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EFFECT OF SUBSURFACE HETEROGENEOUS CORRELATED HYDRAULIC PROPERTIES ON THE STOCHASTI BEHAVIOR OF THE UNSTEADY WELL DRAWDOWN IN A CONFINED AQUIFER

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Groundwater is considered to be one of the most important water resources especially in areas where surface water resource, is not sufficient. However, utilizing this important groundwater resource requires a complete understanding of the groundwater flow behavior within the subsurface formations. The assumption of considering the subsurface formation as homogeneous media has been proven to be un-realistic. Therefore, the subsurface formation has to be considered as a heterogeneous structure in investigating the groundwater flow behavior. In addition, full understanding of the impact of the subsurface heterogeneous formation on the groundwater flow behavior is very essential in safe utilization of this important water resource.

The current study investigates the impact of two subsurface heterogeneous correlated hydraulic properties on the stochastic behavior of a two-dimensional unsteady well drawdown in a confined aquifer. In addition, the presented study compares the impact of two subsurface heterogeneous cases. The first case considers two subsurface correlated hydraulic properties (current study), while the second case considers the heterogeneous subsurface hydraulic conductivity only. The two hydraulic properties considered in the current study are the subsurface hydraulic conductivity and storativity. The Monte Carlo stochastic approach is used in this study to perform various numerical computations required to assess the investigated problem. The results obtained from this study showed how important it was to consider and understand the heterogeneous subsurface formation and its impact on the groundwater flow behavior, since the two-dimensional unsteady well draw-down was greatly affected by the degree of heterogeneity and the correlation structure of the subsurface hydraulic properties.

Key words: stochastic, Monte Carlo, well drawdown, correlated subsurface properties.

1. Introduction

Water scarcity all over the world urged the scientific community to investigate the various most accurate approaches for exploitation of any water resources. Ground water is very important water resource in the areas where surface water is not available or sufficient. Egypt is wasting no time or effort in using all available water resources in its national projects. Several areas and cities in Egypt depend mainly on the ground water as the only available source for water. Therefore, understanding of the actual behavior of ground water flow, especially during the pumping process, is very essential for better usage of this important water resource. It is now very well recognized that the assumption of homogeneous subsurface formation is not valid and the groundwater flow in any aquifer should be investigated basing on heterogeneous subsurface formation, cf. Chang *et al.* [4, 5]. Several researches have been directed mainly towards investigation of the ground water flow and its characteristics during the pumping process. Streltsova [14] presented a comprehensive analysis of time-dependent drawdown alternations caused by the heterogeneous properties of stratified formations based on solutions developed under different assumptions on interlayered crossflow. On the other hand, Sen [12] presented a study to predict the drawdown variations in a fully penetrating large diameter well discharging from a leaky aquifer, using a derived non-equilibrium formula in terms of the depression cone volume. Later Sen [13] presented a comprehensive review of the validity of Darcy's law with the emphasis on the necessity for a nonlinear flow law, especially for Reynolds numbers greater than one. In additions, the author developed the nonequilibrium formula for nonlinear flow that is useful either in determining the aquifer parameters from the time of drawdown observations or in predicting the drawdown variations in a confined aquifer tapped by a fully penetrating well. Chu [6] used the concept that the cone of depression volume is related to the amount of water removed from the aquifer to obtain transient solutions of well drawdown and radius of influence for a well in an unconfined aquifer under a constant recharge rate. Indelman *et al.* [9] presented a first-order sensitivity analysis of drawdown prediction to the uncertainty in estimating the hydraulic properties of multiple leaky aquifer systems. In this study, a new set of dimensionless parameters was suggested to reduce the amount of sensitivity coefficient calculations. Recently, Oliver [11] presented a study to investigate the influence of nonuniform transmissivity and storativity on the drawdown. The author used the perturbation approach to derive the Frechet derivatives and kernels for the effect of two-dimensional areal variations in transmissivity and storativity on the drawdown at an observation well. Oliver's study [11] showed that observation of the well drawdown is relatively sensitive to near-well transmissivity variation, especially if the nonuniformity is not radially symmetric about the well.

Stochastic approach has prove its applicability in understanding the groundwater flow behavior in heterogeneous subsurface formations. Several groundwater researchers have utilized the stochastic approach in their studies. Due to space limitations, only the most recent studies will be mentioned in this paper.

ABDIN and ABDEEN [1] presented a study to investigate the impact of subsurface heterogeneous hydraulic conductivity on the unsteady two-dimensional pumping well drawdown in a confined aquifer. The authors utilized the well-known Monte Carlo technique to evaluate the variability of the unsteady drawdown as a result of several variable heterogeneous hydraulic conductivity realizations. The results of this study showed the importance of considering the variability of subsurface hydraulic conductivity in designing the pumping rate from a confined aquifer well, and how the well drawdown was very much affected by changing the subsurface properties. ZHANG and LU [18] presented a study for the stochastic analysis of flow in heterogeneous unsaturated-saturated system. In this study, the authors developed a stochastic model for transient unsaturated-saturated flow in randomly heterogeneous media using the method of moment equations. The developed stochastic model in the ZHANG and LU [18] study utilized the perturbation and the finite difference approaches for the solution of the partial differential equations. The authors stated that their stochastic model was applicable to the entire domain of a bounded, multidimensional unsaturated-saturated system in the presence of random or deterministic recharge and sink/source and in the presence of multi-scale, nonstationary medium features. In addition, ZHANG and LU [18] in their study found that the presence of water table rendered the flow moments strongly nonstationary even in the absence of medium nonstationary, features and this finding was confirmed by Monte Carlo simulations. On the other hand, HOLT *et al.* [7] presented a study for the utilization of Monte Carlo error analysis to illustrate the impact of measurement errors in field-estimated hydraulic properties on prediction made with 1D and 3D unconditional stochastic models of unsaturated flow and transport. In this study the hydraulic properties were re-estimated by simulating tension infiltrometer measurements in the presence of small simple errors. HOLT *et al.* [7] stated that two types of observation errors were considered, along with one inversion-model error resulting from poor contact between the instrument and the medium. The authors in this study reached an overall result that the errors in the spatial statistics of hydraulic properties caused critical stochastic model assumptions to be violated, limiting the usable parameter space for model predictions. HUANG *et al.* [8] presented a three-dimensional, geostatistically based iterative inverse method for mapping spatial distributions of the hydraulic conductivity and sorption, partitioning the coefficient fields by sequential conditioning on both the nonreactive and reactive tracer breakthrough data. Specifically HUANG *et al.* [8] adopted, in their study, a streamline-based semi-analytical simulator to simulate chemical movement in a physically and chemically heterogeneous field, and to serve as the forward modeling. In addition, the authors utilized the adopted semianalytical simulator to calculate sensitivity efficients of the reactive chemical concentration with respect to the changes of conductivity and sorption coefficient. The results of this study indicated that the iterative stochastic inverse method was able to identify and produce the large-scale physical and chemical heterogeneity features. ABDIN [2] presented a study to investigate the impact of the subsurface heterogeneous storativity on the two-dimensional unsteady well drawdown in a confined aquifer, compared with the impact of the subsurface hydraulic conductivity. This study clearly demonstrated the direct impact of any subsurface property on the spatial and unsteady stochastic behavior of the well draw. In addition, it showed that the impact of subsurface hydraulic conductivity only was much higher than the impact of the subsurface storativity only on the unsteady drawdown distribution and its variability.

From what was mentioned previously about the recently reviewed literature, it is clearly shown that, consideration of the heterogeneous characteristics of the subsurface properties of the aquifer is essentially important for the correct and accurate understanding of the groundwater flow behavior. However, a full understanding of how the subsurface heterogeneous formation properties are affecting the groundwater flow behavior (especially during the pumping process) is still very limited. Therefore, the current manuscript aims towards investigating the impacts of the subsurface heterogeneous properties and their correlation on the two-dimensional unsteady well drawdown from a confined aquifer.

2. Problem description

The current study presented in this manuscript investigates the impacts of the subsurface heterogeneous correlated hydraulic conductivity and storativity on the two-dimensional unsteady well drawdown resulting from pumping water from the center of a confined aquifer. Specifically, the current study investigates the impacts of the correlation structure between the two subsurface hydraulic properties (hydraulic conductivity and storativity) on the unsteady well drawdown. To get more insight knowledge and understanding about the stochastic behavior of the groundwater flow for the investigated application, the current study performs comprehensive comparison between two heterogeneous cases. The first case considers the two hydraulic properties (hydraulic conductivity and storativity) to be heterogeneously correlated while the second case considers only the hydraulic conductivity to be the heterogeneous parameter and the storativity is assumed to have a deterministic pattern all over the studied domain. The adopted geometric and hydraulic assumptions; and the governing equations used in solving the investigated problem are profoundly described in the following subsections.

2.1. *Assumptions*

In this study, a full penetrating well that is discharging water from the center of a confined aquifer is considered, as shown in Fig. 1. The aquifer thickness is assumed to be 100.0 m. The porous medium and the water are considered to be incompressible and under isothermal conditions.

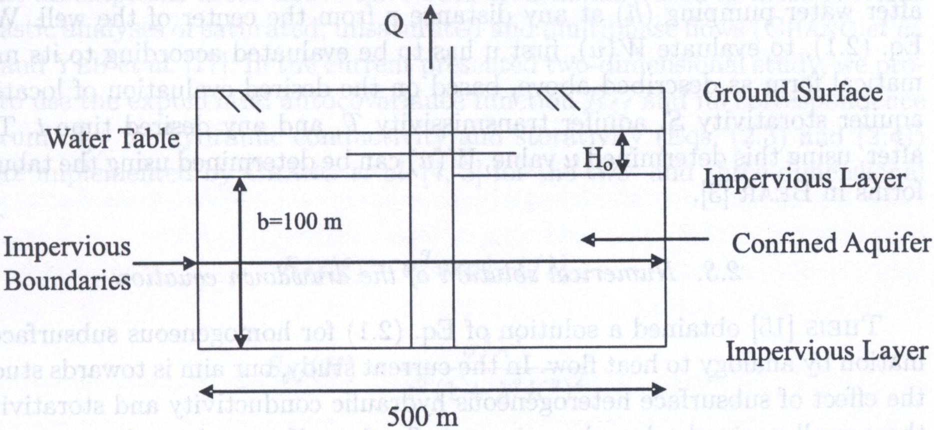

FIG. 1. Schematic diagram for the investigated problem.

2.2. *Governing equations*

The governing equations describing the transient well drawdown, when the well is discharging at a constant rate Q from a two-dimensional confined aquifer where the potentiometric surface is initially horizontal and equal to h_0, can be written as follows (BEAR [3]):

$$s = h_0 - h = \frac{Q}{4\pi T} W(u) \tag{2.1}$$

where

$$W(u) = \int_u^\infty \frac{e^{-\varphi}}{\varphi} d\varphi \quad \text{and} \quad u = \frac{r^2 S}{4Tt}.$$

$W(u)$ is called the well function and can be found in table format in most introductory textbooks in hydrology such as e.g. BEAR [3]. S is the aquifer storativity and h is the time-dependent water head in the aquifer after the pumping starts. r is the radius of the well where the drawdown $(h_0 - h)$ is evaluated, t is the time parameter, and φ is the integral parameter. However, T is the aquifer transmissivity and it can be simply written as follows:

$$T = bk, \tag{2.2}$$

where b is the confined aquifer thickness and k is the hydraulic conductivity. It is probably worth mentioning here that Eq. (2.1) is the standard groundwater equation to evaluate the transient drawdown (s) in the groundwater level during the water pumping process, as it is fully described in BEAR [3]. In addition, this drawdown (s) is normally evaluated as the difference between the original groundwater head (h_0), before pumping, and the groundwater head at any time after water pumping (h) at any distance r from the center of the well. Within Eq. (2.1), to evaluate $W(u)$, first u has to be evaluated according to its mathematical form as described above, based on the desired evaluation of location r, aquifer storativity S, aquifer transmissivity T, and any desired time t. Thereafter, using this determined u value, $W(u)$ can be determined using the tabulated forms in BEAR [3].

2.3. Numerical solution of the drawdown equation

THEIS [15] obtained a solution of Eq. (2.1) for homogeneous subsurface formation by analogy to heat flow. In the current study, our aim is towards studying the effect of subsurface heterogeneous hydraulic conductivity and storativity on the overall unsteady drawdown in a confined aquifer resulting from a pumping well. Therefore, we propose to use the numerical solution utilizing the finite difference approximation to solve Eq. (2.1) spatially and at any time increment. The fully implicit technique is used in this study to guarantee the highest accuracy. The elementary computer program, developed by WANG and ANDERSON [16] to solve Eq. (2.1) for the homogeneous case, is used in this manuscript as the core element for the comprehensive program developed by the authors to study the effect of subsurface heterogeneity on the aquifer drawdown and to perform the Monte Carlo simulations for the same purpose.

2.4. Random generation of the heterogeneous subsurface parameters

As mentioned earlier, the spatially correlated stochastic input processes were the subsurface hydraulic conductivity (k) and storativity (S). In addition to the spatial correlation of each one of the heterogeneous parameters, they are both considered to be statistically inter-correlated. Several assumptions are used in the generation of these stochastic processes and they can be outlined as follows:

- Subsurface hydraulic conductivity and storativity are considered as second-order stationary stochastic processes, and are assumed to be log-normally distributed.
- The fluctuations of each of these two stochastic processes are assumed to have correlation scales significantly smaller than the scale of the flow domain.

- In all performed simulations, ergodic hypothesis is considered to be valid so that a single realization represents the ensemble average at all representative realizations.
- The spatial structure of the fluctuations of S and k will be described by the exponential autocovariance function (YEH *et al.* [17]).

The assumptions listed above have been traditionally used in most previous stochastic analyses of saturated, unsaturated and multiphase flows (CHANG *et al.* [4, 5] and YEH *et al.* [17]. In the current presented two-dimensional study, we propose to use the exponential autocovariance function R_{ff} and its correspondence spectrum, S_{ff} for hydraulic conductivity and storativity (Eqs. (2.3) and (2.4)) as were implemented by CHANG *et al.* [4, 5] for the two- and three-dimensional cases.

(2.3) $$R_{ff}(\xi) = \sigma_i^2 \exp\left[-\xi/\lambda\right],$$

(2.4) $$S_{ff}(K) = \frac{\sigma_i^2 \lambda^3}{\pi^2 \left(1 + \lambda^2 K^2\right)^2}.$$

Equations (2.3) and (2.4) are applied twice, one for hydraulic conductivity and one for storativity; and subscript i refers to S (storativity), one time, and f (hydraulic conductivity) – the other time. However, ξ denotes the separation distance between the generated values, λ – covariance length for the generated values, σ_i – standard deviation for the generated values of S, one time, and f (hydraulic conductivity process) the other time; and K is the wave number. The random generation of the process is performed using the turning bands method and the spectral approach described by MANTOGLOU and WILSON [10]. On the other hand, the statistical inter-correlation between the two considered stochastic heterogeneous parameters (hydraulic conductivity, k, and storativity, s), can be described as follows:

(2.5) $$s = \bar{s} + \left\{ \frac{\sigma_s}{\sigma_f} * \left(k - \bar{k} \right) \right\}$$

where: s – storativity random process, $\bar{s}$ – average of the generated storativity random process, σ_s – standard deviation of s, k – hydraulic conductivity random process, $\bar{k}$ – average of the generated hydraulic conductivity random process, σ_f – standard deviation of k.

3. STOCHASTIC APPLICATION OF THE DEVELOPED MODEL

In this section, the application of the developed model to simulate the pumping from a centered well in a confined aquifer with heterogeneous subsurface formations is demonstrated. The geometric and hydraulic properties of the investigated problem are presented in Table 1. To investigate the impact of the subsurface heterogeneous parameters on the unsteady well drawdown and its statistical variations, several simulations for the developed model are applied. These simulations can be divided into two categories. The first one is concerned with the evaluation of the unsteady two-dimensional drawdown and the question how it is affected by the heterogeneous nature of the two stochastic properties. The second category are the Monte Carlo simulations and they are performed for the evaluation of the unsteady two-dimensional drawdown variability (represented by the standard deviation of the drawdown) and how this variability is affected by the heterogeneous nature of the two stochastic properties. In addition, within these two categories of modeling application and simulations, profound comparison between the two heterogeneous cases considered in this study and described in the section, are performed. The results of all these simulation are discussed in the following section.

Table 1. Geometric and hydraulic parameters for the example simulation

Parameter	Definition	Value
NX	No. of nodes in the X-direction	50
NY	No. of nodes in the Y-direction	50
DX	Increment spacing in the X-direction	10.0 (m)
DY	Increment spacing in the Y-direction	10.0 (m)
LX	Correlation length in the X-direction	30.0 (m)
LY	Correlation length in the y-direction	30.0 (m)
H_0	Initial water head in the aquifer	10.0 (m)
S	Average storitivity (S)	0.002
B	Confined aquifer thickness	100.0 (m)
K	Average hydraulic conductivity (k)	3.0 (m/day)
Q	Well pumping rate	300 (m^3/day)

4. Results and discussions

As mentioned previously, several analyses were performed to understand the stochastic behavior of the drawdown and its variability in the confined aquifer and the impact of the subsurface heterogeneity, represented by the two-dimensional random field hydraulic conductivity and storativity, on this behavior. First category of these analyses utilized the ergodic hypothesis and used one realization for each of the subsurface hydraulic conductivity (f) and storativity (S) in the simulation to represent the ensemble average. In this type of analysis, the effect of subsurface heterogeneity and the correlation length of the two generated processes on the unsteady stochastic behavior of the drawdown all over the entire aquifer were examined.

The second category of analysis includes the Monte Carlo simulations to capture all the random generations of subsurface heterogeneity and to help in understanding the physical behavior of the drawdown variations over the entire aquifer based on the population possibilities. In addition, comparison between the impact of two heterogeneous parameters (hydraulic conductivity and storativity) and heterogeneous hydraulic conductivity only (reported by ABDIN and ABDEEN, [1]) on the unsteady stochastic behavior of the drawdown and its variability ,were also investigated in the two categories of analysis. In all, later presented figures, when σ_S is not given a value, this means that it is equal zero and storativity S has a deterministic pattern in the entire confined aquifer and this is the case used by ABDIN and ABDEEN [1] for comparison.

4.1. Stochastic behavior of the drawdown

Two analyses are performed in this category to determine the stochastic behavior of the drawdown over the entire confined aquifer. These analyses are the subsurface heterogeneity, represented by the standard deviation of the two subsurface properties, and the correlation length of these two parameters. The results of these two analyses are described in what follows.

4.1.1. Effect of subsurface heterogeneity. Degree of subsurface heterogeneity is a very important parameter that affects the behavior of groundwater flow (CHANG *et al.* [4, 5]). Subsurface heterogeneity in this study is represented by the standard deviation of k and S processes. The simulation of this type of analysis was performed for several degrees of heterogeneity (various σ_f and σ_S values). Figure 2 shows the drawdown at the pumping well for three heterogeneous cases and the subsurface homogeneous case as a function of the pumping time. The results presented in this figure showed that as the subsurface heterogeneity increased from $\sigma_f = \sigma_S = 0.0$ (homogeneous case) to $\sigma_f = \sigma_S = 0.5$, the drawdown values at the pumping well were increased. On the other hand, Fig. 3 shows the spatial distribution of the drawdown along the X-direction from the well after 38 days of continuous pumping for two heterogeneous cases and the homogeneous one. The results, presented in this figure, indicated also that as the subsurface heterogeneity increased from $\sigma_f = \sigma_S = 0.0$ (homogeneous case) to $\sigma_f = \sigma_S = 0.5$, the spatial distribution of the drawdown values were increased. The spatial and unsteady increase in the drawdown values, as the subsurface heterogeneity increases, shows the direct physical dependence of

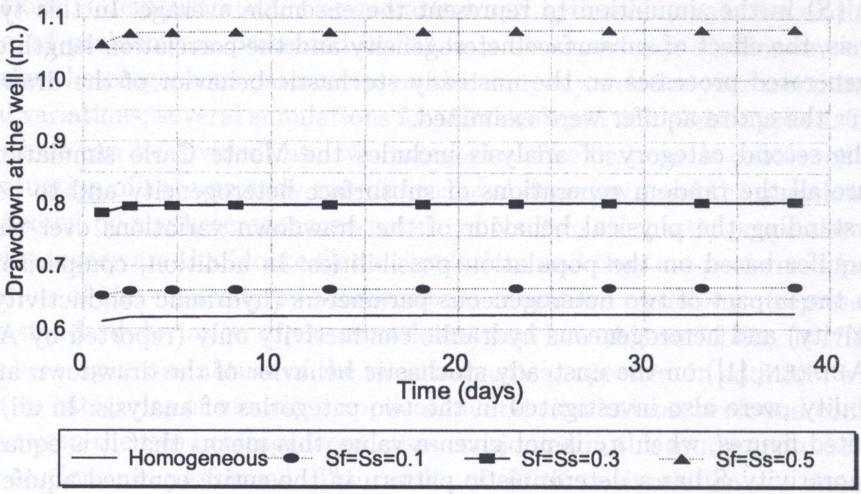

FIG. 2. Drawdown at the pumping well for three heterogeneous cases and the homogeneous one; Sf and Ss refer to σ_f and σ_S respectively.

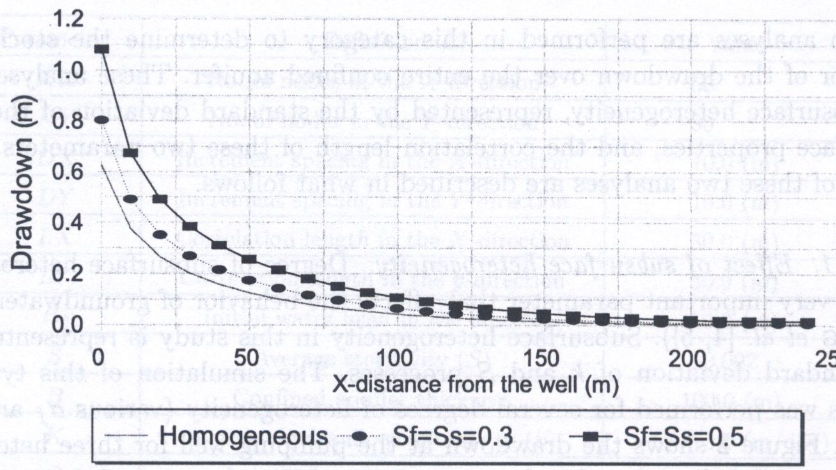

FIG. 3. Drawdown along the X-direction after 38 days of pumping for two heterogeneous cases and the homogeneous one; Sf and Ssrefer to σ_f and σ_S; respectively.

the drawdown on the heterogeneous characteristics of the subsurface formation. On the other hand, to investigate which subsurface heterogeneous parameter (s) has the greatest effect on the unsteady well drawdown, comparison between the drawdown resulting from the correlated heterogeneous k and S; and the cases of heterogeneous k only (ABDIN, and ABDEEN, [1]) were performed and the results are presented in Figs. 4 and 5 as functions of time and distance after

38 days of pumping, respectively. These results indicated that the case of two correlated subsurface heterogeneous parameters have the greatest impact on the unsteady well, compared with one heterogeneous parameter (k only) as well, as the homogeneous case. This result shows that any increase in the subsurface heterogeneity implies an increase in the spatial and unsteady well drawdown, which supports the previously mentioned physical conclusion of direct dependence of the drawdown values on the heterogeneous characteristics of the subsurface.

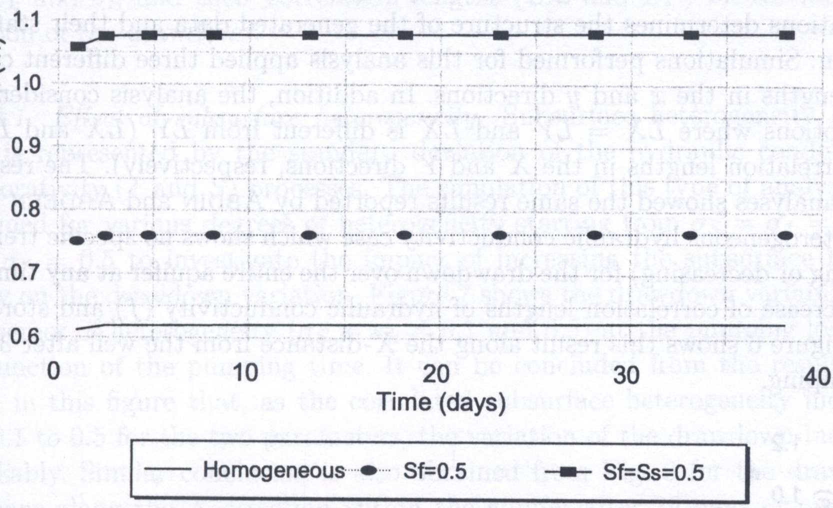

FIG. 4. Drawdown at the pumping well for two heterogeneous cases and the homogeneous one; Sf and Ss refer to σ_f and σ_S, respectively.

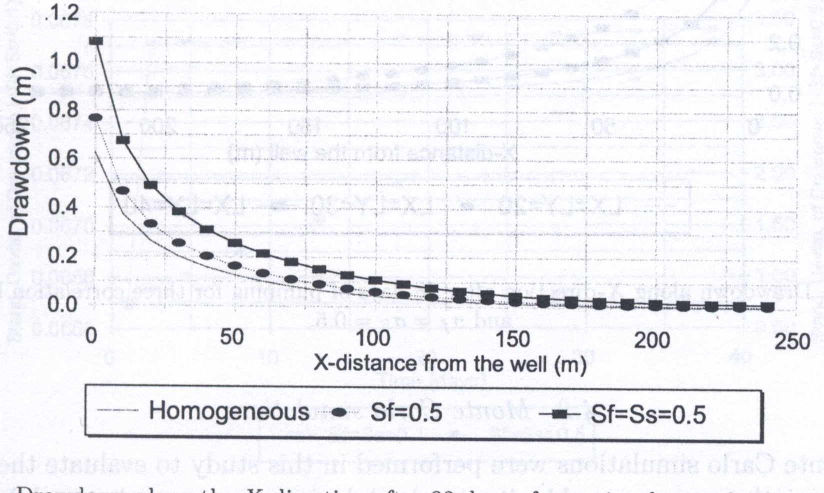

FIG. 5. Drawdown along the X-direction after 38 days of pumping for two heterogeneous cases and the homogeneous one Sf and Ss refer to σ_f and σ_S, respectively.

All the results presented in this section provide an insight into the impact of subsurface heterogeneity on the stochastic behavior of the drawdown and reminds the field engineers not to neglect any variations in the subsurface properties during the design of pumping water from this type of aquifers.

4.1.2. Effect of correlation length. Correlation length of any stochastic parameter's realization is defined as the length where this parameter's realization is correlated within its range. This property of the two stochastic parameters' realizations determines the structure of the generated data and their statistical pattern. Simulations performed for this analysis applied three different correlation lengths in the x and y directions. In addition, the analysis considered the two options where $LX = LY$ and LX is different from LY (LX and LY are the correlation lengths in the X and Y directions, respectively). The results of these analyses showed the same results reported by ABDIN and ABDEEN [1], for the heterogeneous hydraulic conductivity case which shows no specific trend (increasing or decreasing) for the drawdown over the entire aquifer at any time with the increase of correlation lengths of hydraulic conductivity (f) and storativity (S). Figure 6 shows this result along the X-distance from the well after 38 days of pumping.

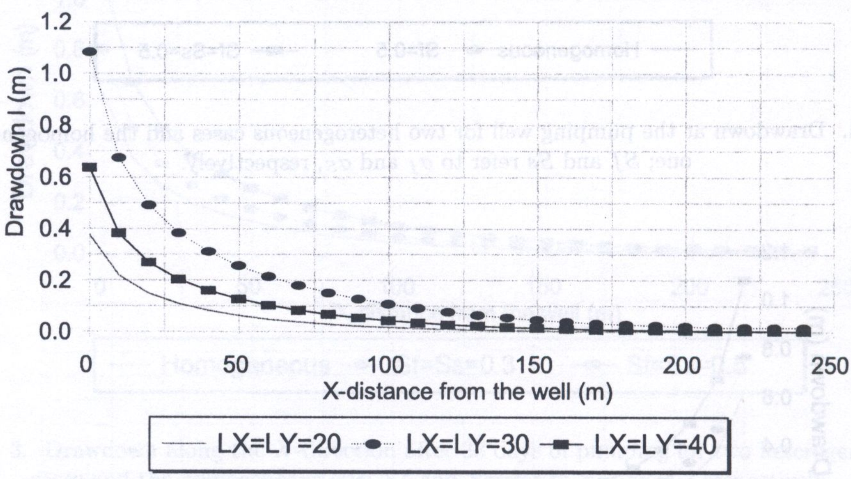

FIG. 6. Drawdown along X-direction after 38 days of pumping for three correlation lengths; and $\sigma_f = \sigma_S = 0.5$.

4.2. Monte Carlo simulations

Monte Carlo simulations were performed in this study to evaluate the drawdown variations represented by its standard deviation computed at each location over the entire aquifer at any time during the pumping process. The number of Monte Carlo simulations was determined to guarantee the capturing of all the statistical properties of the generated correlated hydraulic conductivity and storativity; and this occurred when the standard deviation of the drawdown had almost stabilized with the increase of the Monte Carlo simulations number. This case has occurred with the number of Monte Carlo simulations equal to 40. To fully investigate the influence of subsurface heterogeneity on the stochastic behavior of the drawdown, this study includes the investigation of the impact of both σ_k and σ_S and their correlation lengths (LX and LY) on the standard deviation of the drawdown.

4.2.1. Effect of subsurface heterogeneity. Subsurface heterogeneity in this study is represented by the standard deviation of the hydraulic conductivity and storativity (f and S) processes. The simulation of this type of analysis was performed for various degrees of heterogeneity starting from $\sigma_S = \sigma_f = 0.1$ to $\sigma_S = \sigma_f = 0.5$ to investigate the impact of increasing the subsurface heterogeneity on the drawdown variation. Figure 7 shows the drawdown variability for two degrees of heterogeneity ($\sigma_S = \sigma_k = 0.1$ and 0.5) at the pumping location, as a function of the pumping time. It can be concluded from the results presented in this figure that, as the correlated subsurface heterogeneity increases from 0.1 to 0.5 for the two parameters, the variation of the drawdown increases remarkably. Similar conclusion is also obtained from Fig. 8 for the drawdown variations along the X-direction within the aquifer after 38 days of pumping. This conclusion shows that the degree of subsurface heterogeneity has a direct physical impact on the unsteady and spatial variability (standard deviation) of the well drawdown.

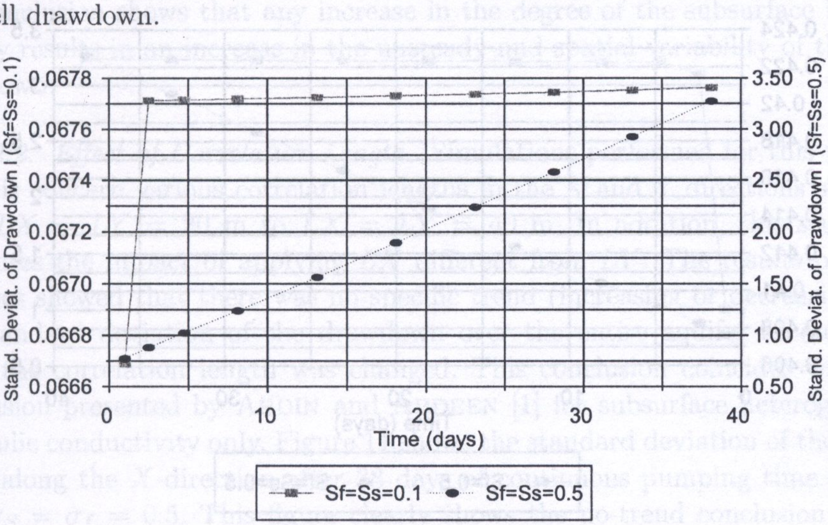

FIG. 7. Drawdown variability at the pumping well with time for two heterogeneous cases; Sf and Ss refer to σ_f and σ_S, respectively.

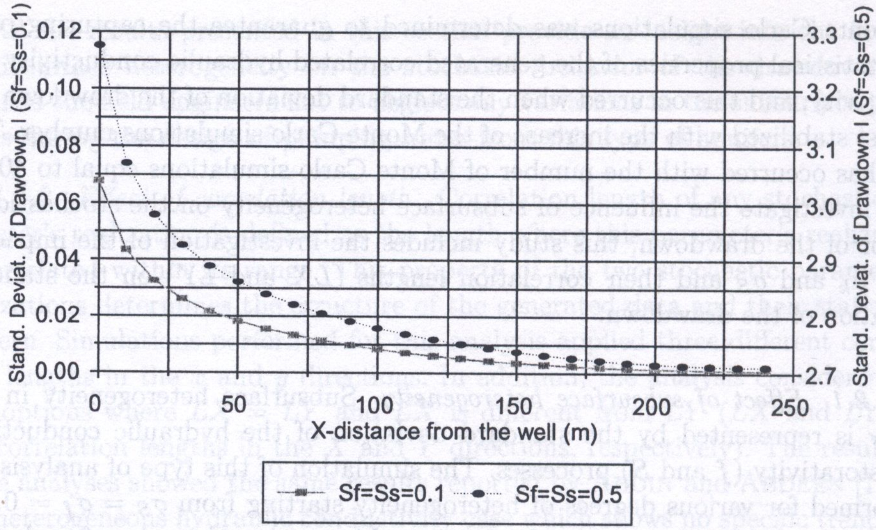

FIG. 8. Draw down variability along X-direction after 38 days of pumping for two heterogeneous cases; Sf and Ss refer to σ_f and σ_S, respectively.

To fully understand how the subsurface heterogeneous properties affect the drawdown variations, comparison between the current study results for two properties and the results presented by ABDIN and ABDEEN [1] for only the hydraulic conductivity as the heterogeneous parameter is performed. Figures 9 and 10

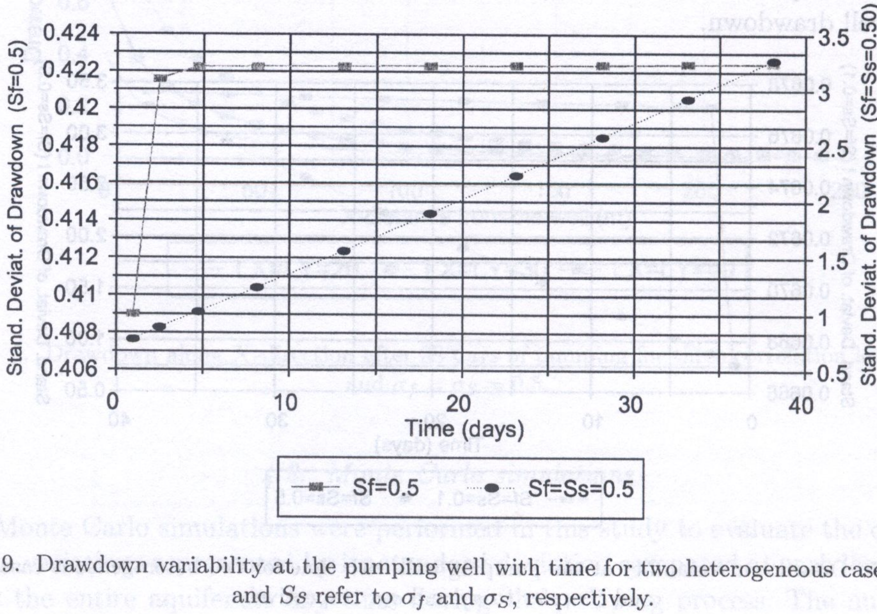

FIG. 9. Drawdown variability at the pumping well with time for two heterogeneous cases; Sf and Ss refer to σ_f and σ_S, respectively.

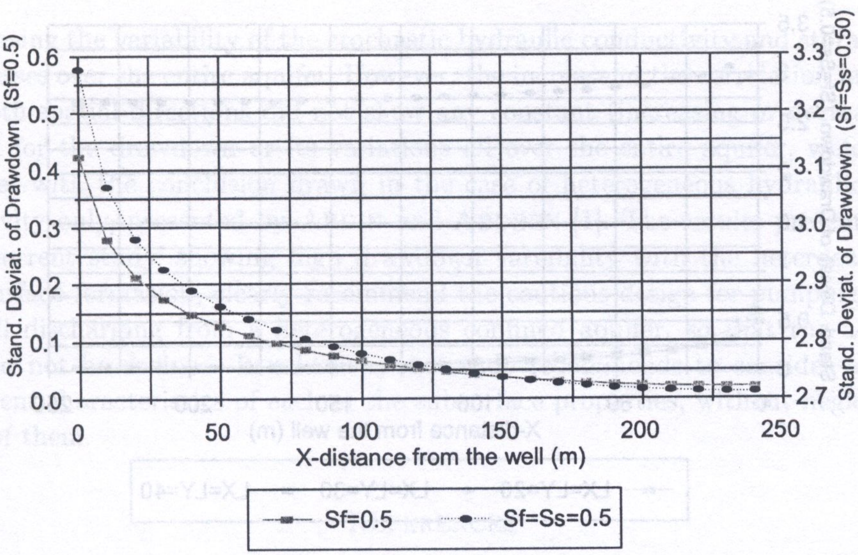

FIG. 10. Drawdown variability along X-direction after 38 days of pumping for two heterogeneous cases; Sf and Ss refer to σ_f and σ_S, respectively.

present the results of this comparison as a function of time at the pumping location and along the X-direction after 38 days pf pumping, respectively. It can be concluded from these results that considering two subsurface parameters to be heterogeneously correlated produces much higher drawdown variations than considering only the hydraulic conductivity to be heterogeneous. Once again, this conclusion shows that any increase in the degree of the subsurface heterogeneity results in an increase in the unsteady and spatial variability of the well drawdown.

4.2.2. Effect of Correlation Length. Simulations performed for this type of analysis concern various correlation lengths in the X and Y directions starting from $LX = LY = 20$ m to $LX = LY = 40$ m. In addition, this study investigates the impact of applying LX different from LY. The results of these analyses showed that there was no specific trend (increasing or decreasing) for the standard deviation of the drawdown over the entire aquifer at any time when the correlation length was changed. This conclusion coincides with the conclusion presented by ABDIN and ABDEEN [1] for subsurface heterogeneous hydraulic conductivity only. Figure 11 shows the standard deviation of the drawdown along the X-direction after 38 days of continuous pumping time for the case $\sigma_S = \sigma_f = 0.5$. This figure clearly shows the no-trend conclusion drawn from this analysis. This no-trend conclusion was observed over the entire aquifer at any time step.

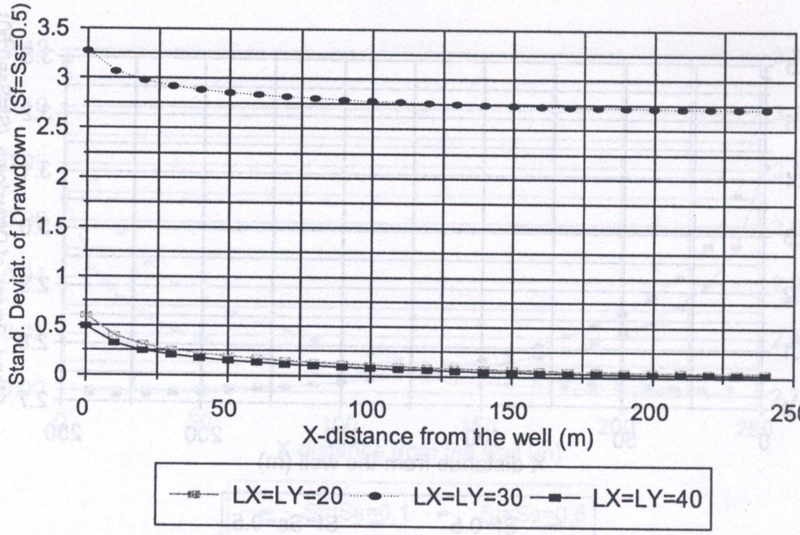

FIG. 11. Drawdown variability along the X-direction after 38 days of pumping for three correlation lengths; and $\sigma_f = \sigma_S = 0.5$.

5. Summary and conclusions

The study presented in this paper investigated the impact of the heterogeneous subsurface properties on the stochastic behavior of the drawdown resulting from pumping water in the center of a confined aquifer. The subsurface heterogeneity was represented by two-dimensional log-normal correlated hydraulic conductivity and storativity processes generated using the Turning Bands method. Various analyses were performed in this study to note all the impacts of these two heterogeneous parameters. In addition, comparison between two stochastic cases was performed. The first case considered the hydraulic conductivity and storativity being the heterogeneous correlated subsurface parameters. In the second case considered, the hydraulic conductivity represented the only heterogeneous parameter and the storativity was assumed to have a deterministic value in the entire confined aquifer (ABDIN and ABDEEN [1]).

The overall conclusion that may be drawn from the results obtained from the study is the direct impact of the properties of the subsurface heterogeneity on the confined aquifer groundwater head variations. The increase in the subsurface heterogeneous hydraulic conductivity and storativity variations clearly resulted in a remarkable increase of the unsteady drawdown, as compared with the case of subsurface heterogeneous hydraulic conductivity presented by ABDIN and ABDEEN [1]. On the other hand, the Monte Carlo analysis performed in this study showed a remarkable increase in the drawdown variability as a result of increasing the variability of the stochastic hydraulic conductivity and storativity processes over the entire aquifer. However, the increase in the correlation lengths in both spatial directions did not show any constant (increasing or decreasing) trend for the drawdown or its variations all over the entire aquifer, which coincides with the conclusion drawn in the case of heterogeneous hydraulic conductivity only, presented by ABDIN and ABDEEN [1]. The results presented in the current study, showing high drawdown variability with the heterogeneous subsurface formation, clearly recommend the cautious design for pumping from a well discharging from a heterogeneous confined aquifer, so that the aquifer should not be drained. In addition, the study recommends to consider all the different characteristics of each of the subsurface properties, without neglecting any of them.

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