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Thermal Instability of a Rivlin-Ericksen Nanofluid Saturated by a Darcy-Brinkman Porous Medium: a More Realistic Model

G.C. RANA[1], Ramesh CHAND[2], Veena SHARMA[3]

[1] *Department of Mathematics*
NSCBM Government P. G. College
Hamirpur – 177005, Himachal Pradesh, India
e-mail: drgcrana15@gmail.com

[2] *Department of Mathematics*
Government College Nurpur
Himachal Pradesh, India
e-mail: rameshnahan@yahoo.com

[3] *Department of Mathematics and Statistics*
Himachal Pradesh University
Shimla – 171 005, Himachal Pradesh, India
e-mail: veena_math_hpu@yahoo.com

In this paper, we study thermal instability in a horizontal layer of Rivlin-Ericksen elastico-viscous nanofluid in porous medium. Brinkman model is used as a porous medium and Rivlin-Ericksen fluid model is used to describe the rheological behavior of nanofluid. In the earlier model (Chand and Rana [18]), we constrained both temperature and nanoparticle volume fractions at the boundaries of Rivlin-Ericksen nanofluid layer. In this paper, we assume that the value of temperature can be constrained on the boundaries, while the nanoparticle flux is zero on the boundaries. The considered boundary condition neutralizes the possibility of oscillatory convection due to the absence of two opposing forces, and only stationary convection occurs, in which Rivlin-Ericksen elastico-viscous nanofluid behaves like an ordinary nanofluid. The effects of Lewis number, medium porosity, modified diffusivity ratio, Darcy-Brinkman number and concentration Rayleigh number in stationary convection are discussed analytically and numerically. The results of this study are in good agreement with the results published earlier [11–21].

Key words: nanofluid, Rivlin-Ericksen thermal instability, viscosity, viscoelasticity, porous medium.

Notations

a – wave number,
D_{B} – diffusion coefficient [m^2/s],
D_{T} – thermophoretic diffusion coefficient,
$\widetilde{\mathrm{D}}\mathrm{a}$ – Darcy-Brinkman number,
F – kinametic viscoelasticity parameter,
g – acceleration due to gravity [m/s^2],
k – medium permeability,
k_m – thermal conductivity [W/(m · K)],
k_{B} – Boltzmann constant [J/K],
Le – Lewis number,
n – growth rate of disturbances [s^{-1}],
N_{A} – modified diffusivity ratio,
N_{B} – modified particle density increment,
p – pressure [Pa],
p' – pressure,
Pr – Prandtl number,
q – Darcy velocity vector [m/s],
Ra – thermal Rayleigh number,
Ra_c – critical Rayleigh number,
Ra_m – density Rayleigh number,
Ra_n – concentration Rayleigh number,
t – time [s],
t' – time,
T – temperature [K],
T' – temperature,
Va – Vadasz number,
n – dimensional frequency,
u, v, w – velocity components,
(x, y, z) – space coordinates [m],
(x', y', z') – space coordinates.

Greek symbols

α – thermal expansion coefficient [1/K],
μ – viscosity [kg/(m · s)],
μ' – kinematic viscoelasticity [kg/(m · s)],
ε – porosity,
ρ – density of nanofluid [kg/m^3],
$(\rho c)_m$ – heat capacity in porous medium,
$(\rho c)_p$ – heat capacity of nanoparticles,
φ – volume fraction of nanoparticles,
ρ_p – density of nanoparticles [kg/m^3],
ρ_f – density of base fluid [kg/m^3],

κ – thermal diffusivity [m^2/s],
σ – thermal capacity ratio.

Superscripts

$'$ – non-dimensional variables.

Subscripts

p – particle,
f – fluid,
0 – lower boundary,
1 – upper boundary,
b – basic state,
H – horizontal plane.

1. Introduction

Thermal instability problems have attracted considerable interest during the last few decades because of their importance in various applications such as geophysics, soil sciences, ground water hydrology, astrophysics, food processing, oceanography, limnology, engineering, etc. Many researchers have investigated thermal instability problems by studying different types of fluids. A detailed account of the thermal instability of a Newtonian fluid under varying assumptions of hydrodynamics and hydromagnetics was given by Chandrasekhar [1]. Bhatia and Steiner [2] studied the thermal instability of a Maxwellian viscoelastic fluid in the presence of magnetic field, while the thermal instability in a viscoelastic fluid in hydromagnetics was considered by Sharma [3].

In all of the above studies, the considered medium was non-porous. The investigation in porous media began with the simple Darcy model and was gradually extended to Darcy-Brinkman model. A good overview of instability problems in a porous medium was given by Ingham and Pop [4] as well as Nield and Bejan [5]. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One type of such fluids is Rivlin-Ericksen elastico-viscous fluid, which finds applications in chemical technology and petroleum industry. Rivlin and Ericksen [6] proposed a theoretical model for such an elastico-viscous fluid. Rana and Sharma [7] and Rana and Thakur [8] studied the onset of convection in a Rivlin-Ericksen fluid heated from below and saturating a Brinkman porous medium.

In recent years, considerable interest has been given to the study of nanofluids. They have become innovative materials used in thermal engineering as well as in automotive industries, energy saving, nuclear reactors, etc. Furthermore, nanoparticle suspensions are being developed to be used in medical applications including cancer therapy. The porous media heat transfer problems have several engineering applications such as geothermal energy recovery, crude oil extraction, ground water pollution, thermal energy storage, etc. Choi [9] was the first one who described the term 'nanofluid'. Convection of nanofluids was studied by Buongiorno in [10] and Buongiorno's model has attracted great interest in the recent years. He insisted that the anomalous heat transfer occurs due to particle migration in fluids, and assumed seven slip mechanisms such as inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus forces, fluid drainage and gravity. Furthermore, he maintained that out of these seven, only Brownian diffusion and thermophoresis are important slip mechanisms in a nanofluid. Buongiorno's model was later studied by Tzou [11, 12], Nield and Kuznetsov [13], Alloui *et al.* [14], Sheu [15], Chand and Rana [16] and Chand *et al.* [17] in their papers on thermal instability in a porous medium layer saturated by a nanofluid. Chand and Rana [18] studied thermal instability of Rivlin-Ericksen elastico-viscous nanofluid saturated by a porous medium assuming that nanoparticle flux can be controlled on the boundaries in the same manner as the temperature can be controlled. However, Nield and Kuznetsov [19] found that it may be difficult to control the nanoparticle volume fraction on the boundaries. Therefore, they developed a more realistic boundary condition by assuming that there is no flux at the plate and the nanoparticle flux value adjusts accordingly. Chand *et al.* [20] studied the revised model of thermal instability in a Rivlin-Ericksen elastico-viscous nanofluid in a porous medium while thermal convection in a rotating nanofluid layer saturating a Darcy-Brinkman porous medium was studied by Rana and Chand [21].

Our aim in this paper is to study the thermal instability of a horizontal layer of Rivlin-Ericksen elastico-viscous nanofluid in a Darcy-Brinkman porous medium with more realistic boundary conditions.

2. Mathematical formulations

In Fig. 1, we consider an infinite horizontal layer of Rivlin-Ericksen elastico-viscous nanofluid heated from below of thickness d bounded by planes $z = 0$ and $z = d$. Each boundary wall is assumed to be impermeable and perfectly thermally conducting. Fluid layer is acted upon by gravity force $g(0, 0, -g)$. The temperature T of nano particles at $z =$ is T_0 and T_1 at $z = d$, $(T_0 > T_1)$. The reference temperature is T_1.

Let $q(u, v, w)$, p, φ, ρ_p, ρ_f, μ, μ' and α respectively denote the Darcy velocity vector, hydrostatic pressure, the volume fraction of the nanoparticles, density of nanoparticles, density of the base fluid, viscosity, kinematic viscoelasticity, and α is the coefficient of thermal expansion. Then, the equations of conservation of mass and momentum for Rivlin-Ericksen elastico-viscous nanofluid in porous medium (Buongiorno [10], Nield and Kuznetsov [13], Sheu [15], Chand

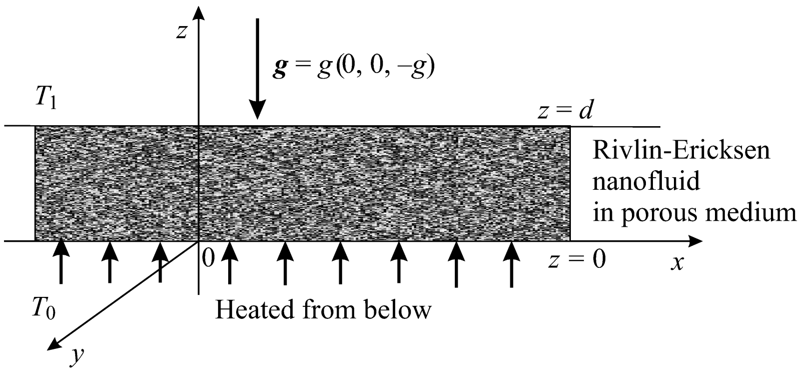

FIG. 1. Physical configuration of the problem.

and RANA [18], NIELD and KUZNETSOV [19], CHAND *et al.* [20], and RANA and CHAND [21]), after employing Oberbeck-Boussinesq approximation, are

$$\nabla \cdot \mathbf{q} = 0, \tag{2.1}$$

$$\frac{\rho}{\varepsilon}\frac{d\mathbf{q}}{dt} = -\nabla p + \left(\varphi\rho_p + (1-\varphi)\left\{\rho_f\left(1-\alpha\left(T-T_0\right)\right)\right\}\right)\mathbf{g} + \widetilde{\mu}\nabla^2\mathbf{q} - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\mathbf{q}, \tag{2.2}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}\left(\mathbf{q}\cdot\nabla\right)$ stands for convection derivative.

The equation of energy in a nanofluid is

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f\, \mathbf{q}\cdot\nabla T = k_m \nabla^2 T + \varepsilon\,(\rho c)_p \left(D_{\mathrm{B}}\nabla\varphi\cdot\nabla T + \frac{D_{\mathrm{T}}}{T_1}\nabla T\cdot\nabla T\right), \tag{2.3}$$

where $(\rho c)_m$ is effective heat capacity of the fluid, $(\rho c)_p$ is heat capacity of nanoparticles and k_m is the effective thermal conductivity of the porous medium.

The equation of conservation of mass for nanoparticles is

$$\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}\cdot\nabla\varphi = D_{\mathrm{B}}\nabla^2\varphi + \frac{D_{\mathrm{T}}}{T_1}\nabla^2 T, \tag{2.4}$$

where D_{B} is the Brownian diffusion coefficient given by Einstein-Stokes equation, and D_{T} is the thermophoretic diffusion coefficient of nanoparticles.

Assuming that the temperature is constant and the thermophoretic nanoparticle flux is zero at the boundaries (NIELD and KUZNETSOV [19]), the boundary conditions are

(2.5) $$w = 0, \qquad \frac{\partial w}{\partial z} = 0, \qquad T = T_0, \qquad D_{\mathrm{B}} \frac{\partial \varphi}{\partial z} + \frac{D_{\mathrm{T}}}{T_1} \frac{\partial T}{\partial z} = 0 \qquad \text{at} \ \ z = 0,$$

(2.6) $$w = 0, \qquad T = T_1, \qquad \frac{\partial w}{\partial z} = 0, \qquad D_{\mathrm{B}} \frac{\partial \varphi}{\partial z} + \frac{D_{\mathrm{T}}}{T_1} \frac{\partial T}{\partial z} = 0 \qquad \text{at} \quad z = d.$$

Equations (2.1)–(2.6) in a non-dimensional form (after dropping the dashes (′) for simplicity) can be written as

(2.7) $$\nabla \cdot \mathbf{q} = 0,$$

(2.8) $$\frac{1}{\mathrm{Va}} \frac{\partial \mathbf{q}}{\partial t} = -\nabla p + \widetilde{\mathrm{D}}\mathrm{a}\, \nabla^2 \mathbf{q} - \left(1 + F \frac{\partial}{\partial t}\right) \mathbf{q} - \mathrm{Ra}_m + \mathrm{Ra}\, T - \mathrm{Ra}_n \varphi \widehat{e}_z,$$

(2.9) $$\frac{1}{\sigma} \frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon} w = \frac{1}{\mathrm{Le}} \nabla^2 \varphi + \frac{N_{\mathrm{A}}}{\mathrm{Le}} \nabla^2 T,$$

(2.10) $$\frac{\partial T}{\partial t} + \mathbf{q} \cdot \nabla T = \nabla^2 T + \frac{N_{\mathrm{B}}}{\mathrm{Le}} \nabla \varphi \cdot \nabla T + \frac{N_{\mathrm{A}} N_{\mathrm{B}}}{\mathrm{Le}} \nabla T \cdot \nabla T,$$

(2.11) $$w = 0, \qquad \frac{\partial w}{\partial z} = 0, \qquad T = 1, \qquad \frac{\partial \varphi}{\partial z} + N_{\mathrm{A}} \frac{\partial T}{\partial z} = 0 \qquad \text{at} \quad z = 0,$$

(2.12) $$w = 0, \qquad \frac{\partial w}{\partial z} = 0, \qquad T = 0, \qquad \frac{\partial \varphi}{\partial z} + N_{\mathrm{A}} \frac{\partial T}{\partial z} = 0 \qquad \text{at} \quad z = 1,$$

where we have introduced non-dimensional variables as

$$(x', y', z') = \left(\frac{x, y, z}{d}\right), \qquad (u', v', w') = \left(\frac{u, v, w}{\kappa}\right) d, \qquad t' = \frac{t\kappa}{\sigma d^2},$$

$$p' = \frac{k_1 p}{\rho \kappa^2} d^2, \qquad \varphi' = \frac{(\varphi - \varphi_0)}{\varphi_0}, \qquad T' = \frac{(T - T_1)}{(T_0 - T_1)},$$

and the non-dimensional parameters denote, respectively, thermal diffusivity κ, thermal capacity ratio σ, Prandtl number Pr, Darcy number Da, Brinkman-Darcy number $\widetilde{\mathrm{D}}\mathrm{a}$, Vadasz number Va, Lewis number Le, kinematic viscoelastic parameter F, Rayleigh number Ra, density Rayleigh number Ra_m, concentration

Rayleigh number Ra_n, modified diffusivity ratio N_A, N_B, and they are defined as follows:

$$\kappa = \frac{k}{\rho c}, \qquad \sigma = \frac{(\rho c_p)_m}{(\rho c_p)_f},$$

$$\mathrm{Pr} = \frac{\mu}{\rho\kappa}, \qquad \mathrm{Da} = \frac{k}{d^2},$$

$$\widetilde{\mathrm{Da}} = \frac{\widetilde{\mu} k}{\mu d^2}, \qquad \mathrm{Va} = \frac{\varepsilon \mathrm{Pr}}{\mathrm{Da}},$$

$$\mathrm{Le} = \frac{\kappa}{D_\mathrm{B}}, \qquad F = \frac{\mu' \kappa}{\mu \sigma d^2},$$

$$\mathrm{Ra} = \frac{\rho g \alpha d \left(T_0 - T_1\right)}{\mu\kappa}, \qquad \mathrm{Ra}_m = \frac{\rho_p \varphi_0 + \rho\left(1 - \varphi_0\right) g d}{\mu\kappa},$$

$$\mathrm{Ra}_n = \frac{\left(\rho_p - \rho\right) \varphi_0 g d}{\mu\kappa}, \qquad N_\mathrm{A} = \frac{D_\mathrm{T}\left(T_0 - T_1\right)}{D_\mathrm{B} T_1 \varphi_0},$$

$$N_\mathrm{B} = \frac{(\rho c)_p \varphi_0}{(\rho c)_f}.$$

The basic state is assumed to be quiescent and is given by $u = v = w = 0$, $p = p(z)$, $T = T_b(z)$, $\varphi = \varphi_b(z)$.

Equations (2.8)–(2.10) reduce to

$$0 = -\frac{dp_b}{dz} - \mathrm{Ra}_m + \mathrm{Ra} T_b + \mathrm{Ra}_n \varphi_b, \tag{2.13}$$

$$\frac{d^2 T_b}{dz^2} + \frac{N_\mathrm{B}}{\mathrm{Le}} \frac{d\varphi_b}{dz} \frac{dT_b}{dz} + \frac{N_\mathrm{A} N_\mathrm{B}}{\mathrm{Le}} \left(\frac{dT_b}{dz^2}\right)^2 = 0, \tag{2.14}$$

$$\frac{d^2 \varphi_b}{dz^2} + N_\mathrm{A} \frac{d^2 T_b}{dz^2} = 0. \tag{2.15}$$

Using boundary conditions in (2.11) and (2.12), Eq. (2.15) gives

$$\frac{d\varphi_b}{dz} + N_\mathrm{A} \frac{dT_b}{dz} = 0. \tag{2.16}$$

When we substitute this value into Eq. (2.14), we obtain

$$\frac{d^2 T_b}{dz^2} = 0. \tag{2.17}$$

When we integrate Eq. (2.17) with respect to z and use boundary conditions (2.11) and (2.12), we obtain

$$T_b = 1 - z. \tag{2.18}$$

When we integrate Eq. (2.16) with respect to z and use boundary conditions (2.11) and (2.12), we obtain

$$\varphi_b = \varphi_0 + N_{\mathrm{A}} z. \tag{2.19}$$

These results are identical to the results obtained by NIELD and KUZNETSOV in [19].

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, which are as follows:

$$\begin{aligned} q(u,v,w) &= q'(u,v,w), & T &= T_b + T', \\ \varphi &= \varphi_b + \varphi', & p &= p_b + p'. \end{aligned} \tag{2.20}$$

Using Eqs. (2.18), (2.19) and (2.20) in Eqs. (2.7)–(2.12) and after linearization by neglecting the product of the prime quantities, we obtain (after dropping the dashes ($'$) for simplicity) the following equations:

$$\nabla \cdot \mathbf{q} = 0, \tag{2.21}$$

$$\frac{1}{\mathrm{Va}} \frac{\partial \mathbf{q}}{\partial t} = -\nabla p + \widetilde{\mathrm{D}}\mathrm{a} \nabla^2 q - \left(1 + \frac{\partial}{\partial t} F\right) \mathbf{q} + \mathrm{Ra}\, T - \mathrm{Ra}_n \varphi, \tag{2.22}$$

$$\frac{1}{\sigma} \frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon} w = \frac{1}{\mathrm{Le}} \nabla^2 \varphi + \frac{N_{\mathrm{A}}}{\mathrm{Le}} \nabla^2 T, \tag{2.23}$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_{\mathrm{B}}}{\mathrm{Le}} \left(\frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z} \right) - 2 \frac{N_{\mathrm{A}} N_{\mathrm{B}}}{\mathrm{Le}} \frac{\partial T}{\partial z}, \tag{2.24}$$

$$w = 0, \qquad T = 0, \qquad \frac{\partial w}{\partial z} = 0, \qquad \frac{\partial \varphi}{\partial z} + N_{\mathrm{A}} \frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 0 \quad \text{and at} \quad z = 1. \tag{2.25}$$

The six unknowns u, v, w, p, T and φ can be reduced to three by combining the operating Eq. (2.22) with $\nabla^2 e_z$, we therefore obtain

$$\frac{1}{\mathrm{Va}}\frac{\partial}{\partial t}\nabla^2 w - \widetilde{\mathrm{D}}\mathrm{a}\,\nabla^4 w + \left(1 + \frac{\partial}{\partial t}F\right)\nabla^2 w = \mathrm{Ra}\,\nabla_H^2 T - \mathrm{Ra}_n \nabla_H^2 \varphi, \tag{2.26}$$

where ∇_H^2 is a two-dimensional Laplacian operator.

3. Normal mode analysis method

Analyzing the disturbances in the normal modes and assuming that the perturbed quantities are as follows:

$$[w, T, \varphi] = [W(z), \Theta(z), \Phi(z)]\exp\left(ik_x x + ik_y y + nt\right), \tag{3.1}$$

where k_x, k_y are wave numbers in x and y directions, and n is the growth rate of disturbances.

Using Eq. (3.1), Eqs. (2.22)–(2.25) can be written as

$$\left(1 + nF + \frac{n}{\mathrm{Va}} - \mathrm{Da}\left(D^2 - a^2\right)\right)\left(D^2 - a^2\right)W + \mathrm{Ra}a^2\Theta - \mathrm{Ra}_n a^2\Phi = 0, \tag{3.2}$$

$$\frac{W}{\varepsilon} - \frac{N_\mathrm{A}}{\mathrm{Le}}\left(D^2 - a^2\right)\Theta - \left(\frac{1}{\mathrm{Le}}\left(D^2 - a^2\right) - \frac{n}{\sigma}\right)\Phi = 0, \tag{3.3}$$

$$W + \left(D^2 - a^2 - n + \frac{N_\mathrm{A}}{\mathrm{Le}}D - \frac{2N_\mathrm{A}N_\mathrm{B}}{\mathrm{Le}}D\right)\Theta - \frac{N_\mathrm{B}}{\mathrm{Le}}D\Phi = 0, \tag{3.4}$$

$$w = 0, \qquad T = 0, \qquad \Theta = 0, \qquad DW = 0, \qquad D\varphi + N_A D\Theta = 0 \quad \text{at} \quad z = 0 \quad \text{and at} \quad z = 1, \tag{3.5}$$

where $D = \frac{d}{dz}$ and $a^2 = k_x^2 + k_y^2$ is a dimensionless resultant wave number.

We assume the solution to W, Θ and Φ is in the form

$$W = W_0 \sin\pi z, \qquad \Theta = \Theta_0 \sin\pi z, \qquad \Phi = \Phi_0 \sin\pi z, \tag{3.6}$$

which satisfies the boundary conditions (3.5).

Substituting solution (3.6) into Eqs. (3.2)–(3.4), integrating each equation from $z = 0$ to $z = 1$, and performing some integrations by parts, we obtain the following matrix equation:

$$\begin{bmatrix} J\left(1 + \dfrac{n}{\mathrm{Va}} + nF + \widetilde{\mathrm{D}}\mathrm{a}J\right) & -a^2\mathrm{Ra} & a^2\mathrm{Ra}_n \\ 1 & -(J+n) & 0 \\ \dfrac{1}{\varepsilon} & \dfrac{N_\mathrm{A}J}{\mathrm{Le}} & \dfrac{J}{\mathrm{Le}} + \dfrac{n}{\sigma} \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

where $J = \pi^2 + a^2$.

The non-trivial solution of the above matrix requires that

$$\text{Ra} = \frac{1}{a^2}\left(J(J+n)\left(1+nF+\frac{n}{\text{Va}}+\widetilde{\text{D}}\text{a}J\right)\right) - \frac{N_\text{A}J+\dfrac{\text{Le}}{\varepsilon}(J+n)}{J+\dfrac{n\text{Le}}{\sigma}}\text{Ra}_n. \tag{3.7}$$

4. The stationary convection

Due to the absence of two opposing buoyancy forces, the oscillatory convection does not exist, therefore only stationary convection occurs. For stationary convection we use $n = 0$, and Eq. (3.7) reduces to

$$(\text{Ra})_s = \frac{\widetilde{\text{D}}\text{a}\left(\pi^2+a^2\right)^3+\left(\pi^2+a^2\right)^2}{a^2} - \text{Ra}_n\left(\frac{\text{Le}}{\varepsilon}+N_\text{A}\right). \tag{4.1}$$

Equation (4.1) is identical to the equation obtained by Nield and Kuznetsov [13], Sheu [15], Chand and Rana [16, 18] and Chand *et al.* [20]. From Eq. (4.1), we notice that the kinematic viscoelasticity parameter F vanishes with n so the Rivlin-Ericksen elastico-viscous nanofluid behaves like an ordinary nanofluid.

The critical cell size at the onset of instability is obtained from the condition

$$\left(\frac{\partial \text{Ra}}{\partial a}\right)_{a=a_c} = 0.$$

The corresponding critical Rayleigh number Ra_c for steady onset is

$$(\text{Ra}_c)_s = \frac{27\pi^2}{4} - \text{Ra}_n\left(\frac{\text{Le}}{\varepsilon}+N_\text{A}\right). \tag{4.2}$$

This is the same critical wave number as obtained by Chand and Rana [16] and Chand *et al.* [20]. Consequently, the critical value for a_c will be the same as the well-known result for Bénard instability in the regular fluid.

In the absence of nanoparticles ($\text{Ra}_n = \text{Le} = N_\text{A} = 0$) i.e., for ordinary fluid, we have

$$\text{Ra}_c = \frac{27\pi^2}{4}. \tag{4.3}$$

This is the exactly the same result for Bénard instability in the regular fluid as obtained by Chandrasekhar [1]. We observe that the parameter N_B is not involved in the above equations. This means that the average contribution of the nanoparticle flux to the thermal energy equation is zero.

5. Results and discussion

The critical thermal Rayleigh number for stationary convection was given in Eq. (4.2). The stationary critical thermal Rayleigh number was found to be independent of elastico-viscous parameters and Rivlin-Ericksen nanofluid behaved like an ordinary Newtonian fluid. For heavy nanoparticles, the value of Ra_n was negative according to the definition of Ra_n. It was also noted that a negative value of Ra_n indicates a bottom-heavy case, while a positive value indicates a top-heavy case. In the following discussion, we have used the positive value of Ra_n. Now, we will study the effects of Lewis number, medium porosity, modified diffusivity ratio, Darcy-Brinkman number, and concentration Rayleigh number on thermal instability of a Rivlin-Ericksen nanofluid in stationary convection by examining the behavior of

$$\frac{\partial(\mathrm{Ra})_s}{\partial \mathrm{Le}}, \qquad \frac{\partial(\mathrm{Ra})_s}{\partial \varepsilon}, \qquad \frac{\partial(\mathrm{Ra})_s}{\partial N_{\mathrm{A}}}, \qquad \frac{\partial(\mathrm{Ra})_s}{\partial \widetilde{\mathrm{D}}\mathrm{a}}, \qquad \frac{\partial(\mathrm{Ra})_s}{\partial \mathrm{Ra}_n}$$

analytically and numerically.

From Eq. (4.1), we obtain

$$\frac{\partial(\mathrm{Ra})_s}{\partial \mathrm{Le}} = -\frac{\mathrm{Ra}_n}{\varepsilon}, \tag{5.1}$$

which implies that Lewis number has a destabilizing influence on the stationary convection of the system for top-heavy arrangement, which is in an agreement with the results obtained by Kuznetsov and Nield [13], Chand and

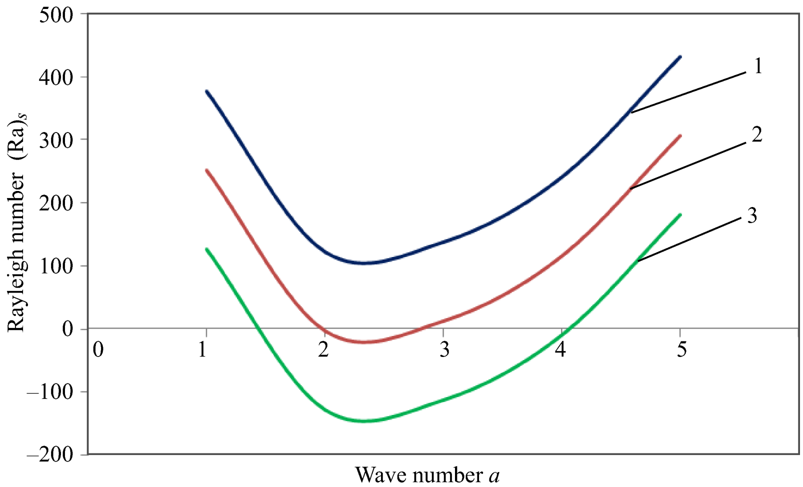

Fig. 2. Variation of Rayleigh number with wave number for different values of Lewis number ($\varepsilon = 0.4$, $\mathrm{Ra}_n = 0.5$, $N_{\mathrm{A}} = 2$, $\widetilde{\mathrm{D}}\mathrm{a} = 0.3$); 1 – Le = 100, 2 – Le = 200, 3 – Le = 500.

Rana [16], Chand *et al.* [20], and Rana and Chand [21]. In Fig. 2, the Rayleigh number is plotted against dimensionless wave number for different values of Lewis number. This shows that as Lewis number decreases, the Rayleigh number increases. Thus, Lewis number has a destabilizing effect on the stationary convection, which is in good agreement with the result obtained analytically from Eq. (5.1).

From Eq. (4.1), we obtain

$$\frac{\partial(\mathrm{Ra})_s}{\partial\varepsilon} = \frac{\mathrm{Ra}_n \mathrm{Le}}{\varepsilon^2}, \tag{5.2}$$

which implies that medium porosity has a stabilizing influence on the stationary convection of the system for top-heavy arrangement, which is in agreement with the results derived by Kuznetsov and Nield [13], Chand and Rana [16], Chand *et al.* [20], and Rana and Chand [21]. In Fig. 3, the Rayleigh number is plotted against dimensionless wave number for different values of medium porosity. This shows that as medium porosity increases, the Rayleigh number also increases. Thus, medium porosity has a stabilizing effect on stationary convection, which is in good agreement with the result obtained analytically from Eq. (5.1).

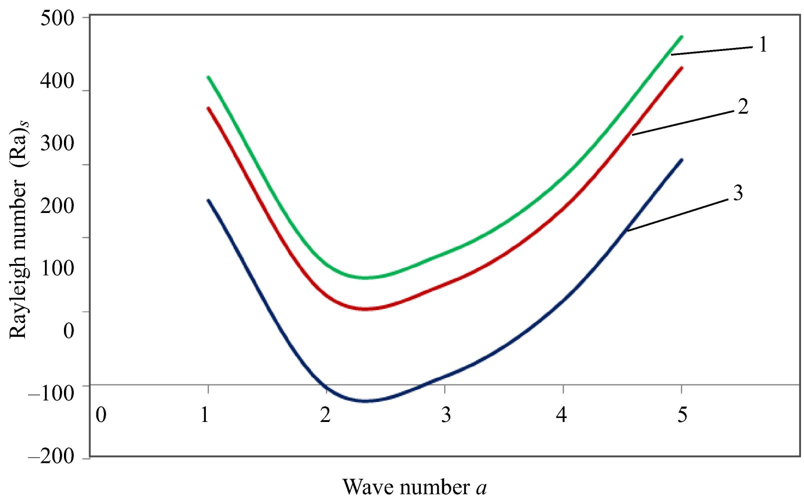

Fig. 3. Variation of Rayleigh number with wave number for different values of medium porosity (Le = 100, $\mathrm{Ra}_n = 0.5$, $N_\mathrm{A} = 2$, $\widetilde{\mathrm{Da}} = 0.3$); 1 – $\varepsilon = 0.6$, 2 – $\varepsilon = 0.4$, 3 – $\varepsilon = 0.2$.

From Eq. (4.1), we obtain

$$\frac{\partial(\mathrm{Ra})_s}{\partial N_\mathrm{A}} = -\mathrm{Ra}_n, \tag{5.3}$$

which implies that modified diffusivity ratio has a destabilizing influence on the stationary convection of the system for bottom-heavy arrangement, which is in agreement with the results derived by KUZNETSOV and NIELD [13], CHAND and RANA [16], CHAND *et al.* [20], and RANA and CHAND [21]. In Fig. 4, the Rayleigh number is plotted against dimensionless wave number for different values of modified diffusivity ratio. This shows that as modified diffusivity ratio increases, the Rayleigh number decreases. Thus, modified diffusivity ratio has a destabilizing effect on stationary convection, which is in good agreement with the result obtained analytically from Eq. (5.3).

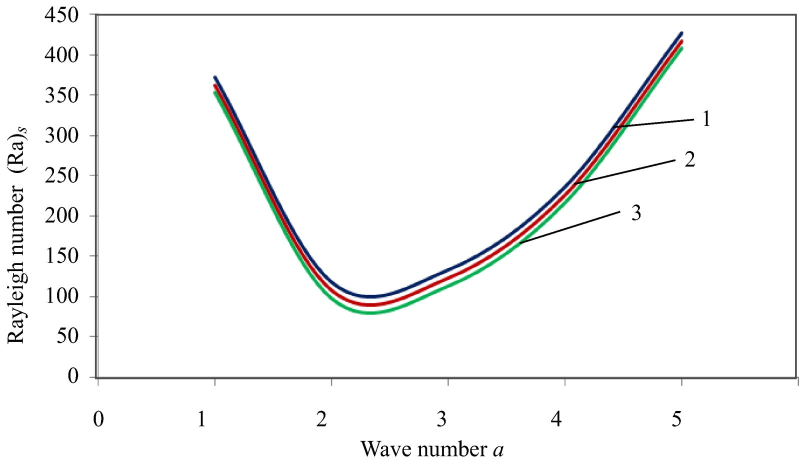

FIG. 4. Variation of Rayleigh number with wave number for different values of modified diffusivity ratio (Le = 100, $\mathrm{Ra}_n = 0.5$, $\varepsilon = 0.4$, $\widetilde{\mathrm{Da}} = 0.3$); 1 – $N_{\mathrm{A}} = 10$, 2 – $N_{\mathrm{A}} = 30$, 3 – $N_{\mathrm{A}} = 50$.

From Eq. (4.1), we obtain

$$\frac{\partial(\mathrm{Ra})_s}{\partial\widetilde{\mathrm{Da}}} = \frac{\left(\pi^2 + a^2\right)^3}{a^2}, \tag{5.4}$$

which implies that Brinkman-Darcy number has a stabilizing influence on the stationary convection of the system, which is in agreement with the results derived by KUZNETSOV and NIELD [13], CHAND and RANA [16], CHAND *et al.* [20], and RANA and CHAND [21]. In Fig. 5, the Rayleigh number is plotted against dimensionless wave number for different values of Brinkman-Darcy number. This shows that Rayleigh number increases with an increase of Darcy-Brinkman number. Thus, Darcy-Brinkman number has a stabilizing effect on stationary convection, which is in good agreement with the result obtained analytically from Eq. (5.4).

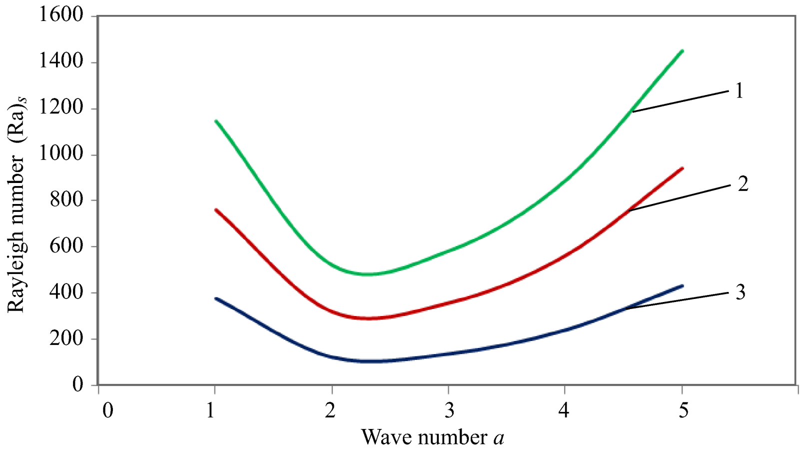

FIG. 5. Variation of Rayleigh number with wave number for different values of Darcy-Brinkman number ($\mathrm{Le} = 100$, $\mathrm{Ra}_n = 0.5$, $\varepsilon = 0.4$, $N_{\mathrm{A}} = 2$); 1 – $\widetilde{\mathrm{Da}} = 0.9$, 2 – $\widetilde{\mathrm{Da}} = 0.6$, 3 – $\widetilde{\mathrm{Da}} = 0.3$.

From Eq. (4.1), we obtain

$$\frac{\partial(\mathrm{Ra})_s}{\partial \mathrm{Ra}_n} = -\left(\frac{\mathrm{Le}}{\varepsilon} + N_{\mathrm{A}}\right), \tag{5.5}$$

which implies that concentration Rayleigh number has a destabilizing influence on the stationary convection of the system, which is in an agreement with the results derived by KUZNETSOV and NIELD [13], CHAND and RANA [16], CHAND *et al.* [20], and RANA and CHAND [21]. In Fig. 6, the Rayleigh number

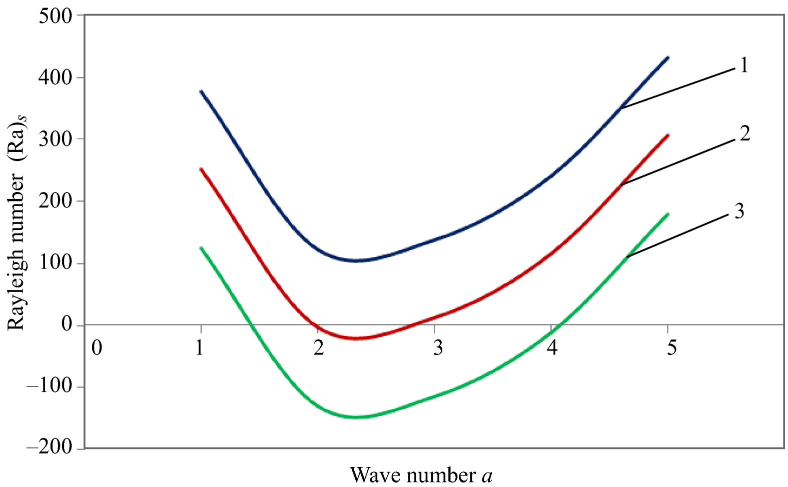

FIG. 6. Variation of Rayleigh number with wave number for different values of concentration Rayleigh number ($\mathrm{Le} = 100$, $\widetilde{\mathrm{Da}} = 0.3$, $\varepsilon = 0.4$, $N_{\mathrm{A}} = 2$); 1 – $\mathrm{Ra}_n = 0.5$, 2 – $\mathrm{Ra}_n = 1$, 3 – $\mathrm{Ra}_n = 1.5$.

is plotted against dimensionless wave number for different values of concentration Rayleigh number. This shows that as concentration Rayleigh number increases, the Rayleigh number decreases. Thus, concentration Rayleigh number has a destabilizing effect on stationary convection, which is in good agreement with the result obtained analytically from Eq. (5.5).

6. Conclusions

In this paper, thermal instability in a Rivlin-Ericksen nanofluid saturated by a Darcy-Brinkman porous medium under more realistic boundary conditions was studied by employing a model that incorporated the effects of Brownian motion, thermophoresis and viscoelasticity. In the stationary convection, it was found that the Rivlin-Ericksen elastico-viscous nanofluid behaves like an ordinary nanofluid. Medium porosity and Darcy-Brinkman number have a stabilizing influence, while Lewis number, modified diffusivity ratio and concentration Rayleigh number have a destabilizing influence on the stationary convection of the system. The principal difference is that Ra_n involves different scaling (a typical nanofluid fraction instead of the difference between two fractions) and Ra_n cannot be negative, so the oscillatory convection does not exist and only stationary convection occurs.

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