions. The basis for this criterion is the Huber–Mises yield criterion for anisotropy. It is known that mechanical properties of wood change during its moistening in the range from dry state to the fiber saturation point. Therefore, we have assumed that in the case of wood, which undergoes simultaneous loading and wetting, the physical relations for mechanical strains are in the form of Hook's law, with material coefficients being a function of moisture content. Thus, we have automatically included the effect of coupling the stresses with the moisture content in these relations. The values of material constants appearing there are taken from the experiments (Kowal and Kowalski [4]). The values of the coefficients such as the plastic strain ratio and the factors of proportionality represent the anisotropic qualities of softwood. On the basis of the proposed yield criterion, the critical load or critical moisture may be calculated.

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Received July 19, 2000.