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A DYNAMIC PROBLEM OF A CRACK IN A PLATE STRIP

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In this paper discussed is the quasi-static problem of displacement and stress distribution in an infinite elastic strip containing a semi-infinite crack located in its middle plane. The crack is assumed to propagate at a constant velocity along the straight line lying in the middle plane of the strip. Using the integral Fourier transforms, the problem is reduced to a corresponding Wiener-Hopf equation.

The value of the stress intensity factor at the tip of the crack is accurately determined. Numerical evaluation of the inverse Fourier transforms yields the distribution of stress and displacement components at an arbitrary point of the strip. The results are illustrated by graphs.

1. Formulation of the problem

Let us consider an infinite elastic strip of width $2h$, containing in its middle plane a rectilinear, semi-infinite crack (Fig. 1a). The surfaces of the crack are stress-free, and the external edges of the plate strip satisfy the following boundary conditions: horizontal (tangential) displacements vanish, and vertical (normal)

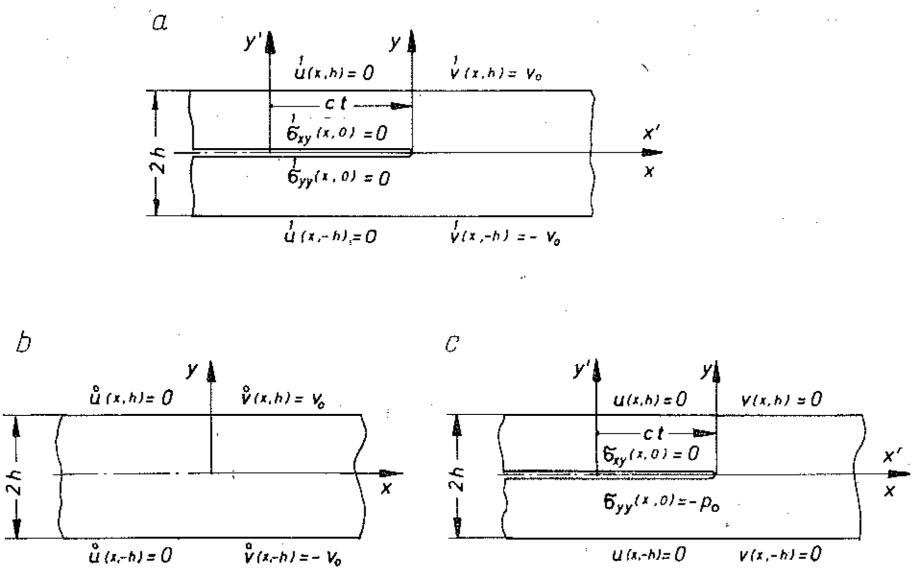


Fig. 1

(*) The paper was prepared during the second author's stay at the Munich Technological University, Institute A for Mechanics, sponsored by the Alexander von Humboldt Foundation displacements are constant and assume a prescribed value $v=\pm v_0$ (plus sign for the upper, and minus — for the lower edge).

The static version of the same problem was solved by W. G. KNAUSS in [1]. Here we shall tackle the more general problem: it will be assumed that the crack moves at a constant velocity c along the horizontal axis x' of a fixed, rectangular coordinate system (x', y').

Pursuant to the results obtained by several authors [2 – 5] who considered the critical value of crack propagation velocity in elastic media, the velocity will be assumed to be less than the Rayleigh surface wave propagation velocity. The plate strip will be assumed to satisfy the conditions of plane strain.

Applying the superposition principle, the solution may be represented in the form of a sum of the elementary solution of a strip without the crack (Fig. 1b), and the solution of a strip with a crack (Fig. 1c). The state of displacements and stresses in the case illustrated by Fig. 1b is given by the formulae

$$\overset{\circ}{u}(x, y)=0, \quad \overset{\circ}{v}(x, y)=\frac{v_0 y}{h},$$

(1.1)
$$\overset{\circ}{\sigma}_{xx}(x, y)=\frac{2\mu\nu}{1-2\nu}\frac{v_0}{h},$$
$$\overset{\circ}{\sigma}_{yy}(x, y)=\frac{2\mu(1-\nu)}{1-2\nu}\frac{v_0}{h},$$
$$\overset{\circ}{\sigma}_{xy}(x, y)=0;$$

ν denoting Poisson's ratio, and μ — the elastic Lamé constant.

In the case illustrated by Fig. 1c, the edges of the strip are rigidly clamped, and the crack surfaces are subject to the normal loads $\sigma_{yy}(x, 0)=-p_0$, with p_0 equal to

(1.2)
$$p_0=\overset{\circ}{\sigma}_{yy}(x, 0)=\frac{2\mu(1-\nu)}{1-2\nu}\frac{v_0}{h},$$

This is the problem which will be dealt with in this paper; owing to its quasi-static character, a convective reference frame (x, y) will be introduced according to the transformation formula

(1.3)
$$x'=x+ct, \quad y'=y,$$

Here c denotes the constant velocity of motion of the new system with respect to the fixed one (x', y').

The conditions of symmetry following from the assumptions illustrated by Fig. 1c reduce the problem considered to the determination of displacements and stresses in an infinite plate strip of width h with the following boundary conditions written in the convective coordinate system (x, y):

(1.4)
$$\begin{aligned} u(x, h)=v(x, h)&=0 && \text{for} \quad |x|<\infty, \\ \sigma_{xy}(x, 0)&=0 && \text{for} \quad |x|<\infty, \\ v(x, 0)&=0 && \text{for} \quad x>0, \\ \sigma_{yy}(x, 0)&=-p_0 && \text{for} \quad x<0. \end{aligned}$$

The equations of motion written for the plane state of strain are known to have the form

$$(1.5)\qquad \mu\nabla'^2 u+(\lambda+\mu)\theta'_{,x'}=\rho\ddot{u}, \qquad \mu\nabla'^2 v+(\lambda+\mu)\theta'_{,y'}=\rho\ddot{v}.$$

Here λ, μ are Lamé constants, ρ — material density, and the respective operators ∇'^2 and θ' denote the Laplacean and dilatation referred to the immobile coordinate system, $(x'\, y')$.

Expressing the displacements u, v in terms of the scalar and vector potentials φ, ψ,

$$(1.6)\qquad u=\varphi_{,x}+\psi_{,y}, \qquad v=\varphi_{,y}-\psi_{,x},$$

and using the Eq. (1.3), we obtain the equations of motion (1.5) transformed to the convective coordinate system (x, y), namely:

$$(1.7)\qquad \beta_1^2\varphi_{,xx}+\varphi_{,yy}=0, \qquad \beta_2^2\psi_{,xx}+\psi_{,yy}=0.$$

Here,

$$\beta_1^2=1-c^2/c_1^2, \qquad \beta_2^2=1-c^2/c_2^2,$$

and c_1, c_2 denote the respective velocities of longitudinal and transversal elastic waves; $c_1^2=(\lambda+2\mu)/\rho$, $c_2^2=\mu/\rho$. By means of the Eqs. (1.6), (1.7), we obtain then the expressions for the stresses and displacements in the rectangular coordinate system (x, y),

$$(1.8)\qquad \begin{aligned} \sigma_{xx}(x, y)&=\mu[(1+2\beta_1^2-\beta_2^2)\varphi_{,xx}+2\psi_{,xy}],\\ \sigma_{yy}(x, y)&=-\mu[(1+\beta_2^2)\varphi_{,xx}+2\psi_{,xy}],\\ \sigma_{xy}(x, y)&=\mu[2\varphi_{,xy}-(1+\beta_2^2)\psi_{,xx}]. \end{aligned}$$

The considerations to follow, consisting in the determination of functions φ and ψ satisfying the boundary conditions, (1.4), are based on the two-sided integral Fourier transform defined by the formulae [6]

$$(1.9)\qquad F(\alpha, y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x, y)\, e^{i\alpha x}\, dx, \qquad f(x, y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty+i\tau_0}^{\infty+i\tau_0} F(\alpha, y)\, e^{-i\alpha x}\, d\alpha,$$

with the integration parameter being a complex variable $\alpha=\sigma+i\tau$, and the path of integration in the Eq. $(1.9)_2$ lying, within the strip $\tau_-<\mathrm{Im}\,\alpha<\tau_+$ of regularity of the function $F(\alpha, y)$. In addition, observe that the function $F(\alpha, y)$ may be represented in the form:

$$(1.10)\qquad F(\alpha, y)=F^-(\alpha, y)+F^+(\alpha, y),$$

where the one-sided Fourier transforms

$$(1.11)\qquad F^-(\alpha, y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} f(x, y)\, e^{i\alpha x}\, dx, \qquad F^+(\alpha, y)=\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} f(x, y)\, e^{i\alpha x}\, dx$$

are functions analytical within the respective halfplanes $\mathrm{Im}\,\alpha<\tau_+$ and $\mathrm{Im}\,\alpha>\tau_-$.

Applying the Fourier transform (1.9) to the fundamental set of Eqs. (1.7) we obtain the following two equations:

$$\Phi_{,yy}-\alpha^2\beta_1^2\Phi=0, \qquad \Psi_{,yy}-\alpha^2\beta_2^2\Psi=0.$$

Solutions of these are then represented in the form:

$$\begin{aligned}\Phi(\alpha, y)&=C_1(\alpha)\,\mathrm{sh}\,\alpha\beta_1 y+C_2(\alpha)\,\mathrm{ch}\,\alpha\beta_1 y,\\ \Psi(\alpha, y)&=C_3(\alpha)\,\mathrm{sh}\,\alpha\beta_2 y+C_4(\alpha)\,\mathrm{ch}\,\alpha\beta_2 y,\end{aligned} \tag{1.12}$$

which, by means of the transforms (1.9) applied to the Eqs. (1.6), (1.8), yields the F-transforms of displacements and stresses:

$$\begin{aligned}U(\alpha, y)&=-i\alpha\{C_1(\alpha)\,\mathrm{sh}\,\alpha\beta_1 y+C_2(\alpha)\,\mathrm{ch}\,\alpha\beta_1 y+i\beta_2[C_3(\alpha)\,\mathrm{ch}\,\alpha\beta_2 y+\\ &\qquad +C_4(\alpha)\,\mathrm{sh}\,\alpha\beta_2 y]\},\\ V(\alpha, y)&=\alpha\{\beta_1[C_1(\alpha)\,\mathrm{ch}\,\alpha\beta_1 y+C_2(\alpha)\,\mathrm{sh}\,\alpha\beta_1 y]+i[C_3(\alpha)\,\mathrm{sh}\,\alpha\beta_2 y+\\ &\qquad +C_4(\alpha)\,\mathrm{ch}\,\alpha\beta_2 y]\},\\ \Sigma_{xx}(\alpha, y)&=-\mu\alpha^2\{(1+2\beta_1^2-\beta_2^2)[C_1(\alpha)\,\mathrm{sh}\,\alpha\beta_1 y+C_2(\alpha)\,\mathrm{ch}\,\alpha\beta_1 y]+\\ &\qquad +2i\beta_2[C_3(\alpha)\,\mathrm{ch}\,\alpha\beta_2 y+C_4(\alpha)\,\mathrm{sh}\,\alpha\beta_2 y]\},\\ \Sigma_{yy}(\alpha, y)&=\mu\alpha^2\{(1+\beta_2^2)[C_1(\alpha)\,\mathrm{sh}\,\alpha\beta_1 y+C_2(\alpha)\,\mathrm{ch}\,\alpha\beta_1 y]+\\ &\qquad +2i\beta_2[C_3(\alpha)\,\mathrm{ch}\,\alpha\beta_2 y+C_4(\alpha)\,\mathrm{sh}\,\alpha\beta_2 y]\},\\ \Sigma_{xy}(\alpha, y)&=-i\mu\alpha^2\{2\beta_1[C_1(\alpha)\,\mathrm{ch}\,\alpha\beta_1 y+C_2(\alpha)\,\mathrm{sh}\,\alpha\beta_1 y]+\\ &\qquad +i(1+\beta_2^2)[C_3(\alpha)\,\mathrm{sh}\,\alpha\beta_2 y+C_4(\alpha)\,\mathrm{ch}\,\alpha\beta_2 y]\}.\end{aligned} \tag{1.13}$$

The unknown functions $C_i(\alpha)$ must be determined from the boundary conditions of the problem considered.

2. Solution

Let us now apply the F-transform (1.9) to the boundary conditions (1.4) and use the Eqs. (1.10). We obtain

$$\begin{aligned}&U(\alpha, h)=V(\alpha, h)=\Sigma_{xy}(\alpha, 0)=0,\\ &V(\alpha, 0)=V^-(\alpha, 0),\\ &\Sigma_{yy}(\alpha, 0)=\Sigma_{yy}^-(\alpha, 0)+\Sigma_{yy}^+(\alpha, 0),\end{aligned} \tag{2.1}$$

Here,

$$\begin{aligned}V^-(\alpha, 0)&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} v(x, 0)\,e^{i\alpha x}\,dx, \quad \text{reg. for } \operatorname{Im}\alpha<\tau_1,\\ \Sigma_{yy}^+(\alpha, 0)&=\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}\sigma_{yy}(x, 0)\,e^{i\alpha x}\,dx \quad \text{reg. for } \operatorname{Im}\alpha>-\tau_2,\end{aligned} \tag{2.2}$$

$$\Sigma_{yy}^-(\alpha, 0)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0}\sigma_{yy}(x, 0)\,e^{i\alpha x}\,dx=-\frac{p_0}{\sqrt{2\pi}}\begin{cases}-\dfrac{i}{\alpha} & \text{reg. for } \operatorname{Im}\alpha<0,\\ -\dfrac{i}{\sigma}+\pi\delta(\sigma) & \text{for } \operatorname{Im}\alpha=0;\end{cases}$$

τ_1 and τ_2 are certain small, positive numbers; the common region of regularity of the transforms (2.2) is initially assumed to be the strip $-\tau_2 < \operatorname{Im} \alpha < 0$.

The transform Σ_{yy}^- may be expressed by means of the Dirac delta-function,

$$\Sigma_{yy}^-(\alpha, 0) = -\frac{p_0}{\sqrt{2\pi}}\left[-\frac{i}{\alpha} + \pi\delta(\alpha)\right] \tag{2.3}$$

and then the strip of regularity $-\tau_2 < \operatorname{Im} \alpha < 0$ of the transforms (2.2) may be supplemented by the real axis $\operatorname{Im} \alpha = 0$, except the origin $\alpha = 0$ of the coordinate system. Owing to the regularity strip thus defined, we avoid certain difficulties in numerical evaluation of the inverse F-transform, since the path of integration in the Eq. (1.9) may be assumed to be the real axis $\operatorname{Im} \alpha = 0$.

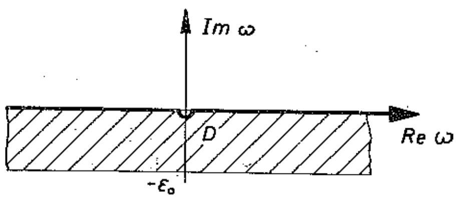


Fig. 2

By means of the formulae (1.13), (2.1), (2.3), the problem of determining the unknown functions $C_i(\alpha)$ is reduced to the solution of a Wiener-Hopf type equation; with the notations

$$\omega = \alpha h = \lambda + i\varepsilon, \qquad V^-(\alpha, 0) = V^-(\omega) \quad \text{and} \quad \Sigma_{yy}^+(\alpha, 0) = \Sigma_{yy}^+(\omega),$$

the corresponding equation takes the form:

$$V^-(\omega) = -\frac{\gamma h}{4\mu\beta_2} H(\omega)\left[\Sigma_{yy}^+(\omega) - P(\omega)\right] \tag{2.4}$$

Here,

$$H(\omega) = \frac{(1-\beta_2^2)\operatorname{ch}\omega\beta_1 \operatorname{ch}\omega\beta_2(\operatorname{th}\omega\beta_2 - \beta_1\beta_2\operatorname{th}\omega\beta_1)}{\omega\left[(\gamma+\beta_1\beta_2)\operatorname{ch}\omega\beta_1\operatorname{ch}\omega\beta_2 - (1+\gamma\beta_1\beta_2)\operatorname{sh}\omega\beta_1\operatorname{sh}\omega\beta_2 - \gamma(1+\beta_2^2)\right]},$$
$$P(\omega) = \frac{hp_0}{\sqrt{2\pi}}\left[-\frac{i}{\omega} + \pi\delta(\omega)\right], \qquad \gamma = \frac{4\beta_1\beta_2}{(1+\beta_2^2)^2}, \tag{2.5}$$

its region of existence being the strip D shown in Fig. 2. The equation is solved by the factorization method [7]; to this end, the function $(2.5)_1$ must be factorized first. The corresponding procedure described in [8] requires the function $H(\omega)$ to be represented in the form

$$H(\omega) = \bar{H}(\omega)\, H_1(\omega), \tag{2.6}$$

satisfying the following three requirements:

(1) Functions $H(\omega)$ and $\bar{H}(\omega)$ must behave identically at infinity, $|\omega| \to \infty$, and at zero, $|\omega| \to 0$;

(2) Functions $H(\omega)$ and $\bar{H}(\omega)$ should behave similarly within the region of existence of the Wiener-Hopf equation; in the case considered this concerns the neighbourhood of the real axis $\operatorname{Im} \alpha = 0$;

(3) Function $H_1(\omega)$ cannot have any zeros or singularity points in the strip $|\operatorname{Im} \omega| < \varepsilon_1$, with $0 < \varepsilon_1 < \varepsilon_0$.

A function satisfying the requirements presented above may be assumed in the form:

$$\bar{H}(\omega)=\frac{1-\beta_2^2}{\gamma-1}\frac{1}{\sqrt{\omega^2+A^2}}\left[1+\frac{2\lambda_0\vartheta\omega^2}{\omega^4+\lambda_0^4}\right], \tag{2.7}$$

with

$$A=\frac{\gamma(1-\beta_2^2)^2}{4\beta_2(\gamma-1)(1-\beta_1^2)}, \tag{2.8}$$

and the remaining two magnitudes $\lambda_0=\lambda_0(c)$ and $\vartheta=\vartheta(c)$ are certain functions of the crack propagation velocity c.

In the case of the Poisson ratio $\nu=0.25$, the approximations for λ_0 and ϑ are

$$\lambda_0(c)=2.5+5(c/c_2)^4;$$
$$\vartheta(c)=0.2-0.0211(c/c_2)^2+0.5345(c/c_2)^4-0.2279(c/c_2)^6+0.9494(c/c_2)^8,$$

and the values of $H(\omega)$ and of the relative errors $\Delta F=[(H-\bar{H})/H]\ 100\%$ are given in Table 1.

Table 1

ω	$c/c_2=0.2$		$c/c_2=0.4$		$c/c_2=0.6$		$c/c_2=0.8$	
	$H(\omega)$	ΔF	$H(\omega)$	ΔF	$H(\omega)$	ΔF	$H(\omega)$	ΔF
0	1.29	0	1.16	0	0.96	0	0.70	0
1	1.27	1.23	1.16	1.32	0.98	1.74	0.74	2.34
2	1.14	−0.81	1.08	−0.21	0.97	0.99	0.82	4.55
3	0.93	−0.83	0.91	−0.24	0.88	−1.03	0.89	3.52
5	0.60	0.38	0.59	1.00	0.61	−0.52	0.79	−2.23
10	0.30	0.15	0.30	0.36	0.31	−0.10	0.42	−2.49
50	0.06	0.01	0.06	0.02	0.06	0.00	0.08	−0.09
∞	0	0	0	0	0	0	0	0

Prior to factorization of $\bar{H}(\omega)$ let us introduce the following notations:

$$M(\omega)=M^-(\omega)M^+(\omega)=\omega^4+2\lambda_0^2\vartheta\omega^2+\lambda_0^4,$$
$$L(\omega)=L^-(\omega)L^+(\omega)=\omega^4+\lambda_0^4,$$
$$R(\omega)=R^-(\omega)R^+(\omega)=\sqrt{\omega^2+A^2},$$

and

$$\begin{aligned} &M^-(\omega)=(\omega-\omega_1^M)(\omega-\omega_2^M), \quad M^+(\omega)=(\omega-\bar{\omega}_1^M)(\omega-\bar{\omega}_2^M),\\ &L^-(\omega)=(\omega-\omega_1^L)(\omega-\omega_2^L), \quad L^+(\omega)=(\omega-\bar{\omega}_1^L)(\omega-\bar{\omega}_2^L),\\ &R^\pm(\omega)=\sqrt{\omega\pm iA}. \end{aligned} \tag{2.9}$$

In the case $0 \leqslant \vartheta < 1$, the zeros $\omega^M_{1,2}$ of $M(\omega)$ are found from the formulae

$$(2.10)\quad \begin{aligned} \omega^M_1 &= \frac{\lambda_0}{\sqrt{2}}\left(\sqrt{1-\vartheta}+i\sqrt{1+\vartheta}\right), & \omega^M_2 &= \frac{\lambda_0}{\sqrt{2}}\left(-\sqrt{1-\vartheta}+i\sqrt{1+\vartheta}\right),\\ \bar{\omega}^M_1 &= \frac{\lambda_0}{\sqrt{2}}\left(\sqrt{1-\vartheta}-i\sqrt{1+\vartheta}\right), & \bar{\omega}^M_2 &= \frac{\lambda_0}{\sqrt{2}}\left(-\sqrt{1-\vartheta}-i\sqrt{1+\vartheta}\right),\end{aligned}$$

and the zeros $\omega^L_{1,2}$ of $L(\omega)$ follow directly from the Eqs. (2.10) by assuming in them $\vartheta=0$.

Using the same notations, the function (2.7) may be represented finally in the form

$$(2.11)\qquad \bar{H}(\omega)=\frac{1-\beta_2^2}{\gamma-1}K^-(\omega)K^+(\omega),$$

Here,

$$(2.12)\qquad K^\pm(\omega)=\frac{M^\pm(\omega)}{R^\pm(\omega)L^\pm(\omega)},$$

Functions $K^\pm(\omega)$ determined in this manner are analytical in the respective halfplanes $\operatorname{Im}\omega > -\varepsilon_0$ and $\operatorname{Im}\omega \leqslant 0$.

Assumption of the function $\bar{H}(\omega)$ in the form (2.7) satisfies the requirement concerning $H_1(\omega)$ and owing to the fact that $H_1(\omega)$ tends to unity within the strip $|\operatorname{Im}\omega| < \varepsilon_1$ for $|\omega| \to \infty$, the function may be written in the form [7]:

$$(2.13)\qquad H_1(\omega)=\frac{H_1^+(\omega)}{H_1^-(\omega)},$$

Here,

$$\ln H_1^+(\omega)=\frac{1}{2\pi i}\int_{-\infty+i\gamma_2}^{\infty+i\gamma_2}\frac{\ln H_1(z)}{z-\omega}dz,\qquad \ln H_1^-(\omega)=\frac{1}{2\pi i}\int_{-\infty+i\gamma_1}^{\infty+i\gamma_1}\frac{\ln H_1(z)}{z-\omega}dz,$$

and $-\varepsilon_1<\gamma_2<\gamma_1<\varepsilon_1$.

Such functions $H_1^\pm(\omega)$ have no zeros and singularities in the respective halfplanes $\operatorname{Im}\omega>\gamma_2$ and $\operatorname{Im}\omega<\gamma_1$ and, in view of $H_1(0)=H_1(\infty)=1$, they satisfy the additional condition $H_1^\pm(0)=H_1^\pm(\infty)=1$.

Consequently, making use of the relations (2.11), (2.13), the function $\bar{H}(\omega)$ may be written, by means of the Eqs. (2.6), in the form:

$$(2.14)\qquad \bar{H}(\omega)=\frac{1-\beta_2^2}{\gamma-1}\,\frac{K^-(\omega)K^+(\omega)H_1^+(\omega)}{H_1^-(\omega)}.$$

After substituting the relation (2.14) into the Eq. (2.4), we find:

$$(2.15)\qquad -\frac{4\mu\beta_2(\gamma-1)}{\gamma h(1-\beta_2^2)}\,\frac{V^-(\omega)H_1^-(\omega)}{K^-(\omega)}=K^+(\omega)H_1^+(\omega)\Sigma^+_{yy}(\omega)-E(\omega),$$

with the notation

$$E(\omega)=K^+(\omega)H_1^+(\omega)P(\omega).$$

Factorization of the function $E(\omega)$ — i.e., its representation in the form

$$E^{+}(\omega)=E^{+}(\omega)-E^{-}(\omega)$$

in which

$$\begin{aligned} E^{+}(\omega)&=[K^{+}(\omega)H_1^{+}(\omega)-K^{+}(0)H_1^{+}(0)]P(\omega),\\ E^{-}(\omega)&=-K^{+}(0)H_1^{+}(0)P(\omega), \end{aligned} \tag{2.16}$$

enables us to write the Eq. (2.15) in the form:

$$-\frac{4\mu\beta_2(\gamma-1)}{\gamma h(1-\beta_2^2)}\frac{V^{-}(\omega)H_1^{-}(\omega)}{K^{-}(\omega)}-E^{-}(\omega)=K^{+}(\omega)H_1^{+}(\omega)\Sigma_{yy}^{+}(\omega)-E^{+}(\omega).$$

Applying now the generalized Liouville theorem we obtain the solution of the Eq. (2.4):

$$\begin{aligned} V^{-}(\omega)&=-\frac{\gamma h(1-\beta_2^2)}{4\mu\beta_2(\gamma-1)}\frac{E^{-}(\omega)K^{-}(\omega)}{H_1^{-}(\omega)} \qquad \text{reg. for } \operatorname{Im}\omega<0,\\ \Sigma_{yy}^{+}(\omega)&=\frac{E^{+}(\omega)}{K^{+}(\omega)H_1^{+}(\omega)} \qquad \text{reg for } \operatorname{Im}\alpha>-\varepsilon_0. \end{aligned} \tag{2.17}$$

These formulae enable us to determine analytically the exact value of the stress intensity factor, as also to calculate the approximate distribution of the displacement v along the crack edges, and of the stress σ_{yy} along the crack extension.

The formulae (2.17) also yield the functions $C_i(\omega)$,

$$\begin{aligned} C_1(\omega)&=hC_1^{*}(\omega)V^{-}(\omega), \qquad C_2(\omega)=hC_2^{*}(\omega)V^{-}(\omega),\\ C_3(\omega)&=-ihC_3^{*}(\omega)V^{-}(\omega), \qquad C_4(\omega)=-ihC_4^{*}(\omega)V^{-}(\omega), \end{aligned} \tag{2.18}$$

Here

$$\begin{aligned} C_1^{*}(\omega)&=-\frac{1+\beta_2^2}{\beta_1(1-\beta_2^2)}\frac{1}{\omega}, \qquad C_4^{*}(\omega)=\frac{2}{1-\beta_2^2}\frac{1}{\omega},\\ C_2^{*}(\omega)&=-\frac{2\beta_2}{(1-\beta_2^2)\omega D(\omega)}\left[(1+\beta_2^2)\operatorname{ch}\omega\beta_1-2\operatorname{ch}\omega\beta_2+\frac{2(1-\beta_2^2)\operatorname{sh}\omega\beta_2}{\gamma\omega H(\omega)}\right],\\ C_3^{*}(\omega)&=\frac{1+\beta_2^2}{(1-\beta_2^2)\omega D(\omega)}\left[(1+\beta_2^2)\operatorname{ch}\omega\beta_1-2\operatorname{ch}\omega\beta_2+\frac{(1-\beta_2^4)\operatorname{sh}\omega\beta_1}{\omega H(\omega)}\right],\\ D(\omega)&=(1+\beta_2^2)\operatorname{sh}\omega\beta_2-2\beta_1\beta_2\operatorname{sh}\omega\beta_1. \end{aligned} \tag{2.19}$$

By introducing these relations into the Eqs. (1.13), performing the numerical evaluation of the inverse F-transform $(1.9)_2$ along the real axis $\operatorname{Im}\omega=0$, we obtain the complete solution of the problem under consideration.

3. Stress intensity factor

According to the well-known Abel theorem [9], once the asymptotic behaviour of the F-transforms at $|\alpha|\to\infty$ is known, we can determine the behaviour of the corresponding inverse transforms for $|x|\to 0$. By means of the Eqs. (2.12), (2.16), (2.17), and making use of the fact that $H_1^{\pm}(\infty)=1$ it may be demonstrated that with $|\alpha|\to\infty$

$$V^-(\alpha) = -\frac{ip_0\gamma\sqrt{h}\,(1-\beta_2^2)}{4\mu\beta_2(\gamma-1)\sqrt{2\pi i A}}\,\frac{1}{\alpha\sqrt{\alpha}}, \qquad \Sigma_{yy}^+(\alpha) = \frac{ip_0\sqrt{h}}{\sqrt{2\pi i A}}\,\frac{1}{\sqrt{\alpha}}.$$

The Abel theorem cited above yields now the conclusion that for $|\xi|=|x/h|\to 0$ the displacement v and stress σ_{yy} assume the form

$$(3.1)\qquad \begin{aligned} v(\xi,0) &= \frac{\gamma h(1-\beta_2^2)N(c)}{2\mu\beta_2(\gamma-1)}\sqrt{-\xi} && \text{for } \xi\to -0,\\ \sigma_{yy}(\xi,0) &= \frac{N(c)}{\sqrt{\xi}} && \text{for } \xi\to +0, \end{aligned}$$

with the notation

$$(3.2)\qquad N(c) = \frac{p_0}{\sqrt{\pi A}} = \frac{2\mu v_0(1-\nu)}{h(1-2\nu)\sqrt{\pi A}},$$

the value of A being given by the Eq. (2.8).

From the Eq. (3.2) it follows that increasing crack propagation velocities and increasing width of the strip makes the stress intensity factor decrease, and for c equal to the velocity of propagation of the surface Rayleigh waves, the stress intensity factor $N(c)$ becomes equal to zero.

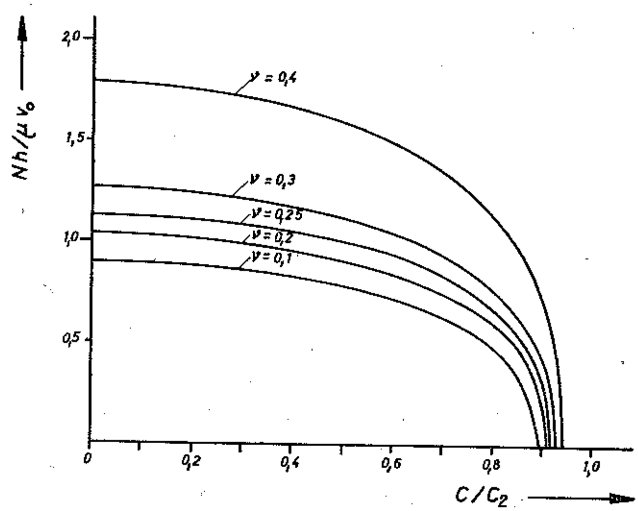


Fig. 3

In the case of $c \to 0$ we obtain the solution of the static problem, the stress intensity factor assuming the value:

$$N_s = \frac{2\mu v_0}{h\sqrt{2\pi(1-2\nu)}}.$$

The graph showing the dependence of $N(c)$ on the crack propagation velocity as given by the Eq. (3.2) is shown in Fig. 3; Poisson's ratio ν is assumed to be equal to 0.25.

4. Displacements and stresses in the plane $y=0$.

Performing the inverse F-transforms on the Eqs. (2.17), and assuming $H_1^{\pm}(\omega)=1$, we may find the approximate distribution of displacements v on the surface of the crack, and the stresses σ_{yy} along its extension;

$$(4.1)\qquad \begin{aligned} v(\xi, 0) &= -\frac{\gamma(1-\beta_2^2)}{4\mu\beta_2(\gamma-1)\sqrt{2\pi}} \int_{-\infty-i\varepsilon_1}^{\infty-i\varepsilon_1} E^-(\omega) K^-(\omega) e^{-i\omega\xi} d\omega \quad \text{for } \xi<0, \\ \sigma_{yy}(\xi, 0) &= \frac{1}{h\sqrt{2\pi}} \int_{-\infty-i\varepsilon_1}^{\infty-i\varepsilon_1} \frac{E^+(\omega)}{K^+(\omega)} e^{-i\omega\xi} d\omega \quad \text{for } \xi>0, \end{aligned}$$

here $\xi = x/h$.

With the notations

$$Q_0(\xi) = \int_{-\infty-i\varepsilon_1}^{\infty-i\varepsilon_1} \frac{\sqrt{\omega+iA}}{\omega} e^{-i\omega\xi} d\omega, \qquad Q(z, \xi) = \int_{-\infty-i\varepsilon_1}^{\infty-i\varepsilon_1} \frac{\sqrt{\omega+iA}}{\omega - z} e^{-i\omega\xi} d\omega,$$

$$K(z, \xi) = \int_{-\infty-i\varepsilon_1}^{\infty-i\varepsilon_1} \frac{e^{-i\omega\xi}}{(\omega-z)\sqrt{\omega-iA}} d\omega, \qquad \kappa_1 = \sqrt{1+\vartheta} - 1, \qquad \kappa_2 = \frac{\kappa_1}{\sqrt{1-\vartheta}},$$

and by substituting by the Eqs. (2.12), (2.16) into (4.1), use being made of certain properties of the complex variable functions and the delta-function, the displacement $v(\xi, 0)$ and stresses $\sigma_{yy}(\xi, 0)$ assume the form

$$(4.2)\qquad \begin{aligned} v(\xi, 0) &= \frac{ip_0\gamma h(1-\beta_2^2)}{8\pi\mu\beta_2(\gamma-1)\sqrt{iA}} \{K(0, \xi) - i\kappa_1[K(\omega_1^L, \xi) - K(\omega_2^L, \xi)]\} \quad \text{for } \xi<0, \\ \sigma_{yy}(\xi, 0) &= -\frac{ip_0}{2\pi\sqrt{iA}} \{Q_0(\xi) - i\kappa_2[Q(\overline{\omega}_1^M, \xi) - Q(\overline{\omega}_2^M, \xi)]\} \quad \text{for } \xi>0. \end{aligned}$$

The values of ω_i^L and ω_i^M are given by the Eqs. (2.10). Integrations in the Eqs. (4.2) performed, we may find the displacements $\overset{1}{v}(\xi, 0)$ and stresses $\overset{1}{\sigma}_{yy}(\xi, 0)$ on the surfaces of the crack and along its extension according to the original formulation of the problem (Fig. 1a),

$$(4.3)\qquad \overset{1}{v}(\xi, 0) = \begin{cases} v(\xi, 0) & \text{for } \xi<0, \\ 0 & \text{for } \xi>0, \end{cases} \qquad \overset{1}{\sigma}_{yy}(\xi, 0) = \begin{cases} 0 & \text{for } \xi<0, \\ p_0 + \sigma_{yy}(\xi, 0) & \text{for } \xi>0, \end{cases}$$

Here

$$v(\xi, 0)=v_0\left\{\operatorname{erf}\sqrt{-A\xi}+i\kappa_1\sqrt{A}\left[\frac{e^{-i\omega_1^L\xi}}{\sqrt{A+i\omega_1^L}}\operatorname{erf}\sqrt{-\xi(A+i\omega_1^L)}+\right.\right.$$

$$\left.\left.-\frac{e^{-i\omega_2^L\xi}}{\sqrt{A+i\omega_2^L}}\operatorname{erf}\sqrt{-\xi(A+i\omega_2^L)}\right]\right\}, \tag{4.4}$$

$$\sigma_{yy}(\xi, 0)=\frac{p_0}{\pi\sqrt{A}}\left\{\sqrt{\frac{\pi}{\xi}}\,e^{-A\xi}-\pi\sqrt{A}\operatorname{Erfc}\sqrt{A\xi}+i\pi\kappa_2\times\right.$$

$$\left.\times\left[\sqrt{A-i\varpi_1^M}\,e^{-i\varpi^M\xi}\operatorname{erf}\sqrt{\xi(A-i\varpi_1^M)}-\sqrt{A-i\varpi_2^M}\,e^{-i\varpi^M\xi}\operatorname{erf}\sqrt{\xi(A-i\varpi_2^M)}\right]\right\}.$$

The graph of boundary values of the displacement $\overset{1}{v}(\xi, 0)$ for various crack propagation velocities and $\nu=0.25$ is shown in Fig. 4. The boundary values of stresses $\overset{1}{\sigma}_{yy}(\xi, 0)$ for $c/c_2=0.8$ and $\nu=0.25$ is demonstrated in Fig. 5.

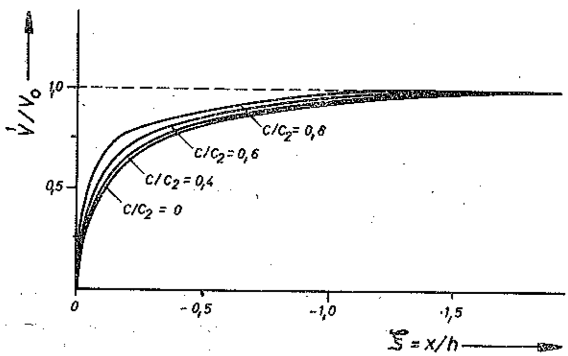


Fig. 4

5. Displacement and stress distribution inside the strip

Inserting the relations (2.18) into the Eq. (1.13), performing the inverse F-transform $(1.9)_2$, we obtain the distribution of displacements and stresses at an arbitrary point of the strip.

Analytical evaluation of the corresponding integrals in the case considered here is not possible and hence the inverse F-transform is calculated numerically along the real axis $\omega=\lambda$ (Fig. 2). To this end we put $\eta=y/h$, $\omega=\lambda$ and $H_1^{\pm}(\lambda)=1$, substitute the relations (2.18) into the Eqs. (1.13), and perform the inverse integral transform prescribed; we obtain:

$$u(\xi, \eta)=\frac{i}{h\sqrt{2\pi}}\int_{-\infty}^{\infty}Z_u(\lambda, \eta)\,V^-(\lambda)\,e^{-i\lambda\xi}\,d\lambda,$$

$$v(\xi, \eta)=\frac{1}{h\sqrt{2\pi}}\int_{-\infty}^{\infty}Z_v(\lambda, \eta)\,V^-(\lambda)\,e^{-i\lambda\xi}\,d\lambda, \tag{5.1}$$

(5.1) [c.d.]

$$\sigma_{xx}(\xi,\eta)=\frac{\mu}{h^2\sqrt{2\pi}}\int_{-\infty}^{\infty}Z_{xx}(\lambda,\eta)V^-(\lambda)e^{-i\lambda\xi}d\lambda,$$

$$\sigma_{yy}(\xi,\eta)=\frac{\mu}{h^2\sqrt{2\pi}}\int_{-\infty}^{\infty}Z_{yy}(\lambda,\eta)V^-(\lambda)e^{-i\lambda\xi}d\lambda,$$

$$\sigma_{xy}(\xi,\eta)=\frac{i\mu}{h^2\sqrt{2\pi}}\int_{-\infty}^{\infty}Z_{xy}(\lambda,\eta)V^-(\lambda)e^{-i\lambda\xi}d\lambda,$$

with the following notations:

(5.2)

$$Z_u(\lambda,\eta)=-\lambda\{C_1^*(\lambda)\operatorname{sh}\lambda\beta_1\eta+C_2^*(\lambda)\operatorname{ch}\lambda\beta_1\eta+ \\ +\beta_2[C_3^*(\lambda)\operatorname{ch}\lambda\beta_2\eta+C_4^*(\lambda)\operatorname{sh}\lambda\beta_2\eta]\},$$

$$Z_v(\lambda,\eta)=\lambda\{\beta_1[C_1^*(\lambda)\operatorname{ch}\lambda\beta_1\eta+C_2^*(\lambda)\operatorname{sh}\lambda\beta_1\eta]+ \\ +C_3^*(\lambda)\operatorname{sh}\alpha\beta_2\eta+C_4^*(\lambda)\operatorname{ch}\lambda\beta_2\eta\},$$

$$Z_{xx}(\lambda,\eta)=-\lambda^2\{(1+2\beta_1^2-\beta_2^2)[C_1^*(\lambda)\operatorname{sh}\lambda\beta_1\eta+C_2^*(\lambda)\operatorname{ch}\lambda\beta_1\eta]+ \\ +2\beta_2[C_3^*(\lambda)\operatorname{ch}\lambda\beta_2\eta+C_4^*(\lambda)\operatorname{sh}\lambda\beta_2\eta]\},$$

$$Z_{yy}(\lambda,\eta)=\lambda^2\{(1+\beta_2^2)[C_1^*(\lambda)\operatorname{sh}\lambda\beta_1\eta+C_2^*(\lambda)\operatorname{ch}\lambda\beta_1\eta]+ \\ +2\beta_2[C_3^*(\lambda)\operatorname{ch}\lambda\beta_2\eta+C_4^*(\lambda)\operatorname{sh}\lambda\beta_2\eta]\},$$

$$Z_{xy}(\lambda,\eta)=-\lambda^2\{2\beta_1[C_1^*(\lambda)\operatorname{ch}\lambda\beta_1\eta+C_2^*(\lambda)\operatorname{sh}\lambda\beta_1\eta]+ \\ +(1+\beta_2^2)[C_3^*(\lambda)\operatorname{sh}\lambda\beta_2\eta+C_4^*(\lambda)\operatorname{ch}\lambda\beta_2\eta]\};$$

the values of $V^-(\lambda)$ and $C_i^*(\lambda)$ being determined by the respective formulae $(2.17)_1$ and (2.19).

Prior to the numerical evaluation of integrals in the Eqs. (5.1) the integrands should be decomposed into the real and imaginary parts. Let us observe that $Z(\lambda,\eta)$ are real functions of λ and η, and thus it suffices to decompose the function $V^-(\lambda)\exp(-i\lambda\xi)$, which may be written in the form:

$$V^-(\lambda)e^{-i\lambda\xi}=\frac{hv_0}{2\sqrt{2\pi}}[X_0(\lambda)\delta(\lambda)e^{-i\lambda\xi}+X(\lambda)-iY(\lambda)], \tag{5.3}$$

with the notations

$$X_0(\lambda)=\frac{2\pi\sqrt{A}}{\sqrt{A+i\lambda}}\,\frac{M^-(\lambda)}{L^-(\lambda)},$$

$$X(\lambda)=h(\lambda)\cos\lambda\xi+g(\lambda)\sin\lambda\xi,\qquad Y(\lambda)=h(\lambda)\sin\lambda\xi-g(\lambda)\cos\lambda\xi,$$

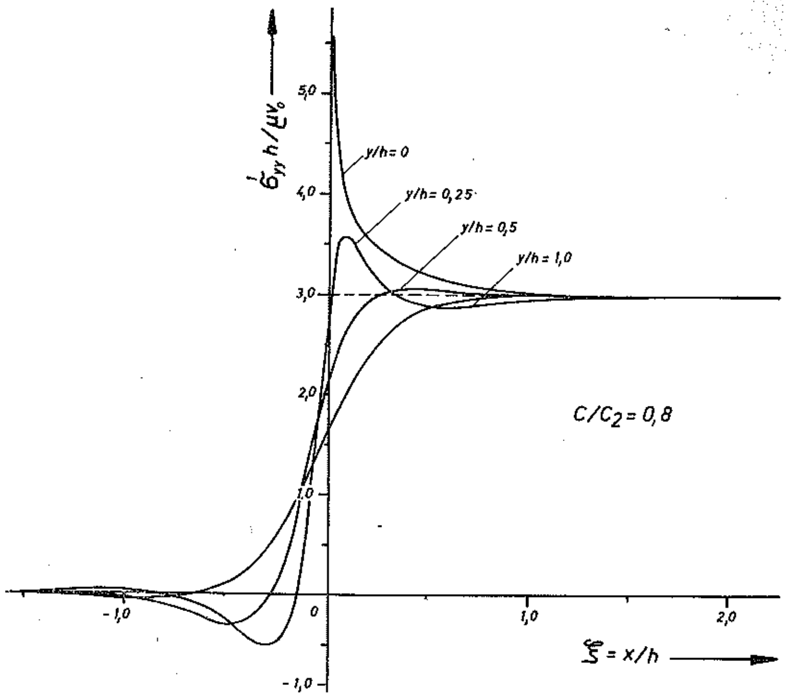
$$h(\lambda)=-\frac{\sqrt{A}}{\rho(\lambda^4+\lambda_0^4)}[(\sqrt{\rho+\lambda}-\sqrt{\rho-\lambda})T(\lambda)+(\sqrt{\rho+\lambda}+\sqrt{\rho-\lambda})S(\lambda)],$$

$$g(\lambda)=-\frac{\sqrt{A}}{\rho(\lambda^4+\lambda_0^4)}[(\sqrt{\rho+\lambda}+\sqrt{\rho-\lambda})T(\lambda)-(\sqrt{\rho+\lambda}-\sqrt{\rho-\lambda})S(\lambda)],$$

$$T(\lambda)=\frac{1}{\lambda}[(\lambda^2-\lambda_0^2)^2+2\sqrt{1+\vartheta\lambda_0^2\lambda^2}],$$

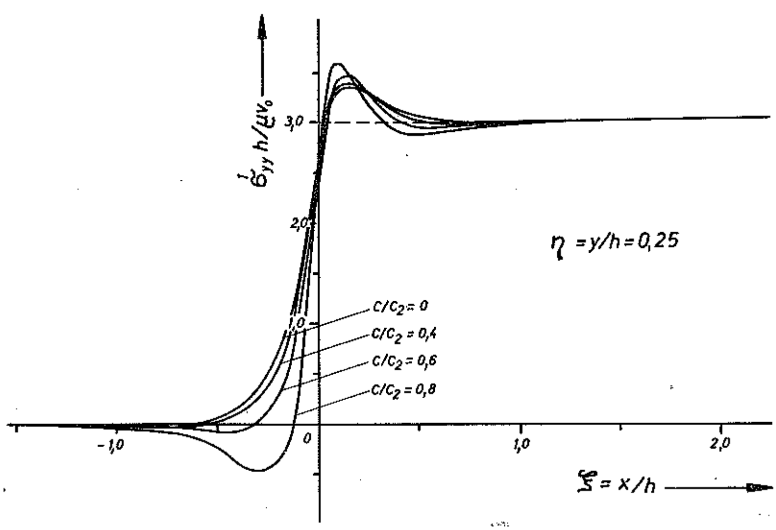
$$S(\lambda)=\lambda_0\sqrt{2}(\lambda^2-\lambda_0^2)(\sqrt{1+\vartheta}-1),$$

$$\rho=\sqrt{\lambda^2+A^2}.$$



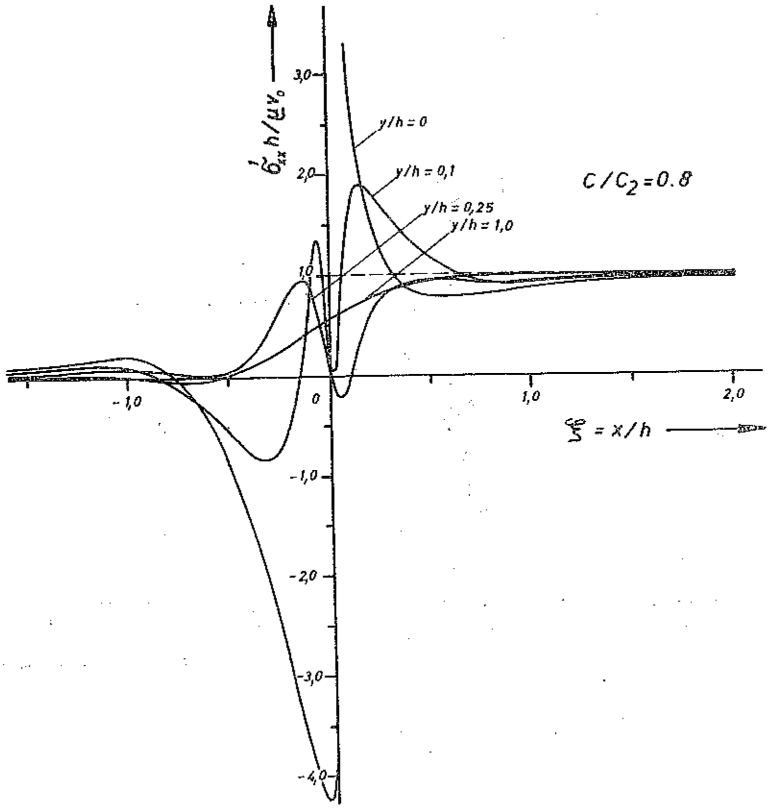
$\overset{1}{\sigma}_{yy}\, h/\mu v_0$
5,0
4,0
3,0
2,0
1,0
0
-1,0
y/h = 0
y/h = 0,25
y/h = 0,5
y/h = 1,0
$C/C_2 = 0,8$
-1,0
1,0
2,0
$\xi = x/h$

Fig. 5



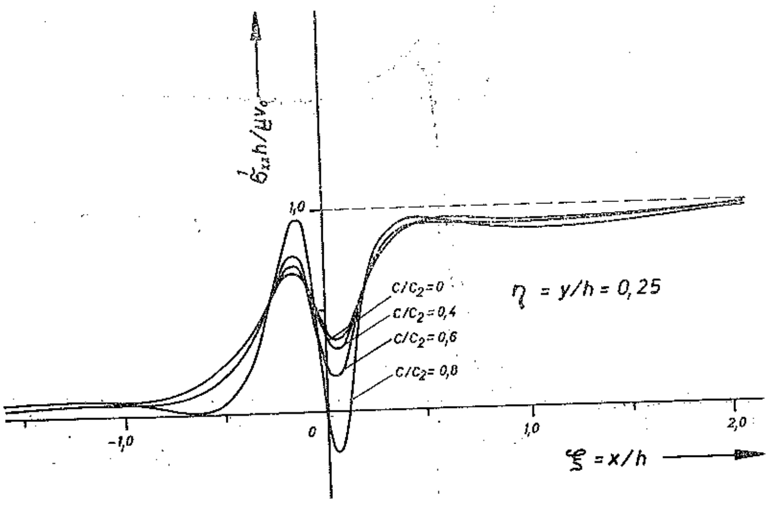
$\overset{1}{\sigma}_{yy}\, h/\mu v_0$
3,0
2,0
1,0
0
$\eta = y/h = 0,25$
$C/C_2 = 0$
$C/C_2 = 0,4$
$C/C_2 = 0,6$
$C/C_2 = 0,8$
-1,0
1,0
2,0
$\xi = x/h$

Fig. 6



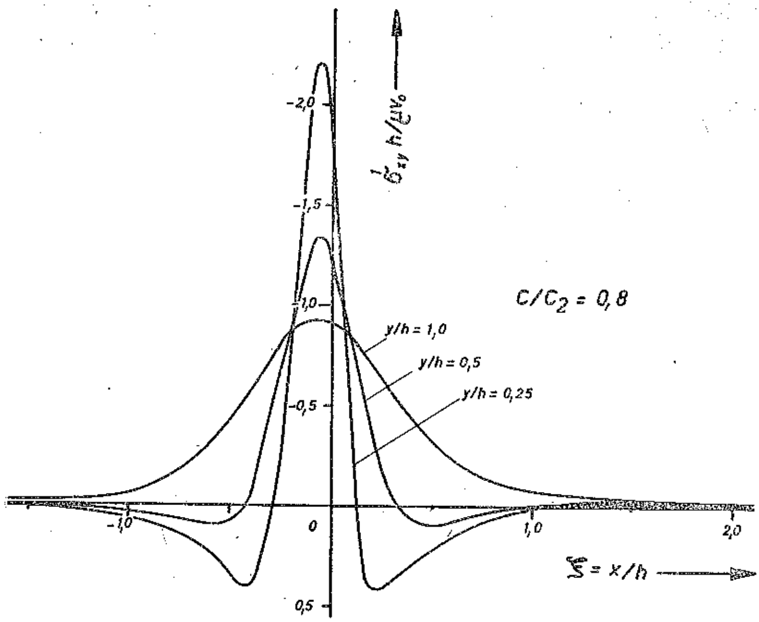
$\overset{1}{\sigma}_{xx} h/\mu v_0$
3,0
2,0
1,0
0
-1,0
-2,0
-3,0
-4,0
y/h = 0
y/h = 0,1
y/h = 0,25
y/h = 1,0
C/C2 = 0.8
-1,0
1,0
2,0
ξ = x/h

Fig. 7



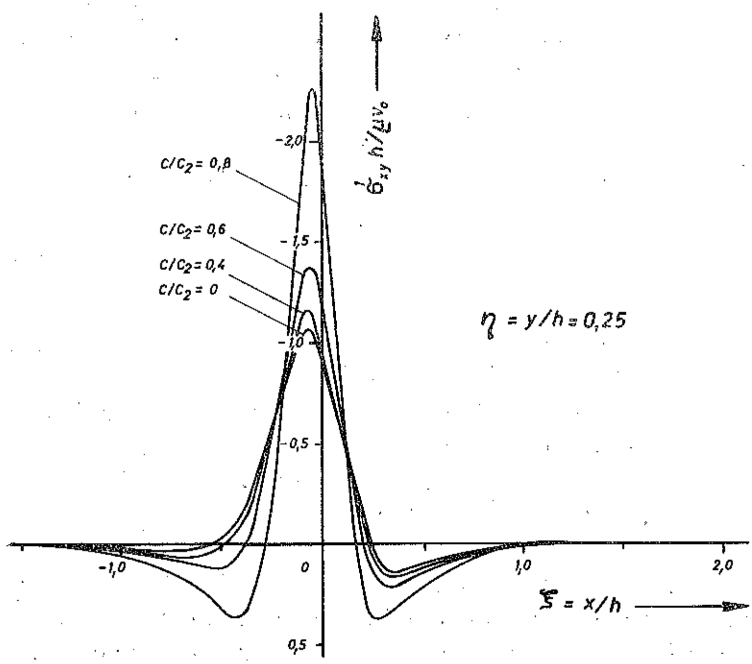
$\overset{1}{\sigma}_{xx} h/\mu v_0$
1,0
C/C2 = 0
C/C2 = 0,4
C/C2 = 0,6
C/C2 = 0,8
η = y/h = 0,25
-1,0
0
1,0
2,0
ξ = x/h

Fig. 8



-2,0
-1,5
-1,0
-0,5
0,5
$\overset{1}{\sigma}_{xy}\, h/\mu v_0$
C/C2 = 0,8
y/h = 1,0
y/h = 0,5
y/h = 0,25
-1,0
0
1,0
2,0
ξ = x/h

Fig. 9



-2,0
-1,5
-1,0
-0,5
0,5
$\overset{1}{\sigma}_{xy}\, h/\mu v_0$
C/C2 = 0,8
C/C2 = 0,6
C/C2 = 0,4
C/C2 = 0
η = y/h = 0,25
-1,0
0
1,0
2,0
ξ = x/h

Fig. 10

Inserting the Eqs. (5.3) into (5.1), making use of the properties of delta-functions and of the fact that $X(-\lambda)=X(\lambda)$, $Y(-\lambda)=-Y(\lambda)$, and adding together the Eqs. (1.1) and (5.3), we determine the stresses and displacements at an arbitrary point of the strip shown in Fig. 1a:

$$\overset{1}{u}(\xi,\eta)=\frac{v_0}{2\pi}\int_0^\infty Z_u(\lambda,\eta)\,Y(\lambda)\,d\lambda,$$

$$\overset{1}{v}(\xi,\eta)=\frac{v_0}{2\pi}\left[\pi(1+\eta)+\int_0^\infty Z_v(\lambda,\eta)\,X(\lambda)\,d\lambda,\right.$$

$$\text{(5.5)}\qquad \overset{1}{\sigma}_{xx}(\xi,\eta)=\frac{\mu v_0}{2\pi h}\left[\frac{2\pi\nu}{1-2\nu}+\int_0^\infty Z_{xx}(\lambda,\eta)\,X(\lambda)\,d\lambda,\right.$$

$$\overset{1}{\sigma}_{yy}(\xi,\eta)=\frac{\mu v_0}{2\pi h}\left[\frac{2\pi(1-\nu)}{1-2\nu}+\int_0^\infty Z_{yy}(\lambda,\eta)\,X(\lambda)\,d\lambda,\right.$$

$$\overset{1}{\sigma}_{xy}(\xi,\eta)=\frac{\mu v_0}{2\pi h}\int_0^\infty Z_{xy}(\lambda,\eta)\,Y(\lambda)\,d\lambda,$$

the functions $Z(\lambda,\eta)$, $X(\lambda)$ and $Y(\lambda)$ being defined by the Eqs. (5.2) and (5.4).

The improper integrals occurring in the Eqs. (5.5) converge fairly well in the region $0<\eta\leqslant 1$. Certain difficulties may be encountered in calculating the values of $\overset{1}{\sigma}_{xx}$ for $\eta=0$, convergence of the integral $(5.5)_3$ being insufficiently rapid. In order to avoid these difficulties let us observe that the function $Z_{xx}(\lambda,0)$ appearing in the integrand of $(5.5)_3$ may be represented (according to the Eq. (5.2) in the form:

$$Z_{xx}(\lambda,0)=-Z_{yy}(\lambda,0)-2\lambda^2(\beta_1^2-\beta_2^2)\,C_2^*(\lambda).$$

By inserting this expression into the Eq. $(5.5)_3$, we obtain:

$$\overset{1}{\sigma}_{xx}(\xi,0)=-\overset{1}{\sigma}_{yy}(\xi,0)+\frac{\mu v_0}{2\pi h}\left[\frac{2\pi}{1-2\nu}-2(\beta_1^2-\beta_2^2)\int_0^\infty \lambda^2\,C_2^*(\lambda)\,X(\lambda)\,d\lambda\right],$$

Here the analytically determined stress $\overset{1}{\sigma}_{yy}(\xi,0)$ is described by the Eq. $(4.3)_2$, and the corresponding integral is suitable for numerical calculations.

The graphs of stresses (5.5) in various cross-sections of the strip at a constant velocity of crack propagation (with $\nu=0.25$) are shown in Figs. 5, 7, 9. The variation of stresses (5.5) in the case of a fixed cross-section and for various crack velocities ($\nu=0.25$) is demonstrated in Figs. 6, 8, 10.

The autors have to confess that, in the process of writing and preparing the pa for print, they were not aware of existence of a paper by V. D. KULIEW [10] ...ling with a similar problem, though confined to the evaluation of the stress ...sity factor.

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Streszczenie

DYNAMICZNY PROBLEM SZCZELINY W PASMIE SPRĘŻYSTYM

W pracy omówiono quasi-statyczne zagadnienie rozkładu składowych stanu przemieszczenia i naprężenia w nieskończonym pasmie sprężystym, osłabionym w jego środkowej płaszczyźnie półnieskończoną szczeliną. Założono, że szczelina propaguje się ze stałą prędkością wzdłuż prostej, leżącej w środkowej płaszczyźnie rozważanego pasma. Stosując całkową transformację Fouriera, rozwiązanie tego zagadnienia sprowadzone zostało do rozwiązania odpowiedniego równania typu Wienera-Hopfa.

Znaleziono ścisłą wartość współczynnika intensywności naprężenia w końcu szczeliny. Wykonując numerycznie odwrotną transformację całkową Fouriera znaleziono rozkład składowych stanu przemieszczenia i naprężenia w dowolnym punkcie pasma. Otrzymane wyniki zilustrowano wykresami.

Резюме

ДИНАМИЧЕСКАЯ ЗАДАЧА ЩЕЛИ В УПРУГОЙ ПОЛОСЕ

В работе обсуждена квазистатическая проблема распределения составляющих состояния перемещения и напряжения в бесконечной упругой полосе, ослабленной в ее срединной плоскости полубесконечной щелью. Предположено, что щель распространяется с постоянной скоростью вдоль прямой, находящейся в срединной плоскости рассматриваемой полосы. Применяя интегральное преобразование Фурье, решение этой проблемы сведено к решению соответствующего уравнения типа Винера-Хопфа.

Найдено точное значение коэффициента интенсивности напряжения в конце щели. Производя численно обратное интегральное преобразование Фурье найдено распределение составляющих состояния перемещения и напряжения в произвольной точке полосы. Полученные результаты иллюстрирвоаны графиками.

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