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Some Reliability Issues of the Corrugated I-Beam Girder

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The main aim of this paper is reliability analysis of the corrugated-web I-girder carried out to verify its susceptibility to the random corrosion of the web and to make a comparison of the results of the first and of the second order reliability analysis. The methodology implemented in the study is based on the stochastic finite element method related to the generalized stochastic perturbation technique, where a discretization of the entire structure is carried out with four-node quadrilateral shell finite elements. This is numerically implemented using the FEM engineering system ROBOT and computer algebra system MAPLE, where all probabilistic procedures are programmed. The perturbation-based results are compared with these coming from the Monte-Carlo simulation and with an analytical solution obtained via symbolic integration carried out in MAPLE also. The indices of reliability are determined for the maximum deflections of the beam as the function of an input coefficient of variation of the web's thickness whose meaning is the extent of a corrosion process.

Key words: stochastic finite element method, perturbation method, reliability analysis, steel structures, weighted least squares method.

Notations

b – input random variable (web thickness),
$E[b]$ – expected value of the input random variable,
$Var(b)$ – variance of the input random variable,
$\sigma(b)$ – standard deviation of the input random variable,
$\alpha(b)$ – coefficient of variation of the input random variable,
$\mu_k(b)$ – k-th central moment of the input random variable,
$p_b(x)$ – probability density function of the input random variable,
$\mathbf{u}(b)$ – structural displacements vector,
ε – perturbation parameter,
$\mathbf{K}(b)$ – stiffness matrix,
$\mathbf{q}$ – external load vector,

D_i – real unknown coefficients in the least squares method approximation,
$r_{(\alpha)}$ – residuals in the least squares method,
$w_{\alpha\alpha}$ – the weights in the least squares method approximation,
S – least squares method functional,
$\mathbf{J}$ – Jacobian in the least squares method,
β_{FORM} – reliability index according to FORM (First Order Reliability Method),
f_{all} – admissible deflection of the plate girder,
$f_{\max}$ – maximum deflection of the plate girder,
t_w – thickness of the girder's web,
β_{SORM} – reliability index according to SORM (Second Order Reliability Method),
P_{f2} – probability of the failure,
κ – curvature approximating the primary limit surface,
Φ – cumulative probability density function.

1. Introduction

An importance in role of the corrugated webs has been increasing in civil engineering practice since their appearance in the 1990s principally due to their high transverse rigidity. They tend to replace the classical I-beams and columns with straight webs and are extensively used as the homogeneous steel large span bridge girders as well as the hybrid bridge girders. A spread of the corrugated web is dictated by its technological advancement allowing a superior material usage by saving of up to 30% of steel itself. This is achieved by intense reduction of web thickness, contributing to approximately 30–40% of the overall weight of the I-beam; an innovative shape of the web ensures modifications within the stress distribution in this element. The web serves solely for transfer of the shear and as a stiffener of flanges, where it resembles diagonals and verticals of a lattice girder. An additional asset introduced by the corrugated web is an increase of bending resistance around the weak axis followed by better resistance to torsion and exceedingly high buckling resistance (even without additional ribs).

The major asset of the corrugated web – its high slenderness – also constitutes its biggest problem, while the structures with the SIN type webs are exceedingly predisposed to corrosion. Each instance of such a process is dangerous for the elements having small thickness. Moreover a number of theoretical and computational problems connected with the corrugated web beams are still unresolved; some of them may lead to local buckling of the web close to the support. It may be one of the reasons of an excessive conservativeness and ambiguousness of design standards in ambient temperature and strictly limited range of the beams covered by the manufacturer's guidelines (restricted span, flange width and web thickness of the beam). Situation is even worse for elevated temperatures as currently the response of the structure in fire conditions is a bit accidental. Furthermore, there is no actual research concerning stochastic reliability of such structures which are predisposed to random perturbations, in particular (but not exclusively) those connected with a corrosion of the web.

Therefore, the major aim of this work is presentation of an effective numerical time-independent method for validation of a stochastic reliability of the SIN-type girder with possible fluctuations of the web thickness. Our numerical analysis could be relatively easily replaced with the time-dependent case study while engaging some time series representation of the corrosion process itself.

2. Computational analysis

The analysis is focused on the SIN-type girder with span of 40 m (Fig. 1), height equal to 2.5 m and width of 1.6 m. The beam is designed strictly according to the Polish version of the Eurocode (PN-EN 1993-1-5, 2008) and optimized to reach 90% of its Ultimate Limit State (ULS) by manual modification of the girder's geometry implemented in PTC Mathcad prime engineering calculation software. Analysis of the ULS consists in verification of the maximum bending and shear as well as the Serviceability Limit State (SLS) related to the maxi-

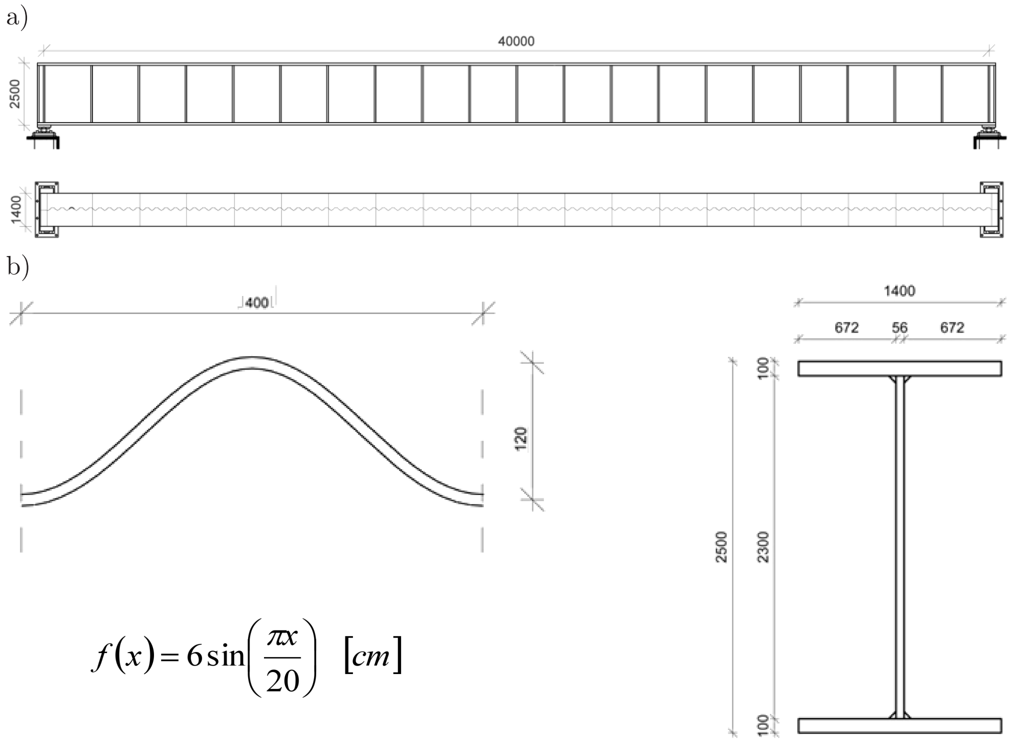

Fig. 1. Geometry of the girder: a) layout, view and static scheme [mm], b) corrugation and cross section geometry [mm].

mum deflection in a middle of the girder span. The girder carry all the loads of a half width of a 20 m wide bridge, which have a magnitude of 150 kN/m and is distributed all over its upper flange.

The girder is modeled by using ROBOT FEM software with 24 000 of its rectangular four-node thin shell finite elements distributed uniformly in both flanges and the web as well as over 40 000 nodal points with six degrees of freedom each. Half-waves of the SIN web were discretized with 20 straight elements each in their longitudinal direction to achieve a smooth discretization of the actual shape of the web. All the simulations involve web thickness spectrum of $t_w \in \langle 51, 52, \ldots, 61 \rangle$ mm and serve for a determination of the ultimate stresses and deflections for each web thickness of the girder (Table 1). These ultimate values are used to determine analytical response functions of the structure to gradual changes in web thickness (Fig. 2) and are based on the weighted least squares method (WLSM) with a distribution of weights similar to the Dirac function (discrete maximum in the middle is many times larger than the uniform distribution of the unit weights elsewhere within a computational domain). The WLSM allows for selection of the polynomial corresponding to the analyzed continuous relationship of two variables (even if these are not available analytically). In this particular case it approximates deflections of a beam as a function of its web's thickness. Subsequently, the response function is implemented in three independent and parallel probabilistic procedures – analytic method (AM), stochastic perturbation theory (SPT) and Monte Carlo simulation (MCS) methods – to calculate all the basic probabilistic characteristics for

Table 1. Comparison of analytical results versus ROBOT FEM simulation.

t_w	Analytical results			Results from the FEM model						
	f [cm]	$\sigma_{\max}$ [MPa]	$\tau_{\max}$ [MPa]	f [cm]	$\sigma_{\max}$ [MPa]	σ_{Mises} [MPa]			$\tau_{\max}$ [MPa]	
						Middle of span	Support 1	Support 2	Support 1	Middle of span
51	7.12	169.8	54.2	6.83	148.84	182.2	272.38	170.15	117.18	172.40
52	7.12	167.5	53.4	7.52	148.43	177.98	267.41	166.73	120.79	178.29
53	7.11	167.1	52.8	73.0	148.24	177.65	263.32	164.14	119.37	174.37
54	7.13	167.1	52.1	7.11	148.05	177.07	259.35	161.65	117.97	170.63
55	7.14	167.0	51.4	6.86	147.82	176.51	255.52	159.24	116.58	167.06
56	7.14	167.0	50.7	6.43	147.78	177.15	252.03	157.17	113.27	160.37
57	7.15	166.9	50.1	6.42	147.55	176.0	248.30	154.78	112.89	158.78
58	7.16	166.8	49.5	6.40	147.33	174.88	244.69	152.47	112.51	157.30
59	7.16	166.8	48.9	6.25	147.15	174.36	241.29	150.36	111.19	154.33
60	7.17	166.7	48.3	6.05	146.97	173.86	237.98	148.37	109.89	151.48
61	7.17	166.7	47.8	5.96	146.8	173.36	234.78	146.32	108.64	148.75

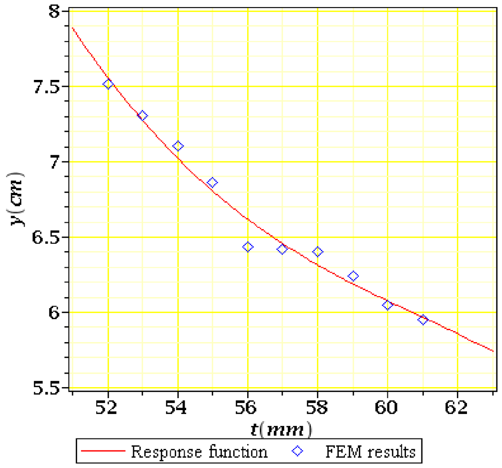

FIG. 2. Maximum deflection vs. web thickness.

the expected value of web thickness. This allows for a comparison of the efficiency of these three basic probabilistic methods and simultaneous validation of their results. An exemplary graph of the probabilistic moment is presented in Fig. 3. The output reliability indices determined according to FORM and SORM (as well as the exemplary probabilistic moment) are presented as a function of the web thickness corresponding to the intensity or advancement of a corrosion process.

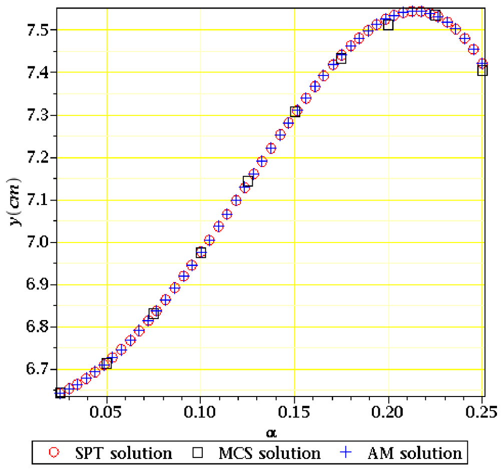

FIG. 3. Expectations of maximum deflection vs. coefficient of variation of web thickness.

3. Theoretical background

Current design standard Eurocode 0 recommends calculations of the reliability of a structure by an application of the first order probabilistic methods. Therefore, it is necessary to introduce the theoretical fundamentals for calculations of this index.

3.1. Probabilistic calculations

First, the following definitions concerning the given input random variable b and its algebraic functions are introduced in the context of the probability theory. The equivalent statistical estimators are also proposed below (M stands for the total number of the Monte-Carlo trials) [1, 3, 4]

• the expected value of b

$$E[b] = \int_{-\infty}^{+\infty} b p_b(x) dx \equiv \frac{1}{M} \sum_{i=1}^{M} b^{(i)}, \tag{3.1}$$

• its variance

$$Var(b) = \int_{-\infty}^{+\infty} (b - E[b])^2 \, p_b(x) dx \equiv \frac{1}{M-1} \sum_{i=1}^{M} \left(b^{(i)} - E[b] \right)^2, \tag{3.2}$$

and standard deviation

$$\sigma(b) = \sqrt{Var(b)}. \tag{3.3}$$

The coefficient of variation is calculated with use of the above formulae as

$$\alpha(b) = \frac{\sqrt{Var(b)}}{E[b]}, \tag{3.4}$$

while a relation describing k-th central probabilistic moment (for $k > 2$) has the following form:

$$\mu_k(b) = \int_{-\infty}^{+\infty} (b - E[b])^k p_b(x) dx \equiv \frac{1}{M} \sum_{i=1}^{M} \left(b^{(i)} - E[b] \right)^k. \tag{3.5}$$

The probability density function used in computations is the Gaussian one, i.e.

$$p_b(x) = \frac{1}{\sigma(b)\sqrt{2\pi}} \exp\left(- \frac{(b - E[b])^2}{2\sigma^2(b)} \right); \quad x \in \Re. \tag{3.6}$$

Once the truncated Gaussian random fields are to be analyzed (most of material and geometrical parameters take only positive values), an integration process must be limited using three-sigma rule or taking into account some other physical limitations of the given random variables.

Analytical formulas are further employed in the stochastic perturbation-based approach based on the Taylor expansion of all random input and output variables with random coefficients around their expectations. This is done on the example of the random function $\mathbf{u}(b)$ with respect to the given input parameter b in the following manner [1]:

$$\mathbf{u}(b) = \mathbf{u}^0(b^0) + \varepsilon \left.\frac{\partial \mathbf{u}(b)}{\partial b}\right|_{b=b^0} \Delta b + ... + \frac{\varepsilon^n}{n!} \left.\frac{\partial^n \mathbf{u}(b)}{\partial b^n}\right|_{b=b^0} \Delta b^n, \tag{3.7}$$

where ε is the given perturbation parameter (taken in all engineering computations as equal to 1), while the n-th order variation of b itself about its expectation is given as

$$\varepsilon^n \Delta b^n = (\delta b)^n = \varepsilon^n (b - b^0)^n. \tag{3.8}$$

The expected values are determined with this expansion in the tenth order approach as

$$E[\mathbf{u}(b)] = \mathbf{u}^0(b^0) + \frac{\varepsilon^2}{2} \frac{\partial^2 \mathbf{u}(b)}{\partial b^2} \mu_2(b) + ... + \frac{\varepsilon^{10}}{10!} \frac{\partial^{10} \mathbf{u}(b)}{\partial b^{10}} \mu_{10}(b). \tag{3.9}$$

Additionally, the Gaussian distribution allows for further recursive simplification of the central probabilistic moments

$$\mu_p(b) = \begin{cases} 0; & p = 2k+1 \\ (p-1)!!\,(\sigma(b))^p\,; & p = 2k \end{cases} \tag{3.10}$$

for any natural $k \geq 1$. We apply similar expansions and procedures to recover higher order statistics of the structural response. Further, this technique is implemented in conjunction with the finite element method, where the elastostatics require a solution of the following matrix equation [8]:

$$\mathbf{K}(b)\mathbf{u}(b) = \mathbf{q}, \tag{3.11}$$

where $\mathbf{K}(b)$ represents the stiffness matrix of the system including some uncertainty source, $\mathbf{q}$ includes the boundary conditions imposed onto the system and is assumed to be deterministic in the proposed computer analysis, while $\mathbf{u}(b)$ represents random structural response of the girder.

A traditional method for a solution of this problem is the direct differentiation method [2] where one includes Taylor expansion from Eq. (3.7) to form increasing order hierarchical equations of the static equilibrium and to insert higher order partial derivatives of $\mathbf{u}(b)$ into the formulas such as Eq. (3.9). In this paper we employ another method that is based on the initial following polynomial approximation of the output displacements:

$$\mathbf{u}(b;m)=\sum_{i=0}^{m} D_i b^i, \tag{3.12}$$

where D_i, $i = 1, ..., n$ are the real unknown coefficients to be determined. Quite similarly to the polynomial chaos technique, these coefficients are sought through the series of FEM experiments with varying design parameter inside the domain of its assumed uncertainty (equidistant division in-between its upper and lower bounds). It gives the necessary condition of a probabilistic convergence for $m < n$ (as higher order terms in additional expansions simply vanish), while the real numerical efficiency is found with the use of a comparison with both statistical and semi-analytical stochastic calculus.

One non-trivial aspect of stochastic analysis is the correlation of coefficient of variation and stochastic variable. The correspondence is not straightforward, as governed by the probability (density) function. Therefore a proper choice of this function is a key factor in each stochastic analysis. In the case of the conducted research, generally the increase of coefficient of variation corresponds to decrease of web thickness (stochastic variable) and by this can be perceived as advancement of the corrosion process.

3.2. The weighted least squares method (WLSM)

Some details of the weighted least squares method (WLSM) are shown, where a polynomial basis of the sth order (indexed by β here) in the numerical tests is used and solved around the mean value of the given input random parameter b. As a result n different pairs $\left(b_\alpha, u^{(\alpha)}\right)$ for $\alpha = 1, ..., n$ are obtained, where the arguments belong to the close neighborhood of expectation of b itself. Then, we use the following polynomial approximation:

$$u(b) \cong D_\beta b^\beta = f(\mathbf{D}, b), \qquad \beta = 1,\, ...,\, s, \qquad s < n. \tag{3.13}$$

The residuals in each trial point are introduced to get an algebraic condition for these expansion coefficients, i.e.

$$r_{(\alpha)} = u^{(\alpha)} - f(\mathbf{D}, b_\alpha), \qquad \alpha = 1,\, ...,\, n. \tag{3.14}$$

Approximation of this function is done by minimization of the weighted residuals functional

$$S=\sum_{\alpha=1}^{n} w_{\alpha\alpha} r_{(\alpha)}^2, \qquad \alpha = 1,\, ...,\, n, \tag{3.15}$$

so that

$$\frac{\partial S}{\partial D_\beta} = -2\sum_{\alpha=1}^{n} w_{\alpha\alpha} r_{(\alpha)} \frac{\partial f(\mathbf{D}, b_\alpha)}{\partial D_\beta}, \qquad \beta = 1, ..., s. \tag{3.16}$$

Further, the following notation is adopted:

$$\mathbf{J} = J_{\alpha\beta} = \frac{\partial f(\mathbf{D}, b_\alpha)}{\partial D_\beta}, \qquad \alpha = 1, ..., n, \qquad \beta = 1, ..., s, \tag{3.17}$$

modified as

$$\sum_{\alpha=1}^{n}\sum_{\beta=1}^{s} J_{\alpha\beta} w_{\alpha\alpha} J_{\alpha\beta} D_\beta = \sum_{\alpha=1}^{n} J_{\alpha\beta} w_{\alpha\alpha} u^{(\alpha)}, \quad \alpha = 1, ..., n, \quad \beta = 1, ..., s \tag{3.18}$$

and converted into the matrix normal equations

$$\left((\mathbf{J})^T \mathbf{wJ}\right) \mathbf{D} = (\mathbf{J})^T \mathbf{wu}. \tag{3.19}$$

This system of equations (with the dimensions $n \times s$) is solved symbolically in MAPLE.

3.3. Reliability indices β_{FORM} and β_{SORM}

All the indices collected and discussed further are determined via the first and the second order reliability methods for the maximum deflections representing the serviceability limit state (SLS) of the girder. The index β_{FORM} is calculated with use of the following formula:

$$\beta_{\mathrm{FORM}} = \frac{E[b]}{\sigma(b)}, \tag{3.20}$$

where $E[g]$ is the expectation of a random limit function g and $\sigma(g)$ represents its standard deviation. This statement is further simplified in case when the displacements measure is used to approximate structural reliability as

$$\beta_{\mathrm{FORM}} = \frac{E\left[f_{\mathrm{all}} - f_{\max}\right]}{\sigma\left(f_{\mathrm{all}} - f_{\max}\right)} = \frac{E\left[f_{\mathrm{all}}\right] - E\left[f_{\max}\right]}{\sqrt{\mathrm{Var}\left(f_{\mathrm{all}}\right) + \mathrm{Var}\left(f_{\max}\right)}} = \frac{E\left[f_{\mathrm{all}}\right] - E\left[f_{\max}\right]}{\sigma\left(f_{\max}\right)}, \tag{3.21}$$

where f_{all} and $f_{\max}$ (after numerical recovery) are given using the following formulas:

$$f_{\mathrm{all}} = \frac{l}{350}, \tag{3.22}$$

$$f_{\max} = 90.451 - 2.413 t_w + 0.000346 t_w^3 - 3.073 \cdot 10^{-10} t_w^6, \tag{3.23}$$

where l is a span of the girder. The maximum deflections $f_{\max}$ are approximated here using the above presented WLSM assuming polynomial representation, whereas f_{all} is assumed directly from Eurocode 3 (devoted entirely to the steel structures basics).

Reliability index β_{FORM} assumes the Gaussian probability distribution of a given random (response) function. Therefore, it is irrelevant to check a reliability of the structure using FORM method for a factor, whose random dispersion is different than the Gauss function; the second order reliability analysis shall be applied in these cases. The general formula for the reliability index β_{SORM} is the following one:

$$\beta_{\mathrm{SORM}} = -\Phi^{-1}(P_{f2}), \tag{3.24}$$

where P_{f2} denotes the probability of failure related to the index β_{FORM} in the following way:

$$P_{f2} = \frac{\Phi\left(\beta_{\mathrm{FORM}}\right)}{\sqrt{1+\beta_{\mathrm{FORM}}\kappa}}, \tag{3.25}$$

κ is the curvature approximating the primary surface defined by the following formula

$$\kappa = \frac{\dfrac{\partial^2 u}{\partial b^2}}{\left(1+\left(\dfrac{\partial u}{\partial b}\right)^2\right)^{3/2}} \tag{3.26}$$

and

$$\kappa > \begin{cases} \dfrac{-1}{\Phi\left(-\beta_{\mathrm{FORM}}\right)}, \\ \dfrac{-1}{\beta_{\mathrm{FORM}}}. \end{cases} \tag{3.27}$$

A general formula of probability of failure is the following one:

$$P_{f2} = \Phi_0(-\beta)\prod_{i=1}^{n-1}\left(1+\beta\kappa\right)^{-1/2}. \tag{3.28}$$

The above given formulae allow calculation of the reliability index for an arbitrary probability density function describing the influence of the phenomenon on the considered random function.

4. Numerical results

The scope of numerical analysis included in this paper is a comparison of both ULS and SLS coming from analytical calculations with these obtained in computer simulations involving normal, shear reduced von Mises' stresses and deflections of the girder as well as analysis of the reliability index computed according to both FORM and SORM techniques.

4.1. Comparison of analytical calculations and computer simulations

Analytical calculations based on the formulas valid for profiles with straight webs (Table 1) correspond to the results of FEM simulations only in a limited way i.e. for deflections and normal stresses (Figs. 10–11). Maximum values of both variables and methods are localized in the middle of the girder span and difference of magnitude does not exceed 10%. Furthermore, it is worth to mention that the analytical method (AM) gives the results more disadvantageous (on the "safe" side). This leads to conclusion that theoretical analysis based on a straight web is quite sufficient for a rational approximation of SIN type beams in this particular case. The hypothesis is invalid however for the shear stresses determining the ULS of the girder. These calculations underestimate the magnitude over three times, which undoubtedly would lead at least to deplanation of the web in a vicinity of the support [6]. Therefore, it is strongly advised to perform the additional FEM simulation during the design process of the SIN type plate girders, especially considering shear stresses on the support [7]. One more remark considering the results presented in Table 1 should be raised, i.e. a dependence of an ultimate deflection on the thickness of the web according to analytical calculus. This correlation shows an increase of a deflection of the beam with increasing web thickness, which is tentatively converse to the engineering expectations. It really means a higher dependence of deflection on the mass of the girder than the change of an inertia moment of the girder connected with an increase of its cross-sectional area. Secondly, we notice some fluctuations of a function $f(t_w)$ for simulations with thicknesses 51 mm and 57 mm that may be connected with a discretization density with the finite elements applied in the ROBOT's computer model. An additional comment is required for the two ultimate values for support von Mises' stress. The difference in their magnitude is caused by different boundary conditions on each side of the beam. Support 2 has two degrees of freedom, i.e. rotation and horizontal displacement, while the first only rotation.

The results collected in Table 1 enable to provide some polynomial representations for the maximum deflection (Fig. 2), for the maximum normal stresses (Fig. 4) and maximum shear stresses (Fig. 6) as well as for the maximum re-

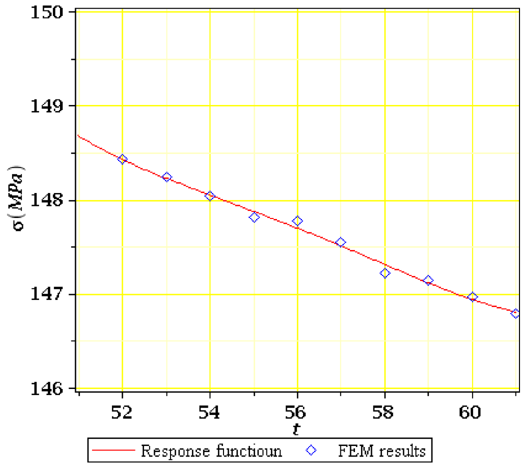

FIG. 4. Maximum normal stress vs. web thickness.

duced stresses (Fig. 8) – all via the weighted least squares method taking as the independent parameter the web's thickness. It is apparent that the very regular set of points is obtained only for the reduced stresses, but all polynomial approximations are found as very smooth functions with no local oscillations which guarantees reliable determination of their higher order partial derivatives with respect to this parameter. These functions enable relatively easy determination of the basic probabilistic characteristics for these state functions and this is demonstrated by using the expectations only for brevity of further presentation. Then, we have in turn expected values of maximum deflection in Fig. 3, and analogously for maximum normal stresses (Fig. 5), maximum shear stresses

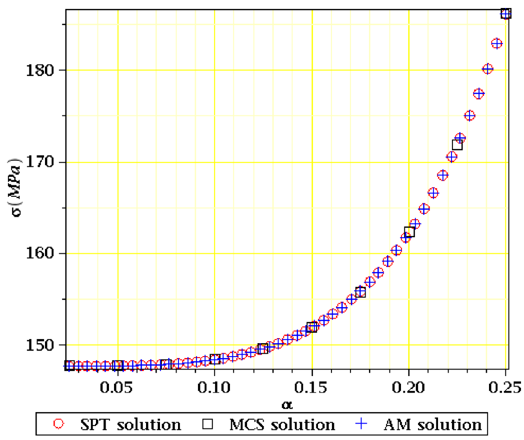

FIG. 5. Expectations of maximum normal stress vs. coefficient of variation of web thickness.

(Fig. 7) as well as maximum reduced stresses (Fig. 9) – all determined as the functions of the input coefficient of variation of t_w ($\alpha(t_w) \in [0.00, 0.25]$). Such a wide interval of the input stochastic fluctuations is driven by the real corrosion mechanisms and accompanying statistical parameters, even at the very beginning of the exploitation period for steel structures. We employ here three different stochastic computational strategies, namely in turn – stochastic perturbation technique (SPT), Monte-Carlo simulation (MCS) and, finally, the analytical method (AM) all based on the same polynomial representation adjacent to the WLSM.

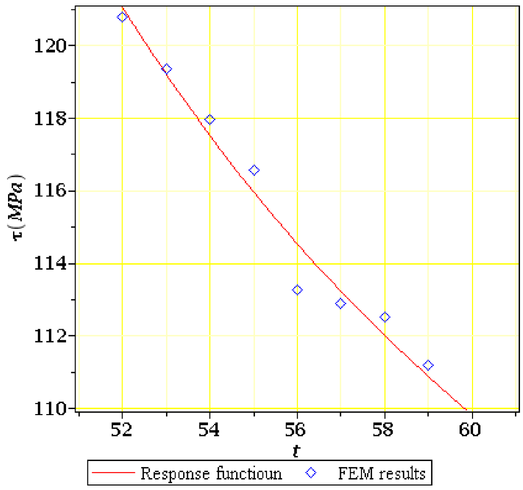

FIG. 6. Maximum shear stress vs. web thickness.

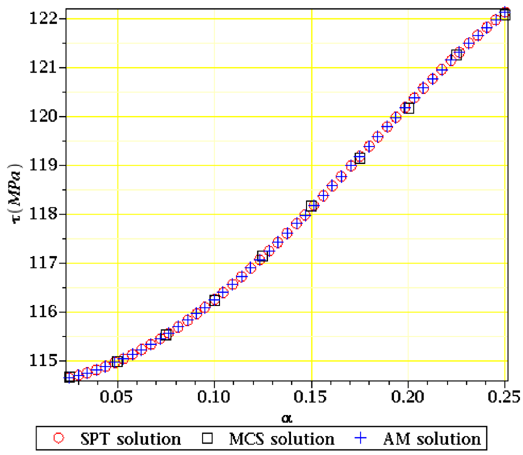

FIG. 7. Expectations of maximum shear stress vs. coefficient of variation of web thickness.

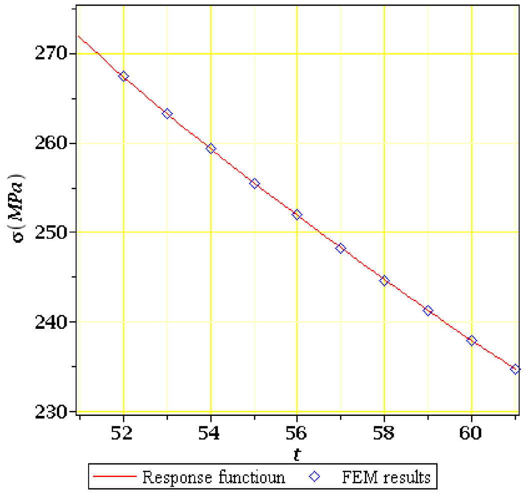

FIG. 8. Maximum von Mises stress vs. web thickness.

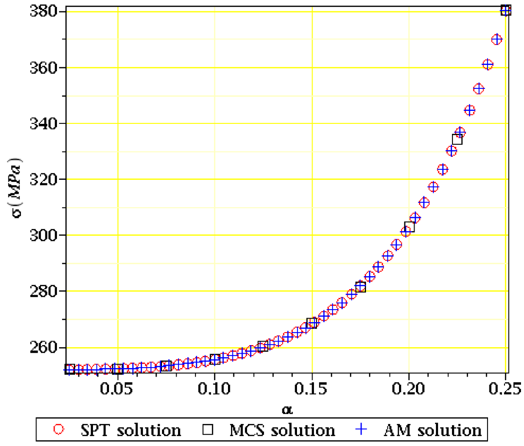

FIG. 9. Expectations of maximum von Mises stress vs. coefficient of variation of web thickness.

FIG. 10. Deflection map of the girder for $t_w = 56$ mm.

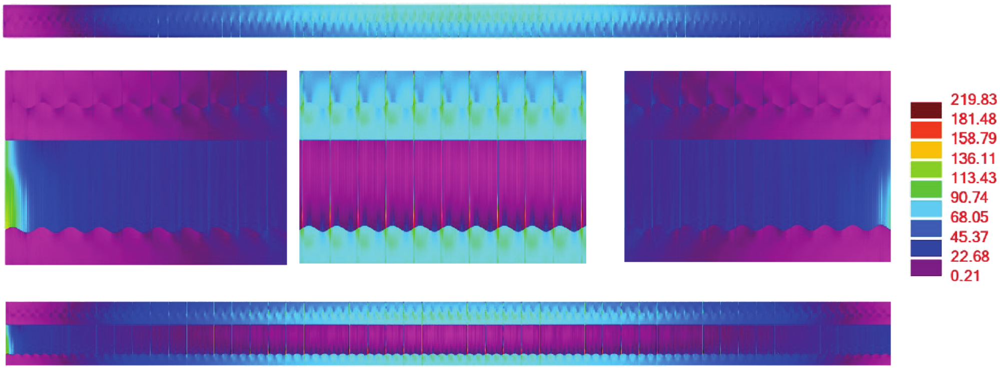

FIG. 11. Normal stress pattern for $t_w = 56$ mm.

Fundamental observation is that all the numerical techniques coincide perfectly with each other in the entire domain of input uncertainty. It is also remarkable that the vertical ranges of Figs. 3, 5, 7 and 9 exhibit significant variations of all these state parameters caused by the web thickness uncertainty fluctuations. It proves the paramount importance of this specific design parameter on the overall strength and safety of the SIN-web girder (and remains true for all plate girders with the webs that are very slender). Generally, the larger the input coefficient of variation, the larger the resulting expectations of the given state parameter. The maximum deflection of the girder noticed in a half of its span is somewhat out of this trend as it increases up to the certain local extreme maximum and then starts to decrease. The reason of this complex behavior is that the increasing web thickness enlarges the mass of a structure and its stiffness at the same time and their combination gives such a result.

4.2. *Reliability indices* β_{FORM} *and* β_{SORM}

The next part concerns entirely the reliability indices determination and this is carried out by using two methods – the first- (FORM) and the second-order reliability method (SORM) and also three concurrent numerical techniques, namely SPT, MCS and AM, as above. This is related to the maximum deflection at the girder center (SLS, Figs. 12, 13) as well as to the maximum normal stresses (ULS, Figs. 14, 15), while the input uncertainty level remains exactly the same. All these graphs show a characteristic exponential decrease of structural reliability together with an increasing input randomness and, furthermore, a perfect coincidence of all the numerical methods, which will enable for some time savings in large scale structures while using only the SPT.

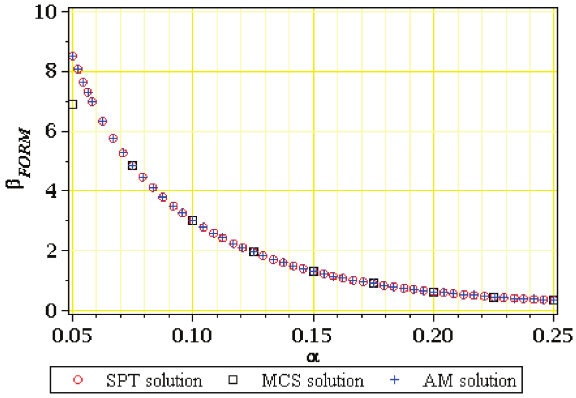

Fig. 12. FORM index for the maximum deflection (SLS).

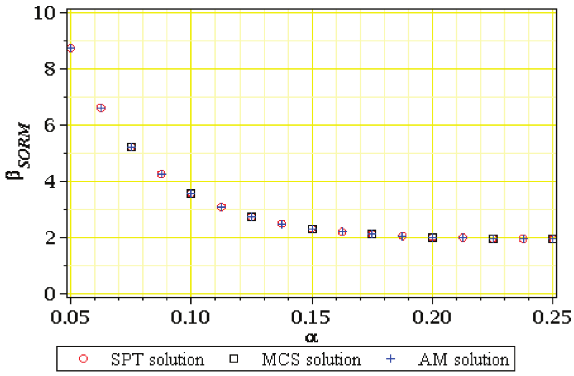

Fig. 13. SORM index for the maximum deflection (SLS).

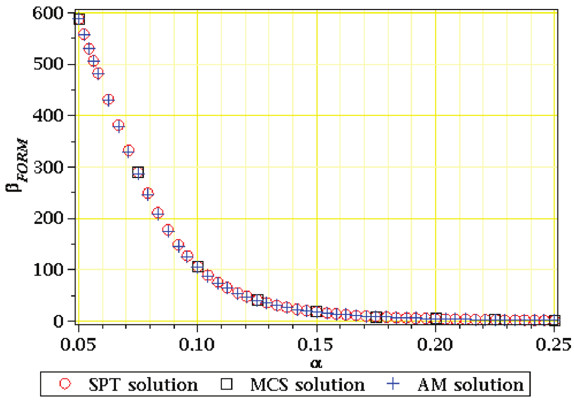

Fig. 14. FORM index for maximum normal stress.

Analysis of reliability of the SIN-type beam indicates that its internal stress state is close to the reliability limits that is allowed in various structures with

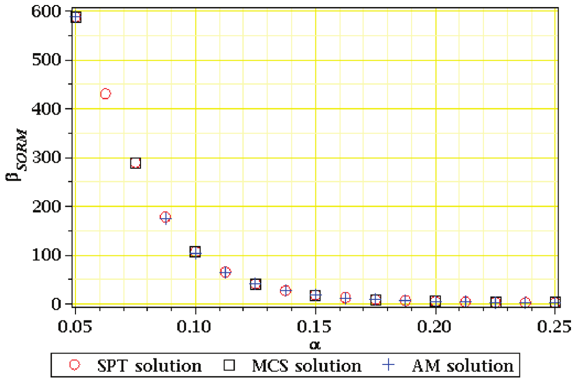

FIG. 15. SORM index for maximum normal stress.

the highest risk class ($\beta > 5.0$) for the elements or groups of elements designed just below their ULS. The major difference between the FORM and SORM approaches is the limit value, where the curves stabilize for the variations higher than 0.20; for FORM it is 0, while for SORM it equals 2. This is connected with an existence of the condition for the minimal curvature κ in calculations of β_{SORM}. The most suitable computational method in terms of the studied girder is SPT, which allows determination of the continuous function in-between the reliability index and input variance [7]. Quite analogous opportunity is behind the analytical method, but it is known that integration of various responses together with exponential density function not always is available in traditional application of the symbolic algebra environment and, therefore, the SPT appears to be the most efficient. Furthermore, the input coefficient of variation is equivalent to a progress of the corrosion process, but is not directly correlated with age of the structure (usually some power laws of corrosion in steel structures are introduced after the experimental evidence). Nevertheless, it is quite easy to transform the below given graphs of reliability into time-dependent ones by engagement of some time-series representation of the corrosion process itself and to compute the probability of its failure in the given time with use of the formulas presented in theoretical introduction. Stochastic computational analysis of this girder takes into consideration solely the influence of a single random variable, which is the web's thickness. Generally its reliability depends upon some other random variables such as the load, dimensions of the girder, material and strength parameters and other as well as upon their cross-correlations, which should be studied further with the SPT Some other important observation concerning thin-walled structures is that their numerical modeling by using traditional beam finite elements also in the stochastic context brings a lot of unpredictable numerical discrepancies and, therefore, should be entirely carried out with the shell finite elements.

5. Conclusions

Engineering calculations concerning the corrugated web girder and its reliability shall be supported by the FEM and SFEM simulations. An analytical approach based on the formulas originating from the theory adjacent to the straight webs is valid only in a limited manner, i.e. for normal stresses and deflections. The method is however completely irrelevant for the shear state on the support which in this case constitutes the ULS of the girder.

Reliability of the corrugated web girder in the context of corrosion should be an important aspect of its design process. This type of a structure is highly susceptible to such phenomena and, therefore it is to be subjected to a more detailed inspections and precise conservation than the traditional steel elements

The results of the reliability analysis according to the first-order reliability method are almost identical to these coming from te SORM for small input coefficients of variation. They start to diverge from about $\alpha = 0.07$ with an increasing manner – the higher the variance, the higher difference between these methods. The index β_{FORM} approaches 0 for high initial variances, while β_{SORM} equals almost 2 and this is caused by the condition for minimum curvature κ while calculating β_{SORM}.

The first-order reliability method is sufficient solely for the random variables, whose influence on the strength or deformation of the beam is of the Gaussian or almost Gaussian character. In all other cases it is indispensable to use the second-order reliability method to gain a proper result and to correct tendencies valid for the reliability index.

The reliability indices engaged here with the stochastic finite element method are almost identical according to the three probabilistic methods compared in this analysis i.e. analytical, stochastic perturbation and the Monte-Carlo simulation techniques. In this case it is advised to use the stochastic perturbation method, which allows for a notable reduction of computation time and for obtaining of the continuous and sufficiently smooth output function versus a set of the discrete values determined via the MCS.

It is recommended further to extend the reliability analysis of the corrugated web girders to the time-dependent one, e.g. by verification of dependence of this index on exposure time of the girder to external conditions and to determine its response function to fire conditions also. It may be done by establishing both bending and shear capacities of the girder in a relation to its temperature.

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