

## INFLUENCE OF THE STATE OF THE MATING FRICTION ELEMENTS OF THE DRUM BRAKE ON THE OUTER THERMAL FIELD

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The analysis presented below concerns the thermal field of the outer surface of the drum brake registered by means of an AGEMA Series@900LW thermovision camera and the LINY software. Roundness deviations of the inner surface of the drum measured with a TALYROND precise measurement device were compared with the distribution of temperature on the outer surface along the circumference. The results show that these quantities are interdependent and the coefficients of correlation calculated for them are always greater than 0.8. What is more, they are not affected by the type of pressure (i.e. shoe). The distribution of temperature along the generating line of the drum represents the character of the contact between the shoe and the drum. Tests carried out for various arrangements of friction members of the drum brake confirmed the interdependence of the thermal field and the geometry of mating surfaces. A maximum temperature rise of the drum has also been discussed.

### 1. INTRODUCTION

During the braking of an automotive vehicle, there is a change of kinetic energy into thermal energy and the heat is a result of rubbing of the brake linings against the drum or disk. Great amounts of heat may be produced then, especially during intensive or frequent cyclic braking (e.g. when driving in the town). The resulting increase in temperature has various negative results.

For most friction materials, an increase in temperature causes a decrease in the coefficient of friction, and in consequence, worse efficiency of braking [1, 2]. Also, due to thermal stresses, faster wear of friction linings and metal surfaces (drum or shoe) is observed. In extreme cases it can lead to the occurrence of bubbles of the vapour of the brake fluid in the pipes, which is a serious disturbance of the conditions of correct operation of the system. For these reasons, the thermal

phenomena occurring in automotive friction brakes are studied thoroughly. The temperature of the drum can be used to diagnose the brakes of a vehicle [3].

The ability to determine a proper value of temperature during the braking process is particularly important in the development and selection of friction materials and also optimal designs of brake parts. The brake thermal analysis based on "rub thermodynamics" discussed in Refs. [4, 5] and others includes forecasting the average bulk temperature of the disk or drum. A temperature distribution of the mating friction elements is calculated either theoretically [6, 7] or numerically [8, 9]. However, the analysis does not take into account the character of co-operation of the friction elements of brakes caused by e.g. their usage wear, which results in non-uniform character of normal pressure [10, 11].

This work is concerned with the analysis of the influence of the wear type and selected construction factors on the emission of heat in an automotive drum brake. On the basis of the temperature field, the quality of co-operation of friction elements is diagnosed.

Therefore, the temperature field is a result of various phenomena: the state of the surface of mating elements, changes in the shape due to wear, vibrations of the brake shoes and others. The aim of this work is to show the relations between the distribution of temperature and the occurrence of some of the phenomena.

## 2. DESCRIPTION OF THE STAND AND APPARATUS USED FOR TESTING

The measurements were carried out using an inertial stand for testing braking systems, where one brake tested at a time represents the whole braking system of a vehicle. The inertial testing is based on a simulation of the braking process for a speeding vehicle accelerated to a certain initial speed and loaded with a certain mass, which involves replacing the kinetic energy of the moving vehicle with the kinetic energy of the inertial masses.

The complementary controlling-and-measuring system enables computer-aided registration of such parameters of a tested brake as braking torque, rotational velocity of the drum, ambient temperature, pressure forces on the brake pedal, pressure in the hydraulic (braking) system, etc. The controlling-and-measuring system consists of a group of sensors connected with an analog-digital card co-operating with a PC and appropriate software. Figure 1 shows a block diagram of the discussed test stand. More details about its testing and measuring capabilities can be found in [11].

An uncovered brake drum can be observed on the stand through a THERMOVISION 900LW set thermovision camera, registering long-wave radiation with a spectral range of 8-12  $\mu\text{m}$ .

The infrared radiation sent by the tested object reaches the scanner, where it is focused by a system of elements forming the lens. Next, it is directed onto a

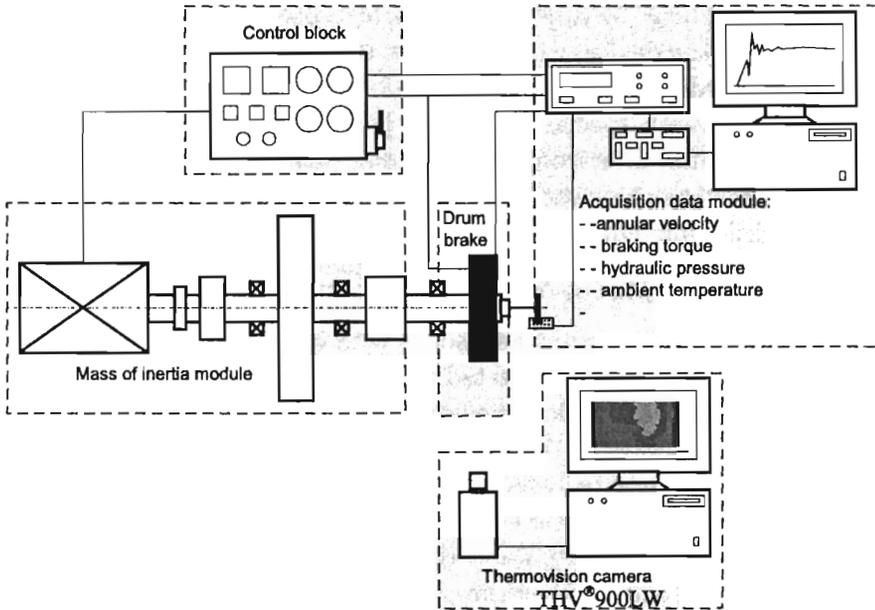


FIG. 1. Block diagram of the stand for testing brakes.

system of two rotating mirrors, one moving horizontally, the other vertically. The signal is then collected from various points on the surface of the object being in the field of view of the scanner. The exclusion of the vertical deviation enables collecting the data for a single line of the image, which is particularly useful in the case of objects rotating with high angular velocities. In such cases, the frequency of line scanning can be over 2.5 kHz. This enables the development of the surface of the rotating drum and accurate registration of the thermal field for one or more revolutions.

In order to investigate the interdependence of the distribution of temperature round the rotating brake drum and the form deviation, the form of its inner surface was measured. The measurement was carried out with a TALY-ROND device, which applies a non-reference method for precise measurements of roundness and waviness deviations. This instrument is equipped with a rotary sensor unit. The sensor fitted at the lower end of the vertically-positioned and motor-driven spindle runs round the measured object, and its axis is in line with the axis of rotation of the spindle. The measuring tip mounted at the end of the sensor touches the surface of the measured object and the differences in the radius are changed into motions of the measuring tip, which are of radial character in relation to the axis of rotation. The changes in the position of the sensor are processed into electric signals, which are then amplified and filtered. On their basis, it is possible to determine the deviations of surface roundness and

waviness, which is done by applying the highly specialised ROFORM software co-operating with the device, which analyses the measured profiles of roundness and waviness according to the international ISO standards [12]. The surface profiles are evaluated with special (analogue, parting, digital, dipolar with phase correction, and Gauss) filters, and the harmonic analysis based on the algorithm of the fast Fourier transformation allows determination of the profile form (oval, trilobing, quadrilobing, etc.).

### 3. METHODOLOGY OF MEASUREMENTS

Several pairs of brake drums and shoes with a different degree of wear and different surface geometry were selected for these tests. Various arrangements of friction pairs representing various operating conditions of their mutual contact were obtained in this way.

Measurements for selected sets of pairs (shoe-drum) were performed on an inertial stand (Fig. 1). Several series of measurements concerned the temperature on the outer surface of the drum and the mechanical properties of the braking process: rotational velocity of the drum, force acting on the brake pedal, pressure in the braking system and rotational torque. The quantities were used to find the repeatability of the measuring conditions of each series.

The tests were carried out on several brake drums with different geometry of wear. They were specially selected to illustrate some of the most frequent operational cases. The tests were carried out on drums dismantled from a vehicle as well as on those previously tested on the stand. Drums illustrating the characteristic operational cases were also checked. A drum with a relatively great axial wear (barrel-shaped surface in the direction of the generating line) was selected because it represented long-lasting co-operation with a shoe whose edges were worn. In that case, the greatest wear of the drum was observed in the central section of the shoe causing its barrel shaping along the generating line. Another drum specially prepared for the tests had the surface rolled at a small angle so that the inner diameter of the obtained cone was approximately 2 mm shorter than the outer diameter. It was a simulation of a frequent case of drum deformation. Additionally, for each set, the shoe with a different degree and character of wear was chosen. The intention was to simulate a different character of the interaction of the friction pair resulting in a different contact of the mating elements.

For each set, three measurement series were registered for various braking torques and various rotational velocities of the drum. For each drum, at least three measurement series were performed for various braking torques and various initial rotational velocities of the drum. Also, each series was registered at various stages of the braking process: at the beginning, in the middle and at the end. This selection was made to verify the work thesis for the whole braking process.

The result of all those measurements was a thermal field on the outer surface of the brake drum. The tests were carried out with the LINY option of the *Erica v.311* software. The distribution of temperatures was registered along the generating lines with a frequency of 2.5 kHz.

Sample thermographs with a distribution of temperature on the outer surface of the drum at certain arrangements of the friction members were presented and discussed in [4].

Measurements of the thermal field allowed the selection of sections characteristic for the analysis of roundness deviations. The selection was made on the basis of the distribution of temperature on the outer surface of the drum along the generating line depending on the character of the contact between the drum and the shoe. The observations of the distribution of temperature made it possible to determine the cylindricity deviation (barrel, saddle or cone). The choice of sections was dependent on the occurrence of the maximum temperature registered for a single revolution of the drum. For the barrel and conical shapes, three sections were analysed. The central one included the area of maximum temperatures, and the other two were situated at the same distance on each side. The saddle-type distribution of temperature with two local maximums required analysing six analogue sections.

A roundness deviation was determined for circles passing through points that are projections along the radius on the inner surface of the drum of points in relation to which the thermal field on the outer surface was analysed. The ROFORM program controlling the instrument analyses the measured profiles according to the international ISO standards.

In order to eliminate systematic and random errors, the measurements were repeated several times in unchanged outer conditions for a given series of tests and with the same setting of the devices.

#### 4. RESULTS

A selected drum-shoe set was fitted on the inertial stand (Fig. 1). After the system was brought up to a certain rotational speed, the drive of inertial masses was switched off and the braking system switched on. At the same time, the characteristic parameters were registered and the recording of the thermal field of the outer surface of the drum started.

Physically similar and repeating results were obtained in all measurements. The repeating feature for each series of a tested set was the same shape of the distribution of temperature along the generating line of the drum as round its circumference.

The outcome of the tests was the thermal field on the outer surface of the drum. A sample thermograph for one of the tested sets of the friction pairs was

given in Fig. 2a showing the direction round the drum  $x$  and along its generating line  $y$ . The direction of the revolution is the same as the direction of the axis  $x$ .

Figure 2b shows the distribution of temperature along the generating line of the drum for six equidistant lines, L1–L6, round the circumference. Similarly, Fig. 2c illustrates the distribution of temperature along the lines on the circumference of the drum that pass through points A, B and C shown in Fig. 2a. Point A lies in the area of occurrence of maximum temperatures. Both diagrams show a single revolution of the drum.

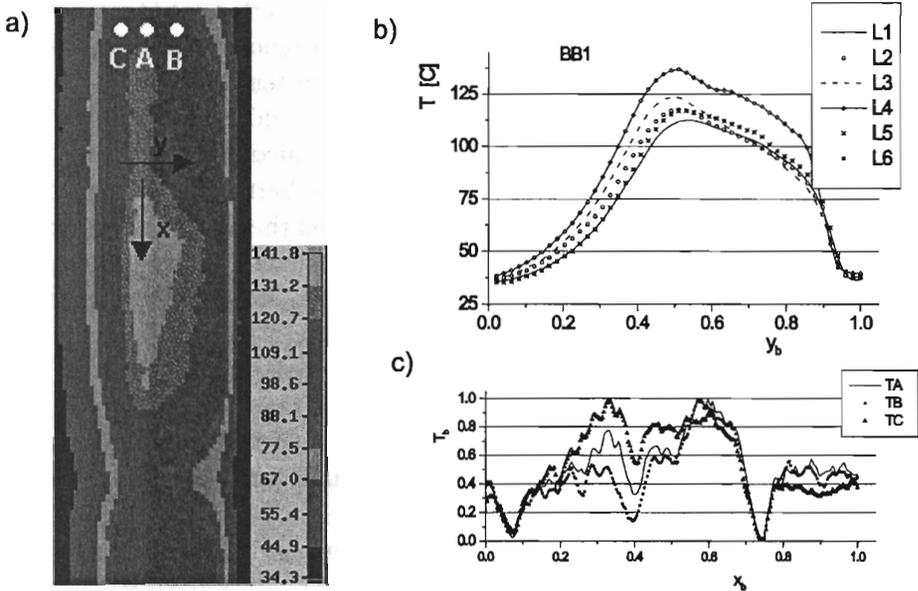


FIG. 2. The BB1 drum (the 21-st revolution): a) thermograph of a segment of the outer surface of the drum, b) distribution of temperatures along the generating line of the drum, along  $y_b$ , c) distribution of temperature on the circumference of the drum, along  $x_b$  for points A, B and C.

For all measuring series, a very good repeatability of results was obtained. A sample distribution of temperature on the circumference for five consecutive revolutions of the drum was shown in Fig. 3a.

To compare the signal of temperature for every next revolution of the drum, the non-dimensional co-ordinate  $T_b$  referring to a single revolution was introduced. The minimum temperature  $T_{\min}$  and the maximum temperature  $T_{\max}$  were assumed to be 0 and 1 respectively, so the relation is

$$(4.1) \quad T_b = \frac{T - T_{\min}}{T_{\max} - T_{\min}}.$$

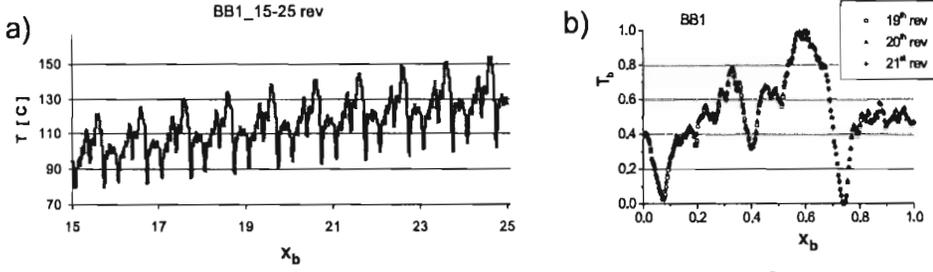


FIG. 3. Temperature on the circumference of the BB1 drum at point A: a) for ten consecutive revolutions (from the 16<sup>th</sup> to 25<sup>th</sup>), b) for three consecutive revolutions (19<sup>th</sup>, 20<sup>th</sup> and 21<sup>st</sup>).

Similarly, the distances round the drum,  $x_b$  and along the generating line,  $y_b$  were represented in non-dimensional co-ordinates. The total value of the co-ordinate  $x_b$  stands for the next revolution of the drum.

The distributions of temperature on the outer surface of the drum shown in Figs. 2b, 3a and 3b are very similar. The relations between the sets of temperatures  $T_i$  and  $T_j$  can be investigated using the properties of the standardised covariance called the coefficient of correlation,  $\rho_{T_i T_j}$ , which is defined as

$$(4.2) \quad \rho_{T_i T_j} = \frac{\text{cov}(T_i, T_j)}{\sigma_{T_i} - \sigma_{T_j}},$$

where  $\text{cov}_{T_i T_j}$  is the covariance between the sets  $T_i$  and  $T_j$ , and  $\sigma_T$  is the standard deviation of the set  $T$ . These quantities are calculated using the equations:

$$(4.3) \quad \begin{aligned} \text{cov}_{T_i T_j} &= \frac{1}{n} \sum (T_i - \mu_{T_i})(T_j - \mu_{T_j}), \\ \sigma_T^2 &= \frac{1}{n} \sum (T - \mu_T)^2, \end{aligned}$$

where  $\mu_T$  is the mean value of temperature.

For the distributions of temperature shown in Fig. 2b, the coefficients of correlation were calculated in relation to the first line, L1. The obtained correlation was very high and its coefficients for particular curves were calculated from Eq. (4.2), and the results are given in Table 1.

The distribution of temperature round the drum for particular single revolutions of the drum is also characterised by very high coefficients of correlation. The values of this coefficient for the section of the drum shown in Fig. 3 calculated in relation to the 19-th revolution were given in Table 2.

A similar analysis was carried out for the relation between certain sets of temperature for all the other measurement series. The values of the appropriate coefficients of correlation resemble those given in Tables 1 and 2.

Table 1.

	$\rho_{TLiTi}$
L1 & L2	0.992
L1 & L3	0.975
L1 & L4	0.986
L1 & L5	0.999
L1 & L6	0.999

Table 2.

	$\rho_{T19Ti}$
19-15 rev	0.997
19-20 rev	0.999
19-21 rev	0.999
19-24 rev	0.997

A non-uniform thermal field registered on the outer surface of the brake drum is connected with non-uniform pressures on the surface of the mutual contact of the drum and the shoe. Each of the mating surfaces has its own geometry, which is different from the designed one. Their current condition is a result of complex processes of wear influenced by design, manufacturing and operational factors. The mutual contact of the two surfaces should be analysed in two directions: circumferential and along the generating line. The interdependence of the non-uniform distribution of temperature in both sections and various pressures results from different shapes of the shoe and the drum.

Figure 4 shows a roundness deviation of the BB1 drum in three sections going through points A, B and C (as designated in Fig. 2a). The deviation is the distance from the least square circle of the inner surface of the drum lying in the plane going through the considered point.

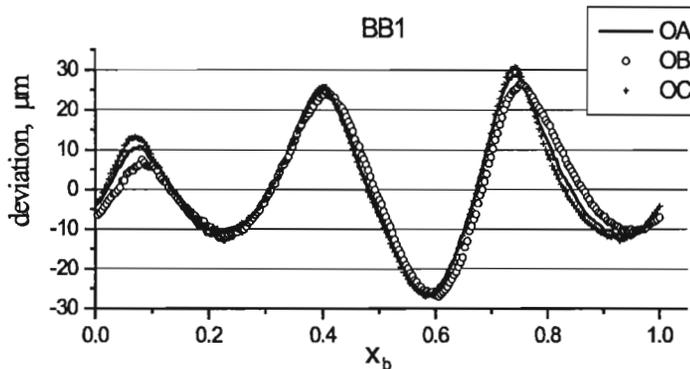


Fig. 4. Roundness deviations for the BB1 drum in sections A, B and C (see Fig. 2a).

The temperature of a body is the measure of the energetic state of the system. For the brake drum, when the conditions of heat exchange are known, it is the measure of the heat transmitted during the braking process and accumulated in the elements of the braking system including the drum sleeve. The energy sent is the friction heat and it depends on the surface normal stresses and, therefore, on the geometry of the mating surfaces. This interdependence can be easily observed in Fig. 2b, where the distribution of temperature along the generating line of the drum represents the character of the contact of the shoe and the drum. Similarly, the distribution of temperature round the drum is related to the geometry of the inner surface of the drum.

Figure 5a shows the relation between the temperature and the form accuracy of the outer surface of the drum in the section going through points A and B, and a roundness deviation of the inner surface of the drum in a section including the considered point.

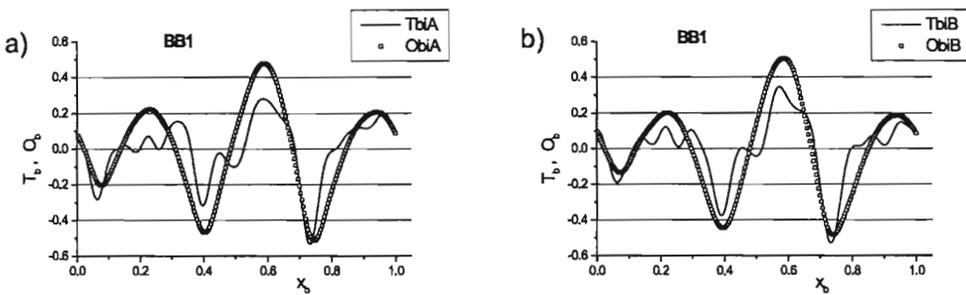


FIG. 5. Temperature and a roundness deviation of the BB1 drum in the section passing through points A and B.

The measured temperature is a determined signal carrying information about the geometry of the system. It refers to the rotating brake drum and is a periodic function with a period of  $2\pi$ , which can be represented as a sum of finite or infinite trigonometric series in the form of a Fourier transform:

$$(4.4) \quad T(\gamma) = T_0 + \sum_{i=1}^k c_i \cos[i(\gamma - \gamma_0)],$$

where:

- $T_0$  mean temperature,
- $c_i$  amplitude of a component of the cosine function of the order of  $i=1, 2, 3, \dots$ ,
- $\gamma$  angle determining the position of the momentous value of temperature,

- $\gamma_i$  phase shift of the function,
- $i$  number of the harmonic.

Each component of the expression (4.4) is a result of a roundness profile. The term with the index  $i = 1$ ,  $i = 2$  and  $i = 3$ , etc, determines the eccentricity, oval and trilobing, etc., respectively [3]. The periodic function of temperature shown in Figs. 2c and 3a can be expanded into a cosine series in the form of (4.4). To compare temperature with a cylindricity deviation, it was necessary to eliminate the zeroth and the first harmonics of the expansion applying the expression (4.4). The obtained function was transformed back by applying the inverse Fourier transformation. A sample result of temperature and an appropriate roundness deviation for points A and B (see Fig. 2a) can be seen in Fig. 5, where dimensionless roundness deviation co-ordinate was defined in the same way as  $T_b$  (see 4.1).

The interdependence of temperature and the form deviation is characterised by the coefficient of correlation, which for sets shown in Figs. 5a and 5b is 0.81 and 0.88 respectively. According to the commonly applied in metrology I. P. Guilford classification, the interdependence for the coefficient of correlation ranging between 0.7 and 0.9 is quite great and the correlation high.

A drum brake mechanism consists of two shoes: concurrent and backward. The analysis of the forces acting on both shoes shows that at identical forces of expansion, the torque transmitted by the backward shoe is greater than that transmitted by the concurrent shoe. As a consequence, the friction linings wear out in a non-uniform way.

One of the arrangements tested had shoes with non-uniform wear of the surface. In operational practice, this type of arrangement occurs, for example, when only one of the linings of the brake is changed. This leads to a non-uniform contact between the drum and each shoe. Since the character of the contact of the mating surfaces is related to pressures, various local streams of heat are generated during braking. The result is a non-uniform thermal field observed from the concurrent and the backward shoes. A sample thermograph with a distribution of temperature in several sections along the generating line of the drum designated as BB4 was shown in Figs. 6 and 7.

The co-operation of the concurrent shoe with the surface of the drum has a saddle character, the illustration of which are two local maximums of temperature along the generating line (Fig. 6c). A completely different contact, i.e. barrel-type, is observed for the backward shoe. Considerable wear of its side edges and the resulting pressures cause that the thermal field is like in Fig. 7c.

In all cases, a comparative analysis of thermal fields round the drum was carried out. The comparison was made for the corresponding distributions of temperature measured on the outer surface of the drum from the side of the concurrent shoe (point A in Fig. 6a) and the backward shoe (point AA in Fig. 7a).

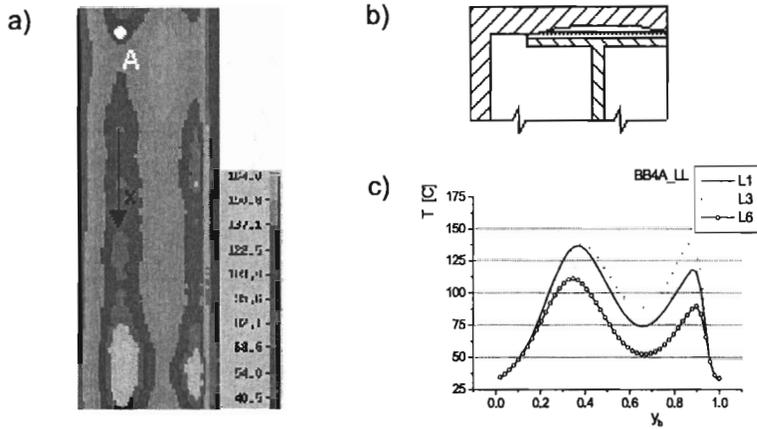


FIG. 6. Concurrent shoe of the BB4 drum: a) thermograph of the surface, b) diagram of the contact of the drum and the shoe, c) distribution of temperature along the generating line of the drum.

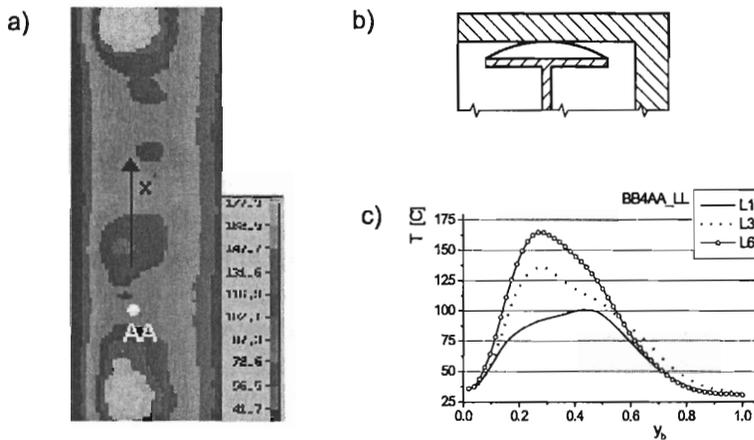


FIG. 7. Backward shoe of the BB4 drum: a) thermograph of the surface, b) diagram of the contact between the drum and the shoe, c) distribution of temperature along the generating line.

A sample distribution of temperature round the drum for both points and for a single revolution was shown in Fig. 8.

In spite of a completely different co-operation of the two shoes with the surface of the drum, it is clear that the circumferential distribution of temperature almost does not change. For the presented section, the coefficient of correlation between the lines  $T_bA$  (for the concurrent shoe) and  $T_bAA$  (for the backward shoe) is 0.959. According to the above-mentioned I. P. Guliford classification, the interdependence of curves is very certain, and the correlation very high, if the coefficient of correlation ranges between 0.9 and 1.

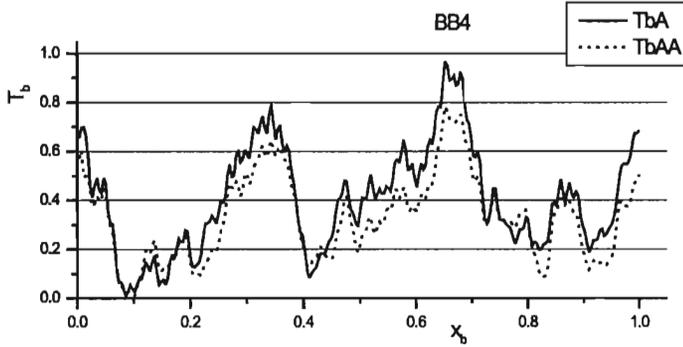


FIG. 8. Distribution of temperature on the outer surface of the BB4 drum from the side of the concurrent ( $T_{bA}$ ) and backward ( $T_{bAA}$ ) shoe.

## 5. ANALYSIS OF THE TEST RESULTS AND CONCLUSION

The measuring system registers also the braking torque and the rotational velocity of the drum. Sample results for one of the examined drums are given in Figs. 9 and 10.

Figure 9 shows a plot of braking torque during the BC2\_B drum test. At the beginning of the test the torque passes through a maximum value that corresponds to the confirmation of the contact surfaces. After the brake process starts it remains nearly constant for a certain period of time. The constant torque causes constant retardation of rotation. It results in a change of the angular velocity shown in Fig. 10, which is nearly linear. A slight increase in the value of the torque at the end of the braking process observed in Fig. 9 indicates an increase of the friction coefficient at low sliding velocity conditions [1]. Constant braking torque results in a steady heat release and a consequent steady increase in temperature during the test (see Fig. 3b).

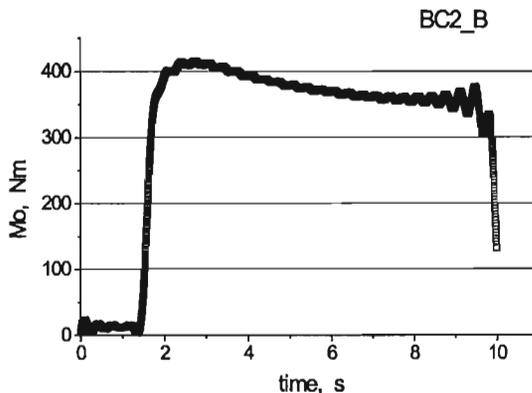


FIG. 9. Braking torque for the BC2\_B drum.

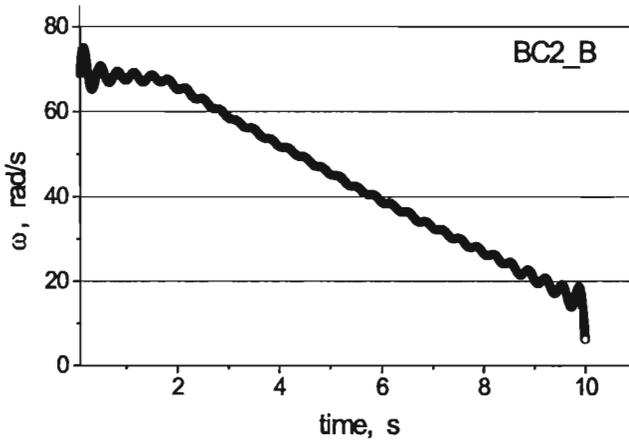


FIG. 10. Rotational speed in the BC2\_B series.

Temperature is measured on the outer surface of the drum, while heat is emitted on the inner one. That is why the unsteady increase in temperature is observed at the beginning of the process. This happens as long as the Fourier Number  $Fo$  remains less than 0.5. In the case of one-dimensional heat conduction, it could be proved that for  $Fo > 0.5$  the difference in temperatures for both sides of the drum stays proportional to the heat flux released and its distribution along the radius of the drum stays steady [5]. This assumption can be applied in the experiment discussed in the present paper. In Fig. 3b one can see that the shape of temperature distribution remains steady for a certain number of successive revolutions. This behaviour can be explained by comparing the heat flux emitted in the radial direction with the heat flux conducted in the tangential direction, the latter one being considerably lower.

The method for bulk temperature estimation is given in [5]. The calculation of the maximum temperature rise is proposed as a sum of the average bulk temperature, its rise due to the average surface temperature and the temperature rise in contact points.

The solution of the one-dimensional heat conduction problem mentioned above, is a basis for the calculation of the increment due to the average surface temperature. Another component of the maximum temperature rises – the flash temperature – may be ignored. It is because of the relatively quick attenuation and preliminary wear-in of the brake shoes before each test [5, 13].

A certain amount of heat is emitted into the atmosphere during the operation of the braking system. This removal of heat from the assembly is caused by conduction, radiation and convection and depends on the relative temperatures of the shield and the environment as well as the flow regime. At the moderate

temperature regime radiation is negligible. Convection heat flux may be estimated with the use of the Newton's law. The necessary Nusselt Number can be calculated from any formula in relation to both Reynold's and Prandl's Numbers [14]. The evaluation shows that the rate of heat transferred by convection does not exceed a few percent of the total heat produced (less than 3% in the experiment considered).

If the law of energy conservation is applied with regard to the results discussed above, the following simplified equation may be written:

$$(5.1) \quad \frac{dT_i}{d\varphi} \approx \alpha_p \frac{R}{\delta \rho c_p} \mu_i p_i$$

where:

- $T_i$  mean temperature of the drum element,
- $\varphi$  angle of rotation,
- $\alpha_p$  portion of heat flux entering the drum shield ( $\alpha_p \approx 1$  [5]),
- $\delta, \rho, c_p$  thickness, density, specific heat of the drum,
- $\mu_i$  coefficient of friction,
- $p_i$  normal pressure.

Equation (5.1) illustrates a simplified relation between the measured temperature and the product  $\mu_i p_i$ . The integration of Eq. (5.1) leads to the interdependence of the drum bulk temperature  $T_{\text{bulk}}$  and the braking torque  $M_o$ , similar to that discussed in [5]:

$$(5.2) \quad \frac{dT_{\text{bulk}}}{d\varphi} = \frac{M_o}{m c_p}.$$

Figure 11 shows the distribution of the mean temperature along the generating line of the BC2\_B drum and the corresponding bulk temperature.

It is easy to notice the linear interdependence of the bulk temperature and the angle of rotation, which is consistent with Eq. (5.2).

On the basis of Figs. 3b and 11 (temperature versus the angle of rotation) it is possible to estimate the temperature slope. According to the distribution presented in Fig. 11, the slopes for bulk temperature and the two lines connecting the points of the top ( $T_{\text{max}}$ ) and the bottom ( $T_{\text{min}}$ ) temperatures are calculated. In the case of the BC2\_B drum the results are given in Table 3.

Applying Eq. 5.2 and the temperature derivative given in Table 3, it was possible to calculate the value of rotational torque. In the analysed case it was about 360 Nm and it was consistent with its mean value given in Fig. 9.

A rough analysis, which enables determining the bulk temperature rise, is run according to the schema given in [5]. It allows correct determination of some

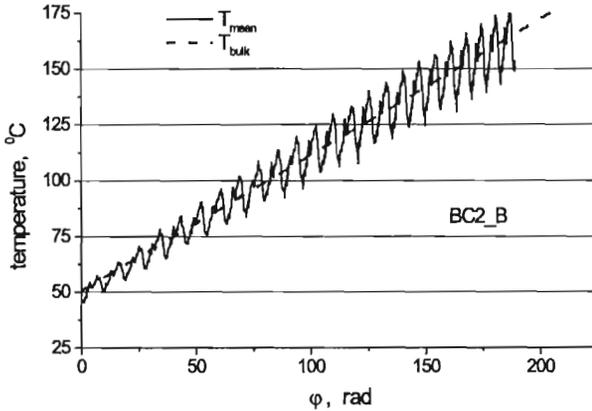


FIG. 11. Mean and bulk drum temperature changes in the BC2\_B braking process.

Table 3.

$\frac{dT_{\text{bulk}}}{d\varphi} = 0,608$	$\frac{dT_{\text{max}}}{d\varphi} = 0,667$	$\frac{dT_{\text{min}}}{d\varphi} = 0,534$
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of the mean values. The temperature derivatives given in Table 3 point to a 20% non-uniformity of normal pressure on the inner surface of the drum. This non-uniformity is caused by roundness deviations along the circumference of the drum discussed in Section 4.

The geometry of the mating friction elements of drum brakes has a considerable influence on the accuracy of motion transmittance and their dynamic state. The properties of this system are defined by the state of the contact of the drum and the shoe in the circumferential direction and along the generating line.

The distribution of temperature along the generating line illustrates the character of the contact of the shoe with the drum. The distribution round the drum is very much dependent on the roundness deviation. The coefficients of correlation in all the analysed drum-shoe arrangements were greater than 0.8.

As shown in Fig. 8, the relations between the cylindricity deviation of the drum and the temperature round the drum do not depend on the geometry of the mating shoe.

The temperature measured on the outer surface of the drum provides information about the form accuracy and the operational state of the mating friction elements. The relations between these quantities can be effectively utilised in diagnosing devices for testing the state of the whole device or its particular elements during operation.

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