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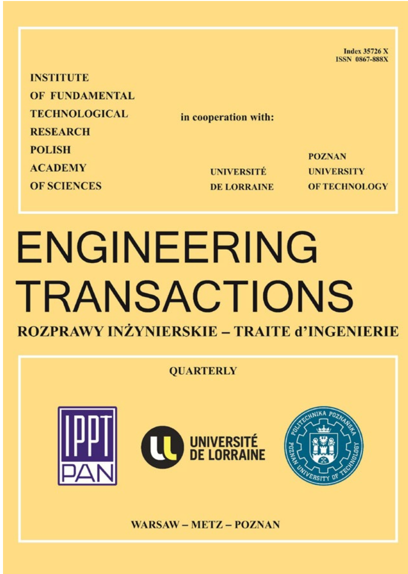

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Simulation Study of Direct-Shear Test on FRP-to-Concrete Bonded Joints by Means of XFEM

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A proper numerical modelling of FRP-to-concrete bonded joints is crucial for determining their strength. In this paper the results of numerical analyses performed by XFEM on such joints in direct-shear test are presented. The study uses a fracture mechanics approach based on the traction-separation law for definition the FRP-concrete interface. It includes definition of damage initiation as well as damage evolution, taking advantage of the fracture energy for plain concrete as well as the interfacial fracture energy of analysed joint. The interfacial fracture energy of bonded joint is essential for accurately describing the local bond–slip behaviour. The numerical study is aimed to investigate the sensitivity of direct-shear test models to magnitude of fracture energies, material strengths, type of adhesive and the length of FRP-to-concrete joint. Some general results and conclusions of performed analyses are presented.

Keywords: FRP strengthening; XFEM analysis; modes of fracture energy; FRP-to-concrete joints; interfacial fracture energy; traction-separation law.

List of notation:

d_t – damage parameter (in tension)

σ_n – the normal bond strength of the interface

τ_s – the shear bond strength of the interface

σ – the normal stress of the contact under mixed-mode loading

τ – the shear bond stress of the contact under mixed-mode loading

G_I – the fracture energy component of Mode I

G_{II} – the fracture energy component of Mode II

G_{FI} – the critical fracture energy in pure Mode I loading

G_{FII} – the critical fracture energy in pure Mode II loading

P – reaction force at the FRP-concrete interface in direct shear test on FRP-to-concrete bonded joints

s – the relative movement (slip) between the FRP and the concrete under shear stress in direct shear test on the FRP-concrete bonded joints

s_0 – local slip at τ_s

s_{max} – maximum bond separation slip

u – the displacement at the end of the FRP strip due to the reaction force P in direct shear test

L – lengths of FRP-concrete joint

G_{cr} – the interfacial fracture energy; value equal to the value of fracture energy G_{FII} for Mode II

ε_{11} – axial strain along the FRP-concrete joint

1. Introduction

Externally bonded fiber-reinforced polymer (FRP) materials have been widely used for many years now as an alternative method to traditional techniques of strengthening of reinforced concrete (RC) structural members [1, 2]. The principles of application of this type of strengthening are well recognized and described in the literature [3, 4]. However, the aspects related to failure of FRP-strengthened concrete members are still a subject of research works. They are mainly focused on the FRP strip debonding from the concrete surface. This debonding is initiated at the toe of flexural or flexural/shear cracks of concrete members (interfacial debonding – IC) and leads usually to a premature and brittle member failure [5, 6]. Therefore, to design of RC beams against FRP debonding failures a proper modelling of FRP-to-concrete interface is required and the results obtained from direct-shear tests provide very valuable inputs in that.

There are many available results of laboratory tests performed for direct-shear FRP-concrete joints [7-10]. The tests consider influence of main parameters on the strength of bonded joint, such as the length, width and thickness of FRP strip and its modulus of elasticity, and shear span-to-depth ratio. They have been implemented within the development of design methods for RC beam strengthening [11, 12]. These results are very useful also for validation of analytical and numerical models which have been proposed in recent years. The analytical models, often simplified, presented in [7, 13, 14] are based on assumed stress-slip relation between FRP strip and concrete; they use elements of LEFM (linear elastic fracture mechanics), or CCM (cohesive crack model) as well as FFM (finite fracture mechanics). The different bond-slip models for FRP-to-concrete bonded joints have been proposed and presented for example in [8]. They are based on different τ-slip relation and on interfacial fracture energy [7]. It was also pointed out that the type of adhesive layer (rigid or flexible) can influence the behaviour of FRP-to-concrete joint, mainly on the effective bond length and the bond strength itself [9].

The application of FEM to analysis of FRP-concrete joints has allowed then to create much more detailed models of FRP-concrete interface. For example, in [6] modelling of debonding failure in FRP-strengthened RC beams used the concept of cohesive zone model and for concrete cover separation the special cohesive elements were implemented.

Using the classical FEM it is very difficult or impossible to address fracture mechanics problems, such as modeling material separation and fracture processes with realistic crack initiation and propagation, which is particularly valuable for concrete [16-19].

The extended finite element method (XFEM) allows to model cracks with functions that are not continuous, like the Heaviside step function, without needing to mesh the crack itself explicitly; cracks can occur arbitrarily in the interior of finite elements. XFEM enables detailed analysis of cracks, material interfaces, and multiple crack interactions within a single model."

The strip debonding failure has been observed in laboratory tests on RC beam elements strengthened with FRP strips performed and described extensively by the author in [5]. In these experiments the main goal was to recognise the overall response of strengthened concrete beams in the form of their load carrying capacity and stiffness increase. The local effect as debonding and its impact on the behaviour of the entire system was not considered at that time. The concrete damaged plasticity material model [5, 6, 15] was employed in analyses which allowed estimation of the ultimate load carrying capacity in enough accurate manner. These results have been validated then by laboratory tests. The very good agreement with experimental results were observed. It was concluded that if the ultimate load capacity estimation is desired only, the XFEM does not need to be applied.

The present study is aimed on detailed modelling of direct-shear test which is highly discontinuous nonlinear phenomenon. In the study the XFEM was applied. The parametric analysis for the main characteristics that affect the performance of these joints has been carried and the results of it will be used for preparing the laboratory tests planned.

2. Application of XFEM for crack modelling of FRP-concrete bonded joints

The XFEM was proposed first in the context of fracture by Belytschko and Black [20], then the method was developing [21, 22] and is till now successfully used for analysis of cracking phenomena of concrete elements. Comprehensive study of a fracture process in concrete and reinforced concrete by means of constitutive models formulated within continuum mechanics where a continuous and discontinuous (using a cohesive crack model and XFEM) modelling approaches were used are presented in [23]. It has been proven that the XFEM seems to be an alternative method to the FEM which extends the allowable basis functions known as partition of unity methods [23, 24]. This method can be used not only to model cracks in homogenous material but also cracking occurring between two different materials (bimaterial interface cracks) [25]. Numerical modelling of crack propagation in plain concrete under Mode II in mixed-mode condition with successful comparison with experiment results are given in [26, 27].

When applied to crack propagation problems, XFEM introduces two extra sets of functions in addition to standard FEM nodal basis functions: a step function *H(x)* to capture the discontinuity in displacement across a crack and a set of functions $F_i(x)$, typically expressed in polar coordinates centered on the crack tip, that capture stress singularity in that region. The extra basis functions are defined as the product of these enrichment functions with the standard nodal basis function to ensure that the basis functions remain mesh-based and local to the enriched nodes.

This method does not require the mesh to match the geometry of discontinuities. It can be used to simulate initiation and propagation of a discrete crack by using fracture energy criterion along an arbitrary, solution-dependent path in the bulk material without the requirement of remeshing (crack propagation is not tied to the element boundaries in a mesh) [23, 25].

In the XFEM modelling of the FRP-concrete shear test the failure mechanism including degradation and eventual separation between two surfaces consists of two components: a damage initiation criterion and a damage evolution law.

The damage is initiated (either an additional crack is introduced, or the crack length of an existing crack is extended after an equilibrium increment) when the contact stresses and/or contact separations satisfy the damage initiation criterion, it means when the fracture criterion reaches the value 1,0 within a given tolerance.

In 2D analysis concerning phenomena of direct shear test of FRP strips from the concrete surface the quadratic nominal stress criterion [15, 28] was assumed as a damage initiation criterion:

$$\left(\frac{\langle\sigma\rangle}{\sigma_n}\right)^2 + \left(\frac{\tau}{\tau_s}\right)^2 = 1 \tag{2.1}$$

where σ_n and τ_s are the normal and shear bond strengths of the interface, respectively; σ and τ are the normal and shear stresses of the contact under mixed-mode loading, respectively. The symbol $\langle\ \rangle$ is used to signify that a purely compressive stress does not initiate damage [i.e., $\langle\sigma\rangle = 0$ if $\sigma < 0$ and $\langle\sigma\rangle = \sigma$ if $\sigma \geq 0$.

The damage evolution law defines the post damage-initiation material behaviour and describes the rate of degradation of the material stiffness once the initiation criterion has been reached. In analysis concerning phenomena of direct shear test of FRP strips from the concrete surface the damage evolution is described using the linear softening model expressed in terms of fracture energy. The adopted power law fracture energy criterion (with power of 2) [15, 28, 29] is defined as a function of mode mix using normal mode fracture energy G_I (Mode I) for plain concrete and shear mode fracture energy G_{II} (Mode II) for interfacial FRP-concrete joint:

$$\left(\frac{G_I}{G_{FI}}\right)^2 + \left(\frac{G_{II}}{G_{FII}}\right)^2 = 1 \tag{2.2}$$

where G_I and G_{II} are the fracture energy components of Modes I and II, respectively. G_{FI} and G_{FII} are the critical fracture energies in pure Mode I and pure Mode II loadings, respectively.

The interfacial fracture energy which is crucial in the analysis of the FRP-concrete bonded joint represents the total external energy required to create, propagate and fully open crack along FRP–concrete interface. Finding the right value of interfacial fracture energy of analysed interface is still an open issue because of number of parameters which govern the local bond–slip behaviour as well as the bond strength of FRP-concrete joint itself. They are such as the tensile, the compressive and the shear strengths of concrete, the adhesive and the FRP strip tensile strengths, modules of elasticity of all components of such joint as well as main dimensions of it.

It is worth to mention that in the case of mixed-mode condition the Mode I (tension) of fracture energy of concrete-FRP joints can be approximated as the Mode I fracture energy of plain concrete provided that debonding in the concrete-FRP bonded joint takes place within concrete. A second observation is that the Mode II (shear) of fracture energy of both plain concrete and concrete-FRP joint appears to be an order of magnitude higher than for the Mode I [14].

3. Numerical analysis of direct-shear test

The subject of the paper is simulation of the single direct-shear test on FRP-to-concrete bonded joint presented in Fig. 1 where the main dimensions are sketched. The applied constraints are intended to prevent the concrete prism from up-lifting and shifting under loading. The FRP strip is glued with adhesive layer to the top surface of concrete prism. The three lengths of joint – 10 cm, 15 cm and 20 cm, and two kinds of adhesive layers of different stiffnesses were considered. The displacement-controlled loading was used by incrementing displacement *u* of the end of the FRP strip (Fig.1). The plots of reaction *P* and slip *s* at the FRP-concrete interface were then created.

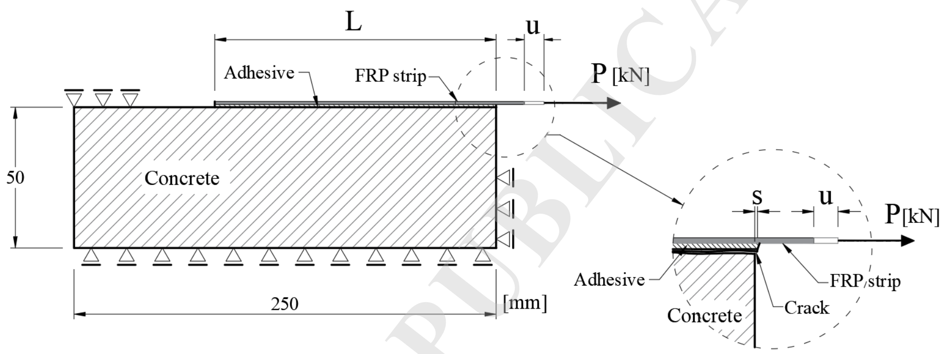


Fig. 1. Setup of analysed direct-shear test.

In the numerical analyses the XFEM method employed in Abaqus ver. 2024 code [15] was used. The failure mechanism definition includes damage initiation as well as damage evolution which are based on the fracture mechanics approach and on assumed bilinear traction-separation law for the FRP-concrete interface (Fig. 2). It was not necessary to predefine in layout of mesh beginning of crack for further propagation. This is one of the most convenient capabilities provided by the XFEM method.

When analysing the traction-separation law for the FRP-concrete interface the area under the entire curve (Fig. 2) represents the interfacial fracture energy G_{cr} which is equal to the value of fracture energy G_{FII} for Mode II.

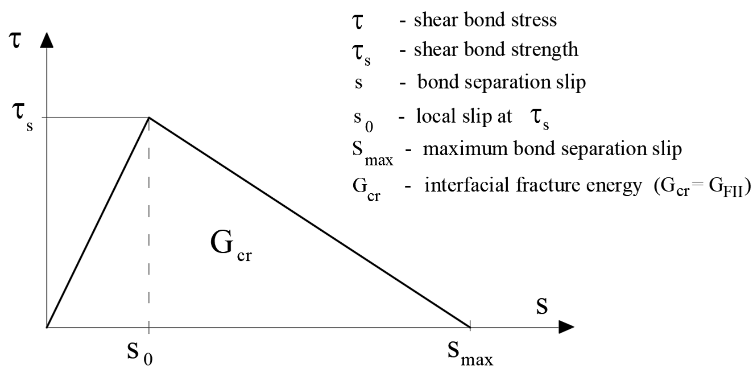


Fig. 2. Bilinear traction-separation law for the FRP-concrete interface.

All parts of the numerical model are defined as elastic materials. The concrete prism as well as the adhesive layer and FRP strip are modelled as 4-node 2D plane strain elements (Fig. 3) of 1 x 1mm dimensions. Although XFEM is often described as mesh-independent because cracks can propagate through elements without mesh conforming to the crack geometry, it still requires a sufficiently fine mesh to obtain accurate results, especially near crack tips where stresses are singular. That is why the convergence study was performed by changing the element size. The analyses with different elements size were performed, namely with 1 x 1 mm, 1. 5 x 1.5 mm and 2 x 2 mm meshes. Finally, a mesh of 1 x 1 mm elements size was used in all calculations for which the peak load has not been changing more than 0.7 % from that of 1.5 x 1.5 mm mesh.

It was observed in laboratory tests [5] that the quasi-brittle behaviour of concrete influences the FRP debonding process. Fracture propagation during IC debonding starts within a thin concrete layer underlying the adhesive layer [5, 18, 30]. It was assumed that in the cases of shear test for all tested bond lengths delamination starts within a thin concrete layer as well, starting close to the loaded end of FRP strip [16, 30]. For that reason, one-element layer of finite elements in concrete part of the numerical model was given additional enrichment with additional degrees of freedom according to XFEM.

The material data used in the numerical analyses were taken from the previously performed research program concerning strengthening under flexure the simply supported RC beams using FRP strips [5]. For concrete the modulus of elasticity and the Poisson's ratio before cracking were determined as 29.98 GPa and 0.164, respectively. The concrete compressive cylinder strength was assumed as 44.40 MPa while the tensile strength was determined as 3.467 MPa. The modulus of elasticity and the Poisson's ratio of the FRP strips were taken as 158.95 GPa and 0.2, respectively. For flexible adhesive the modulus of elasticity and the Poisson's ratio were equal to 7.10 GPa and 0.3, respectively. To check the influence of the stiffness of the adhesive layer on the behaviour of the entire joint, the case of a rigid adhesive with an elastic modulus 10 times the stiffness modulus assumed for a flexible adhesive was also considered [9].

The proper definition of the traction-separation law for FRP-concrete interface is the key issue in delamination analysis by XFEM (Fig. 2). It concerns mainly the stress value τ_s that is used to define the damage initiation criterion as well as the interfacial fracture energy G_{cr} which is equal to the value G_{FII} for the FRP-concrete joint. This value is in turn used in the definition of crack evolution. In our case of 2D analysis, in (1) two components of stresses, namely σ_n and τ_s, must be defined as component normal to the likely cracked surface and as shear component on the likely cracked surface, respectively. While the first component corresponds to the tensile strength of concrete, the second component cannot be determined directly. According to [8] the value of shear strength τ_s is dependent on both the tensile strength of concrete as well as on the geometry of the FRP-concrete joint. Taking into account the width of FRP strip and width of concrete prism τ_s was calculated as 5.721 MPa [8].

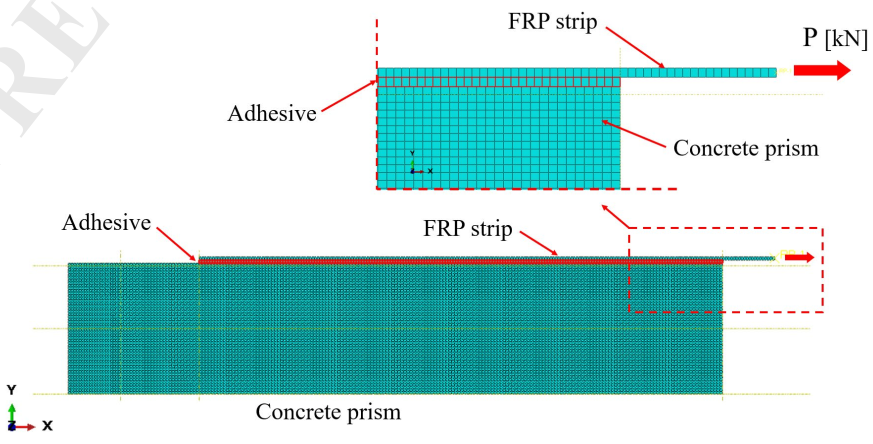


Fig. 3. 2D mesh of concrete prism with FRP strip.

The other important issue in delamination analysis is to assume correct and reliable values of fracture energy G_{Fi} parameters [14, 30]. While finding the fracture energy value G_{FI}, which refers to fracture energy for Mode I (tensile) is rather straightforward, then finding the proper fracture energy value for Mode II (shear) which refers to the interface failure of the FRP-concrete joint is not easy. In this study the value of G_{FI} is assumed as 0.090 kN/m as a function of both maximum aggregate size and the strength class of concrete [31]. Whereas the value of fracture energy for the FRP-concrete interface G_{FII} depends strongly on the geometry of a joint as well as one of two basic strengths of concrete. According to [6, 8, 10] the value of fracture energy G_{FII} for Mode II is calculated as 0.694 kN/m as a function of tensile strength of concrete, but in accordance to [14] it could be also assumed as 1.524 kN/m as a function of compressive strength of concrete. There is a significant discrepancy between the two calculated values, which confirms the need for future laboratory verification of that interface fracture energy value. In the present numerical analyses to estimate the influence of different parameters to the behaviour of the FRP-concrete bonded joint, both above mentioned values of G_{FII} are used in definition of damage evolution law.

4. Numerical results

The series of numerical analyses were performed and some general conclusions are drawn below.

The influence of different values of the interfacial fracture energies G_{FII} on the strength of FRP-concrete bonded joint was examined first. It was noticed earlier that the value of such energy is a crucial in modelling the FRP-concrete joint and it has a significant effect on its strength. The results of the study for bond length equal to 20 cm are presented in Fig. 4. They are in good agreement with those proposed in [6], where for considered fracture energies the ultimate load capacity of bonded joints are calculated as approximately 26 kN and 38 kN, respectively. It is worth to notice that in the simplified analytical formula the ultimate load does not depend on the bond length.

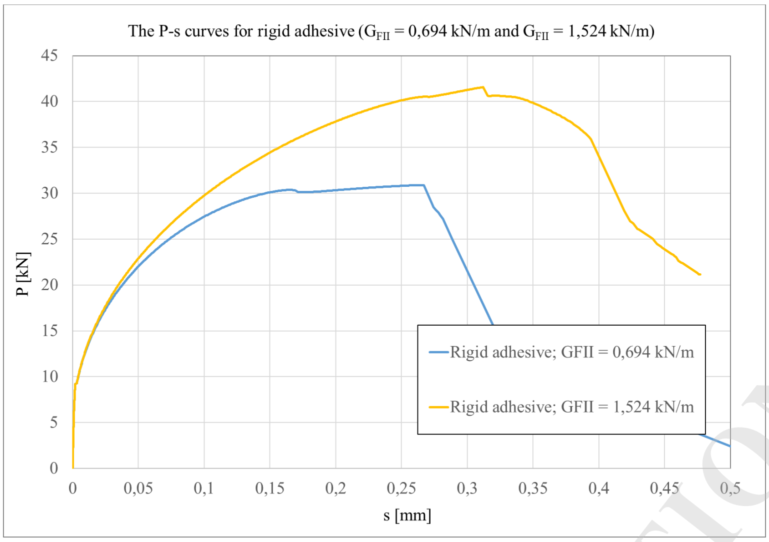


Fig. 4. The load-slip curves for 20 cm joint for different values of interfacial fracture energy.

Most of debonding models neglect the effect of adhesive stiffness. In reliable modelling it should be taken into consideration because, as it was shown for example in [9], the adhesive stiffness may affect the ultimate bond capacity of the FRP-concrete interface. The results of analyses for one bond length, for rigid and flexible adhesive and for two considered interfacial fracture energies, are presented in Fig. 5. It is shown that usage of rigid adhesive increases the bond strength of the FRP-concrete joint – the lower the value of interfacial fracture energy, the greater increase of the bond strength of the joint. It is also seen that flexible adhesive shifts horizontally the bond capacity and makes the FRP-concrete interface more ductile.

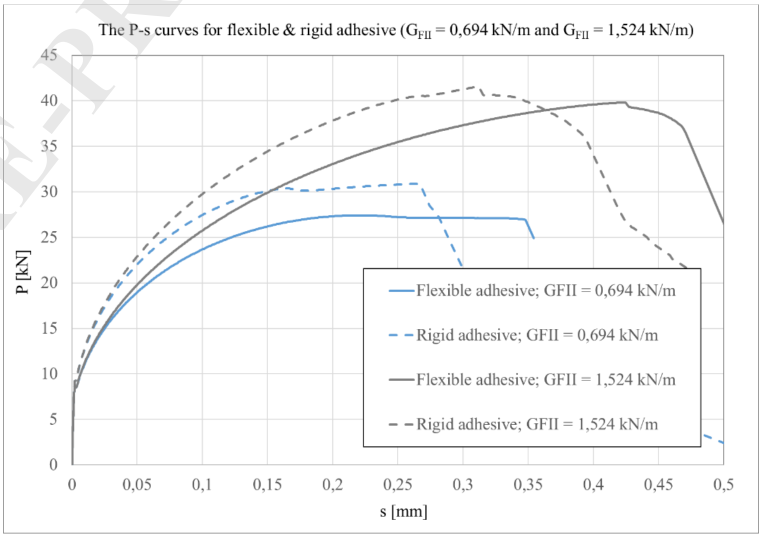
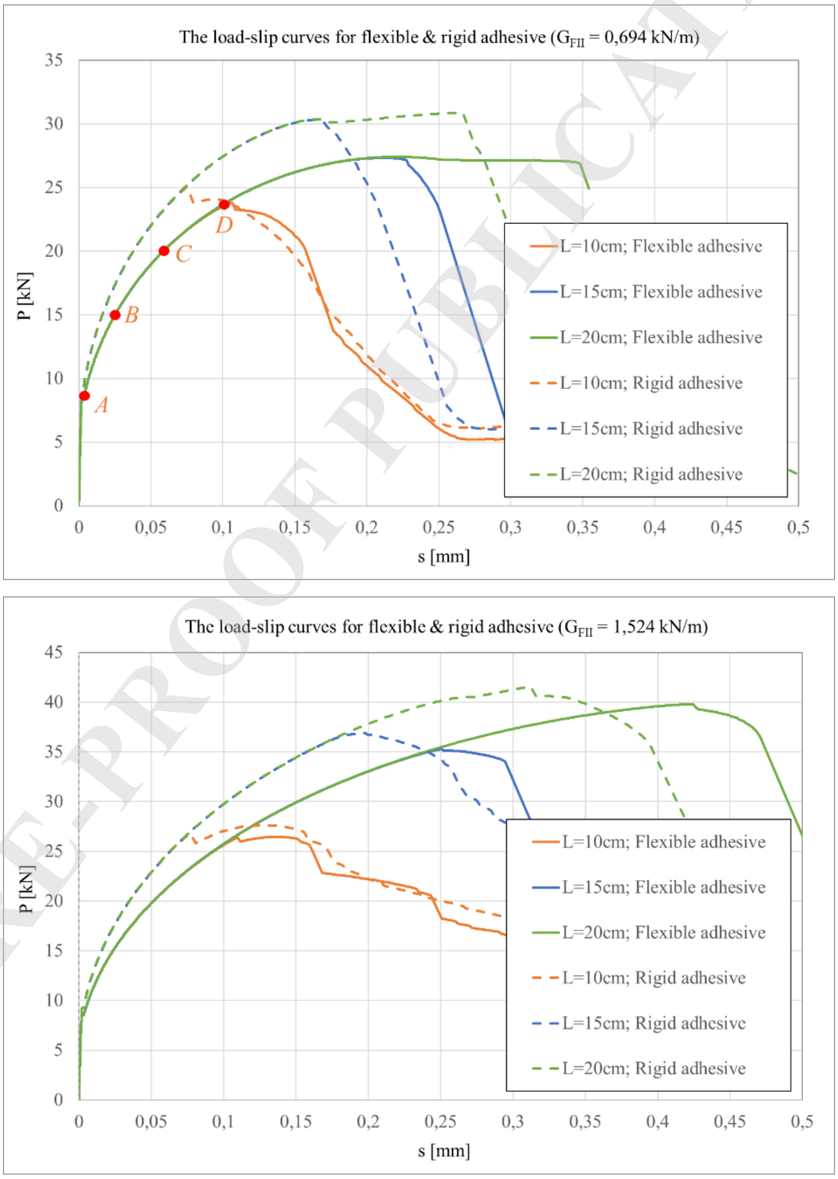


Fig. 5. The load-slip curves for 20 cm joint for different types of adhesives and different values of interfacial fracture energy.

The effective bond length is defined as the length of composite strip that if exceeded, there would be no increase in the force transferred between concrete and FRP strip. So, a proper assumed effective length of FRP strip is another key parameter in modelling of such joints. It was examined next in the study.

The analyses have been performed for three different bonding lengths: 10 cm, 15 cm and 20 cm. The results of analyses are presented in Fig. 6 where it is seen that for longer bond length the bond strength is higher.



a)

b)

Fig. 6. The load-slip curves for different bond lengths and different types of adhesive layer:

a) G_{FII} = 0.694 kN/m; b) G_{FII} = 1.524 kN/m.

To find out the value of effective length of FRP strips the analysis of axial strain ε_{11} along the joint of strip and concrete for different level of loading can be carried out. In Fig. 7 and Fig. 8 the results of such analyses are shown where the strain ε_{11} paths along the strip for two cases of joint lengths (10 cm and 20 cm) are presented. It can be noticed from Fig. 7b that the strain ε_{11} approaches zero at about x = 3.5 cm for load close to the ultimate one. It means that the effective bond length value is about 16.5 cm which corresponds very well to that obtained also in [31]. For the strip length equal to 10 cm similar does not happen, what is clearly seen in the Fig. 7a. It confirms the length of 10 cm turns out to be underestimated.

The strain paths show the higher strain ε_{11} levels and better utilization of the FRP strip for strip length of 20 cm, regardless the kind of adhesive layer.

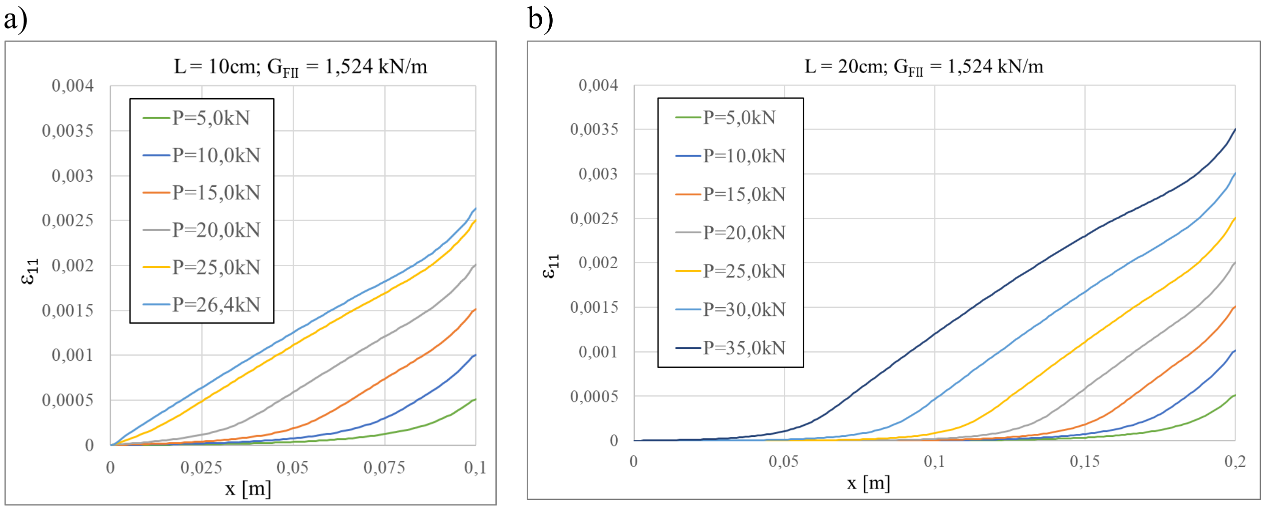


Fig. 7. Strain ε_{11} along the strips for flexible adhesive for G_{FII} = 1.524 kN/m and for different bond lengths: a) L = 10 cm; b) L = 20 cm.

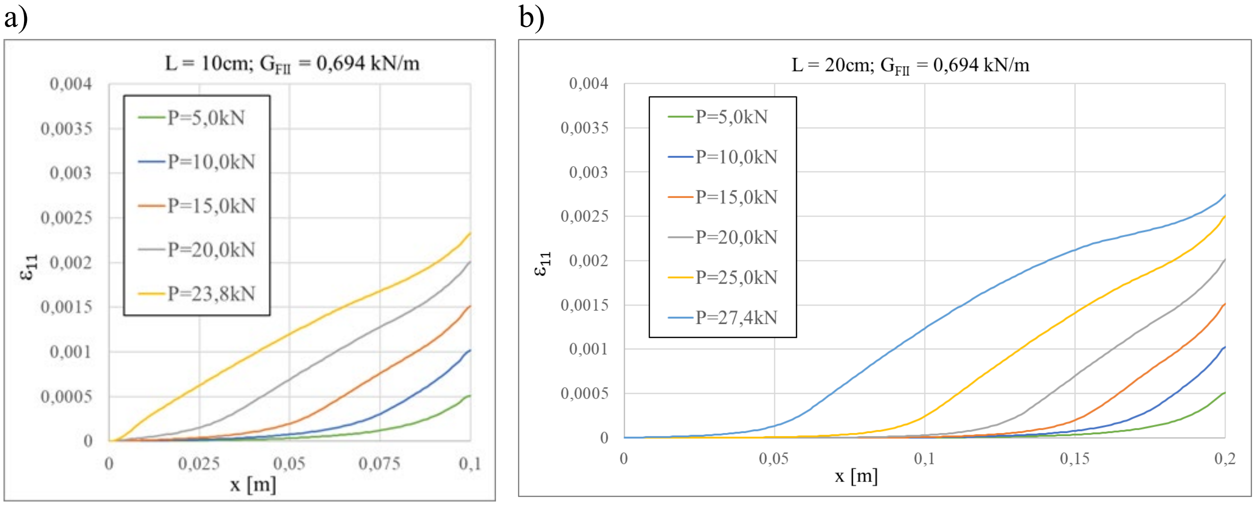
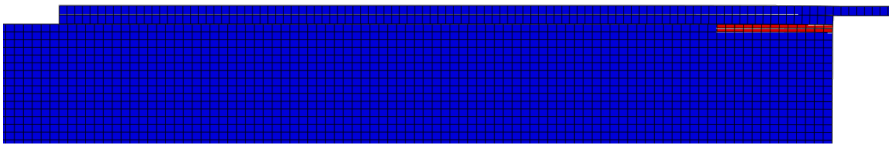
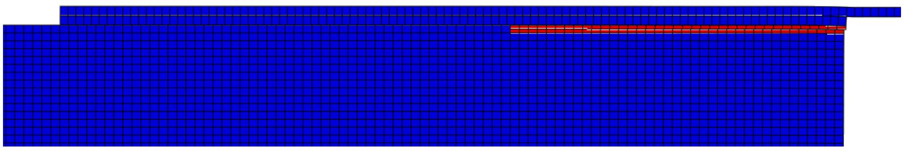


Fig. 8. Strain ε_{11} along the strips for flexible adhesive for G_{FII} = 0.694 kN/m and for different bond lengths: a) L = 10 cm; b) L = 20 cm.

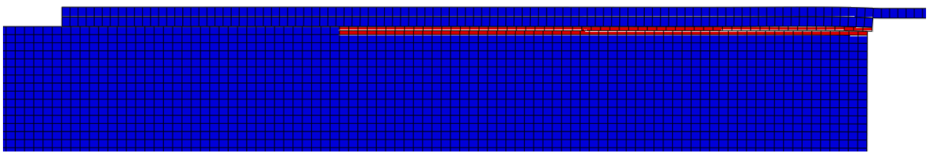
To illustrate the crack progression during the shear test, the contour plots of crack progression (with scale x10) for different load levels indicated in Fig. 6a are shown in Fig. 9, for the case of flexible adhesive and G_{FII} = 0,694 kN/m for bond lengths equal to 10 cm.



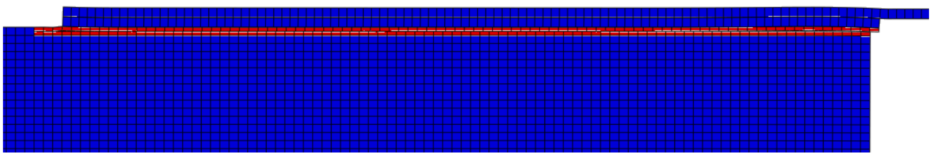
Point A (8.5 kN)



Point B (15.0 kN)



Point C (20.0 kN)



Point D (23.8 kN)

Fig. 9. Contour plots of crack progression (flexible adhesive; G_{FII} = 0.694 kN/m; bond lengths 10 cm) for different load levels (Fig. 6a).

5. Final conclusions

The paper presents the major results of the study of direct-shear test on the FRP-concrete bonded joints using the XFEM. The applied method successfully allowed to analyze the FRP-concrete joint which is highly discontinuous nonlinear phenomenon, and which is not possible to run by standard FEM.

Based on the performed numerical analyses it was found that the key issue in modelling is to adopt the appropriate interfacial fracture energy.

The effective bond length is the other important parameter which does not only determine the effectiveness of the joint but also affects its strength.

The consideration in modelling the stiffness of adhesives is important because it impacts on the local bond-slip behaviour of the FRP-concrete interface and then affects its load carrying capacity.

These above-mentioned parameters are the most important ones which govern the bond–slip behaviour as well as the bond strength of the FRP-concrete joints. They will be the subject of planned laboratory tests to validate them.

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