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Analysis of the Nonstability States During Bending Processes of Metallic Tubes at Bending Machines Part I. Derivation of the Basic Expressions and Relationships

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In this paper the derivation of expressions for admissible values of strains and stresses for vertex points of layers subjected to tension during tube bending at bending machines is presented. The conditions of the dispersed and located loss of stability of the bent tube and the cracking criterion based on the technological index A_5 (five fold sample) were assumed as criteria of instability. The original element of this paper is the extension of the criterion of strain location in a form of possible initiation of a neck or furrow (introduced by Marciniak for thin plates [1]) to bending thin- and thick-walled metal tubes at bending machines. Occurrence of loss of stability (especially that in a localised form) during tube bending can strongly reduce the service life. Thus, it is recommended to avoid such states during tube bending for elbows for pipelines or pipe installations.

Key words: allowable strains and stresses, bending angles, neutral layer, wall thickness of elbows.

1. Introduction

Tube bending (see e.g. [2–25]), as a technological problem appeared in the end of the 19th century when production of tubes started on industrial scale. Tubes were delivered mainly to industry of steam engines and boilers, gas engineering, power engineering, civil engineering. At present tubes and elbows are purchased by almost all branches of industry and tube bending is a typical activity in many technological processes in metal industry. Production of tubes and elbows is increasing more rapidly than production of steel because tubes and elbows are made also of other materials, for example, plastics. Higher requirements concerning the quality of produced tubes and elbows are set. The choice of a tube bending method is dependent on a kind of material, thickness of the tube, bending radius, the required accuracy and quality of bending, work conditions, bend angle, serial production, and others.

In paper [18] a generalised model of strain during metal tubes bending on bending machines was derived. In the considered case, the tubes were bent with the wrapping method at a rotating template with the use of a lubricated steel mandrel. The model contains three strain components in an analytical form, including displacement of the neutral axis. The derived strain scheme satisfies initial and boundary kinematic conditions of the bending process, conditions of continuity and compatibility of strains. The obtained analytical expressions can be classified as kinematically admissible. The present paper is a further development of [14, 18]. Tube bending with bending machine is usually made with the method of wrapping at the rotating template using a suitable mandrel, see Fig. 1.

Pipelines and tube installations can be operated during a definite time of life and safe work. The pipelines and tube installations contain straight parts, elbows, pipe fittings (tees, four-way pieces, reducing pipes, nozzles, etc.) and connecting elements, for example, welds, screw joints, and others. Their lifetimes are different. The lifetime of the straight parts is the longest, next, there are elbows and pipe fittings, while the lifetime of the connecting elements such as welds is the shortest, see [2, 3, 6, 7, 14–18, 26–30].

Premature damages of the elbows occurring during operation of the elbows can be also caused by application of unsatisfactory method of strength calculations, see e.g. [6, 7, 14–16, 19, 20, 26, 30]. Such a situation can be caused by the lack of precise methods of determination of permissible distribution of the wall thickness in the points of the maximum strains during the elbow bending. It concerns especially the top parts of the elbow bending zone, where this thickness is minimal. If the strain components and intensity in the bending zone, especially in the top part are well known, better calculations of strength of elbows and their better manufacturing will allow to improve reliability of machines and devices.

At present, tube bending with bending machines is usually performed applying the method of wrapping at the rotating template with the use of a suitable mandrel, see Fig. 1.

Tube bending with the considered method always causes reduction of the wall thickness in the layers subjected to tension, increase of wrinkling and the wall thickness in the layers subjected to compression, and deformation (ovalisation) of the cross section. Those unfavourable phenomena should be included into the tolerance limits given in the European and Polish regulations [20, 25, 31]. The reduction of the wall thickness and negative influence of large strains in the top points of the elbow are the most important factors influencing the operation life of the elbow [6, 7, 11, 12, 14, 16, 19, 26–29].

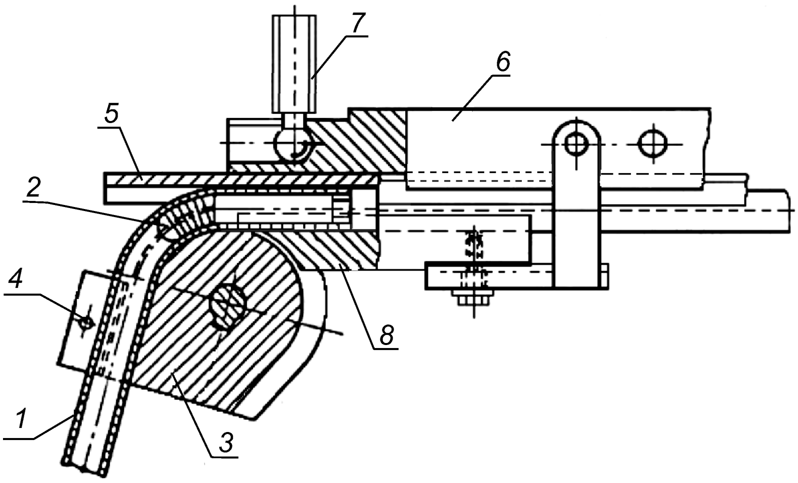

FIG. 1. Scheme of a tube bending machine with a rotating template and a flexure mandrel. *1* – bent tube, *2* – flexible segment mandrel, *3* – rotating template, *4* – clamping jaws, *5* – clamping strip, *6* – guide, *7* – screw for regulation of clamping force of the strip and the planisher, *8* – planisher.

In this paper previous ideas and concepts [31–37] of the principal author are developed. The development consists of extension of the stability loss condition in the localised form (possible initiation of the furrow) derived by Marciniak [1] for uniaxial and biaxial tension of sheets to the cases of bending thin- and thick-walled metal tubes at the bending machines, in particular, with the use of the rotational template and the mandrel, Fig. 1.

In [2, 3, 14–18] it was assumed that the bending of thin-walled metal tubes at bending machines in the layers subjected to tension (tube bending with the method of wrapping at the rotational template using the mandrel) is a complex process of heterogeneous curvilinear tension (curvilinear wrap forming) under the biaxial stress state. In the case of the layers subjected to compression, it is something like a combination of heterogeneous curvilinear tension and bounded upsetting, respectively. In this paper the author analyses also the influence of parameters of hardening and normal anisotropy on admissible values of the bending angle, strains, and stresses occurring in the layers subjected to tension during cold bending of thin- and thick-walled metal tubes at bending machines in the range of the bending angle $\alpha_b \in \langle 0^\circ; 180^\circ \rangle$ and for the case when $y_0 \geq 0$, where y_0 is the displacement of the neutral axis of plastic bending, see [2, 14, 16–19, 21] and Fig. 2. Moments of possible occurrence of the stability loss in the dispersed form for the case of uniaxial tension and occurrence of the localised stability loss (for example in the form of local initiation of the external furrow) under biaxial stress state were assumed as the criteria [1, 14, 16, 28, 29, 32–36].

For the case of thin-walled metallic tubes, the problem of occurrence of the plane state of deformation (PSD) under the plane stress state (PSS) was also considered [1, 14, 16, 31–41]. Analytical calculations (see Part II) were realised for two extreme cases, i.e., for a generalised scheme of strain and simplification of the 3rd type, because the calculation results resulting from simplifications of the 1st and 2nd types [2, 3, 14, 16, 17, 18] will be included between those two extreme cases. The problem of cracking was considered according to a technological index A_5.

The author concentrates only on analysis of the top points of the layers subjected to tension because from the previous experiments and operating tests it appears that the process of the elbow damage in most pipelines (especially those used in energy engineering) usually starts and develops in external top points of the layers subjected to tension in the bending zone. From the collected statistical data [25–29] it appears that an average life time of the elbows of pipelines loaded by internal pressure and operating at the elevated temperatures is shorter than that of straight intervals of pipelines, and creep strength is lower even by about 30% and more. Thus, occurrence of stability loss states (especially those localised) during tube bending causes further drop of the operating life. It is recommended in this paper to prevent such states in technology of metal tube bending for the pipeline elbows (depending on their application.

In the second part of the paper one can find simplified methods of solving simple examples in order to determine admissible values of strains, bending angles and the appropriate stresses). At the end of the second part there are an expression for displacement and a new position of the neutral axis of plastic bending y_0 for the considered elements of the stability loss and for the bending angles included in the range $k\alpha_b \leq 180°$.

2. Fundamental assumptions and relationships

The analytical-geometrical description and analysis of the process concerning tube bending with the use of the wrapping method at the rotating template and with or without the mandrel, by assuming that $d_{\text{int}} \cong \text{const}$ (admissible ovalisation is below 6%, according to [25, 31]), is presented. The analytical description of deformation is limited to the plastic strain state, because elastic strains are very small and they can be neglected [2, 8, 15, 17, 18].

Let us take into account the experimental data presented in [8] and the author's data [16, 18]. The generalised logarithmic components of the strain state including also displacement of the neutral axis of plastic bending y_0 [19] were derived. They are also adapted to calculations of strains at external points of each N-th layer included in the wall of the bent tube (it concerns the ring-shaped division for thick-walled tubes for the purposes of Finite-Elements-Method (FEM) [16, 18]) in the layers subjected to tension and compression. Then

$$(2.1)_1 \qquad \varphi_1 \cong \lambda_i \ln \frac{2(R-y_0) \pm (d_i \cos\beta_i \pm 2y_0)\left(\cos(k\alpha) - \cos\left(k\dfrac{\alpha_b}{2}\right)\right)}{2(R-y_0)},$$

$$(2.1)_2 \qquad \varphi_2 \cong \ln \frac{d_i}{d_{\text{ext}}},$$

$$(2.1)_3 \qquad \varphi_3 \cong \ln \frac{g_i}{g_0}$$

and for the purposes of FEM

$$\varphi_1^{(N)} = \lambda_i \ln \frac{2R \pm d_i^{(N)} \cos\beta \left(\cos(k\alpha) - \cos\left(k\dfrac{\alpha_b}{2}\right)\right)}{2R},$$

$$\varphi_2^{(N)} = \ln \frac{d_i^{(N)}}{d_{\text{ext}}^{(N)}}, \qquad \varphi_3^{(N)} = \ln \frac{g_{li}^{(N)}}{g_{l0}^{(N)}}, \qquad g_i = \sum_{N=1}^{N=n} g_{li}^{(N)},$$

where $i = 1$ for the elongated layers, $i = 2$ for the compressed layers.

In the strain model (2.1) we can introduce three simplifications of the physical sense. Simplification 1 is obtained by introduction of $d_i = d_{\text{ext}}$ into Eq. $(2.1)_1$, in simplification 2 we introduce $d_i = d_{\text{ext}}$ into Eq. $(2.1)_2$ and in the case of simplification 3 we introduce $d_i = d_{\text{ext}}$ into Eqs. $(2.1)_1$ and $(2.1)_2$, see [3, 16, 17]. The simplifications are denoted adequately by one, two, or three signs: ($'$), ($''$), or ($'''$) respectively. The expression for strain intensity and plastic incompressibility of the material takes the following form (see [1, 2, 16, 37, 38]):

$$(2.2) \qquad \begin{cases} \varphi_{(i)} = \sqrt{\dfrac{2}{3}(\varphi_1^2 + \varphi_2^2 + \varphi_3^2)}, \\ \varphi_1 + \varphi_2 + \varphi_3 = 0. \end{cases}$$

Expressions (2.1) and (2.2) are used for description of the strain state of the tube subjected to bending in the top, $\cos(k\alpha) = 1$, and external, $\cos\beta = 1$ points of the layers subjected to tension or compression, where φ_1, φ_2, φ_3 are logarithmic components of plastic strains. Notations: $\varphi_{(i)} \equiv \varphi_i$ is the intensity of logarithmic plastic strains. For simplifications 1, 2, or 3 order we have that $\varphi_i' \equiv \varphi_{(i)}''$, $\varphi_i'' \equiv \varphi_{(i)}''$, $\varphi_i''' \equiv \varphi_{(i)}'''$.

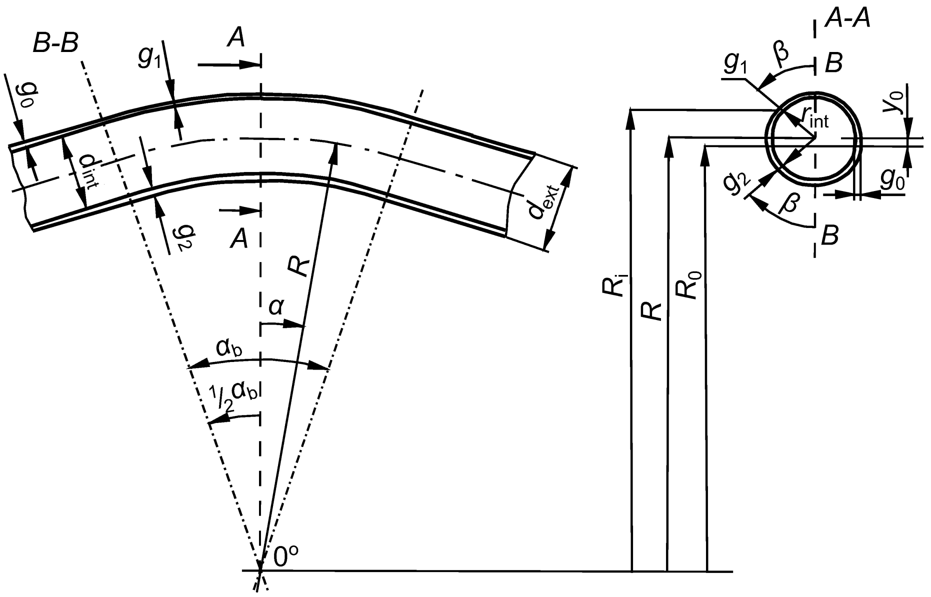

FIG. 2. Geometrical quantities pertaining to pipe-bending processes.

The quantities used in Eqs. (2.1) and in Fig. 2 have the following meanings:

R – nominal radius of tube bending,

R_i and R_0 – "big active actual radius of bending" connected with the longitudinal strain, and radius determining actual position of the neutral layer respectively,

y_0 – displacement of the neutral layer of plastic bending,

r_i – "small active actual radius" of the elbow in the bending zone, $r_i = r_{\text{int}} + g_i$ and $d_i = 2r_i$,

r_{ext} and d_{ext} – external radius and diameter of the tube subjected to bending, respectively, $d_{\text{ext}} = 2r_{\text{ext}}$,

r_{int} and d_{int} – internal radius and diameter of the tube, respectively, $d_{\text{int}} = 2r_{\text{int}}$ and $d_{\text{int}} = \text{const}$,

g_0 and g_i – initial thickness of the tube and actual thickness of the elbow wall in the bending zone,

$\alpha_b \equiv \alpha_g$ – bending angle measured in the bending zone. In the bending zone, the angles of bending and bend are equal, so $\alpha_b = \alpha_0$, where α_0 – bend angle (angle of rotation of the bending machine template),

α, β – angles of the point position in the bending zone,

β – angle of circulation of the layers subjected to tension and compression of the elbow, $\beta \in \langle 0;\ 90° \pm \beta_0 \rangle$ and $\sin\beta_0 = y_0/r_{\text{ext}}$, where β_0 is the angular range of displacement of the neutral axis of bending, index $i = 1$ and sign $(+)$ in Eqs. (2.1) relate to the layers subjected to tension,

index $i = 2$ and sign $(-)$ in Eqs. (2.1) relate to the layers subjected to compression,

k – technological-material coefficient determined from experimental results, which defines a range of the bending zone in the bend zone, so that $k\alpha_b = 180°$. From the theoretical point of view $k \in \langle 1; \infty \rangle$. For practical purposes we can assume that $k \in \langle 1;\, 6 \rangle$. Based on the known test results we can approximately assume that $k \in \langle 1 \div 3 \rangle$, see [2, 3, 8, 11, 12, 14–18].

For the elbows bent at the bending angle $\alpha_0 = 180°$, the coefficient k means the ratio of the bending angle α_0 to the real bend angle α_b, i.e. $k = \alpha_0/\alpha_b$. When $\alpha_0 = 180°$, then $\alpha_0 = k\alpha_b = 180°$. When, for example, $\alpha_b = 90°$, then $k = 2$; when $\alpha_b = 60°$, then $k = 3$ etc.

λ_i – correction coefficient (technological-material) of strain distribution in the layers subjected to tension ($i = 1$) and compression ($i = 2$) of the bending and bend zone, defined from the experimental results so that $\lambda_1 \cong 1$ and $\lambda_2 \in \langle 0; 1 \rangle$. In the case of the most known tests we can approximately assume that $\lambda_2 \approx 0.5$ [2, 8, 14, 1–18].

As it was said, admissible values of the bending angle and the strain intensity are determined for suitable moments of stability loss occurrence, see [1, 14, 16, 32, 33, 35, 36, 38, 40, 41]. It is obvious that range of plastic strains seen from the point of view of application in plastic work processes is limited, because of possibility of the material stability loss, or coherence loss, i.e., cracking (fracture). Stability loss usually occurs as the first one, however, we are able to select suitable material properties, strain conditions, and production technology where the fracture occurrence does not happen before stability loss [1, 7, 16, 38]. In this paper, the author concentrated only on analysis of moments of possible occurrence of stability loss in the dispersed form [1, 14, 16, 33, 36, 38], localised in a local point (for example, a beginning of local initiation of the neck or furrow [1, 14, 16, 32–36, 38, 41]) under uniaxial and biaxial stress state. The case of initiation of the plane state of deformation (PSD) in the plane stress state (PSS) was also considered.

Let as assume that the tube is made of a rigid-plastic metallic material with isothropic hardening, which satisfies the Huber-Mises-Hencky (H-M-H) condition of plasticity and the plastic flow laws formulated by Levy-Mises. Let us also assume the displacement of the neutral axis $y_0 \geq 0$, and that cold tube bending is performed at the bending machine at the ambient temperature. It is a quasi-static and quasi-isothermic process. Thus, dynamic and thermal effects accompanying small and big plastic deformations are not taken into account [42, 43, 45].

In the present considerations the following form of the hardening curve was assumed [1, 14, 40]:

$$\sigma_{\mathrm{p}} = D(\varphi_0 + \varphi_{(i)})^n, \tag{2.3}$$

where σ_{p} is the yield stress in [MPa], n is the coefficient of hardening, D is the material constant in [MPa], φ_0 is the logarithmic initial strain, $\varphi_{(i)} \equiv \varphi_i$ is the logarithmic strain intensity.

For most metals and alloys applied in engineering practice the value of the coefficient n is in the range $\langle 0 \div 0.6\rangle$.

3. The considered cases of loss stability

Let us consider three special cases of the stability loss in the tube bending process.

Case 1. Uniaxial tension [1, 14, 16, 33]. Loss of stability in the dispersed form (maximum drawing force).

Then

$$\varphi_{(i)a} = n - \varphi_0 \quad \text{and} \quad \varphi_{(i)a} = \varphi'_{(i)a} = \varphi''_{(i)a} = \varphi'''_{(i)a}, \tag{3.1}$$

where $\varphi_{(i)a} \equiv \varphi_{ia}$ is the substitute strain corresponding to this stability loss.

Case 2. Biaxial stress state [1, 14, 16, 36]. Stability loss in the form of localised deformation when locally $d(\sigma_{\mathrm{p}} \cdot g_1) = 0$.

According to [1, 16, 32, 33, 35, 40] and taking into account: expressions for principal components of the strain state during tube bending Eq. (2.1), expressions for plastic incompressibility, the strain intensity, and assuming that $d_{\mathrm{int}} \cong$ const. (d_{int} – internal diameter of the bent tube) and that ($d\varphi_3 = dg_1/g_1$), after transformations we obtain

$$\varphi_{(i)b1} \cong \sqrt{\frac{(1+r)\left[8\left(\dfrac{g_1}{d_1}\right)^2 + 4\left(\dfrac{g_1}{d_1}\right) + (1+r)\right]}{(1+2r)}}\,n - \varphi_0, \tag{3.2$_1$}$$

$$\varphi'''_{(i)b2} = \frac{1+r}{\sqrt{1+2r}}\,n - \varphi_0. \tag{3.2$_2$}$$

Assuming in $(3.2)_1$ and $(3.2)_2$ some simplifying expressions, so as $(g_1/d_1 \approx g_0/d_{\mathrm{ext}} = s^*)$ or for cylindrical angular division in FEM [18] $(g_{l1}^{(n)}/d_{l1}^{(n)} \approx$

$g_{l0}^{(n)}/d_{l0}^{(n)} \approx s^{(n)*}$), we can see the influence of pipe geometry parameter s^* on values of admissible strain intensity. Then

$$(3.3)_1 \qquad \varphi_{(i)b1} \approx \sqrt{\frac{(1+r)[8(s^*)^2+4s^*+(1+r)]}{(1+2r)}n} - \varphi_0$$

and

$$(3.3)_2 \qquad \varphi'''_{(i)b2} = \frac{1+r}{\sqrt{1+2r}}n - \varphi_0,$$

where

$\varphi_{(i)b1}$ and $\varphi'''_{(i)b2}$ are the values of the strain intensity corresponding to the generalised model of strain for this form of stability loss and for simplification of the 3rd type (3rd order) appropriately [3, 16], and also $\varphi_{(i)b1} = \varphi'_{(i)b1}$, $\varphi'''_{(i)b2} = \varphi''_{(i)b2}$,

g_1 and d_1 are real thickness and diameter of the elbow in the layers subjected to tension,

s^* is the thin-walled parameter of the bent tube defined as $s^* = g_0/d_{\text{ext}}$, see [12, 16, 18],

s^*_w is the thin-walled parameter of the bent tube defined as $s^*_w = g_0/d_{\text{int}}$, see [6, 16, 18, 20], then $s^*_w = s^*/(1-2s^*)$,

g_0 and d_{ext} are the initial thickness and external diameter of the bent tube.

The parameter r is the coefficient of Lankford normal anisotropy, see [1, 14, 16, 33, 35, 38], which can be written as

$$(3.4)_1 \qquad r = \frac{\varphi_2}{\varphi_3} = \frac{\ln \dfrac{b}{b_0}}{\ln \dfrac{g}{g_0}}$$

and for tubes

$$(3.4)_2 \qquad r = \frac{\varphi_2}{\varphi_3} = \frac{\ln \dfrac{d}{d_{\text{ext}}}}{\ln \dfrac{g}{g_0}},$$

where

b_0 and b are the specimen widths before and after deformation (elongation),

d_{ext} and d are the external diameters of the tube before and after deformation (elongation),

g_0 and g are the specimen thicknesses before and after deformation (elongation).

From the above relationship it appears that when the coefficient r increases, then the reduction of the specimen thickness is lower, i.e., resistance to reduction of thickness of the tube wall increases. The coefficient r is in the range $(1 \div 2.5)$ for most steels used for tube manufacturing.

The cases of stability loss (case 1 and 2) are applied here for estimation of the instability state of tubes of thin-walled parameter $(0 < s_w^* < 0.15)$. In papers [16, 18], the thin-walled parameter $(0 < s^* \leq 0.1)$ was assumed as more suitable from the technological point of view. Parameter s^* is determined as a certain geometric mean of the thin-walled parameter $(0 < s^* < 0.2)$ according to [12] and $(0 < s_w^* < 0.05)$ according to [6, 20]. The case of biaxial stress state considered in Part II concerns bending of thin-walled tubes $(0 < s^* \cong 0.101)$. In most cases (or even in all the cases) technologically thick-walled tubes $(s_w^* > 0.15)$ of big diameters $(d_{\text{ext}} > 160$ mm) are subjected to hot or semi-hot bending [14, 16, 18, 44]. During thick-walled tube bending, there is the triaxial stress state inside the material of the bending tubes, so $(\sigma_1 \geq \sigma_2 \geq \sigma_3$, $\sigma_3(r_{\text{ext}}) \cong \sigma_3(r_1) \cong 0)$. Let us note that during hot bending the coefficient of normal anisotropy $(r \approx 1)$, and there is no material hardening $(n \approx 0)$ [41, 44].

The Huber-Mises-Hencky (H-M-H) condition of plasticity and Levy-Mises equations of plastic flow [12, 13, 16, 19, 21, 29, 35, 36] for an isotropic body expressed in principal stresses take the following forms:

$$\sigma_{\text{p}} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}, \tag{3.5}$$

$$\frac{d\varphi_1}{\sigma_1-\sigma_m} = \frac{d\varphi_2}{\sigma_2-\sigma_m} = \frac{d\varphi_3}{\sigma_3-\sigma_m} = \frac{d\varphi_{(i)}}{\frac{2}{3}\sigma_{\text{p}}}, \tag{3.6}$$

where $\sigma_m = \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)$ is the mean stress, σ_1, σ_2, and σ_3 are principal stresses, and φ_1, φ_2, and φ_3 are logarithmic principal plastic strains.

When the deformation process is proportional, then suitable strain increments can be replaced by overall strains [1, 16, 33–35, 38, 39, 41].

We obtain the stress state components corresponding to the state (case 2.1) and Eq. $(3.3)_1$, for thick-walled tube bending (on hot or semi-hot, $r \approx 1$) from the equations of plastic flow (3.6), taking into account the H-M-H condition of plasticity (3.5) and with no effect of hardening $(\sigma_{\text{p}} \cong \text{const})$. Assuming for simplification as previously that $(g_1/d_1 \approx g_0/d_{\text{ext}})$, we can see here and further the influence of pipe geometry parameter s^* on stresses' states. Taking into account application of a very low value of pressure force of the strip holding down the bent tube to the template, and the resulting low value of the radial stress ($\sigma_3(r_{\text{ext}}) \cong \sigma_3(r_1) \cong 0$) on the external surfaces of the layers subjected to tension, after transformations we obtain

$$
\begin{aligned}
\sigma_1 &\cong \frac{2(1+s^*)}{\sqrt{3[4(s^*)^2+2s^*+1]}}\sigma_{\text{p}}, \\
\sigma_2 &\cong \frac{1-2s^*}{\sqrt{3[4(s^*)^2+2s^*+1]}}\sigma_{\text{p}}, \\
\sigma_3(r_{\text{ext}}) &\cong \sigma_3(r_1) \cong 0.
\end{aligned} \tag{3.7}
$$

The stress state components corresponding to the state (case 2.2), expressed by Eqs. $(3.2)_2$ or $(3.3)_2$, for thick-walled tube bending (made also of the isotropic material, then $r \cong 1$), can be obtained in a similar way, including an additional condition resulting from determination of almost zero stress value σ_3''' on the external (unloaded) surfaces of the layers subjected to tension. Thus

$$
\begin{aligned}
\sigma_1''' &\cong \frac{2}{\sqrt{3}}\sigma_{\text{p}}, \\
\sigma_2''' &\cong \frac{\sigma_p}{\sqrt{3}}, \\
\sigma_3'''(r_{\text{ext}}) &\cong \sigma_3(r_1) \cong 0.
\end{aligned} \tag{3.8}
$$

As it can be seen, the case 2.1 determined for the generalised scheme of strain and simplifications of the 1st type, Eqs. $(3.2)_1$, $(3.3)_1$, and (3.7), depend not only on the material parameters (n, r, φ_0), but also on the geometric ones (g_1 and d_1) of the bent tube (they approximately depend on the value of coefficient s^*, $s^* = g_0/d_{\text{ext}} \approx g_1/d_1$). It appears that when thin-walled character of the tube increases, $s^* \uparrow$, then $\varphi_{(i)b1} \uparrow$, increases too. For the case 2.2, Eqs. $(3.2)_2$, $(3.3)_2$, and (3.8) derived for the scheme of simplifications 2nd and 3rd type, see [3, 14, 16, 17], do not depend on the geometric parameter of the bent tube s^*. In the case when $s^* = 0.5$ (full cross section, full rod), from Eqs. (3.6) and (3.7) it appears that $\sigma_1 = \sigma_{\text{p}}$, $\sigma_2 = \sigma_3 = 0$ and ($\varphi_2 = \varphi_3 = -0.5\varphi_1$), also that $\varphi_{(i)b1f} \cong (2n - \varphi_0)$.

General distribution of principal stresses in all the points of the bending zone can be obtained from Eqs. (2.3), (3.5), and (3.6), suitable equations of equilibrium including friction forces for a case of asymmetric (of variable thickness) thin- and thick-walled closed shell (or asymmetric interval of toroidal closed shell), see [41].

The H-M-H condition of plasticity for PSS and a material with hardening and properties of normal anisotropy has the following form [1, 14, 16, 17, 38]:

$$
(1+r)\sigma_{\text{p}}^2 = (1+r)\sigma_1^2 - 2r\sigma_1\sigma_2 + (1+r)\sigma_2^2. \tag{3.9}
$$

For thin-walled tubes (where there is the biaxial stress state in the layers subjected to tension during the bending process), the Levy-Mises equations of plastic flow expressed in logarithmic measures of strain have the following form [1, 16, 35, 38]:

$$\frac{d\varphi_1}{(1+r)\sigma_1 - r\sigma_2} = \frac{d\varphi_2}{(1+r)\sigma_2 - r\sigma_1} = \frac{d\varphi_3}{-(\sigma_1+\sigma_2)} = \frac{d\varphi_{(i)}}{(1+r)\sigma_{\mathrm{p}}}, \tag{3.10}$$

where

$$d\varphi_{(i)} = \sqrt{\frac{(1+r)(d\varphi_1^2 + d\varphi_2^2 + d\varphi_3^2)}{1+2r}}.$$

The stress state components corresponding to the states in case 2.1 and 2.2, respectively, during thin-walled tube bending for a case of the material showing properties of the normal anisotropy (obtained from the equations of plastic flow (3.10), including the hardening curve (2.3) and the condition of plasticity (3.9)), and after the use the Eqs. $(3.3)_1$ and $(3.3)_2$ have the following forms:

$$(3.11)_1 \qquad \sigma_1 \cong \frac{(1+r)(1+r+2s^*)}{\sqrt{(1+r)(1+2r)[8(s^*)^2 + 4s^* + (1+r)]}}\sigma_{\mathrm{p}}$$

$$\text{and} \qquad \sigma_1''' \cong \frac{1+r}{\sqrt{1+2r}}\sigma_{\mathrm{p}}''',$$

$$(3.11)_2 \qquad \sigma_2 \cong \frac{(1+r)(r-2s^*)}{\sqrt{(1+r)(1+2r)[8(s^*)^2 + 4s^* + (1+r)]}}\sigma_{\mathrm{p}}$$

$$\text{and} \qquad \sigma_2''' \cong \frac{r}{\sqrt{1+2r}}\sigma_{\mathrm{p}}'''.$$

When $s^* = 0.5$ (bent bar, rod), then from Eqs. (3.10), (3.11), and (3.3) it appears, as previously, that $\sigma_1 = \sigma_p$, $\sigma_2 = \sigma_3 = 0$, and $(\varphi_2 = \varphi_3 = -0.5\varphi_1)$ and $\varphi_{(i)b1f} \cong (2n - \varphi_0)$.

Thus, Eqs. $(3.2)_1$, $(3.3)_1$, and $(3.11)_1$ are formal [for thin $(s^* < 0.05)$ and thick-walled tubes $(0.05 \le s^* < 0.1)$] extension of the expressions obtained by Marciniak [1] for isotropic sheets and the state where $(0 \le \sigma_2/\sigma_1 \le 0.5)$. Assuming that for plane sheets their diameters (d_1 and d_{ext}) $\to \infty$, then $s^* \to 0$, so we obtain the Eqs. $(3.2)_2$, $(3.3)_2$, and $(3.11)_2$. The obtained Eqs. $(3.2)_2$, $(3.3)_2$ were already cited in many papers, see [1, 14, 16, 32, 35, 36]. The idea of that extension can be presented on the basis of analysis of the graphs published in [1], and in Fig. 3, obtained for tension of wide plane specimens.

In the case of metal tube bending at bending machines, from the condition $d(\sigma_{\mathrm{p}} \cdot g) = 0$ we have obtained the effects not recognised so far.

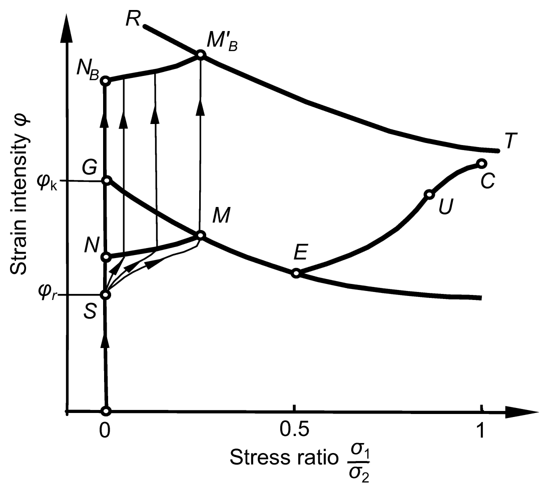

Fig. 3. Course of elongation for wide samples in the background of the limit strain curves [1], where φ_r and φ_k are homogeneous and border deformations.

a) In the case of the generalised model of strains and simplification of the 1st kind, for the external top point of the bent elbow where exists PSS, we can state that
   - when $s^* \neq 0$, then the condition $d(\sigma_{\mathrm{p}} \cdot g) = 0$ corresponds to the stress states occurring in the hyperbolic range on the H-M-H ellipse of plasticity, when (σ_1 and σ_2) > 0 and $d\varphi_2 < 0$. In these points of H-M-H ellipse (points of stresses), a set of quasi-linear partial differential equations of statics have a hyperbolic character [32, 34, 39, 41],
   - when $s^* = 0$ (internal surface of the bent elbow, where $g_1 = 0$ in thickness or the plane specimen when $d_{\mathrm{ext}} \to \infty$), then the condition $d(\sigma_{\mathrm{p}} \cdot g) = 0$ refers to the parabolic point on the ellipse of plasticity where ($d\varphi_2 = 0$) – there is initiation of PSD [32, 34, 39, 41],

b) In the case of simplifications of the 2nd and 3rd kinds [3, 16, 17], for the top point of the bent elbow, we obtain that the condition $d(\sigma_{\mathrm{p}} \cdot g) = 0$ refers to the point s (see Fig. 1, Part II) on the ellipse of plasticity, where ($d\varphi_2 = 0$), it physically means the local initiation of PSD.

In Fig. 3, the line *OSNG* determines strain states in the external layers under uniaxial tension ($\sigma_2/\sigma_1 = 0$), and the line *ES* presents the limit strain states because of stability loss of the specimen, determined from the condition of the maximum tensile force. The line *GMEUC* presents the limit strain line because of the local stability loss (location of the plastic strains, for example as furrow initiation), and the line RT corresponds to the cracking (fracture) limit in the furrow itself. The point S corresponds to the moment of neck formation (initiation of the dispersed stability loss) [1]. The states in the points included into the area SEG, for example on the line MN represent the strain states existing at the given moment in the points of the cross section of the specimen subjected to tension (or in the cross section of the layers of the bent tube subjected to tension).

c) Let us concentrate on the external layers of the bent tube of the dimensions ($\varnothing 44.5 \times 4.5$ mm, $s^* \approx 0.101$) and for the assumed material data for steels: $r = 1$, $n = 0.2$ and $\varphi_0 = 0.016$. Then we can say that for the point S in Fig. 3: $\varphi_{(i)a} \cong 0.184$, see Eq. (3.1), and $\sigma_2/\sigma_1 = 0$.
   - For the point E: $\varphi'''_{(i)b2} \cong 0.215$, see Eqs. $(3.2)_2$ and $(3.3)_2$, and: $\sigma_2/\sigma_1 = 0.5$ also the values obtained from Eqs. $(3.2)_1$ and $(3.3)_1$ for $s^* = 0$.
   - For the point M: $\varphi_{(i)b1} \cong 0.241$, see Eqs. $(3.2)_1$ and $(3.3)_1$ and $\sigma_2/\sigma_1 \cong 0.3622$, see Eqs. (3.7) or (3.11). For other points on the line GE (except for the points G, M, E), the values of the geometric dimensions of the bent tube are different (different coefficients of thin-walled character included into $s^* \in (0; 0.5)$ than $s^* \cong 0.101$. This value of the coefficient was determined for the tube $\varnothing 44.5 \times 4.5$ mm. When the coefficient $s^* \in (0.5; 0)$, then $\sigma_2/\sigma_1 \in (0; 0.5)$ respectively.
   - For the point G: $\varphi_{(i)b1f} \cong 0.384$, this value was obtained from Eqs. $(3.2)_1$ and $(3.3)_1$ when $s^* = 0.5$ (full bar). From Eqs. $(3.7)_1$, $(3.7)_2$ or $(3.11)_1$ we obtain $\sigma_2/\sigma_1 = 0$, it means that $\sigma_2 = 0$.

If in the external points of the layers subjected to tension another strain scheme is represented by the strain components described by the averaged equations where the reference area is the middle (central) layer in the tube wall, like under biaxial tension of sheets [1, 38], then suitable expressions for permissible strain intensity corresponding to Eqs. $(3.2)_1$ and $(3.3)_1$ take the following form:

$$\varphi_{(i)b1m} \cong \sqrt{\frac{(1+r)\left[2\left(\dfrac{g_1}{d_1}\right)^2 + 2\left(\dfrac{g_1}{d_1}\right) + (1+r)\right]}{(1+2r)}}\,n - \varphi_0. \tag{3.12}$$

As previously, assuming some simplifying expressions ($g_1/d_1 \approx g_0/d_z = s^*$) and making some transformations we have

$$\varphi_{(i)b1m} \approx \sqrt{\frac{(1+r)[2(s^*)^2 + 2s^* + (1+r)]}{(1+2r)}}\,n - \varphi_0. \tag{3.13}$$

The stress state components (under the simplifying assumptions that ($g_1/d_1 \approx g_0/d_{\text{ext}}$), corresponding to the strain state (3.12) for the case of thick-walled tubes from the range ($s^* > 0.1$ or $0.2 \leq s^* < 0.5$ [12, 16] or $0.05 < s^* < 0.5$ according to [6, 20]), hot- or semi-hot bend made of an isotropic material for ($r = 1$), take the form

$$\sigma_1 \cong \frac{2 + s^*}{\sqrt{3[(s^*)^2 + s^* + 1]}}\sigma_{\text{p}},$$

(3.14)
$$\sigma_2 \cong \frac{1 - s^*}{\sqrt{3[(s^*)^2 + s^* + 1]}}\sigma_{\text{p}},$$

$$\sigma_3(r_{\text{ext}}) \cong \sigma_3(r_1) \cong 0.$$

During thin-walled tube bending and in the case of a material showing properties of normal anisotropy, when the condition of plasticity (3.9) and equations of plastic flow (3.10) are taken into account, the stress state components corresponding to the strain state (3.13) have the following form:

$$(3.15)_1 \qquad \sigma_1 \cong \frac{(1 + r)(1 + r + s^*)}{\sqrt{(1 + r)(1 + 2r)[2(s^*)^2 + 2s^* + (1 + r)]}}\sigma_{\text{p}},$$

$$(3.15)_2 \qquad \sigma_2 \cong \frac{(1 + r)(r - s^*)}{\sqrt{(1 + r)(1 + 2r)[2(s^*)^2 + 2s^* + (1 + r)]}}\sigma_{\text{p}}.$$

d) According to the procedure applied in the previous (case c), and assuming the same data, $r = 1$, $n = 0.2$ and $\varphi_0 = 0.016$, we obtain (see Fig. 3) that:
   - values of admissible intensities of strains and stresses corresponding to the points S, E, and G, are the same as previously (in case c);
   - now the point M corresponds to the point M_m (located at another line connecting the points GE), the following values correspond to it: $\varphi_{(i)b1m} = 0.227$, see Eqs. (3.12) and $(3.13)_1$, and $\sigma_2/\sigma_1 \cong 0.428$, see Eqs. (3.14) or (3.15). Other points located on that line (except for the points G, M_m, and E), are related to other values of geometric dimensions of the bent tube (for different than $s^* \cong 0.101$ coefficient of the thin-walled character of the tube, included in the range $s^* \in (0; 0.5)$, determined for the dime of dimensions ($\varnothing 44.5 \times 4.5$ mm). When $s^* \in (0.5; 0)$, then $\sigma_2/\sigma_1 \in (0.2; 0.5)$. It means that now in the case of the bar (when $s^* = 0.5$) in the considered central layer of the tube wall, the circumferential stress is already not equal to zero ($\sigma_2 \neq 0$). Substituting $s^* = 0.5$ to Eqs. (3.14) or (3.15), we obtain $\sigma_2/\sigma_1 = 0.2$.

Case 3. Formation of PSD under PSS [1, 14, 16, 17, 32, 35].
In such case

$$\varphi_{(i)e} = nz - \varphi_0. \tag{3.16}$$

We can write in this case, that $(\varphi_{(i)e} \equiv \varphi'''_{(i)e})$, and $\varphi_{(i)e}$ is the value of the strain intensity corresponding to the loss of stability, z is the subtangent including influence of the stress σ_p on the moment of stability loss under conditions of the plane stress state and in the moment of formation of the plane state of deformation, then

$$z = \frac{1+r}{\sqrt{1+2r}}. \tag{3.17}$$

This state of stability loss refers to thin-walled tubes because of the assumed conditions resulting from the plane stress state.

4. Final remarks and conclusions

1. The condition of possible localised stability loss for the case of initiation of the plane (biaxial) state of deformation (PSD) in the plane stress state (PSS) determines higher permissible strain intensities than in the case of stability loss in the dispersed form (maximum drawing force) and lower ones for the localised stability loss $d(\sigma_\mathrm{p} \cdot g) = 0$ during biaxial tension. In the case of stability loss in the dispersed form under uniaxial uniform tension, see [1, 16, 33, 37, 38], admissible strain intensity is comparable to the value of the coefficient of plastic strain hardening of a metal. The important contribution of the present paper is a formal extension of the criterion of strain localisation (formulated for sheets by Marciniak [1]) for the case of tube bending. In the case of the generalised strain scheme and simplification of the 1st type, such an extended criterion (with and without including displacement of the neutral axis of plastic bending y_0) depends additionally on geometric dimensions of the bent tube (approximately on its thin-walled parameter s^*).
2. In the case of metal tube bending at the bending machines, from the condition $d(\sigma_\mathrm{p} \cdot g) = 0$, new effects (unknown in literature) have been obtained, namely:
   a) for the generalised strain model and the simplification of the 1st order, for the external top point of the bent elbow, where PSS exists we obtain that
      - when $s^* \neq 0$, then the condition $d(\sigma_\mathrm{p} \cdot g) = 0$ concerns the stress states included into the hyperbolic range of a set of quasi-linear

partial differential equations of static on the H-M-H ellipse of plasticity, i.e., for the case when (σ_1 and σ_2) > 0 and $d\varphi_2 < 0$,

- when $s^* = 0$ (internal surface of the bent elbow, where $g_1 = 0$ in thickness or the plane specimen when $d_{\text{ext}} \to \infty$), then, the condition $d(\sigma_{\text{p}} \cdot g) = 0$ refers to the parabolic point on the H-M-H ellipse of plasticity, in which ($d\varphi_2 = 0$) is the initiation of PSD. In this point of stresses a set of quasi-linear partial differential equations of statics have a parabolic character [32, 34, 35, 39, 41],
- when $s^* = 0.5$, then tube becames bar or rod (with full cross section), where the condition $d(\sigma_{\text{p}} \cdot g) = 0$ corresponds to the point $\sigma_1 > 0$ and $\sigma_2 = 0$, and $d\varphi_2 = d\varphi_3 = -0.5 d\varphi_1$ on the H-M-H ellipse of plasticity. Thus, if the coefficient $s^* \in (0.5; 0)$, then $\sigma_2/\sigma_1 \in (0; 0.5)$. When in the external points or the point of the layers subjected to tension strain scheme represented by the strain components described by the averaged expressions (when the reference area is the middle (central) layer in the tube wall) is acting, then for $s^* \in (0.5; 0)$, we obtain $\sigma_2/\sigma_1 \in (0.2; 0.5)$. It means that for the bar (when $s^* = 0.5$) in the central layer, the stress is $\sigma_2 \neq 0$, so $\sigma_2/\sigma_1 = 0.2$.

b) for the strain model of deformation resulting from the simplifications of the 2nd and 3rd type [3, 16, 17] we can state that for the external top point of the bent elbow where the PSS occurs, the condition $d(\sigma_{\text{p}} \cdot g) = 0$ refers to the parabolic point s on the H-M-H ellipse of plasticity, where $d\varphi_2 = 0$, which means local initiation of PSD.

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