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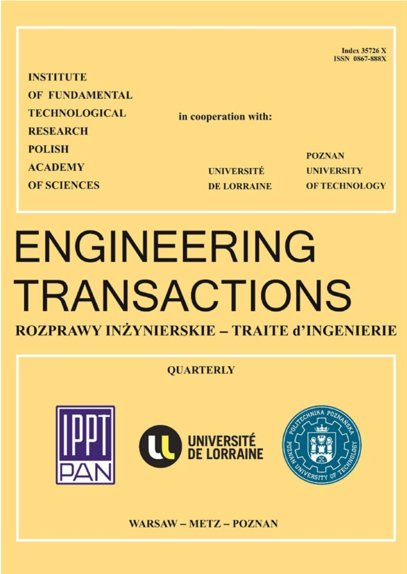


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# Graphical Method for Synthesizing a Four-Bar Linkage with Specified Coupler Angular Reversal Positions

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Graphical methods remain an important tool in the theory of mechanisms due to their ability to visually convey fundamental kinematic principles. They are particularly useful in early design stages and in educational contexts, where intuitive understanding is essential. Among the applications of graphical synthesis methods, mechanisms that require a link to momentarily stop at specific angular positions—commonly referred to as angular reversal positions—are of particular interest. While various analytical and numerical methods exist for designing such mechanisms, they typically focus on dwell positions of rotational or translational links and rely on optimization techniques, often at the cost of geometric transparency. This paper presents a graphical synthesis method for a four-bar linkage designed to achieve two prescribed positions at which the coupler reverses its direction of rotation. This specific problem has not been previously addressed in the literature. It emerges in mechanisms used for emptying containers, where the coupler carries the container and must instantaneously pause at two distinct angular positions to ensure stable discharge. Unlike many graphical methods that involve ambiguity related to trial-and-error selection of geometric parameters, the proposed technique guarantees solution correctness and uniqueness while also satisfying the Grashof conditions. This contrasts with numerical methods, where constraint checking is often deferred until the final stages. The construction proposed here is both practically relevant and theoretically novel, broadening the scope of synthesis methods to encompass mechanisms exhibiting link dwells in planar motion, and reaffirming the relevance of graphical approaches.

**Keywords:** mechanism synthesis; graphical methods; angular reversal; dwell mechanism; instantaneous center of rotation.

## 1. Introduction

The design of a new machine is preceded by an analysis aimed at the optimal realization of its intended utility functions. Mechanism synthesis offers the necessary tools for this process: selecting a suitable structural scheme and determining the link dimensions to achieve the desired motion, subject to structural, geometric, kinematic, and dynamic constraints. When a designer chooses a mechanism type for a given task and seeks optimal dimensions, the problem becomes one of dimensional synthesis. Early synthesis methods were developed graphically. Despite their inherent limitations, they retain both scientific and didactic value—especially in developing geometric intuition crucial to engineering practice. Although numerical methods prevail in modern science, the graphical methods continue to offer insight into underlying steps of the problem-solving process, which makes them foundational in engineering education and design practice.

Historically, many pioneering engineering achievements were the result of gradual evolution and refinement of earlier concepts. Graphical methods have long played a central role in technical problem-solving. In the 17th century, René Descartes introduced analytical geometry, and Galileo Galilei studied motion, laying the groundwork for later graphical representations. Newton's Principia Mathematica employed geometric reasoning to show that the trajectory of a body under an inverse-square central force is a conic section [1, 2]. Gaspard Monge developed descriptive geometry [3], which became essential for mechanism analysis. Other graphical contributions include Culmann's method for structural analysis [4], the Cremona method for trusses [5], and Mohr's circle for stress and strain visualization [6].

The early 20th century was the golden age of graphical methods. During this period, graphical methods for the analysis and synthesis of mechanisms were intensively developed [7, 8, 9, 10, 12]. Designing a mechanism to meet specific motion and functional requirements is inherently more complex than performing a mechanism analysis. In the field of mechanism synthesis and analysis, graphical methods were widely used until they were gradually supplanted by numerical approaches. The complexity and diversity of geometric transformations applied in mechanism design is well illustrated by the graphical techniques used in the synthesis of four-bar linkages for path generation with four or five prescribed precision points. With regard to geometric approaches, Burmester [9, 17] introduced fundamental graphical techniques for the synthesis of linkages in the late 19th century. Other methods were later developed to address motion generation problems. One widely adopted approach is Freudenstein's method, particularly for solving the three-position motion generation problem [15]. Additionally, the Roberts–Chebyshev theorem demonstrated that three distinct four-bar linkages can be constructed geometrically to produce the same coupler curve [7]. A comprehensive overview of graphical synthesis methods can be found in [11, 13-16]. Artobolevsky applied his general theory of mechanism structure to advance graphical methods for the kinematic and kinetostatic analysis of mechanisms. He also developed general methodologies for mechanism synthesis [11]. Contemporary researchers continue to apply graphical methods for the synthesis of mechanisms with significant engineering applications [14, 16]. Lakshminarayana and Rao [18] performed a geometric synthesis of a RSSR crank-rocker mechanism designed for a prescribed oscillation angle and quick-return ratio. Furthermore, the study presented in [19] introduces a geometric synthesis method for function generation in a steering control mechanism with four discrete positions. This steering linkage forms a critical component of the steering systems used in most modern land vehicles.

The enduring inclusion of graphical methods in modern textbooks reflects their unique ability to visually communicate kinematic concepts. Dwell mechanisms—mechanisms that maintain a stationary position at specified points for a finite period of time —are particularly important in industrial applications. Numerous analytical and numerical synthesis methods have been developed for such mechanisms [20–37], primarily focusing on dwell of links that move in purely translational or rotational motion. In most cases, the synthesis methods guide the coupler through several prescribed positions without imposing kinematic constraints. The graphical synthesis of 6-bar dwell linkage mechanism is presented in [38]. Although the core of this approach is a geometric constructions, the analytical equations were derived to optimize the solution.

This study considers the synthesis of a four-bar linkage with two prescribed angular reversal positions of the coupler—a problem not previously reported in the literature. To be precise, the mechanism under consideration is not classified as a dwell mechanism, since the motion stop is instantaneous. The context is a container-emptying mechanism, where the container is mounted on the coupler, and stable discharge requires the coupler to pause at two distinct positions. In the basic case, the coupler rotates by $\frac{1}{2}\pi$ during discharge when the input link rotates by a predetermined angle. This serves as the basis for generalizing to rotation of $\frac{1}{2}\pi \leq \delta < \pi$. The method guarantees uniqueness and correctness of the solution and ensures that the Grashof conditions are satisfied. Unlike many existing graphical methods, where arbitrary choices of selected parameters may lead to invalid results, the proposed construction systematically avoids such ambiguities. In contrast to optimization-based computer methods, where constraint violations are often detected late in the process, this graphical approach integrates all constraints from the outset.

## 2. FORMULATION OF THE PROBLEM

The topic is to construct graphically a four-bar linkage to guide a container so that it rotates exactly by an angle $\frac{1}{2}\pi \leq \delta < \pi$, caused by the rotation of the driving link by a given angle $\pi + \alpha$, where $0 \leq \alpha \leq \frac{1}{2}\pi$. To ensure an effective functionality, it is imposed that the coupler with the container operates between two extreme positions, such as (Fig. 1):

- position (*b*) (mounting the container),
- rotated by an angle $\delta$ (*b*') (unloading the container),

and in these extreme positions the instantaneous angular velocity of the coupler $\omega_b$ is zero. Moreover the crank $O_1A$ has to make a full revolution, therefore crank-rocker mechanism is being designed. The Instantaneous center of rotation (ICR) of the coupler lies at the intersection of the rotating links (crank and rocker) axes. When coupler changes its direction ($\omega_b$ is zero), the ICR lies at infinity. At these extreme positions the crank and rocker are mutually parallel. Given the freedom of choice, it is assumed that in position (*b*) the coupler is perpendicular to the input link. The active link rotates through an angle of $\pi + \alpha$, and as a result, the coupler rotates counterclockwise from position (*b*) to position (*b*'). In position (*b*') the instantaneous angular velocity of the coupler $\omega_b$ is zero, and a return rotation to position (*b*) follows, while the active link completes a full rotation around $O_1$. In position (*b*), the direction of the coupler's velocity changes, meaning its instantaneous angular velocity is also zero. When the coupler is in position (*b*), the passive rotating link (rocker *l*) must be aligned parallel to $O_1A$. Then, the ICR of the coupler – denoted as C - lies at infinity. Similarly, when the coupler is in position (*b*'), the passive rotating link (*l*') must be aligned parallel to $O_1A$'. Then, the ICR of the coupler C' also lies at infinity. Since the transition of the coupler from position (*b*) to position (*b*') is required to occur over a prescribed input crank rotation angle, the task corresponds to a time-prescribed motion synthesis problem. The schematic of the problem is shown in Fig. 1.

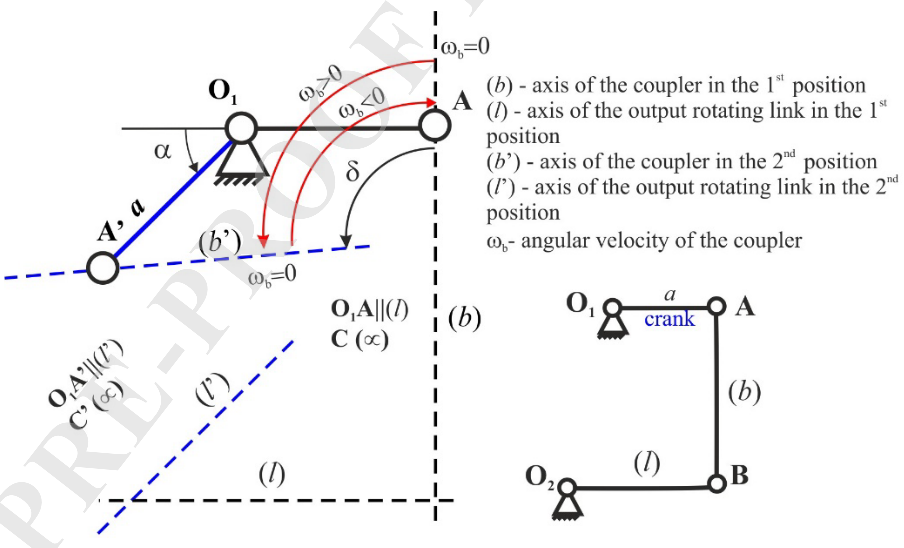



FIG. 1. Geometrical illustration of the problem.

## 3. GEOMETRICAL CONSTRUCTION

### 3.1. Case 1: $\delta = \frac{1}{2}\pi$

The method begins with a base case in which the coupler rotates by $\frac{1}{2}\pi$, then generalizes to the range $\frac{1}{2}\pi \leq \delta < \pi$. In the first approach let us take that $\delta = \frac{1}{2}\pi$. Arbitrarily chosen quantities in a current step are labelled in green. Quantities determined in a current step are marked in blue, quantities to be determined are in red.

1. Let us draw a pivotal joint $O_1$, take an arbitrary length *a* of the driving rotating link, and sketch the link $O_1A$ in its horizontal position $O_1A$ and in the position $O_1A'$, rotated by an angle $\pi + \alpha$ (Fig. 2a).
2. Draw the vertical line (2) passing through point A. Another one (1) rotated by angle $\delta = ½\pi$, passing through A'. Draw the arches of radius of $b = a$ from points A and A' in order to find the locations of points B and B', respectively (Fig. 2b).

The lines (1) and (2) represent the extreme positions (*b*) and (*b*') of the coupler, i.e. positions at which its instantaneous angular velocity is 0.

3. Connect points B and B' (3) and measure the angle $\gamma$ between line BB' and the vertical line (Fig. 2c).

Let us observe that always $\gamma \geq ½\alpha$ (a brief proof is provided in section 4).

4. Shift point B' to the right by $\Delta$ so that segment BB'' (4) forms with the vertical line the angle $½\alpha$ (Fig. 2d).

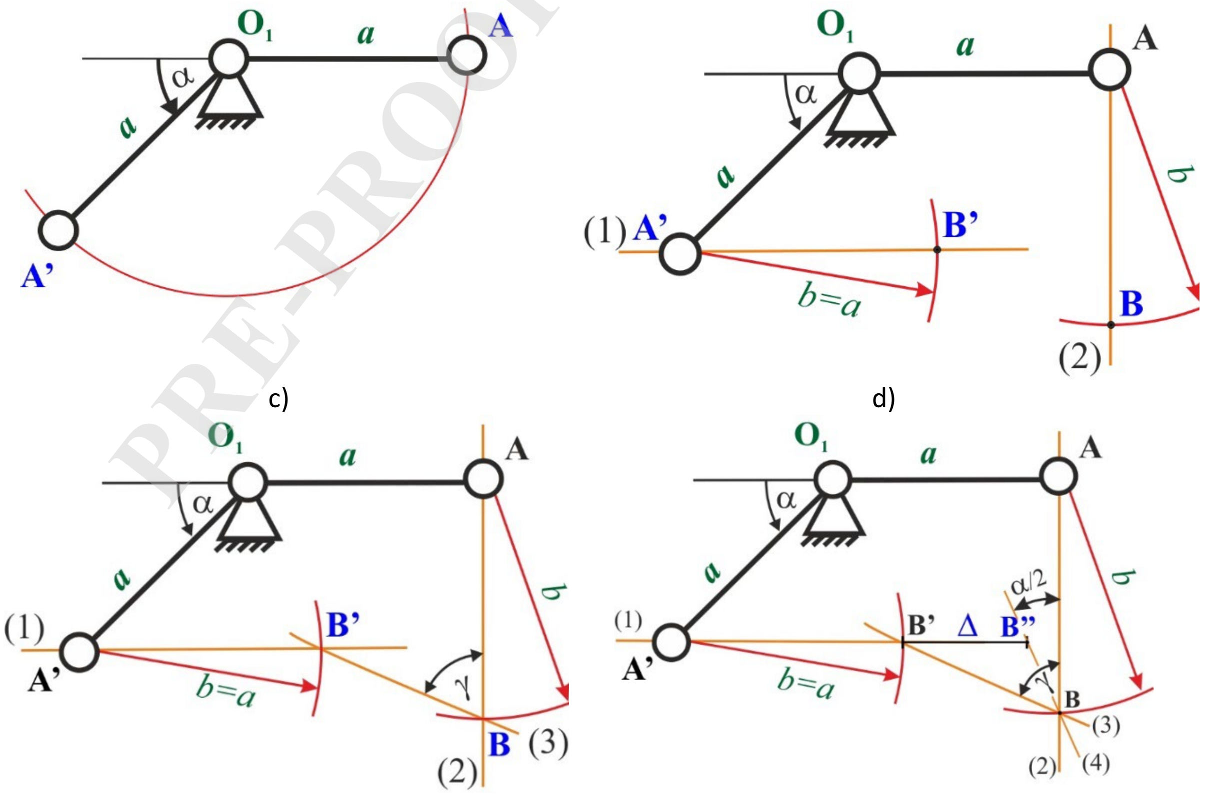
As a consequence there occurs the difference between the coupler lengths |A'B''| and |AB| of $\Delta$. Locations of points B and B'' have to be corrected in order to ensure that the coupler length is equal in both positions. For this purpose, an additional construction is required.

5. Draw a vertical segment of length $\Delta$ (Fig. 2e). From the upper end of this segment draw a line parallel to BB''. From the bottom end draw a line at an angle of $¼\pi$, extending until the both lines intersect. From the intersection point draw the line perpendicular to $\Delta$. The intersection of these perpendiculars divides $\Delta$ into *h* and *y*.

The shaded triangles formed are congruent, ensuring that $\Delta = h + y$.

6. Line BB'' (4) is translated parallelly (line 5) so that point B'' is shifted to the left along A'B'' by *y*, in consequence, B is lowered by *h*. The newly sought positions of the points are labelled as $B^1$ and $B'^1$.

a) b)




c) d)

e) f)

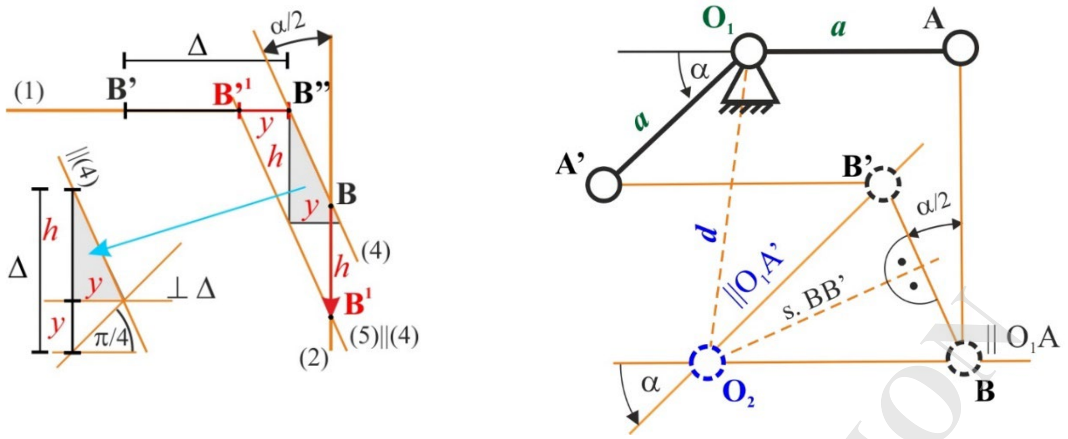



FIG. 2. The following steps of the synthesis procedure in case when δ = ½π; a) step 1; b) step 2; c) step 3; d) step 4; e) steps 5-6; f) step 7.

7. To avoid introducing superfluous notation, we relabel the actual positions of points $B^1$ and $B'^1$ as B and B', respectively. Draw the horizontal line (parallel to $O_1A$) passing through point B, as well as the line symmetrical to the BB' (Fig. 2f). These two lines intersect at the rocker pivot $O_2$. This completes the construction.

It is evident that $|O_2B| = |O_2B'|$. Now observe that BB' must be inclined at ½α to vertical line (step 4) in order that $O_2B'$ is parallel to $O_1A'$ and $O_2B$ is parallel to $O_1A$, as required at the angular reversal positions. The resulting mechanism is shown in Fig. 3.

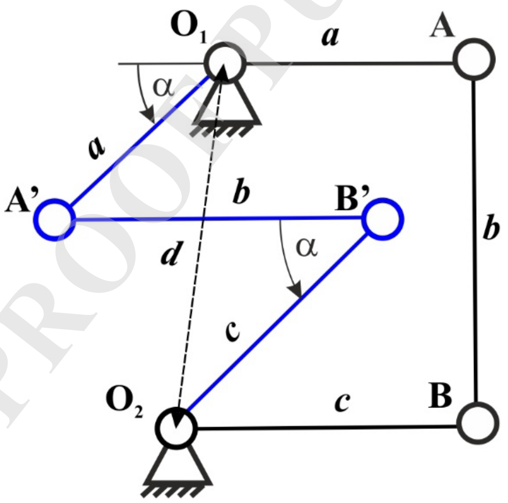



FIG. 3. The resulting mechanism for the case 1.

### 3.2. Case 2: $\delta > ½\pi$

Let us consider the general case in which the coupler rotates by an angle ½π < δ < π, caused by the rotation of the driving link by a given angle π + α.

1. Perform steps (1), (2), (3), and (4) as in the case of the first construction, keeping in mind that line (1) is rotated by an angle δ > ½π (Figs. 4a–b).

The difference between the coupler lengths |A'B''| and |AB| equals Δ. The quantities *h* and *y* are determined from additional constructions.

2. Draw a vertical segment ab of length Δ (Fig. 4c). From the upper end (a) of this segment, draw a line parallel to (4). From the bottom end (b) of the segment, draw a line at an angle π - δ (parallel to line 1) until it intersects the previous line. As a result the auxiliary quantity *x* is determined.

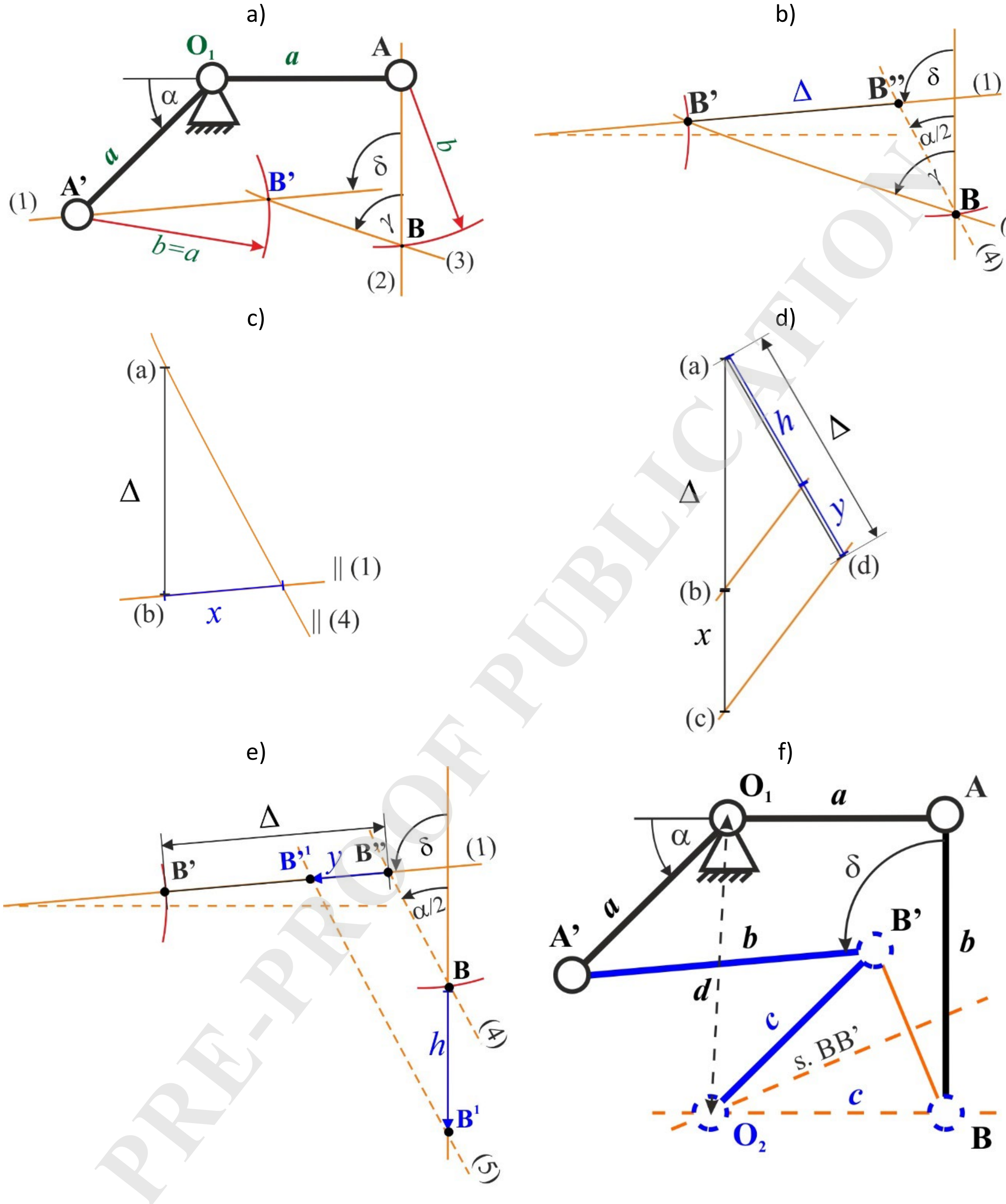



FIG. 4. The following steps of the synthesis procedure in case when $\frac{1}{2}\pi < \delta < \pi$; a) steps 1 (1-3); b) step 1 (4); c) step 2; d) step 3; e) step 4; f) step 5.

3. Then, extend segment Δ by a specified value *x* (Fig. 4d). From the upper point (a), draw a new segment ad of length Δ at an arbitrary angle. Connect points (c) and (d), and then from point (b), draw a segment parallel to cd. In this way the segment ad is divided into parts of lengths *h* and *y*.

4. In the main diagram (Fig. 4e) move line (4) in parallel so that point B is shifted along AB by a distance *h.* As a result point B'' is shifted along A'B'' by *y*. This way we find the correct locations of points B (= $B^1$) and B' (= $B'^1$).

5. The rocker pivot $O_2$ lies at the intersection of the horizontal line passing through point B and the line symmetrical to the BB' (Fig. 4f). The resulting mechanism is shown in Fig. 5.

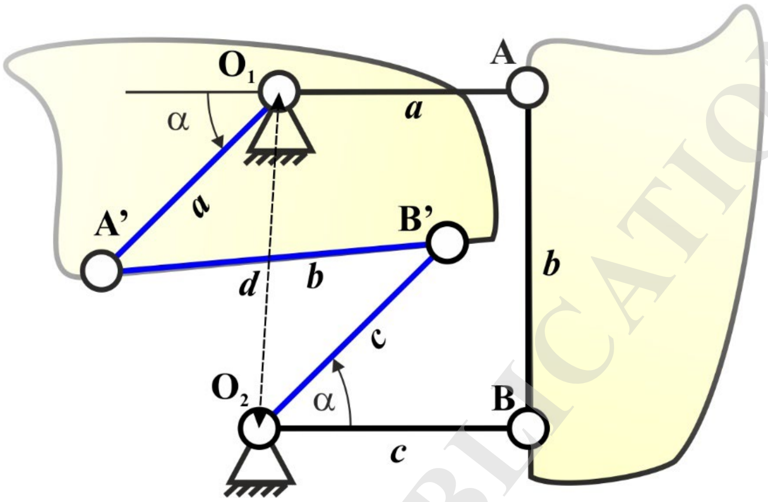



FIG. 5. The resulting mechanism for the case 2: $\frac{1}{2}\pi < \delta < \pi$.

Notice that, by similarity of triangles, the construction shown in Fig. 4c reflects the proportion:

$$\frac{h}{y} = \frac{\Delta}{x} \tag{1}$$

By combining the equality $\Delta = h + y$ with Eq. 1, we obtain the following proportion

$$\frac{\Delta + x}{\Delta} = \frac{\Delta}{h} \tag{2}$$

geometrically presented in Fig 4d. This way, using the Thales theorem we find *h* and *y*.

## 4. ANALYSIS OF THE SOLUTION

It should also be noted that for given values of α and δ, the only dimension that has been selected arbitrarily is the angle between the driving link $O_1A$ and the coupler AB in the initial position. For simplification of the solution procedure and the analysis of the solution's validity conditions, this angle is assumed to be ½π, although other values are also permissible. The dimension *b*, initially chosen arbitrarily, was ultimately determined—similarly to dimensions c and *d*. A change in dimension *a* does not affect the mechanism's structure, as it leads to a proportional change in the remaining dimensions. This results in an affinely transformed mechanism. However, a solution does not exist for all possible values of α and δ. The solution space is constrained by the Grashof conditions, and the condition of avoiding a defective solution due to the so-called branch defect (which is equivalent to circuit defect in case of crank-rocker mechanism) [35].

Let us determine the range of angles α and δ for which the construction guarantees a valid crank-rocker mechanism.

*4.1. The procedure validity*

We first justify that step 3, and consequently the subsequent steps of the algorithm, can be executed because γ is always greater than ½α. Let us observe that after initially assuming *b = a* (step 2), the angle γ is always greater than ¼π (Fig. 6a). Since α lies within the interval (0, ½π), the angle ½α lies within (0, ¼π), then γ is always greater than ½α. In consequence the points B' and $O_2$ lie to the left of AB.

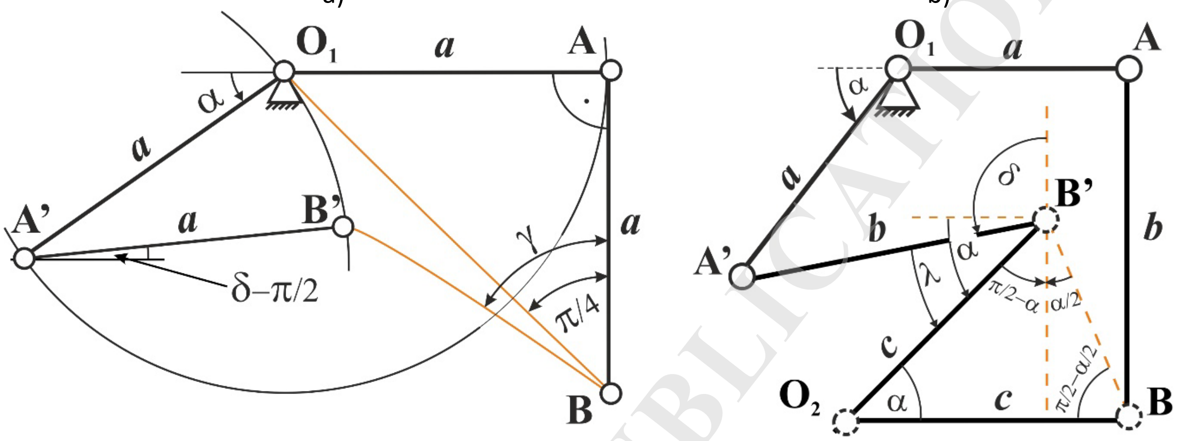



FIG. 6. a) Illustration that γ > ¼π (a); b) Illustration of angle λ for branch defect analysis.

*4.2. Branch defect analysis*

Let us verify whether both positions of the mechanism are achieved within the same configuration of the four-bar linkage. In other words, we are excluding the so-called branch defect (or *circuit defect*). In the same configuration the less angle between links AB and $BO_2$ in each mechanism position is measured in the same direction — either clockwise or counterclockwise. Since angle $\angle ABO_2$ is equal to π/2 and is measured counterclockwise, the less angle $\angle A'B'O_2$ λ must also be measured counterclockwise (Fig. 6b). Since $\delta + \lambda + \frac{\pi}{2} - \alpha = \pi$. Hence $\lambda = \frac{\pi}{2} + \alpha - \delta$ and it must be greater than 0 and less than $\pi$, which occurs when:

$$-\frac{\pi}{2} + \alpha < \delta < \frac{\pi}{2} + \alpha . \qquad (3)$$

Let us note that when $\alpha + \pi/2 = \delta$, the rocker coincides with the coupler, and this represents a singular configuration. If the condition (3) is not satisfied, the crank-rocker mechanism would not be able to transition between the extreme positions without disassembly.

*4.3. Grashof conditions*

For the resulting mechanism to operate as a crank-rocker mechanism, the Grashof conditions must be satisfied. The link $O_1A$, with an arbitrarily chosen length *a*, is the shortest link. The construction ensures that always *b* > *a*. Furthermore, $d^2 = b^2 + (c-a)^2$, hence *d* > *a*. It can be shown analytically that

$$c = a\frac{\sin\frac{\delta}{2} - \sin\left(\alpha - \frac{\delta}{2}\right)}{\sin\frac{\delta}{2} + \sin\left(\alpha - \frac{\delta}{2}\right)}. \tag{4}$$

Let us check when *c* > *a*:

$$a\frac{\sin\frac{\delta}{2} - \sin\left(\alpha - \frac{\delta}{2}\right)}{\sin\frac{\delta}{2} + \sin\left(\alpha - \frac{\delta}{2}\right)} > a,\ \frac{-2\sin\left(\alpha - \frac{\delta}{2}\right)}{\sin\frac{\delta}{2} + \sin\left(\alpha - \frac{\delta}{2}\right)} > 0,\ 2\sin\left(\alpha - \frac{\delta}{2}\right)\left(\sin\frac{\delta}{2} + \sin\left(\alpha - \frac{\delta}{2}\right)\right) < 0.$$

The dimension *c* is greater than *a* when

$$\alpha < \frac{\delta}{2} \tag{5}$$

In the following proofs we assume that Eq. (5) is met. We check whether the sum of the minimal and maximal link lengths is less than the sum of the remaining links.

1. Let us assume that dimension *d* is the largest. We will proceed with an indirect proof. Assume that the sum of the minimal and maximal link lengths is greater than the sum of the remaining link lengths: $a + d > b + c$, $d > b + (c - a)$, $d^2 = b^2 + (c - a)^2 > (b + (c - a))^2$. This leads to a contradiction; therefore, the assumption is false, and it must be that the sum of the extreme link lengths is less than the sum of the remaining link lengths: $a + d < b + c$.

2. Let us assume that the dimension *b* is the greatest. However, $d^2 = b^2 + (c - a)^2$, it follows that *b* is always less than *d*. Consequently, *b* cannot be the greatest.

3. Let us assume that *c* is the maximal. In this case, we provide a direct proof, that the sum of the minimal and maximal link lengths is lower than the sum of the remaining link lengths. Then

$a + c < d + b$, $d > (a + c) - b$, $d^2 = b^2 + (c - a)^2 > ((c + a) - b)^2$.
$d^2 = b^2 + (c - a)^2 > b^2 - 2b(c + a) + (c + a)^2$, $2ac < b(c + a)$, which is always true when Eq. (6) is met.

The Grashof conditions can be also proved geometrically. In summary, by combining condition (4) with condition (5), the following inequality guarantees obtaining a crank-rocker mechanism free of branch defect

$$2\alpha < \delta < \frac{\pi}{2} + \alpha \tag{6}$$

In summary, we also observe that the construction also covers the case in which the driving link rotates through an angle of $\pi - \alpha$. In that case, it is sufficient to reverse the direction of rotation of the link $O_1A$. In the range where dimension *c* is smaller than *a*, the link $O_2B$ becomes the crank and $O_1A$ becomes the rocker. It was shown that for a fixed angle $O_1AB = ½\pi$, the problem has exactly one solution. If, for functional reasons, the angle $O_1AB$ must differ from $½\pi$, the synthesis process must be carried out exactly as presented. However, a separate analysis of the Grashof conditions should be performed, and conditions must be established to ensure that branch defect does not occur.

### *4.4. Uniqueness of the solution*

We demonstrate that, when the angle $ABO_2$ is equal to $\frac{1}{2}\pi$, the presented configuration leads to a unique and correct solution to the synthesis problem. Let us consider alternative potential solution paths. Note that the links AB and A'B' can also be constructed along lines (1) and (2) in the opposite directions — i.e., to the left from point A' and upward from point A.

We now examine the case illustrated in Fig. 7, aiming to identify a position of joint B located in the upper half-plane with respect to A. Due to space limitations, only a proof sketch is presented here, with its essential elements depicted in Fig. 7. The segment $AB'^{1}$ is inclined at an angle of $\frac{1}{2}\alpha$ relative to the vertical axis. We begin with a situation where the length of the coupler in the second position is $|A'B'^{1}|$, whereas in the first position the coupler length in the initial configuration is effectively zero, i.e., $|AB| = |AA| = 0$. However, by translating the line $AB'^{1}$ parallel to itself, it becomes evident that the rate of increase in the length of link AB — denoted $\boldsymbol{n}_1$ — is greater than that of link A'B' — denoted $\boldsymbol{n}_1'$. Therefore, it is guaranteed that positions B and B' can be found such that $|AB| = |A'B'|$. Nevertheless, the configurations $O_1ABO_2$ and $O_1A'B'O_2$ belong to two distinct branches of the mechanism's configuration space (Fig. 8). The angle $ABO_2$ is measured in the opposite direction compared to $A'B'O_2$. This results in a solution that exhibits a branch defect - a condition where the same input motion leads to two distinct configurations. When the input link is rotated to position $O_1A'$, the remaining links will not attain the desired position $A'B'O_2$, but instead will settle into a configuration $A'B''O_2$.

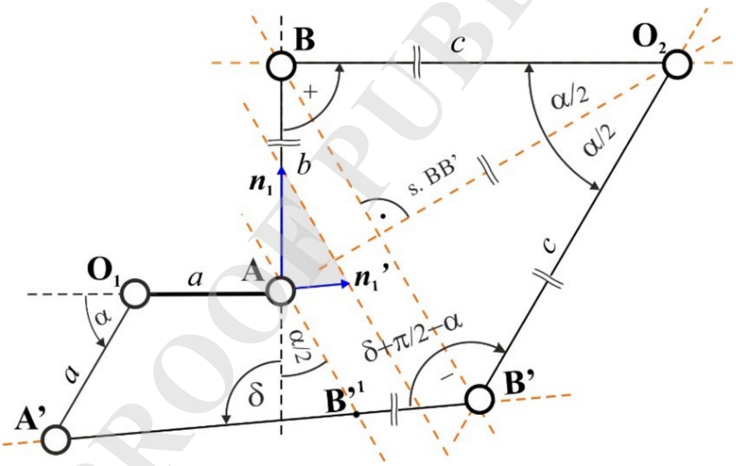



FIG. 7. The second solution of the problem (construction no 2).

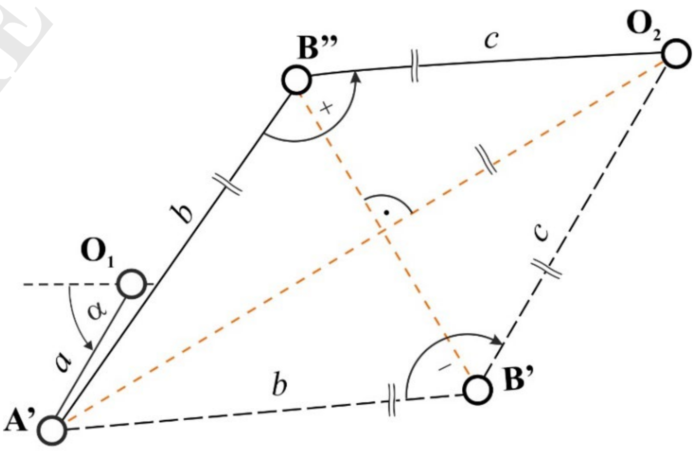



FIG. 8. Illustration for the branch defect of the second solution.

Having excluded the second construction due to branch defect, we now turn to a third possible configuration, illustrated in Fig. 9. We seek a configuration in which joint B is located in the upper half-plane with respect to A, while joint B' lies to the left of joint A'. However, it can be readily demonstrated that, under such conditions, it is impossible to satisfy the constraint |AB| = |A'B'|. The only exception occurs when α = ½π. Even in this special case, however, the resulting configuration would still exhibit a branch defect, rendering it invalid for the intended mechanism behavior.

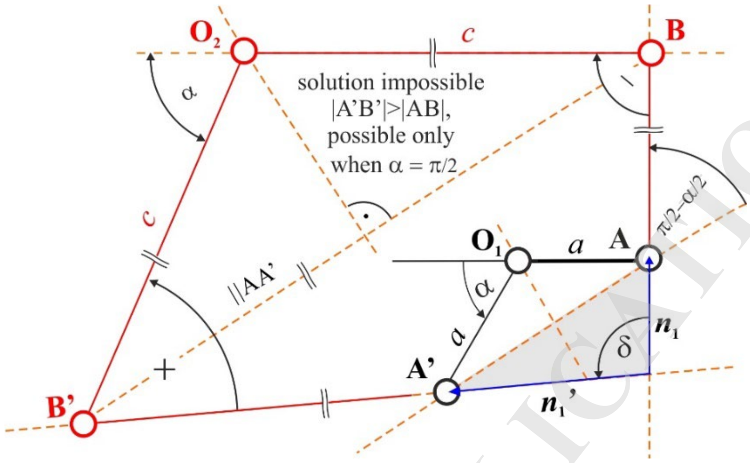



FIG. 9. Lack of the solution in case of construction no 3.

The configuration in which joint B lies in the bottom half-plane with respect to A and joint B' lies to the left of A' also does not yield a valid solution since in this case the condition |AB| < |A'B'| is always satisfied.

Consequently, all geometrically feasible constructions consistent with the imposed constraints have been systematically evaluated.

## 5. CONCLUSIONS

This paper has presented a novel graphical synthesis method for a four-bar linkage designed to achieve two prescribed angular reversal positions of the coupler — a problem not previously addressed in the literature. In contrast to many graphical methods that exhibit ambiguity related to trial-and-error selection of geometric parameters, the proposed approach guarantees solution correctness and uniqueness, while inherently satisfying the Grashof conditions from the outset. This stands in contrast to numerical methods, where constraint verification is often postponed until the final design stages.

The method is straightforward to implement and allows for clear visualization of the solution. It also enables more advanced kinematic analyses when needed. The resulting construction combines practical relevance with theoretical novelty, thereby expanding the range of design possibilities for mechanisms involving angular coupler dwells and reaffirming the continued usefulness of graphical techniques.

Although graphical methods are generally limited to simpler design problems, they provide intuitive insight into the problem-solving process by visually expressing fundamental kinematic relationships. A notable advantage of the proposed approach is its didactic value: it leverages kinematic properties of lever mechanisms — components that are seldom applied in practice and often poorly understood by students — while making them accessible and comprehensible through graphical representation.

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No potential conflict of interest was reported by the author.





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