Piotr Perzyna – Scientific Conductor within Theory of Thermo-Viscoplasticity

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In this paper, authors reminisce on years of scientific cooperation and achievements on the personality of Piotr Perzyna and his Theory of Thermo-Viscoplasticity, which are displayed from remembrance. All the essence of latin words *Master, Doctor, Docent, Professor* are in some kind the reflection of the scientific history that has happened during the last 40 years. For every author the shift of 10 to 20 years shows that Piotr Perzyna served with his personality and knowledge for two to three different generations, working with authors as 20/30/50 years younger man. Now, one year after Piotr Perzyna passed away in 2013 (50 years after publishing the basic paper on Theory of Thermo-Viscoplasticity in 1963), the role he has played for authors has become much clearer and conspicuous as well as in a wider context. Like $Piotr = Peter = Petrus = K\eta\varphi\acute{\alpha}\varsigma$ Professor Perzyna is the *rock on which we build*.
Let it remain not only in memory.

Key words: viscoplasticity, strain localisation, wave effects, damage anisotropy.

1. Introduction

There is no doubt that Professor Piotr Perzyna belonged to the most profound group of scientists working on mechanics in the last few decades. His name is mentioned hundreds of times in exposed monographs and journals. Naturally his works are mainly devoted to the famous Perzyna's Theory of Thermo-Viscoplasticity (PTTV) [27, 28]. The authors of this paper were for many years the beneficiaries of the fruitful cooperation and deep scientific discussions with their mentor – Piotr Perzyna. In particular, as a result of this cooperation it was possible to extend the theoretical background by introducing the numerical tools to check the validity of theoretical assumptions by comparison of the computational results with the laboratory tests. This coupling of theoretical formulations supported by numerical results verified by experiments, influenced the evolution of the mathematical assumptions. One of the

good examples for this is the evolution of understanding of the internal state variables vector, which at the beginning was restricted only to the rate of deformation field and later extended by the scalar volume porosity. At the end, the vector of internal state variables consists also of porosity tensor which evolution introduces the enforced anisotropy during the deformation process.

The other important aspect of the PTTV is the possible way of dealing with the so called softening problems due to temperature rise and growth of localized plastic deformation. Even at the beginning of the eighties of the XX century, scientists struggled with unexpected effects of pathological mesh dependency that appeared in computational results. Several concepts were proposed to overcome this problem and introduce the regularization effects. When using the proposed in this paper formulation for large strain rate problems, the regularization is introduced intrinsically and moreover the places for plastic strain localization zones and their widths are simply the results of the wave interactions in any Initial Boundary Value Problem (IBVP). In this paper, attention is placed on these characteristic aspects of Perzyna's theory. The paper shows the general formulation of the set of governing equations and the discussion of selected problems, including those which can be still under consideration and in focus of future research.

2. General framework of Perzyna's viscoplasticity

Let us follow the general framework of the thermodynamical theory of viscoplasticity in the form proposed by Perzyna in the early 1990's [31] (the one strongly influenced by works of J.E. Marsden group cf. [1, 19]). This description bases on Riemannian manifolds and utilises Lie derivative – both facts are crucial for self consistency and clear physical interpretation.

2.1. Kinematics

The abstract body is a differentiable manifold. To describe the kinematics of the finite elasto-viscoplastic deformations we use the multiplicative decomposition of the total deformation gradient to the elastic and viscoplastic part [18]

$$(2.1) \qquad \mathbf{F}(\mathbf{X}, t) = \mathbf{F}^e(\mathbf{X}, t) \cdot \mathbf{F}^p(\mathbf{X}, t),$$

where $\mathbf{F} = \dfrac{\partial \phi(\mathbf{X}, t)}{\partial \mathbf{X}}$ is the deformation gradient, ϕ describes the motion, $\mathbf{X}$ denotes material coordinates, t represents time and $\mathbf{F}^e$, $\mathbf{F}^p$ are elastic and viscoplastic parts of the deformation gradient, respectively.

We accept the Euler-Almansi strain measure because its Lie derivative defines directly the symmetric part of the spatial deformation gradient, thus spatial

deformation gradient defines truly the rate of strains. So, we have the fundamental relation

$$(2.2) \qquad \mathbf{d}^\flat = L_{\boldsymbol{v}}(\mathbf{e}^\flat) = \frac{1}{2}(\mathbf{l} + \mathbf{l}^T),$$

and simultaneously

$$(2.3) \qquad \mathbf{d}^{e\flat} = L_{\boldsymbol{v}}(\mathbf{e}^{e\flat}), \quad \mathbf{d}^{p\flat} = L_{\boldsymbol{v}}(\mathbf{e}^{p\flat}),$$

where $\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p$ is the symmetric part of the spatial deformation gradient $\mathbf{l}(\mathbf{x}, t) = \dfrac{\partial \boldsymbol{v}(\mathbf{x}, t)}{\partial \mathbf{x}}$, $\boldsymbol{v}$ denotes spatial velocity, $\mathbf{x}$ is the vector of spatial coordinates, $L_{\boldsymbol{v}}$ stands for Lie derivative, $\mathbf{e}$ stands for the Euler-Almansi strain, $\flat$ indicates that a tensor has all its indices lowered [19] while indices e and p denote elastic and viscoplastic parts, respectively.

2.2. Constitutive postulates

Assuming that the balance principles hold, namely: conservation of mass, balance of momentum, balance of moment of momentum and balance of energy and entropy production, we define four constitutive postulates [29, 31]:

(i) Existence of the free energy function ψ. Formally we apply it in the following form

$$(2.4) \qquad \psi = \widehat{\psi}(\mathbf{e}, \mathbf{F}, \vartheta; \boldsymbol{\mu}),$$

where $\boldsymbol{\mu}$ denotes a set of internal state variables governing the description of dissipation effects and ϑ represents temperature. It is important to notice, that we have used semicolon to separate the last variable due to its different nature (it introduces a dissipation to the model), without $\boldsymbol{\mu}$ the presented model describes thermoelasticity [7].

(ii) Axiom of objectivity. The material model should be invariant with respect to diffeomorphism (any superposed motion).

(iii) The axiom of the entropy production. For every regular process the constitutive functions should satisfy the second law of thermodynamics.

(iv) The evolution equation for the internal state variables vector $\boldsymbol{\mu}$ should be of the form

$$(2.5) \qquad L_{\boldsymbol{v}}\boldsymbol{\mu} = \widehat{\mathbf{m}}(\mathbf{e}, \mathbf{F}, \vartheta, \boldsymbol{\mu}).$$

where evolution function $\widehat{\mathbf{m}}$ has to be determined based on the experimental observations.

2.3. Constitutive relations – general form

We can write the so called reduced dissipation inequality in the form [19, 34]

$$
(2.6) \qquad \frac{1}{\rho_{\text{Ref}}} \boldsymbol{\tau} : \mathbf{d} - (\eta \dot{\vartheta} + \dot{\psi}) - \frac{1}{\rho \vartheta} \mathbf{q} \cdot \text{grad} \vartheta \geq 0,
$$

where where ρ denotes actual and ρ_{Ref} reference densities, $\boldsymbol{\tau}$ denotes Kirchhoff stress, ψ is the free energy function, ϑ is absolute temperature, η denotes the specific (per unit mass) entropy and $\mathbf{q}$ is the heat flux. Using postulate (i), Eq. (2.6) can be rewritten to the form:

$$
(2.7) \qquad \left(\frac{1}{\rho_{\text{Ref}}} \boldsymbol{\tau} - \frac{\partial \widehat{\psi}}{\partial \mathbf{e}} \right) : \mathbf{d} - \left(\eta + \frac{\partial \widehat{\psi}}{\partial \vartheta} \right) \dot{\vartheta} - \frac{\partial \widehat{\psi}}{\partial \boldsymbol{\mu}} \mathrm{L}_{\boldsymbol{v}} \boldsymbol{\mu} - \frac{1}{\rho \vartheta} \mathbf{q} \cdot \text{grad} \vartheta \geq 0,
$$

so because of arbitrariness

$$
(2.8) \qquad \boldsymbol{\tau} = \rho_{\text{Ref}} \frac{\partial \widehat{\psi}}{\partial \mathbf{e}},
$$

$$
(2.9) \qquad \eta = -\frac{\partial \widehat{\psi}}{\partial \vartheta},
$$

hence Eq. (2.7) reduces to

$$
(2.10) \qquad -\frac{\partial \widehat{\psi}}{\partial \boldsymbol{\mu}} \mathrm{L}_{\boldsymbol{v}} \boldsymbol{\mu} - \frac{1}{\rho \vartheta} \mathbf{q} \cdot \text{grad} \vartheta \geq 0.
$$

The final form of evolution equations for the specific thermomechanical process requires postulation of a set of internal variables governed in $\boldsymbol{\mu}$.

3. Aspects and mile-stones of theory of thermo-visco-plasticity

3.1. Observation and choice of effects obtained from multiscale experiments

An analysis of experimental results has always been very carefully investigated in research under Perzyna's supervision. His wide contacts and visits in worldwide famous experimental labs were constantly the source of material observations and measurements that were the basis to show that the shear band localization failure in dynamic loading processes is affected by complex cooperative phenomena. From this analysis it was also evident that such cooperative phenomena as the thermo-mechanical process, the instability of the flow process along localized adiabatic shear bands, the microdamage process which consists of the nucleation, growth and coalescence of microcracks and

the final mechanism of failure are the most important for a proper description of the fracture phenomenon under dynamic loading. All these cooperative phenomena might be influenced by different additional effects such as the strain rate sensitivity, the induced anisotropy, the thermo-mechanical couplings and others.

The high rate deformation of face-centered cubic metals, such as copper, aluminum, lead and nickel has been extensively studied. It has been shown that the apparent strain rate sensitivity of such metals has two origins: that associated with the finite velocity of dislocations, and that connected with the evolution of the dislocation substructure. The first of these two components – the instantaneous rate sensitivity – is related to the wait-times associated with thermally activated dislocation motion. The second component has more to do with the relative importance of dislocation generation and annihilation at different strain rates, and shall be referred to as the strain rate history effect. The rate and temperature dependence of the flow stress of metal crystals are explained by different physical mechanisms of dislocation motion. The microscopic processes combine in various ways to give several groups of deformation mechanisms, each of which can be limited to the particular range of temperature and strain rate changes.

In a realistic way, despite the rather complex than simplified form, Perzyna decided to include in the description not all effects observed experimentally. Constitutive modelling was excellently understood by Perzyna, as a reasonable choice of effects which are most important for explanation of the phenomenon described. Experiments from different, macro-, meso-, micro- and in the last part of Perzyna investigations from nano-scale, were the rock foundation of theory and constitutive modeling.

3.2. Mathematically well-posed formulation

Rate independent continuum with introduction of material instability in the Drucker's sense, does not quarantine well-posedness of the problem. Even if it allows for calculations of localization zones (for a wide class of commonly acceptable from point of view of engineering applications materials) both the analytical and numerical solutions become meaningless. The next symptom when computational results are obtained for these materials, one can prove easily that they exhibit so called "pathological mesh dependency" – the result, especially the width of localization zone, is essentially different for models with another discretization (for example double, four more dense meshes).

The resolution for those problems comes from the theory of viscoplasticity, when fundamental research on mathematical modelling problem is taken into account. It was proposed and proved that introducing viscoplastic con-

stitutive model does make the important change. The mathematical problem does now present that no loose of well-posedness and no mesh-dependency is present so far. Also, it is viscosity – material parameter – that plays the role for mathematical regularization of system of equations. The material certain value of viscosity parameter in the defined initial boundary value problem, results in specified characteristic of localization zone with its specified width. According to the values of viscosity distinguish another physically motivated mechanical problem with specified plastic strain localisation process and width of shear band.

The problem of formulation of governing equations is particularly important when starting to describe the fundamental phenomena which appear during fast dynamic processes like the creation and development of localized zones of deformations. Two important qualitative results should be in focus in our interests: (i) the width and directions of plastic localization zones (according to the experiments they are sometimes very narrow but still have finite dimensions), (ii) the shape of concentrated strains depends significantly on the type of loading and its velocity (usually the localization patterns are different for quasi-static and dynamic cases). The levels of possible description of localization phenomena based on experimental observations were discussed in detail in [10, 21, 24]. Among different possible constitutive models which deal with softening plasticity, the one that is chosen is rate dependent. It has to be also emphasized the the existence and possible using the other models also introducing the regularization effects like: smeared crack models, non-local theories models, higher order gradients models, embedded models or Cosserat models. In computations all of them need to introduce explicitly additional parameters like small constitutive or geometric imperfections or width of localization zones (embedded elements) to initiate the place of concentrated plastic strains. In the case of rate dependent models (PTTV) both the width and directions result from wave interaction for the specific IBVP and the nature of loading (the length scale parameter is introduced implicitly to the model).

It is convenient to start the discussion of posedness of the IBVP by writing the governing equations. Herein we assume adiabatic regime and the set of internal state variables of the form

$$
(3.1) \qquad\qquad \boldsymbol{\mu} = (\in^p, \xi),
$$

where $\in^p$ is the equivalent plastic deformation $\dot{\in}^p = \left(\frac{2}{3}\mathbf{d}^p : \mathbf{d}^p\right)^{1/2}$, which describes the dissipation effects generated by viscoplastic deformation, and ξ is porosity (scalar) which takes into account the intrinsic microdamage effects.

We have [10, 23]

$$\dot{\phi} = \boldsymbol{v},$$

$$\dot{\boldsymbol{v}} = \frac{1}{\rho_M^0(1-\xi_0)}\left(\frac{\boldsymbol{\tau}}{\rho_M}\mathrm{grad}\rho_M + \mathrm{div}\boldsymbol{\tau} - \frac{\boldsymbol{\tau}}{1-\xi}\mathrm{grad}\xi\right),$$

$$\dot{\rho}_M = \frac{\rho_M}{1-\xi}\Xi - \rho_M\mathrm{div}\boldsymbol{v},$$

$$\dot{\boldsymbol{\tau}} = \left[\mathcal{L}^e - \frac{1}{c_p\rho_{\mathrm{Ref}}}\vartheta\mathcal{L}^{th}\frac{\partial\boldsymbol{\tau}}{\partial\vartheta}\right] : \mathrm{sym}D\boldsymbol{v} + 2\mathrm{sym}\left(\boldsymbol{\tau}\cdot\frac{\partial\boldsymbol{v}}{\partial\mathbf{x}}\right)$$

(3.2)
$$- \left[\left(\frac{\chi^*}{\rho_M(1-\xi)c_p}\mathcal{L}^{th}\boldsymbol{\tau} + \mathcal{L}^e + \mathbf{g}\boldsymbol{\tau} + \boldsymbol{\tau}\mathbf{g}\right) : \mathbf{P}\right]$$

$$\frac{1}{T_m}\left\langle\left(\frac{f}{\kappa}-1\right)^m\right\rangle - \frac{\chi^{**}\mathcal{L}^{th}}{\rho_M(1-\xi)c_p}\Xi,$$

$$\dot{\xi} = \Xi,$$

$$\dot{\vartheta} = \frac{\vartheta}{c_p\rho_{\mathrm{Ref}}}\frac{\partial\boldsymbol{\tau}}{\partial\vartheta} : \mathrm{sym}D\boldsymbol{v} + \frac{\chi^*}{\rho_M(1-\xi)c_p}\boldsymbol{\tau} : \mathbf{P}\frac{1}{T_m}\left\langle\left(\frac{f}{\kappa}-1\right)^m\right\rangle$$

$$+ \frac{\chi^{**}}{\rho_M(1-\xi)c_p}\Xi,$$

where ρ_M^0 is an initial density of a body without microdamages, ρ_M is a current density of a body without microdamages, $\mathcal{L}^e$ is an elastic constitutive tensor, $\mathcal{L}^{th}$ is a thermal operator, c_p is a specific heat, χ^*, χ^{**} are the irreversibility coefficients, $\mathbf{P}$ denotes the direction of viscoplastic flow, T_m is a relaxation time, f is a yield surface, κ is a work hardening-softening function, and m is non-dimensional material parameter.

This evolution form is even better seen when the operator equation is involved [20]. Hence, the inhomogeneous Abstract Cauchy Problem can be formulated in the following form

(3.3) $\quad \dot{\boldsymbol{\varphi}}(t,\mathbf{x}) = \mathcal{A}(t,\mathbf{x})\cdot\boldsymbol{\varphi}(t,\mathbf{x}) + \mathbf{f}(t,\mathbf{x},\boldsymbol{\varphi}) \quad \text{for} \quad t\in(0,T] \quad \text{and} \quad \mathbf{x}\in\Omega$

with the initial condition

(3.4) $$\boldsymbol{\varphi}(0,\mathbf{x}) = \boldsymbol{\varphi}_0,$$

where

(3.5) $$\boldsymbol{\varphi} = \begin{bmatrix} \phi \\ \boldsymbol{v} \\ \rho_M \\ \boldsymbol{\tau} \\ \xi \\ \vartheta \end{bmatrix},$$

$$(3.6) \quad \mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{\boldsymbol{\tau}\mathrm{grad}}{\rho_M^0(1-\xi_0)\rho_M} & \dfrac{\mathrm{div}}{\rho_M^0(1-\xi_0)} & \dfrac{-\boldsymbol{\tau}\mathrm{grad}}{\rho_M^0(1-\xi_0)(1-\xi)} & 0 \\ 0 & -\rho_M\mathrm{div} & 0 & 0 & 0 & 0 \\ 0 & a^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b^* & 0 & 0 & 0 & 0 \end{bmatrix},$$

where

$$a^* = \left[\mathcal{L}^e - \frac{1}{c_p\rho_{\mathrm{Ref}}}\vartheta\mathcal{L}^{th}\frac{\partial\boldsymbol{\tau}}{\partial\vartheta}\right] : \frac{\partial}{\partial\mathbf{x}} + 2\left(\boldsymbol{\tau}\cdot\frac{\partial}{\partial\mathbf{x}}\right),$$

$$b^* = \frac{\vartheta}{c_p\rho_{\mathrm{Ref}}}\frac{\partial\boldsymbol{\tau}}{\partial\vartheta} : \frac{\partial}{\partial\mathbf{x}},$$

$$(3.7) \qquad \mathbf{f} = \begin{bmatrix} \boldsymbol{\upsilon} \\ \mathbf{0} \\ \dfrac{\rho_M}{1-\xi}\Xi \\ c^* \\ \Xi \\ \dfrac{\chi^*}{\rho_M(1-\xi)c_p}\boldsymbol{\tau} : \mathbf{P}\dfrac{1}{T_m}\left\langle\left(\dfrac{f}{\kappa}-1\right)^m\right\rangle + \dfrac{\chi^{**}}{\rho_M(1-\xi)c_p}\Xi \end{bmatrix},$$

where

$$c^* = -\left[\left(\frac{\chi^*}{\rho_M(1-\xi)c_p}\mathcal{L}^{th}\boldsymbol{\tau} + \mathcal{L}^e + \mathbf{g}\boldsymbol{\tau} + \boldsymbol{\tau}\mathbf{g}\right) : \mathbf{P}\right]\frac{1}{T_m}\left\langle\left(\frac{f}{\kappa}-1\right)^m\right\rangle$$
$$-\frac{\chi^{**}\mathcal{L}^{th}}{\rho_M(1-\xi)c_p}\Xi.$$

Now, it is necessary to split the discussion into two branches. One is so called Abstract Cauchy Problem (ACP), which takes into consideration only the initial conditions, and the other one more significant for responsible computations Initial Boundary Value Problem (IBVP). For the detailed description and discussion please refer to the works [20, 21, 22, 24]. The well-posedness of the ACP or IBVP means that the solution for the components of vector $\dot{\boldsymbol{\varphi}}$ exists, is unique and is stable in Hadamard or Liapunov sense. It seems to be possible to prove the conditions for well-posedness of ACP and using the Lax Theorem, define the conditions for finite element approximation. Unfortunately, for IBVP until now it is extremely difficult in a strict mathematical way to prove the well-posedness

of the system of governing equations (for 1-D case it was done by Ionescu and Sofonea [16]). One cannot count for the general proof for arbitrary IBVP. Fortunately, we are not completely helpless. Using the numerical computations, one is able to discover the pathological mesh dependency (using different meshes eg. more dense the typical convergence is not achieved) which is for softening behaviour the implicit proof that the governing set of equations is not well-posed. For this purpose any FEM computational results has to be verified for different meshes including specific densification in the localized areas.

3.3. Solution existence and uniqueness of softening problem

Well-posedness, as was discussed before does provide the mathematical problem verification at the formulation and its properties. The separate research was resolved for the method of numerical solution and properties of numerical analysis that allow us to end up with verification of computed results. Again having the viscoplastic constitutive model, it was proved in research by Perzyna [31], that a solution exist; method of numerical solution is stable, convergent and obtained results lead to unique results. The rate dependent continuum with the crucial role of viscosity again has been recognised as a valuable element of material constitutive formulations.

3.4. Viscosity defined by material parameter

In Perzyna's Theory of Thermo-Viscoplasticity, the material parameter that describes the viscous property of material is time relaxation T_m (see Eq. (3.2)). In problems where softening is analysed as the problem of material bifurcation, usually the place of localization is in line with the understanding that the zero thickness of shear band is out of computation results and finding the place is not correlated with measuring/verifying the width of shear band. Depending on material taken into consideration, the more brittle are with smaller dimension of shear band width, tending to zero width in limit, and more ductile material with finite dimension of shear band width in reality. One has the hope to reflect that part of plastic strain localization phenomenon by introduction the viscosity in constitutive model. Taking a range of values for material parameter of time of relaxation, the result is like in Fig. 1, presenting the distribution of point velocity along the specimen. The case of change of the displayed value almost in a point is related to brittle failure with thin width of localization band. Small value of relaxation time – correlated with small viscous properties of material – gives the result like cut-off specimen into two pieces. The advantage is that despite brittle properties of material there is no mathematical and numerical problem to go through maximal strength of material and continue computations without

risk of sudden numerical break-off through softening, even if the localization zone does not extend the thickness. The opposite material situation, when ductile material is used into analysis, then greater value of time relaxation is used. Material is more viscous, which means that independently on its real plasticity characterization, its behaviour in localization zone is more ductile expressing a wider zone of plastic strain localization [12, 13]. In Fig. 1 it is represented by quite smooth change of point velocity between 0 m/s and 20 m/s. Estimating what is the width of localization zone is possible taking this same chance of point velocity but looking at the horizontal axis where length of specimen is drawn. In case in Fig. 1 the band width of plastic strain localization is quite wide and takes about 20% of specimen length. Viscosity in such a way reflects engineering important property for plastic strain localization zone.
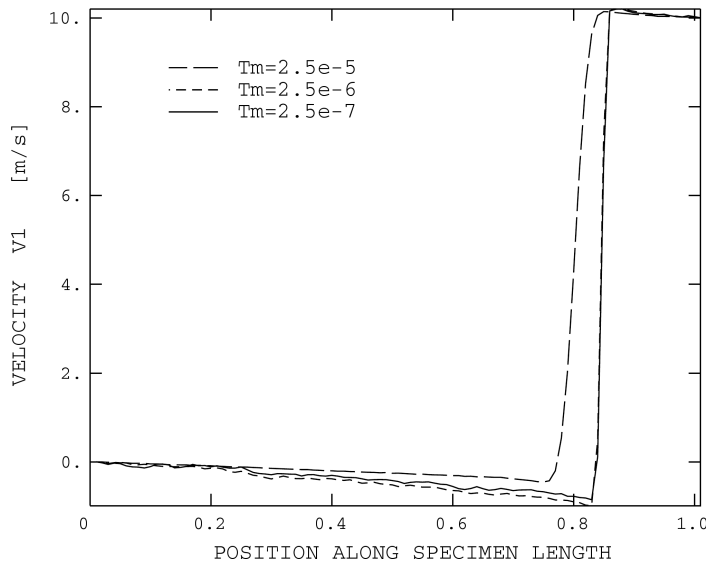

FIG. 1. Distribution of point velocity as a function of dimensionless length of the specimen.

3.5. Propagation of mechanical and thermal waves

Application of the viscous properties in dynamic analysis of very short time processes becomes important in the description and solution of materials behaviour under impulse loading. The propagation and reflection of waves introduces inhomogeneous changes in fields of deformation and stress. The Perzyna viscoplasticity has got one more important physical advantage; that there is no need to introduce initial imperfection, when in other formulations without initial imperfection the localisation does not nucleate and develop [9, 11]. Of course if one would like to reflect the inhomogeneous, not perfect material structure, it is possible to provide analysis initially with material imperfections, but due

to only constitutive relation and the wave propagating in material properties, no initiation of softening development is introduced. It is worth also to mention, that in most of computed problems it would be hard to explain where and why the defined imperfection is considered. Other imperfections result in other deformation development, so other final mechanical state. There is no case for the problem using viscoplasticity, where no imperfection, no imperfection consideration exists. More over using the formulation with viscous material one can analyse, how the consideration of initial imperfection introduces additional mechanical effects, that were out of intention of person who provides structural analysis. The computer simulation provides illustration of waves interactions in the evolution of the plastic strain localization especially in softening states. Observation of the process development by change of point velocity or point acceleration in the adequate time scale always provides far wider understanding of mechanical phenomenon under consideration. When the problem formulation includes viscoplasticity the observation of wave effects is valuable as well (Fig. 2). The value of that observation and the width of perspective is opened with a new dimension – we have not only waves effects observation, but we
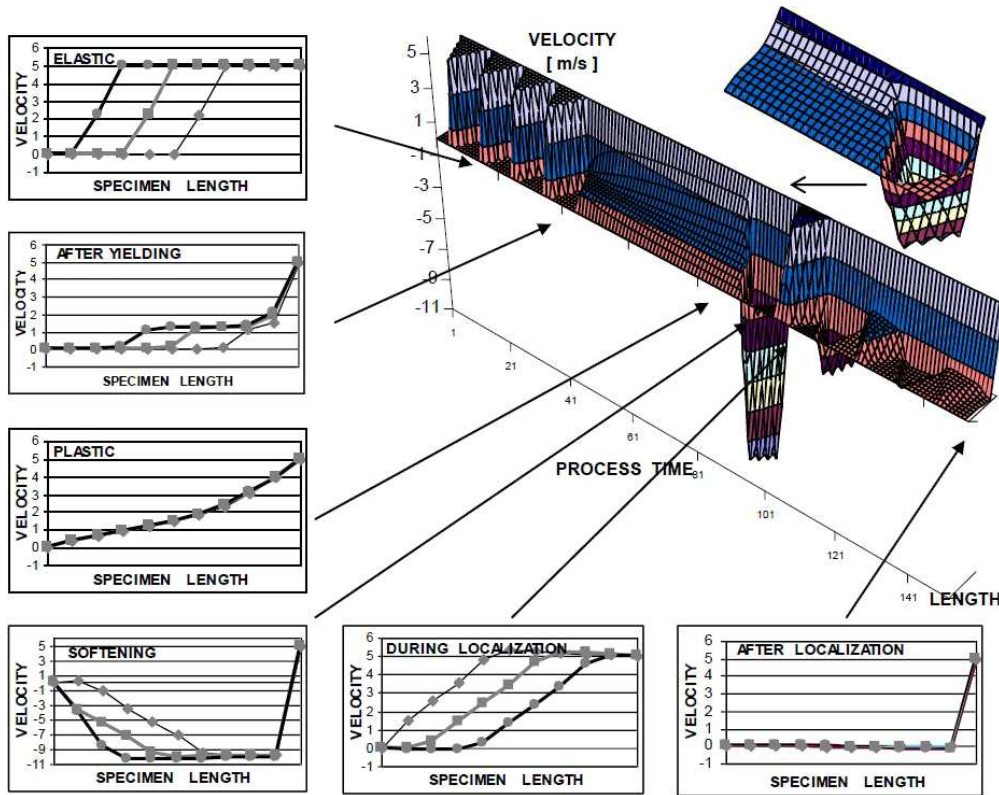

FIG. 2. Point velocity in beam model for the whole and selected moments of process time.

have the influence of viscosity into dispersion, that is mathematically and later physically opening to the new class of problems [12, 13].

3.6. Dispersive material character

Stating once again the scientific adventure based on Perzyna viscosity we are going into the next science territories.

Continuing the usefulness of wave effects observations one meets area of dispersive waves, that are extra separated from non-dispersive waves. The border between in mathematical research is so important that structural scientist and engineer usually does not realize how much different sphere of investigation there are when only a bit of viscosity is introduced in mechanical problem formulation. Observation of non-dispersive elastic-plastic waves is mostly so complicated that it can not be treated as a standard tool in construction engineering [12]. From a mathematical perspective, dealing with non-dispersive waves means doing narrow part of wave phenomena – the very restricted one to some linear relation between frequency and wave number. This opens the problem for non-linear function of group velocity makes it mathematically interesting. It, of course, brings physical opening as wall (Fig. 3). The key for this opening is the presence of viscosity in constitutive law. One again we can state the key-importance and perspective role of viscoplasticity.
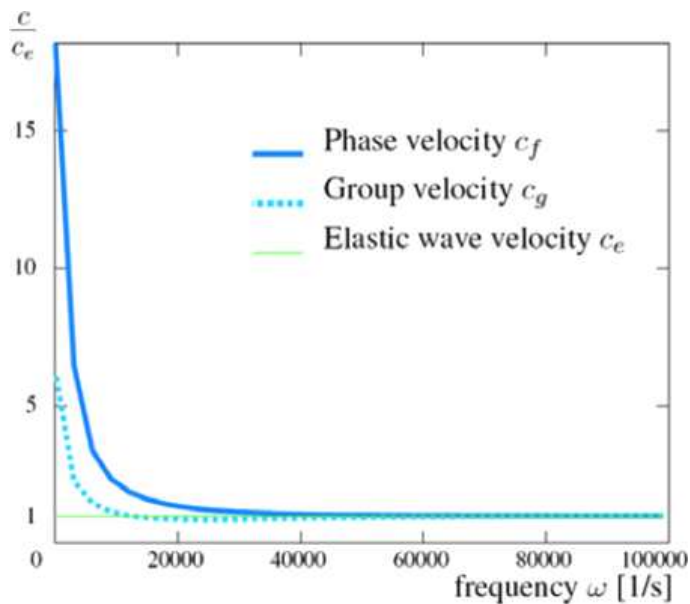

FIG. 3. Phase and group velocity as the function of frequency for elastic-visco-plastic solid (non-dimensional plot with the reference to value of elastic wave velocity).

3.7. Regularization with straight-forward material length-scale parameter

In problems of plastic stain localization or material bifurcation as well, there was the common need to introduce any mathematical regularization of the problem. Having additional problems with mesh-dependency the parameter in relation to width of localization zone was extraordinary introduced to save the problem and its mathematical solution from troubles. The parameter, also defined as length-scale parameter was explicitly introduced in the formulation in different ways within the range of research papers. Some of the proposed enrichments, however, computationally efficient, have no physical justification, so are in a artificial way the source of process control, and at once with no chance of identification. Within the viscoplasticity theory, viscosity itself takes the role of length-scale parameter. It is not just only internal material parameter to be identified, but plays the multifunction role, and one of them is length-scale.

3.8. Diverse way of energy dissipation

Viscoplastic constitutive relation introduces to mechanisms of energy dissipation. One comes from pure plastic flow described by associated or non-associated relation of plasticity criterion and direction of rate of plastic strain. The second mechanism results from dispersive character of wave propagation (Fig. 4). Let us make the observation that both are independent. Plastic model without vis-
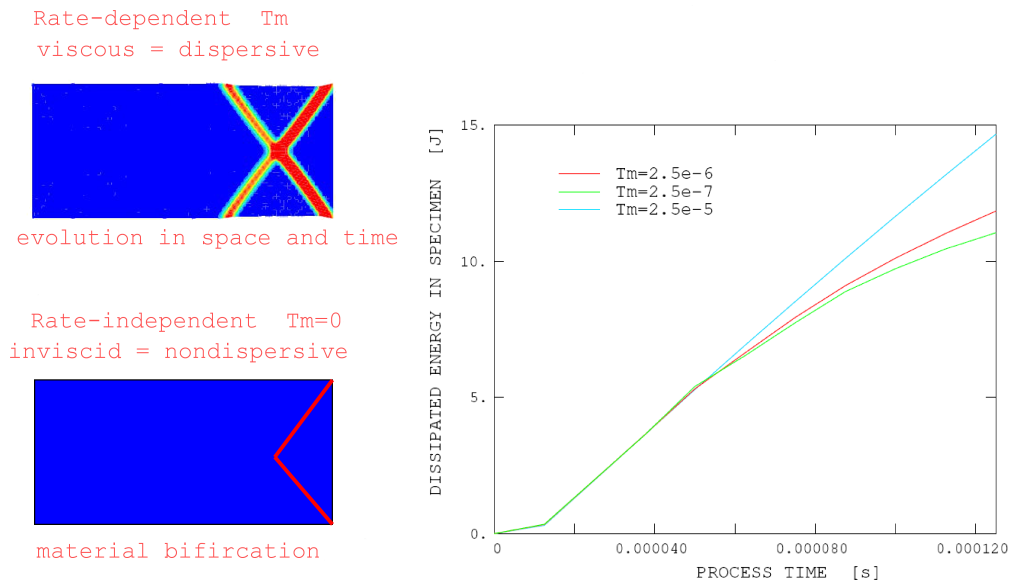

FIG. 4. Plastic equivalent strain for different elastic-visco-plastic material models with significant and un-significant role of viscosity in energy dissipation.

cosity presents plastic dissipation. No matter how complicated or how simple the plastic constitutive law is, it holds the change of external energy delivered to the system into inelastic energy, the part of energy that is not possible to get back. And the second model, but with another mechanism of energy change and dissipation, is non-plastic visco-elastic one, where with no plastic effect, dissipation goes through dispersion, so different change of external energy through material internal mechanism, but different from point of view of wave frequency that carry the energy with different intensity of dissipation [13].

And one more new character of physical mechanism of mechanical phenomenon occurs when we include the particle/package based way of energy exchange. The smooth energy exchange during the plastic flow has got quite opposite energy exchange during wave propagation in viscous media. Package based, non-smooth energy exchange in dispersive media, introduces once again the key-important role of viscous character of constitutive relation, that means crossing the next border in space of material behaviour description. Step-wise change of energy transformation versus continuous in time and smooth character of mechanical phenomena opens new challenges for softening and failure of material.

3.9. Smooth and non-smooth distributions within damage and failure

Development of strain and stress variables and other variables (velocity, temperature) within analysis of thermo-mechanical problem meets some physical observations defining limit. Within Perzyna theory of viscoplasticity some initial or critical limit are formulated. Meeting limits of width strength of material the fracture criterion is formulated. The criterion itself works independently whether the process has got smooth or non-smooth incremental character. As the standard way of explanation of material degradation the smooth distributions of damage and failure is considered. So distribution, tendency of changes, the initiation of failure and its development is rather expected as smooth function. Appearance of non-smooth, step-function or cyclic character changes is unexpected and recognised as the error result at first. But opening the perspective for the properties of the problem resulting from the propagation of dispersive waves, taking only the dispersive character of media, makes it possible to exclude the errors and analyse the non-smooth character of mechanics as adequate and correct description. That does it result from that perspective: the possibility of development of crack in even smooth material as non-smooth in time process. As a package character of energy dissipation, as non-smooth accumulation and change of external energy into dissipative energy, the process of crack growth can develop step-wise in time scale, resulting also step-wise increase of distance in space scale in accordance with wave character during advance of deformation

process [15]. It means that oscillations or in-heterogeneity. Again one can extend the feeling of surprise how far and wide is the influence of application of viscosity in solid mechanics.
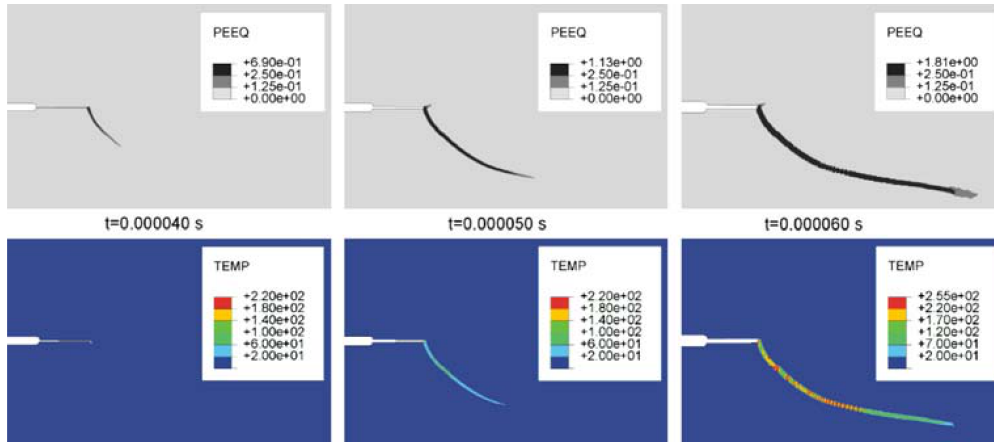

FIG. 5. Plastic equivalent strain and temperature for three time instances of thermo-mechanical impact problem with elastic-visco-plastic material model for description of crack nucleation and growth in pre-notched plate.

4. DAMAGE ANISOTROPY

4.1. Motivation

It is clear, that real materials are never free from defects. Their role in the deformation process is meaningful and spreads through all scales of observations i.e. from macro to atomistic. Concerning metallic materials one can emphasize defects like microcraks, microvoids, mobile and immobile dislocations densities [2, 40].

The existence of defects causes the overall behaviour of material during the deformation is anisotropic. This directional behaviour becomes crucial under extremely dynamic loading. Such results had caused that PTTV (which previously has included scalar damage measure [5, 6]) model has been extended to describe damage anisotropy in [15, 32] and developed in a series of papers discussed shortly in next sections – cf. Fig. 6.

4.2. Governing equations for PTTV including anisotropic damage

Before we start the discussion on selected recent results concerning the role of anisotropic damage description in the framework of PTTV formulation let us focus the attention on some formal aspects. Thus, in the development of PTTV including anisotropic damage description the key point was the introduction of
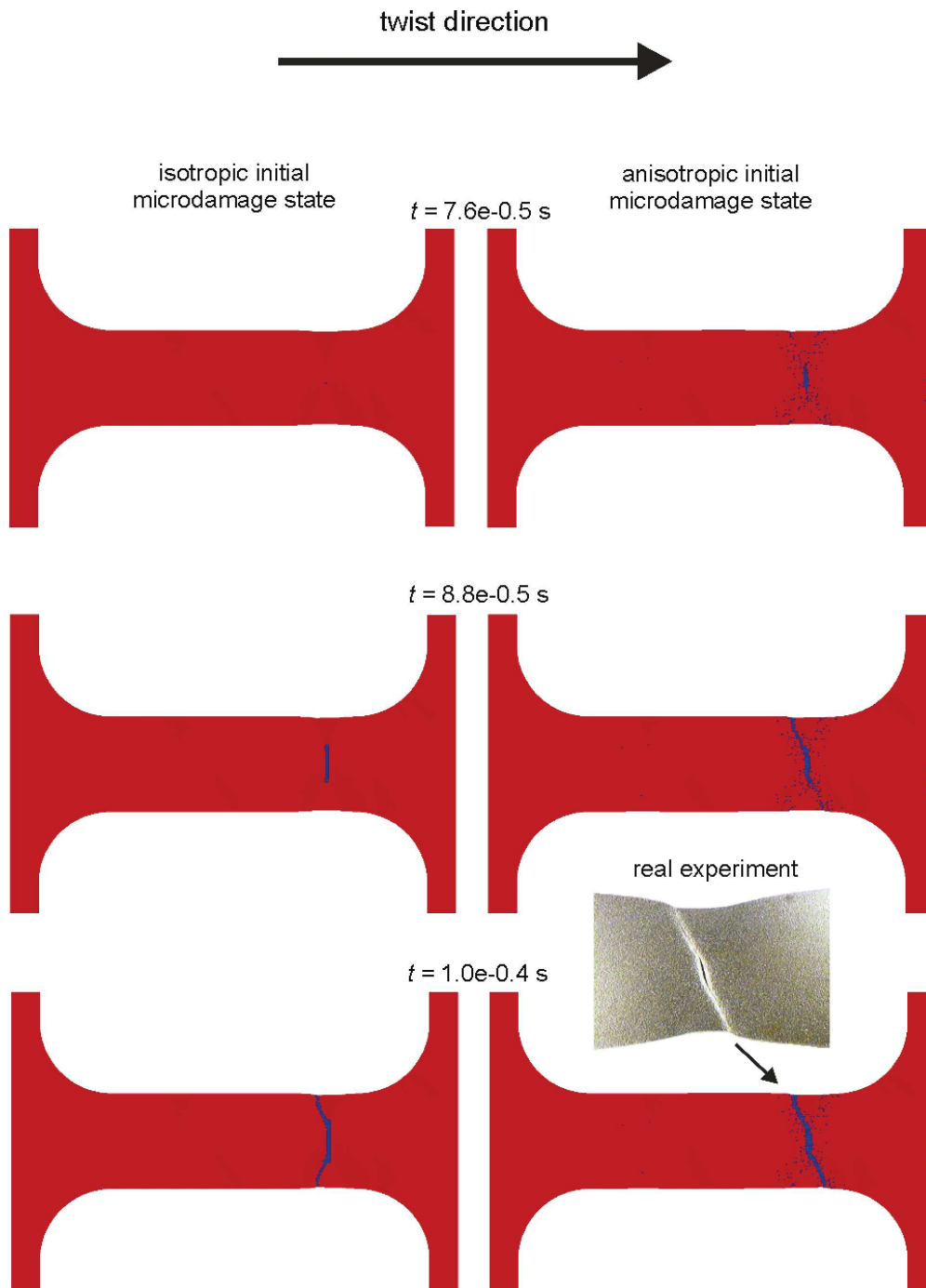
Fig. 6. The role of damage anisotropy in quantitative and qualitative approximation of dynamic tension test cf. [14, 34].

microdamage tensor as an internal variable [15, 32]. Formally the set of internal state variables (in the simplest case) was assumed as

$$\boldsymbol{\mu} = (\in^{p}, \boldsymbol{\xi}), \tag{4.1}$$

where $\boldsymbol{\xi}$ is microdamage tensor which takes into account the anisotropic microdamage effects. It was originally postulated in [32] that the norm of microdamage tensor describes (scalar) porosity, namely

$$\xi = \sqrt{\boldsymbol{\xi} : \boldsymbol{\xi}}. \tag{4.2}$$

Further investigations have shown that the components of $\boldsymbol{\xi}$ can be interpreted as levels of damage on representative volume element planes in macro scale [15, 34]. Final form of governing equations for IBVP including PTTV including anisotropic damage with a set of material functions can be found in [15, 32].

4.3. Global versus local damage approximation

In the paper [37] the notion of global (GDA) and local (LDA) damage approximations by the model were introduced. It was stated that we have GDA if (global) strain-stress curves from experiment and mathematical model are close to each other. Whereas good LDA means that apart from the global we have particularly good coincidence in: macrodamage initiation time, velocity of macrodamage evolution and the geometry of macrodamage pattern.

Based on series of numerical analysis (in agreement with experiments) it was concluded that PTTV model including anisotropic damage enables to reach good LDA. It appeared also that anisotropic measure for damage is a necessary condition to reach good LDA whereas scalar damage models are able to cover GDA only.

4.4. The role of covariance

Another important aspect of PTTV model is its covariant structure assured by consequent utilisation of Lie derivative [19, 30]. This problem was discussed in [35] and gives simultaneously the answer that the selection of objective rate in the constitutive structure is not arbitrary. Based on comparison with logarithmic derivative [41] is was proven that covariant material structure gives better damage (LDA) approximation.

4.5. Transition from ductile to brittle type of damage

It appears that under extreme dynamic loading the mechanism of damage in metallic materials can change from ductile to brittle type [4]. This aspect in terms of PTTV formulation was discussed in [26] and once more the meaningful role of damage anisotropy was proven. It was proposed that the assumed critical

value of porosity causing failure of material depends on velocity of damage evolution – giving analogous interpretation to the cumulative fracture criterion [3, 17].

Based on simulation of flat plate impact test [33] it was presented that for extreme velocities of flyer plate there is no time for porosity evolution in fracture zone on target plate. For such loading target plate spallation had appeared almost in a brittle manner without intensive plastic deformation and for low values of porosity in spallation zone – like in the experiment cf. [4].

4.6. Fractional viscoplasticity

Among other aspects discussed in terms of PTTV formulation including anisotropic damage like reduction of the number of material parameters utilising Artificial Neural Networks[38], demonstration of the role and level of thermal stresses in extremely fast processes [39] or discussing the connection between wave nature of fast dynamic processes with the formation of localised zones of damage [25] it appears that recently proposed concept of fractional calculus application in viscoplasticity [36] can set a new trend. It was shown that substituting the classical integer order derivative in flow rule by a fractional one (real order) unusual flexibility of the model is observed (e.g. non-normality was obtained without additional potential assumption). In this sense, in a perspective PTTV model with fractional derivative application can result in smaller number of material parameters.

5. Conclusions and future perspectives

In this incomplete review the aim was to show several aspects of the development of the Perzyna's viscoplasticity theory and stress in particular those results which came from the long lasting cooperation of the authors with their mentor Professor Piotr Perzyna. Sometimes the theory of viscoplasticity is perceived as very complex and too sophisticated because of its mathematical description. Many engineers prefer using simplified models with faster and easier drives to the solution of models. But of course these models are not able to describe many phenomena on different (micro and meso) scales of observations. In [38] one can find the idea how it is possible to optimize the number of constitutive parameters and in [8] what is the sensitivity of the constitutive properties to these parameters. The problems of identification of the constitutive parameters as well as the number of sufficient number of parameters are still open and could be the interesting field of research.

The general formulation of PTTV was presented and some discussions (well-posedness) were restricted to the formulations which omit the influence of temperature coupling or to adiabatic processes. The development and extensions of

Perzyna's formulations like plastic anisotropy or fractional viscoplasticity were also indicated.

The PTTV is now widely used in computer simulation of many different types of fast processes in which the rates of deformations are extremely high (above 10^4 1/s) like crash tests or estimation of the structures safety under blasts for which the coupling with temperature fields is crucial.

6. List of author's research papers on viscoplasticity leading and inspiring by Piotr Perzyna

6.1. Publications

1. Sumelka W., *Fractional viscoplasticity*, Mechanics Research Communications, **56**, 31–36, 2014.

2. Łodygowski T., Sumelka W., *Anisotropic damage for extreme dynamics*, Handbook of Damage Mechanics, edited by George Z. Voyiadjis, 2014.

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5. Sumelka W., *Recent advances in Perzyna's type viscoplasticity*, 30th anniversary of scientific cooperation between Poznan University and Leibniz Universität Hannover – Bilateral workshop on topics in Civil and Mechanical Engineering, March 14–15, Hannover, Germany, 2013.

6. Sumelka W., Łodygowski T., *Design of steel structures subjected to extrem dynamic loadings*, METNET Annual Seminar, pp. 1, Izmir, October 10–11, Turkey, 2012.

7. Łodygowski T., Sumelka W., *Damage anisotropy*, 6th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS2012), pp. 317, September 10–14, Vienna, Austria, 2012.

8. Sumelka W., *Fractional viscoplasticity*, 37th Solid Mechanics Conference, pp. 102–103, August 27–31, Warsaw, Poland, 2012.

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15. Glema A., Łodygowski T., Sumelka W., *Anisotropic theory of viscoplasticity for spallation and perforation of metal sheets*, 7th International Symposium on Impact Engineering ISIE2010, July 4–7, Warsaw, Poland, 2010.

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