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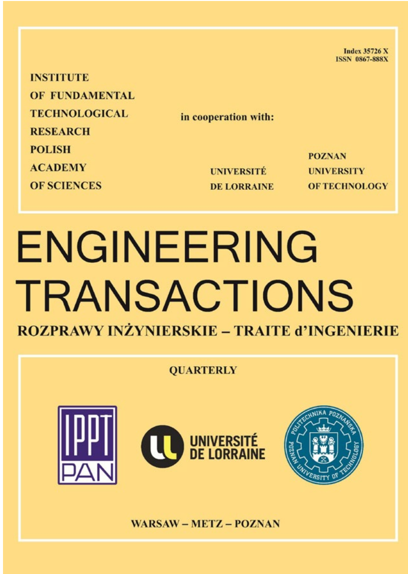


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# Non-linear analysis of wrinkling phenomena in sandwich beams with soft core

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The work is devoted to the local stability analysis of sandwich beams with light core. A linear as well as non-linear numerical analysis is carried out with the use of the finite element method (FEM). Both material and geometrical nonlinearities are taken into account. The goal of the investigation is to examine the influence of the material model on the post-buckling behaviour of a sandwich beam especially on the formation and developing of wrinkles on the compressed face. A pure bending condition is considered for the beam simply supported at both ends. The results for materials of theoretical properties are presented as well as for data from experiments on aluminium sandwich beams. From the results it is seen that the linear analysis of wrinkling phenomenon for non-linear materials gives overestimated results and does not predict correctly the buckling shape.

**Key words**: wrinkling; sandwich beams; bending; buckling; equilibrium path; non-linear analysis.

## 1. INTRODUCTION

The success in the design of a construction partially depends on the correct prediction, analytical or numerical, of the behaviour of this structure. Such prediction is especially difficult when local phenomena play the fundamental role and when the material properties are not uniform or are changing during the loading process. It is the case when sandwich structures are analysed in which at least two substantially different materials are connected together and a local loss of stability may appear in the form of short wrinkles.

The simplest approach to the stability analysis of sandwich structures is the application of methods which describe the problem in a linear way and which allow to determine the value of buckling load and buckling shape. The linear approach to wrinkling problem has been studied e.g. by DOUVILLE and LE GROGNEC [1]. Disparate possible failure modes were presented as well as the values of critical load were attained with the use of analytical solution and finite element method. The influence of geometry and material on buckling behaviour of

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the beam was widely investigated. The wrinkling phenomenon, as a linear problem, has also been described analytically in papers by JASION and MAGNUCKI [2] as well as JASION and MAGNUCKI [3] where it was assumed that the buckling shape has the form of uniform waves distributed equally on the compressed face. More realistic form are the waves with maximum amplitude in the mid-length which fading when moving to the support. Analytical approach to wrinkling of compressed composite facing has been discussed by BIRMAN and BERT [4] where core was treated as an elastic foundation and various models of it were taken into the consideration. Novel analytical solution of buckling of sandwich panels/beams has been formulated by CAO and NIU [5]. The transverse shear deformation of the face has also been considered. The face of sandwich panel was adopted to be a linear elastic beam or plate. The authors stated that the faces should be considered as one beam made of many short ones having the length of one buckling wavelength rather than a whole slender beam.

Short wrinkles mentioned above may appear in sandwich structures with thin faces. However, if the structure is rigid enough as a whole, a global loss of stability may occur. Analytical investigation of both global and local behaviour of sandwich beams has been presented by LÉOTOING *et al.* [6]. Authors, based on analytical solution, provided design indications in the form of diagrams. The global buckling and wrinkling analyses of sandwich plates with anisotropic faces and orthotropic core have been conducted by VESCOVINI *et al.* [7]. The critical loads as well as wrinkling modes were analysed for a foam and honeycomb materials of the core, subjected to uniaxial and multiaxial loads. A new model incorporated of local instabilities and global buckling in sandwich structures has been performed by MHADA and BOURIHANE [8]. The model was based on the technique of slowly varying Fourier coefficients. The main advantage of presented model was accessing results similar to those acquired with the use of another extended models as well as decreasing in computing time. The analysis of elastic buckling of isotropic, laminated composite and sandwich beams and subjected to various loads and boundary conditions has been carried out by KARAMANLI and AYDOGDU [9]. It was acknowledged that the type of applied load has a momentous influence on the values of critical loads and mode shapes which are also closely associated with bounding conditions.

A linear analysis of a structure has some well-known disadvantages. One of them is the assumption of ideal geometrical deformation after the buckling which in the case of sandwich beams means regular waves on the upper face what is rarely possible due to different types of imperfections in actual structures. Another point is that the materials used for the core are often some kind of foam and because of this they may behave in a non-linear way during deformation which cannot be included into linear analysis. For this reason the non-linear analysis should be performed to analyze the behaviour of sandwich structures. A vast investigation of the behaviour of different materials used for cores has been considered by LOLIVE and BERTHELOT [10]. The comparison between the experiment and the numerical simulation for three point bending was presented. The significance of experimental investigation in such problems was pointed out. The non-linear behaviour of a sandwich panel has been investigated by FROSTIG *et al.* [11]. The panel consisted of two faces, upper and lower, as well as functionally graded material of the core. The wrinkling and post-wrinkling response of the construction were taken into the consideration. The analysis affirmed that a functionally graded core can substantially improve wrinkling stability of sandwich structures.

The complexity of stability problem makes it necessary to use the numerical methods of which the most popular is the finite element method. Such approach has been proposed by SJÖLANDER *et al.* [12]. Authors analysed the case of forming the composite laminates and showed that changing stacking sequence the wrinkles can be eliminated in certain locations. Moreover, special finite elements proper for modelling the sandwich beams have been created and studied by SUDHAKAR *et al.* [13]. Furthermore, semi-analytical methods have been implemented by LIU *et al.* [14]. Buckling analysis of functionally graded sandwich beams has been carried out. The material properties of each layer were assumed to be graded along the thickness of the beams. The studies revealed that the boundary conditions have a meaningful influence on the buckling response of beams. In addition, the critical buckling loads increase with an increase in the constraints.

When the local stability is analysed geometrical imperfections of the face play a crucial role since they may initiate the wrinkling or folding phenomena. The problem of imperfections in sandwich structures has been described by EL-SAYED and SRIDHARAN [15]. Wrinkling phenomenon in sandwich panels has been discussed by FAGERBERG and ZENKERT [16]. The importance of initial imperfections was pointed out and the analytical model with such imperfections was provided. The comparison between analytical solution obtained from the model and the results of laboratory and numerical experiments show the correctness of this approach.

The wrinkling phenomenon is still vividly examined and not limited to classical sandwich structures. Stretching of soft shells with variable curvatures has been presented by WANG *et al.* [17]. Numerical analysis of wrinkling behaviour of shell surfaces was performed and phase diagrams on stability boundaries were depicted. The analysis indicated that disparate curvature can withstand wrinkles and control the wrinkling and smoothing responses. Wrinkling analysis of circular membranes has been carried out by HUANG *et al.* [18]. The instability mechanics of an annular membrane under in-plane stretching has also been performed. Furthermore, the analysis of instability of circular thin plate under in-plane compression has been conducted. It was considered, in case of annular plate, that the value of Poisson's ratio and geometric dimensions have superior influence on wrinkling behaviour. In case of compressed plate, the wrinkling phenomenon is associated with boundary conditions.

A new method for predicting the wrinkling stress in sandwich panels has been introduced and employed by SU and LIU [19]. The analysis presented the values of wrinkling stress which were highly dependent on the cell size of foam core. Moreover, the values received with the use of the model were compared to those available in the literature. Good agreement was observed. The extensive study of critical wrinkling and final wrinkling of thin-walled sheet/tube parts under various loading conditions has been accomplished by LI *et al.* [20]. The authors revealed that the wrinkling state mostly occurs under tension-compression stress state. The influence of material properties on the wrinkling behaviour was not taken into the consideration because of variable loading conditions. The effect of wrinkles on the failure of flat laminates has been formulated by HU *et al.* [21]. It was concluded that the local stress concentrations around the wrinkles as well as location of the wrinkles play an important role in failure mechanism of the examined laminates.

In actual structures it is difficult to analyze the wrinkling problem since there may be some interaction between two or more materials which may behave in a different way. When the waves form on a beam one or more materials may enter its plastic range and this may influence further behaviour of the whole structure. In this work the wrinkling phenomenon in sandwich beams is investigated numerically taking into account the changes taking place in the material during deformation. The obtained results provide answers to questions that arise during the modelling of sandwich structures but also help to understand the wrinkling phenomenon and relate the material properties with the buckling behaviour of the beam. First of all it is shown how the elastic/plastic properties of the faces and the core affect the post-buckling behaviour of a beam and how they influence the formation of wrinkles or dents. This knowledge can help in the design of sandwich structures of a desired behaviour. Some knowledge concerning the buckling and limit loads for beams made of materials with different properties is also provided. Finally, the comparison of FE and experimental results confirms that a simplified 2D numerical model can accurately predict the behaviour of the actual structure. An exemplary paper devoted to similar problem is this by STIFTINGER and RAMMERSTORFER [22] in which the influence of plastic properties of the core and the face on the post-buckling behaviour of sandwich beams under pure compression was presented. The post-buckling behaviour of sandwich beams has also been analysed by LÉOTOING *et al.* [23]. Here the influence of the size of imperfections as well as the properties of the material of the core on this behaviour was pointed out.

The content of the article is organised as follows: in chapter 2 the details concerning numerical modelling of the beam are provided including geometry and material; chapter 3 contains the results of analyses and begins with a brief description of the procedures used in the investigation; this is followed by four subsections in which first, the influence of mechanical properties of materials on the post-buckling behaviour of the beam is investigated; secondly, the behaviour of the upper face for different Young moduli is described; thirdly, the influence of imperfections shape and magnitude on the post-buckling behaviour is analysed, and fourthly, the application of the proposed FE model to the analysis of an actual beam is presented; the last subsection also serves as a validation stage. The final 5th chapter is the summary part containing the main conclusions.

## 2. NUMERICAL MODEL OF THE BEAM

### *2.1. Models of materials*

The feature which distinguishes sandwich structures among other ones is that they combine two usually very different materials. The faces are made of tough solid material which carries most of the load whereas the core is usually a very light filler in the form of a foam. The above fact suggests that special attention has to be paid when defining the model of material for numerical analysis. Moreover, the influence of this model on the behaviour of structure has to be carefully investigated.

Three different models of material have been taken into account in this work. The first one is a linear elastic model shown in Figure 1a and b with dashed lines. The simplicity of the model – the linear relation between stress and strain – makes it very effective in numerical calculations. However, small strains assumed in this model are not applicable when the buckling phenomenon appears in a local range. Thus, when the mode of failure of the structure is to be analysed the model which includes the plasticization of material should be considered. The simplest approach is to choose the elastic-perfect plastic model. It assumes the linear stress-strain relation up to the yield strength $R_e$ and then the strains increase infinitely with a constant stress equals $R_e$. Such model is shown in Figure 1a with a solid line.

The models described above are enough for materials like steel or aluminium which are often used as faces of sandwich structures. For manufacturing cores light materials are used like foams made of plastic. Such materials may behave in a non-linear manner from the very beginning of loading process. Thus, in this case a more proper model is a nonlinear plastic one shown in Figure 1b with the solid line. Here the stress-strain relation is given by a series of points and described by Eq. (2.1).

$$\varepsilon = \frac{\sigma}{E}\left[1 + c\left(\frac{\sigma}{E}\right)^{2m}\right]. \tag{2.1}$$

In the above formula $E$ is the Young's modulus of the material at the beginning of the curve, $c$ and $m$ are constants determined based on experimental results. The shape of the curve is similar to this which from the classical formulae proposed by RAMBERG and OSGOOD [24].

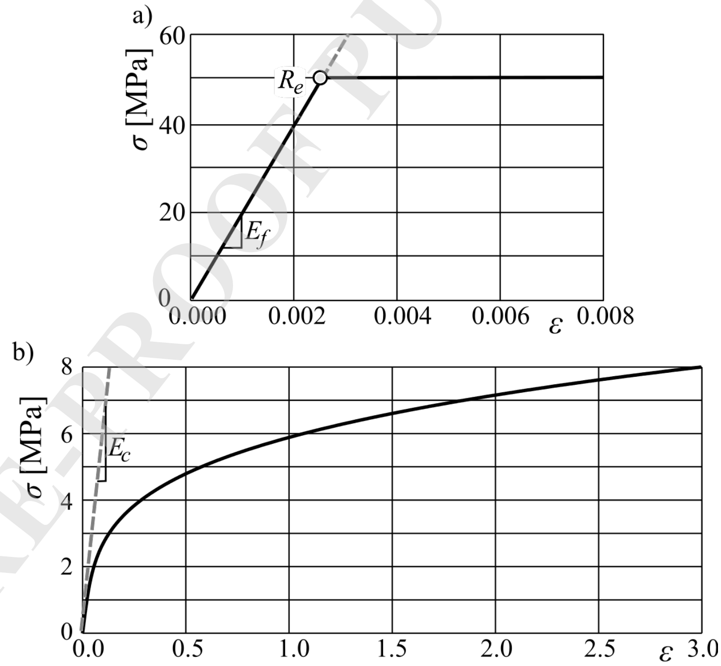


FIG. 1. Models of materials: linear elastic (dashed lines in a) an b)); linear elastic-perfect plastic (a); non-linear plastic (b).

Since the goal of this work is to investigate the influence of materials properties, or in other words the material model, on the behaviour of a beam the mechanical properties of selected materials have been chosen in the way to give the possibility to observe all desired phenomena. For the material of the faces there are: $E_f = 20 \times 10^3$ MPa, $\nu = 0.3$, $R_e = 50$ MPa. The model of the core has been prepared based on the Eq. (2.1) with the following parameters: $E = 50$ MPa, $\nu = 0.3$, $c = 3000$, $m = 1.4$.

## 2.2. *Model of the beam*

All numerical calculations have been performed with the use of ANSYS software. The following dimensions of the model have been assumed: total length of the beam $L = 400$ mm, thickness of the face $t_f = 0.3$ mm, thickness of the core $t_c = 20$ mm, and width of the beam $b = 100$ mm. The total beam thickness equals $t = 20.6$ mm. Since the wrinkling of the upper face is a local phenomenon, a dense finite element mesh is required to represent short wrinkles that may appear under pure bending conditions. Thus it was decided to use a 2D model of the beam, shown in Figure 2, with the assumption of the plane state of stress. Moreover, the symmetry conditions have been used in the mid-length of the beam and consequently only half of the beam has been modelled having in mind that with this approach only symmetrical deformation is possible. The preliminary research has shown that the effective approach to model the pure bending conditions is to connect all three layers with a rigid plate. The plate has the thickness of 1 mm and the Young's modulus $10^3$ times this for a face. The load in the form of moment is achieved by applying the normal forces $F$ to the faces through the rigid plate – compressing force to the upper face and tensile force to the lower one as it can be seen in Figure 2.

The model has been supported at the mid-height of the rigid plate. At the point of support a vertical movement has been removed, and axial displacements were allowed. Such conditions define a movable pin-support giving the possibility to the rigid end of the beam to move horizontally and rotate about the point of support. The displacement of the entire model was prevented by symmetry conditions in the mid-length.

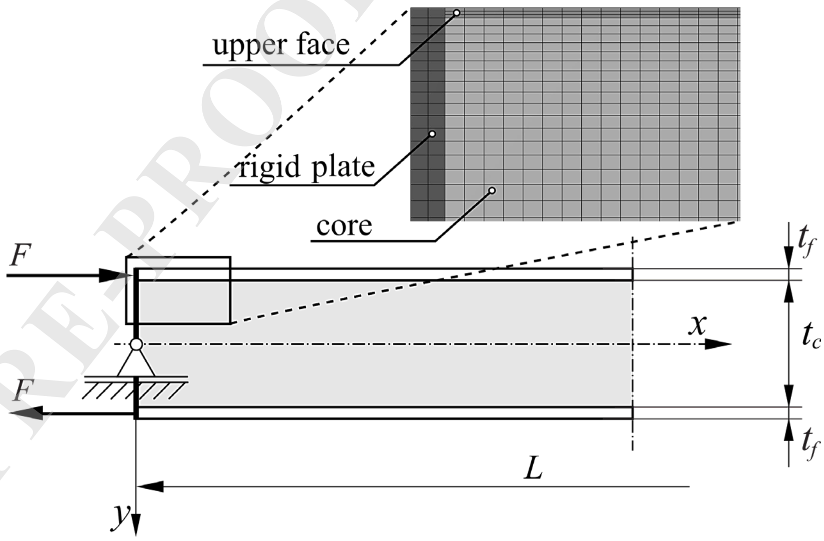


FIG. 2. FE model of the beam.

Higher order elements PLANE183 have been chosen with 8 nodes and 3 degrees of freedom in each node. A non-uniform mesh through the thickness of the core has been applied starting from the mid-height of the core. The mesh was densified with the scaling factor equals 4 which means that the finite element near the upper face is four times thinner than the one in the mid-height of the core. Since the loss of stability has the form of short wrinkles distributed along the length of the beam the most crucial parameter of the mesh is the number of elements along the axis, which has been set to 350. The conducted convergency study showed that both increasing and decreasing this number change the critical load value by only a fraction of a percent. Two other analysed parameters that is the normal stress in the face and the maximum deflection of the beam remained on the same level.

Similar convergency study has been performed as to the number of elements through the thickness of the upper face to prevent excessive stiffening due to an insufficient number of elements. The obtained results revealed that, starting from only two elements through the thickness, the value of the maximum deflection of the beam, the maximum normal stresses as well as the value of the critical moment are the same up to the fifth significant digits, regardless of the number of elements. Having in mind that the wrinkling is a local phenomenon it was decided to use three elements rather than two through the thickness of the upper face in order to improve the representation of the actual beam behaviour in the model.

## 3. RESULTS OF STUDIES

### *3.1. Procedures used in analyses*

Three types of analyses have been conducted to investigate the behaviour of the beam during the whole loading process. First, the static linear analysis has been performed to determine the stress distribution and to obtain the input results for the next step which was the linear buckling analysis. Here the buckling loads, called critical moments, and buckling shapes have been determined. The first buckling mode served then as a geometry disturbance in the last, third type of analysis which was the non-linear buckling one. This last step gave the possibility to include into the model the non-linear behaviour of material as well as geometrical imperfections. As a result the equilibrium paths have been derived describing the post-buckling behaviour of the beam including the limit load and the mode of failure of the beam. The non-linear analyses have been performed with the use of the arc-length method. The procedure available in the system gives the possibility to follow the post-buckling behaviour of a structure even for snap-through problems. The analysis has been divided into 100 substeps with 120 and 80 substeps as a possible maximum and minimum, respectively. The force convergence criterion has been used to control the solution, which is a typical approach for the arc-length method. The available stabilisation option was off. The maximum displacement of the model equal to 60 mm was used as the criterion to stop the calculations.

### *3.2. Influence of mechanical properties of materials on the post-buckling behaviour of the beam*

Typical buckling shape of a sandwich beam under pure bending has the form of short wrinkles in its upper part as shown in Figure 3a. Before wrinkles appear the displacements are very small and for many materials the process takes place in an elastic range. However, even for such small displacements plastic strains may appear in a light core and this may influence a future behaviour of the face. After buckling also in stiff faces plastic strains may appear since the deformation is much higher. For these reasons four different models of the beam with respect of materials will be considered. All models are geometrically non-linear but differ in material formulation. The results of analyses are presented in the form of equilibrium paths and additionally the distribution of elastic or plastic strains depending on material behaviour is provided. The vertical axis of the path corresponds to the applied bending moment normalized by the critical moment, thus the value equal to 1 corresponds to the buckling load, whereas the horizontal axis gives the value of displacement of the lower face measured in the mid-length of the beam. For the results presented in this subsection geometrical imperfections in the form of the first eigenmode have been used, what is a typical approach at least at the initial state of the investigation. According to Figure 3b, they have the shape of a symmetrical fading sine wave with the maximum amplitude in the mid-length of the beam. Since the goal of the investigation is to analyse nearly perfect structures and to observe their response on different material properties, the amplitude equals only to 0.01 mm has been used. The period of the waves was approximately constant and equal to 0.45 of the total beam thickness. It should be noted that the number of waves results from the assumed mechanical properties of the materials and the geometry of the model

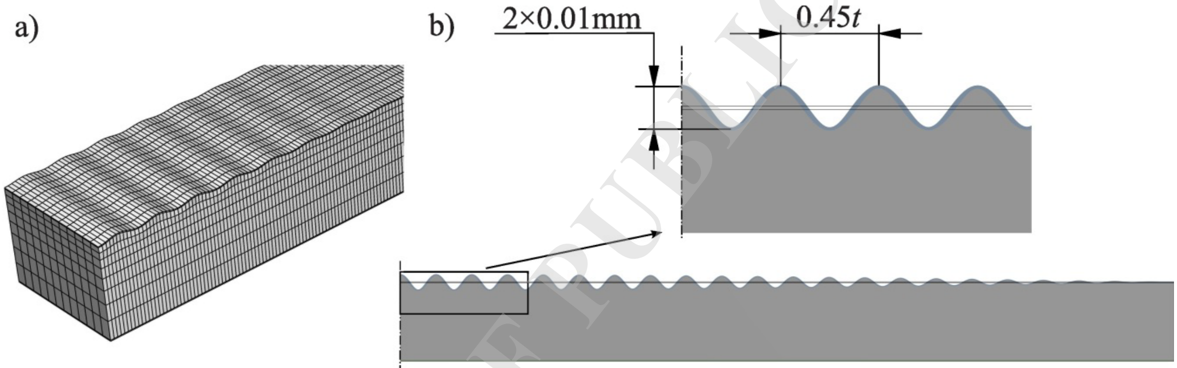


FIG. 3. Behaviour of the upper face of the bended beam: a) wrinkling phenomenon (exemplary FEM result); b) first eigenmode obtained from the linear buckling analysis used as an imperfection pattern (increased scale).

In the first model, an elastic material model has been ascribed to both the faces and the core. Two straight lines are visible on the equilibrium path (Figure 4a). The first one which is in the pre-buckling range describes the linear behaviour of the structure. Here the curvature of the beam increases and the upper face remains plain with the exception of waves being the result of the initial geometrical imperfections. The second line describes the behaviour of the beam in the post-buckling range. The slope of the line, that is the stiffness of the beam, is smaller due to the wrinkles on the upper face. The intersection of both lines which can serves as an estimated buckling load is about 3% lower than the linear buckling load corresponding to the unity on the vertical axis. As it can be seen in Figure 4b, on which elastic strains are shown, just after the moment of buckling the wrinkles appear in the mid-part of the beam on about 2/3 of its length. When the deflection increases the wrinkles spread on the whole length of the structure. Further increase of the deflection results in the increase of the amplitude of wrinkles. It should be noted that the amplitude and the wave-length are the same for the whole beam.

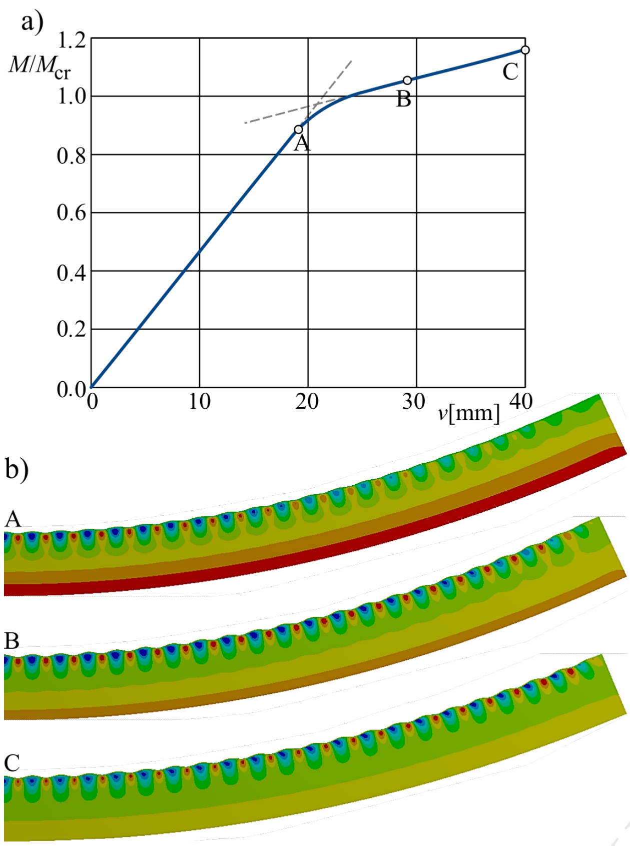


FIG. 4. Results of post-buckling analysis of the beam with elastic faces and elastic core: a) equilibrium path; b) distribution of elastic strain.

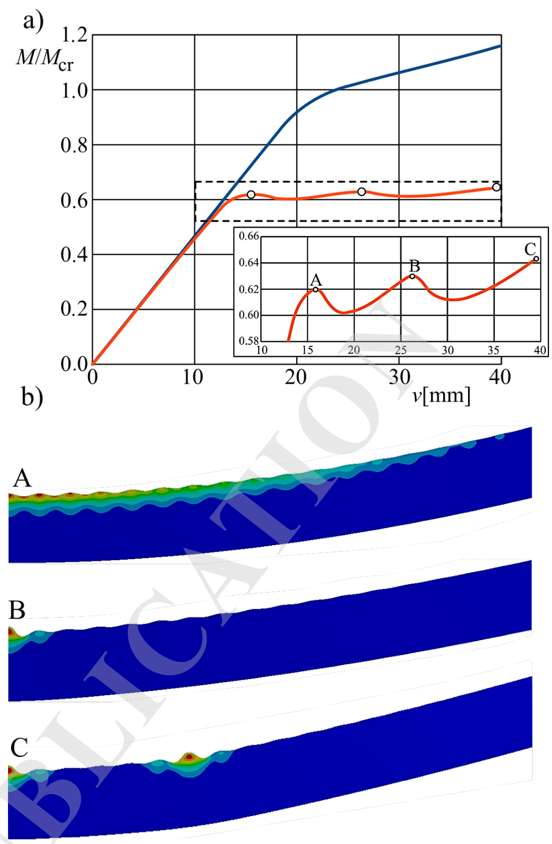


FIG. 5. Results of post-buckling analysis of the beam with elastic faces and plastic core: a) equilibrium path; b) distribution of plastic strain.

In the second model of the beam it is assumed that the core is made of a non-linear material like the one described by the solid line in Figure 1b. The material of the faces remains linear through the whole analysis. According to the equilibrium path shown in Figure 5 initially, before point A, the wrinkles appear in the mid-part of the beam (picture A in Figure 5b). After that the path bends sharply and then follows almost horizontally which indicates that the stiffness of the structure is very small. The reason is the formation of a single fold in the mid-length of the beam (picture B). The amplitude of other wrinkles diminishes. In the enlarged part of the plot it is seen that in the post-buckling range more peaks appear on the equilibrium path. The maximum of each corresponds to the change of buckling mode that is to the start of formation of another single fold as can be seen in picture C. From the plot it can be read that for this FE model that is the plastic core the buckling load is about 40% smaller than this from the linear analysis. Since the plastic strains shown in Figure 5b appear only in the core of the beam the faces are not visible in the figure – they are transparent.

The third model consists of elastic core and elastic-perfectly plastic faces. In this case the buckling load drops considerably and constitutes only 22% of the linear buckling load. As can be seen in Figure 6a after the loss of stability the slope of the path sharply decreases and remains approximately constant. The upper face deforms according to the imperfection shape. The size of wrinkles is very small as can be seen in pictures A and B of Figure 6b even for large deflection of the beam. The distribution of elastic strains in the core at final stage of calculations is uniform.

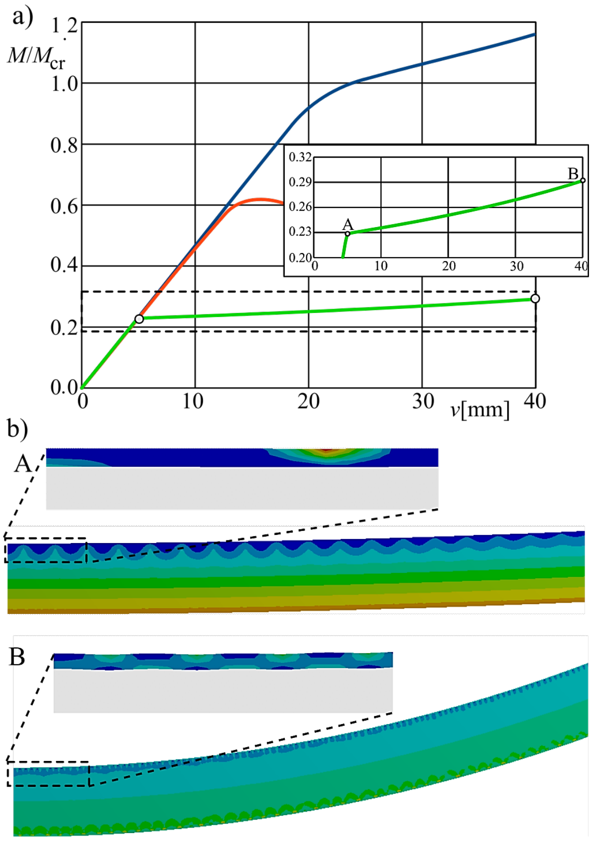


FIG. 6. Results of post-buckling analysis of the beam with plastic faces and elastic core: a) equilibrium path; b) distribution of elastic and plastic strain.

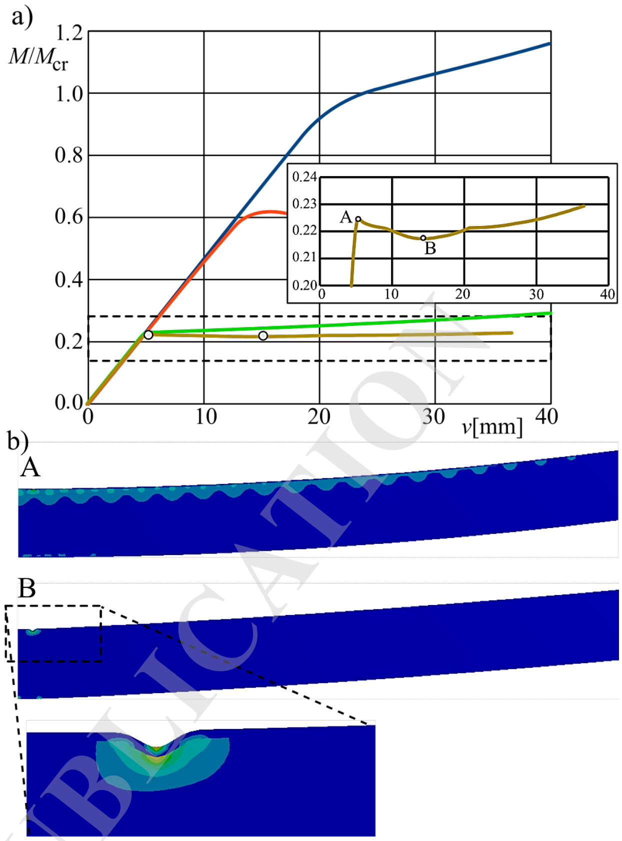


FIG. 7. Results of post-buckling analysis of the beam with plastic faces and plastic core: a) equilibrium path; b) distribution of plastic strain.

The fourth model analysed here is fully plastic. In this case, similar like in a previous one, the buckling load is only about 22% of that obtained in the linear buckling analysis. The equilibrium path shown in Figure 7a collapses sharply and then follows horizontally. At the buckling load a small plastic strain can be observed (Figure 7b) with almost negligible wrinkles and just after that point one large fold directed inward forms near the mid-length of the beam. The enlarged part of the plots reveals that the formation of the fold is related with the drop of the load up to some minimum after which small increase of the load is observed. In the model of the beam both materials behave in a plastic way which can be observed on picture B in Figure 7b. The plastic strains are present in both the core and the faces.

### *3.3. Behaviour of the upper face*

Since the loss of stability of the sandwich beam depends on the behaviour of the upper face it is reasonable to analyze the influence of stiffness of this part as well as the foundation on which it rests, that is the core, on buckling behaviour of the whole structure. Since the investigations are focused on material properties the stiffness will be modified by changing the Young's modulus leaving the thickness of the face and the core unchanged. In the first group of models the values of $E_f$ will range from $1 \times 10^3$ MPa up to $200 \times 10^3$ MPa and $E_c$ will remain unchanged and equal to 50 MPa. In the second group the modulus of the face $E_f$ equals $20 \times 10^3$ MPa and Young's modulus of the core $E_c$ takes values equal to 10, 50, 100, 200, and 300 MPa.

Let’s start the considerations from the linear buckling analysis. As can be expected the stiffness of the face influences the critical buckling load the value of which equals 20.3 Nm for $E_f = 1 \times 10^3$ MPa and 337.5 Nm for $E_f = 200 \times 10^3$ MPa. The mode of buckling is also different depending on the Young’s modulus. For lower values shorter wavelength is observed than for the highest analysed value as can be seen in Figure 8a. From the presented pictures it can be seen that the deformation of the core reaches much deeper if the stiffness of the face is high.

If the stiffness of the core is considered to be variable similar results are seen, that is the higher the Young’s modulus the higher the buckling load. In this case the relation is almost linear as can be seen in Figure 8b. The buckling shape is also influenced but here the increase of stiffness increases the number of waves on the upper face.

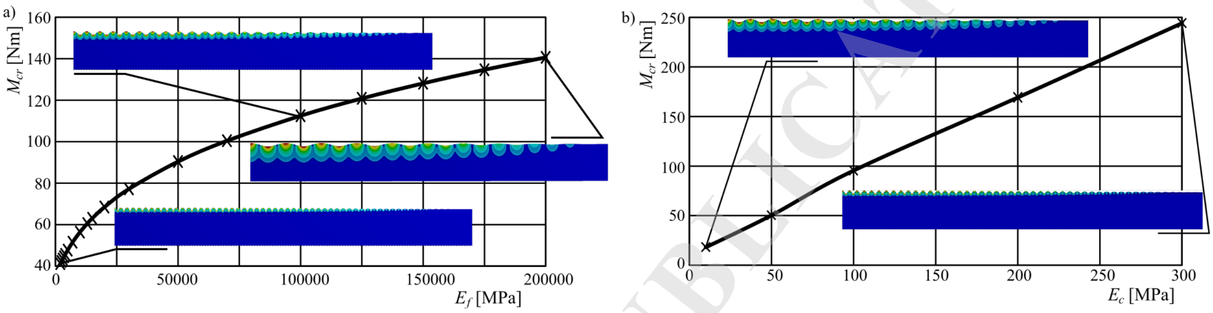

FIG. 8. Influence of Young’s modulus of the face (a) and the core (b) on the critical load.

More interesting information can be drawn when the same models of the beam are analysed with the use of the nonlinear procedure. To observe the formation of wrinkles the model of material of the faces is an elastic one whereas the model for the core is a non-linear one and defined as in previous analyses. The first thing that should be noted is the stiffness of the beam in the initial range of deformation indicated by the slope of the equilibrium paths shown in Figure 9. When the absolute value of load is taken into account the stiffness of beam defined as the ratio of load to the maximum deflection is increased from 0.2 for $E_f = 1 \times 10^3$ MPa to 30 for $E_f = 200 \times 10^3$ MPa, assuming that the stiffness of the core is the same for all models. Of course the limit load also increases with the increase of $E_f$ (see Figure 9b). Different behaviour can be observed when the Young’s modulus of the core is variable. As can be seen in Figure 9d the stiffness of beam is more or less constant and only the limit load increases with the increase of $E_c$.

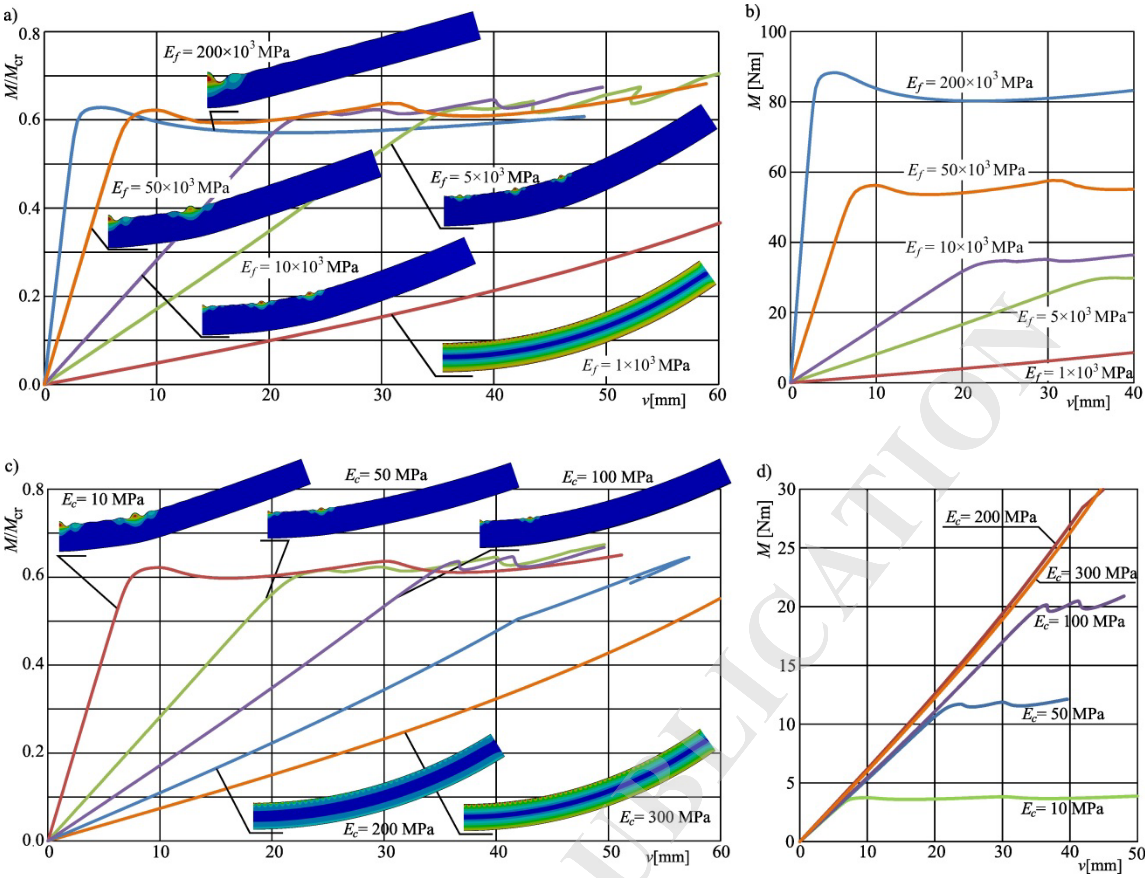


FIG. 9. Comparison of equilibrium paths and failure mode shapes for beams with different Young's modulus of the face (a), (b) and the core (c), (d).

Additional analysis of the obtained results is provided in Figure 9a and 9c. On these plots the vertical axis corresponds to the dimensionless load that is the applied moment divided by the critical moment being the results of linear buckling analysis. What interesting the non-linear critical load for all models is similar and corresponds to about 62% of the linear buckling load. In other words regardless of the stiffness of the face the critical load is decreased about 38%. This observation is valid for both series of models, the one in which the face is varying and the one in which the core is varying.

Aside from the buckling load essential information is how the different models behave under the load and especially how the wrinkles form on the upper face. This is shown on pictures added to the plots in Figure 9a and c. In the first figure it is seen that for the face with the highest stiffness after the loss of stability only one dimple forms and evolves with the increase of deflection. For lower values of the Young's modulus also one dimple appears but with the increase of the deformation further dimples form which is distinguished on the path with additional peak. In the second figure which corresponds to the fixed $E_f$ and variable $E_c$ it is seen that the deformation has always the same form, two folds, the amplitude of which decreases with the increase of the core's stiffness. However for $E_c = 200$ MPa and higher the deformation has the form of regular wrinkles which diminish near the support.

### 3.4. *Investigation of aluminium sandwich beams*

The results presented above are based on an arbitrary selected model of the beam as well as properties of the material. To verify how the proposed models work an additional study is provided below. The data used are based on the results of experimental investigation being a part of scientific grant no. N N502 080738 selected findings of which have been presented in monograph by MAGNUCKI and SZYC [25]. The subject of the grant were aluminium sandwich beams with a metallic foam core as shown in Figure 10c. The beams consist of two solid pure aluminium faces and the core made of aluminium open cell foam called Alporas®. All elements of the beam were glued together.

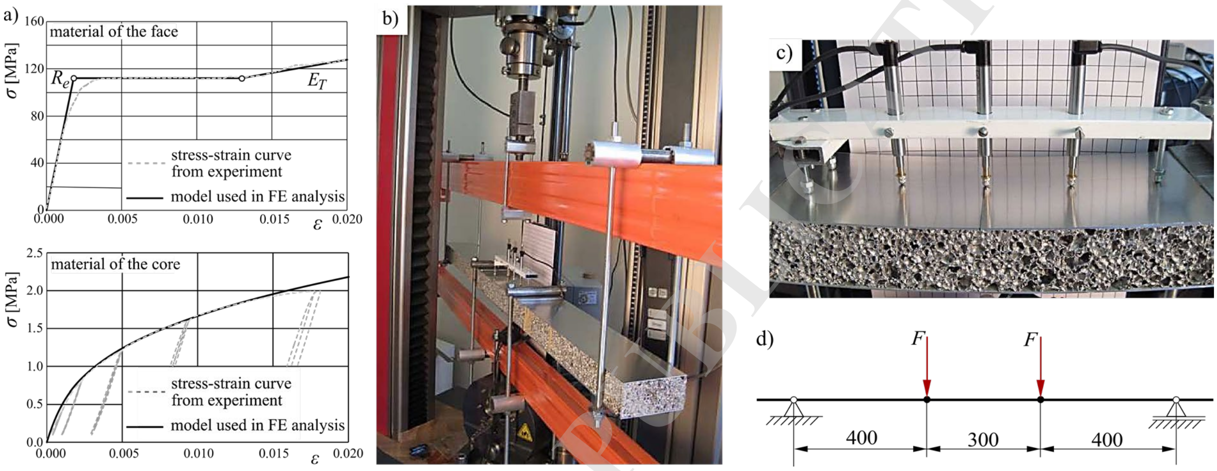


FIG. 10. Samples of structures used in experimental work: a) model of materials for FE analysis; b) and c) test stand; d) load conditions.

As an example a beam with a width of 100 mm and the total thickness of 40 mm has been chosen. The thickness of each face equals 1 mm. Four point bending test has been performed on the universal testing machine Zwick Z100 as shown in Figure 10b-10d. During the experimental test of the beam, the force was measured with a force transducer, and based on this value and the position of the load and support points, the bending moment was determined. The second measured value was the deflection at three points using inductive displacement transducers located in the central part of the beam where the pure bending conditions occurred. The displacement values at these three points were used as a guide to determine the curvature of the beam assuming that cylindrical bending taken place. These two parameters, that is the bending moment and the curvature, were used to plot the results of experiments. The behaviour of the upper face of the beam was also monitored visually. However, the deformation had the character of small waves distributed randomly across the entire surface of the upper face. Therefore, it was not possible to determine critical changes in global behaviour using this approach, unless there was a sudden decrease in stiffness associated with local indentation resulting from delamination.

The material models of the faces and the core used to prepare the FE model are based on the results of static tests. In the case of solid aluminium used to make the faces, a static tensile test was performed. In turn, to determine the mechanical properties of the foam, more convenient approach is to perform a static compression test. This way the problem of clamping the specimen in the machine's jaw is eliminated. Furthermore, since the foam exhibited nonlinear behaviour from the very beginning of the test, the unloading stages were included in the loading process to form hysteresis loops. Based on this loops the Young's modulus can be determined. The results of both tests are shown in Figure 10a as stress-strain curves marked with dashed lines. The models used in the FE analyses are marked with solid lines. It was assumed that both materials behave in the same way under tensile and compressive stresses. The mechanical properties of the faces material are as follows: Young's modulus $E_f = 65.6 \times 10^3$ MPa, Poisson's ratio $\nu = 0.3$, yield point $R_e = 112$ MPa, tangent modulus in a hardening region $E_T = 2.235 \times 10^3$ MPa. The stress-strain relation for the core is described with the formula (1) using the following parameters: $c = 400000$, $m = 1.01$, $E_c = 609$ MPa.

The actual beams were tested until the failure. For this reason a fully-plastic FE model has been used to compare the results of the experiment (blue line) and the behaviour of the beam model (red line) in Figure 11.

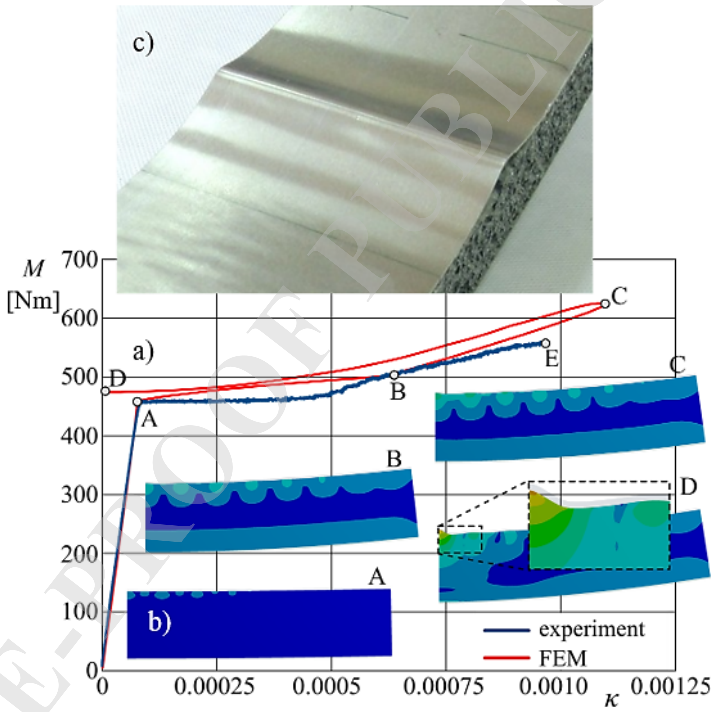


FIG. 11. Comparison of the results of the pure bending test: a) equilibrium paths; b) plastic strain in the FE model; c) failure of the tested beam.

It can be seen that the initial stiffness of the beam and the FE model are identical. What is more important the loss of stability takes place at the same value of the load equal to about 460 Nm. The differences appear after that point. On the curve corresponding to the experiment the plateau region is seen after which the strain hardening phenomenon can be distinguished. In the case of FE simulation after the loss of stability two regions of strain hardening are visible. The final shape mode of failure is similar for both analyses and has the form of one dent near the mid-length and it corresponds to about the same curvature of the beam and the FE model. Because in the FE model neither the separation of the layers nor the material interruption were taken into account the core follows the face's deformation during the whole analysis. In the actual beam the failure have the character of the face separation. As can be seen from pictures A and B in Figure 11 after the loss of stability the plastic deformations start to grow according to the pattern corresponding to the initial imperfections. The deformation of the upper face is very small. Finally a local fold appears in the location were the plastic strains are the highest.

The results of additional analyses are presented below in which different FE models were used to understand how the material formulation may influence the behaviour of the beam's model. The equilibrium paths obtained from all models presented in the previous chapter are compared in Figure 12. The curve obtained in the experiment is marked with the blue line. It can be seen that the crucial parameter is the plasticization of the face. For both models with plastic face the limit load has the same value as the one from the experiment. If the material of the face is an elastic one the limit load increases up to 2.3 kNm for plastic core. For fully elastic model the loss of stability was not observed. However, it should be noted that the value of the linear buckling load obtained in linear analysis equals about 7 kNm which is 15 times higher than the value given by the plastic model and the experiment.

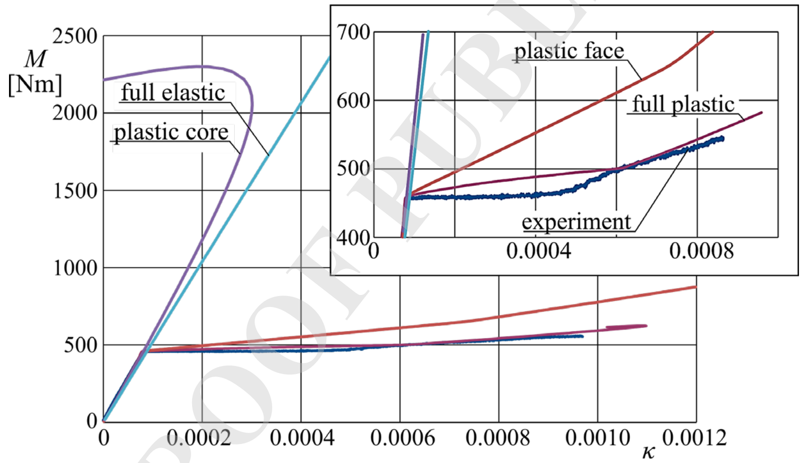


FIG. 12. Comparison of the behaviour of an actual beam and different FE models of the beam.

### *3.5. Influence of imperfections shape and magnitude*

One of the crucial parameter influencing the post-buckling behaviour of the structure is the shape and magnitude of initial geometrical imperfections. These parameters may change both, the value of critical load as well as the shape the structure will take after the loss of stability. The research on this subject has been performed on aluminium beam of the same parameters as in the previous section with the difference that the width of the beam equals 50 mm. Four different shapes of imperfections have been examined corresponding to the first, second, and fourth buckling mode, and the combination of these three. The third mode was omitted to avoid its interference with the first mode since it may result in a large peak in the middle of the beam. The buckling modes are provided in Figure 13a.

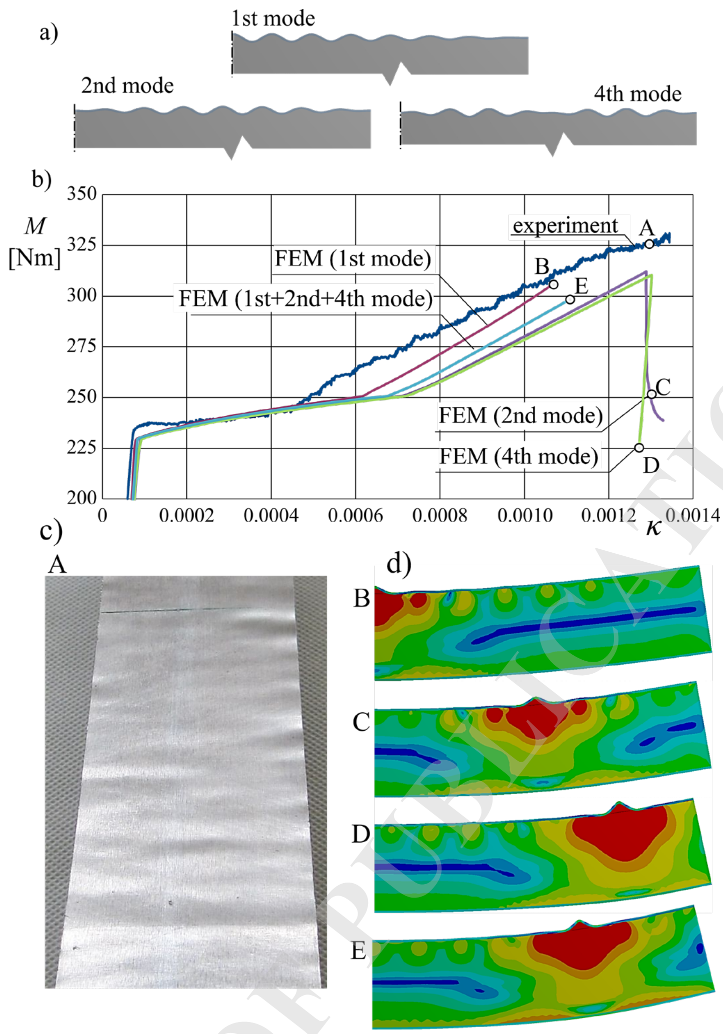


FIG. 13. Comparison of the results of the pure bending test: a) buckling modes; b) equilibrium paths; c) failure of the tested beam; d) elastic strain in the FE model.

They all have the shape of sine waves which in the case of 1st mode have the maximum in the mid-length and fades towards the support. In the case of the 2nd mode, the maximum is in the middle of the half-beam fading towards the mid-length and the support. The last 4th mode is irregular. Since in this subsection the influence of the imperfection on the buckling behaviour is the main goal, a higher amplitude than before has been assumed, that is 0.1 mm. In the case of combination of three different modes each mode was introduced with the value equal to 0.0333 mm.

The results obtained from the FE analyses (fully plastic model) are presented in the form of equilibrium paths in Figure 13b and compared with the plot obtained from the experiment. The comparison looks similar like for the broader beam presented before. The initial stiffness of the FE model and the actual beam is the same. Small discrepancy, about 3%, between the value of load at which the loss of stability takes place is visible. For the beam from experiment the value is slightly higher, equal to 235 Nm, and after that point almost flat region can be distinguished. For the FE model a strain hardening region can be seen. In general the shape of imperfection does not influence on the post-buckling behaviour of the model. When the curves from Figure 13b are compared the difference appears only in the moment at which the local fold starts to form. The slope of the last part of all paths is comparable. The shape of failure mode is also the same with the difference that the location of the final single dent is determined by the initial imperfection. The failure of an actual beam is shown in Figure 13c. Here a number of plastic folds are spread on the whole upper face without a single clearly visible dent.

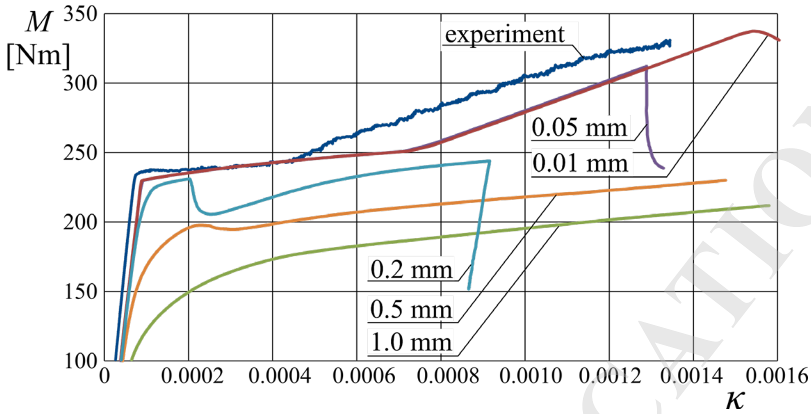


FIG. 14. Influence of the imperfection magnitude (2nd buckling mode) on the post-buckling behaviour of the model.

An exemplary imperfection sensitivity analysis has been performed for the second buckling mode. The magnitudes of imperfection were equal to 0.01, 0.05, 0.2, 0.5 and 1.0 mm. Also in this analysis the equilibrium paths have been generated with the use of FE method and presented in Figure 14. For small imperfections the path looks the same except the point where it collapses. If the imperfection is higher a sudden drop of the path can be observed like for the value equal to 0.2 mm. For the highest analysed imperfection the path is smooth during the whole process of beam deformation.

## 4. CONCLUSSIONS

When analyzing the results presented above the first thing that comes to mind is that the phenomenon of the local loss of stability of a sandwich beam is strongly influenced by the mechanical properties of the materials of particular layers. By comparing the results of analyses for four different models of the beam it can be seen that in the pre-buckling, linear, range there is no difference in the behaviour. Only elastic deformation appears and the stiffness of all models is the same - the same slope of the initial part of equilibrium paths (see. Figure 7a). This means that for deflection analysis in a small deformation range any model should work correctly.

When it comes to the control of the buckling shape it can be seen that stiffer face leads to small number of waves with large amplitude (Figure 8a). On the contrary, by increasing of core stiffness a big number of waves with small amplitude can be obtained (Figure 8a). However, if the plastic behaviour of the material is taken into account the deformation in post-buckling range always takes the form of local folds the number of which depends on the material and the magnitude of deformation.

An important information is that the stiffness of the beam can be barely increased by increasing the Young's modulus of the core. From the example presented in Figure 9d it is seen that increasing the value from 10 to 200 MPa increases the stiffness of the model about 22%.

Since the present investigation was focused on the response of beams with different models of materials during the loss of stability, only one type of initial geometrical imperfections has been considered, namely this corresponding to the eigenmode obtained in the linear buckling analysis. However, further investigations are planned in which natural imperfections can be incorporated based on the measurements of actual specimens. Also, other types of imperfections, like delamination or voids, can be crucial especially for new, complex materials like composites or fomes.

It should be noted that the model presented in this paper is two-dimensional. As a consequence, the wrinkles or folds which appear after the buckling are assumed to spread through the entire depth of the beam. As it was shown in Figure 13c, in actual beam, folds of various sizes spread across the entire upper face which, among others, can be the result of discontinuities of the foamed core. Such simplified model may be the source of discrepancies between the results obtained with the two methods however, when these results are compared, the values analysed in the investigation are similar and the discrepancies are acceptable.

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**Conflict of interest**
The authors declare no conflict of interest.

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