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CONCENTRATION OF THE STRESSES IN SHELLS WITH DISCONTINUITIES OF THE SURFACES

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1. Introduction

The subject of this paper is the concentration of the stresses produced in shells by the angle discontinuities of their middle surfaces. Usually, the construction of large shell structures consists in building the whole structure from smaller elements. Most frequently, the elements are welded together. The inaccuracies of the element shapes give rise to certain additional stress fields which commonly are not taken into account in the calculations.

The deviations of the middle surfaces of the shell from the theoretical surface can be described by continuous and discontinuous functions. In the present paper we shall consider only the cases in which the deviation of the middle surface of the shell made of elements from the ideal element can be described by the function with the discontinuous first derivative in the direction normal to the edge of the element of the shells. Figure 1 presents such a shell with angle discontinuities of the middle surface at the boundary of the rectangular shallow elements. δ denotes the angle between the straight lines tangential to the middle surface and perpendicular to the line of the junction.

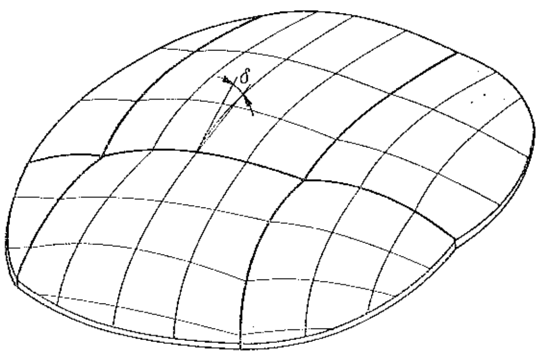


Fig. 1

2. Spherical shell with symmetrical angle discontinuities

Let us consider first the case of a spherical shell constructed of symmetrical toroidal segments. The radii of curvature of the sphere δ_i defines the angle discontinuity of the middle surface of the shell considered. The shell is under internal pressure p. The problem of determination of the stresses in the structure can be solved by considering the conditions of compatibility of displacements at the junctions and equilibrium equations of the segments.

Let us start with the equations of the membrane theory. If we introduce simplifications consisting in disregarding the terms of order $\delta_1^2 \ll 1$, the membrane forces in two neighbouring segments are given by the equations:

$$\left.\begin{matrix} N_{\varphi_g} \\ N_{\varphi_d} \end{matrix}\right\} = \frac{pR}{2} \frac{\sin \alpha_{zi}}{\sin\left(\alpha_{z_i} \pm \frac{\delta_i}{2}\right)}, \quad N_{\vartheta_g} = N_{\vartheta_d} = \frac{pR}{2}, \tag{2.1}$$

where R is the average radius of the spherical shell. The membrane state of stress in the shell is possible only, if we add the equilibrating load P_i (Fig. 3), which can be defined as

$$P_i = \frac{pR}{2} \frac{\delta_i}{\sin \alpha_{zi}}.$$

However, this load does not act on the shell; hence the bending stresses appear in the shell.

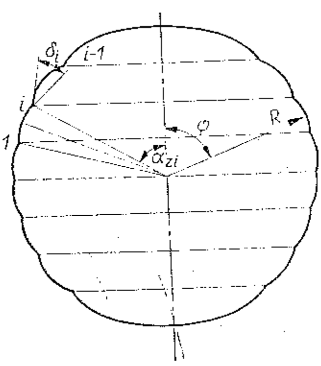


Fig. 2

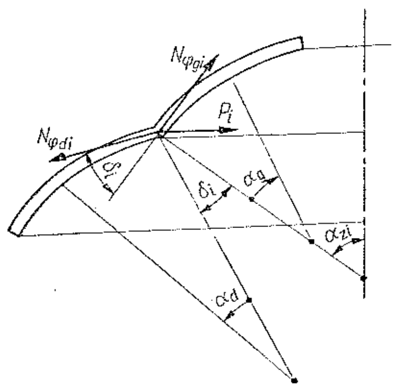


Fig. 3

Let us further consider the deformations of the shell. On the basis of the formulae (2.1) and taking into account Hooke's law, we find the difference in the radial displacements of the neighbouring segments in the following form:

$$\Delta_{r_{0i}} = -\frac{pR^2 \delta_i \nu}{2Eh} \cos \alpha_{zi}. \tag{2.2}$$

The differences of the angles created by the straight lines normal to the middle surface at the junctions of the segments before and after deformation are:

$$\Delta\gamma_{0i} = -\frac{pR^2 \delta_i}{2Eh} \operatorname{ctg}^2 \alpha_{zi}. \tag{2.3}$$

The above formula is valid when the widths of the segments are approximately equal. These quantities are necessary to build the conditions of compatibility of the segments. As already indicated, the equilibrium in the membrane state can be ensured by introducing the load P_i acting, non existent in the circumferential plane. This means that a state of bending stress appears in the shell. These two cases are presented in Fig. 4.

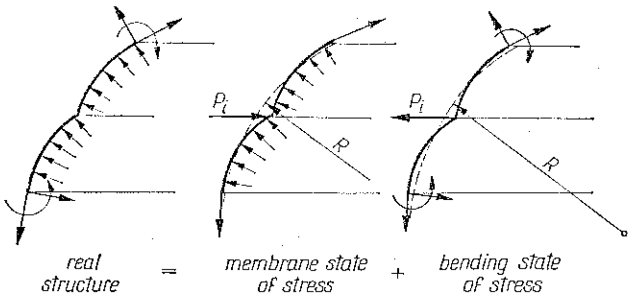


Fig. 4

Considering the bending state of stress, we can replace the real surface of the shell by an ideal spherical surface with average radius R. This is justified by the local character of the disturbances produced by the load P_i. The solution for this case can be obtained on the basis of the equation of the spherical shell loaded symmetrically, which for not very small values of α_{zi} takes the following form [1]:

(2.4) $$LL(RQ_\varphi)+\mu^4 RQ_\varphi=0,$$

where

$$\mu^4=\frac{Eh}{D}-\frac{\nu^2}{R^2}$$

and

$$L=\frac{l}{R}\left[\frac{d^2}{d\varphi^2}+\frac{d}{d\varphi}\operatorname{ctg}\varphi+\operatorname{ctg}^2\varphi\right].$$

The solution of this equation gives the following formulae for the internal forces and moments in two neighbouring segments. The indices g and d correspond to the upper and lower segments, respectively:

$$\left.\begin{matrix}N_{\vartheta_{gi}}\\ N_{\vartheta_{di}}\end{matrix}\right\}=\frac{\lambda e^{-\lambda_{\substack{g\\d}}}}{\sqrt{\sin(\alpha_{zi}\mp\alpha_{\substack{g\\d}})}}\left\{\left[D_{1g}\left(1\mp\frac{1}{2\lambda}\operatorname{ctg}(\alpha_{zi}\mp\alpha_{\substack{g\\d}})-D_{z\substack{g\\d}}\right)\right]\times\right.$$
$$\left.\times\cos\lambda\alpha_{\substack{g\\d}}+\left[D_{1\substack{g\\d}}+D_{z\substack{g\\d}}\left(1\mp\frac{1}{2\lambda}\operatorname{ctg}(\alpha_{zi}\mp\alpha_{\substack{g\\d}})\right)\right]\sin\lambda\alpha_{\substack{g\\d}}\right\},$$

(2.5) $$\left.\begin{matrix}N_{\varphi_{gi}}\\ N_{\varphi_{di}}\end{matrix}\right\}=Q_{\varphi_{\substack{gi\\di}}}\operatorname{ctg}(\alpha_{zi}\mp\alpha_{\substack{g\\d}}),$$

$$\left.\begin{matrix}M_{\varphi_{gi}}\\ M_{\varphi_{di}}\end{matrix}\right\}=\frac{Re^{-\lambda\alpha_{\substack{g\\d}}}}{2\lambda\sqrt{\sin(\alpha_{zi}\mp\alpha_{\substack{g\\d}})}}\left\{\left[D_{1\substack{g\\d}}+D_{2\substack{g\\d}}\left(1\mp\frac{1}{\lambda}\operatorname{ctg}(\alpha_{zi}\mp\alpha_{\substack{g\\d}})\right)\left(\frac{1}{2}-\nu\right)\right]\cos\lambda\,\alpha_{\substack{g\\d}}-\right.$$
$$\left.-\left[D_{1\substack{g\\d}}\left(1\mp\frac{1}{\lambda}\operatorname{ctg}(\alpha_{zi}\mp\alpha_{\substack{g\\d}})\left(\frac{1}{2}-\nu\right)\right)-D_{2\substack{g\\d}}\right]\sin\lambda\alpha_{\substack{g\\d}}\right\},$$

$$\text{(2.5)}\quad \left.\begin{matrix} M_{\vartheta_{gi}} \\ M_{\vartheta_{di}} \end{matrix}\right\} = \frac{Re^{-\lambda_{\substack{g\\d}}}}{2\lambda\sqrt{\sin(\alpha_{zi}\mp\alpha_{\substack{g\\d}})}}\left\{\left[\nu D_{1\substack{g\\d}} + D_{2\substack{g\\d}}\left(\nu\mp\frac{1}{\lambda}\operatorname{ctg}(\alpha_{zi}\mp\alpha_{\substack{g\\d}})\left(\frac{\nu}{2}-1\right)\right)\right]\times\right.$$
[c.d.]
$$\left.\times\cos\lambda\alpha_{\substack{g\\d}} - \left[D_{1\substack{g\\d}}\left(\nu\mp\frac{1}{\lambda}\operatorname{ctg}(\alpha_{zi}\mp\alpha_{\substack{g\\d}})\left(\frac{\nu}{2}-1\right)\right) - \nu D_{2\substack{g\\d}}\right]\sin\lambda\alpha_{\substack{g\\d}}\right\},$$

where

$$D_{1g} = e^{\lambda\alpha_{zi}}(C_1\cos\lambda\alpha_{zi} + C_2\sin\lambda\alpha_{zi}),$$
$$D_{2g} = e^{\lambda\alpha_{zi}}(C_1\sin\lambda\alpha_{zi} - C_2\cos\lambda\alpha_{zi}),$$
$$D_{1d} = e^{\lambda(\pi-\alpha_{zi})}(C_1\sin\lambda\alpha_{zi} - C_2\cos\lambda\alpha_{zi}),$$
$$D_{2d} = e^{\lambda(\pi-\alpha_{zi})}(C_1\sin\lambda\alpha_{zi} + C_2\cos\lambda\alpha_{zi}).$$

The radial displacements of two neighbouring segments are:

$$\text{(2.6)}\quad \left.\begin{matrix} \Delta r_{0gi} \\ \Delta r_{0di} \end{matrix}\right\} = \frac{R\sin(\alpha_{zi}\mp\alpha_{\substack{g\\d}})\lambda e^{-\lambda\alpha_{\substack{g\\d}}}}{Eh\sqrt{\sin(\alpha_{zi}\mp\alpha_{\substack{g\\d}})}}\left\{\left[D_{1\substack{g\\d}}\left(1+\frac{1}{\lambda}\operatorname{ctg}(\alpha_{zi}\mp\alpha_{\substack{g\\d}})\left(\frac{1}{2}+\nu\right)\right) - D_{2\substack{g\\d}}\right]\times\right.$$
$$\left.\times\cos\lambda\alpha_{\substack{g\\d}} + \left[D_{1\substack{g\\d}} + D_{2\substack{g\\d}}\left(1-\frac{1}{\lambda}\operatorname{ctg}(\alpha_{zi}\mp\alpha_{\substack{g\\d}})\left(\frac{1}{2}+\nu\right)\right)\right]\sin\lambda\alpha_{\substack{g\\d}}\right\}.$$

The angles γ_i for two neighbouring segments are:

$$\text{(2.7)}\quad \left.\begin{matrix} \gamma_{gi} \\ \gamma_{di} \end{matrix}\right\} = \frac{2\lambda^2}{Eh\sqrt{\sin(\alpha_{zi}\mp\alpha_{\substack{g\\d}})}}e^{-\lambda\alpha_{\substack{g\\d}}}\left[-D_{2\substack{g\\d}}\cos\lambda\alpha_{\substack{g\\d}} + D_{1\substack{g\\d}}\sin\lambda\alpha_{\substack{g\\d}}\right].$$

Since the segments are joined together, we have to satisfy the following boundary conditions: for $\alpha_g=0$ and $\alpha_d=0$,

$$\text{(2.8)}\quad \begin{gathered} Q_{gi}+Q_{di} = P_i\sin\alpha_{zi}, \qquad M_{\varphi_{gi}} = M_{\varphi_{di}}, \\ \Delta r_{0dim} + \Delta r_{0di_b} = \Delta r_{0gim} + \Delta r_{0gi_b}, \\ \Delta\gamma_{im} - (\gamma_{gi}+\gamma_{di})_b = 0. \end{gathered}$$

The indices m and b denote the membrane and bending states of stress. The solution of the above equations gives values of the constants D_i and, further, the internal forces and moments in the shell. The results of the above calculations are as follows:

The forces and moments resulting from the angle discontinuity of the middle surface are practically independent of the angle α_{zi} if this angle is not very small. If we take into account the membrane forces and bending moments resulting from the solution of the membrane and bending cases, we find the following approximate formula for the maximum stresses at the junction (exact for $\alpha_{zi}=\pi/2$):

$$\text{(2.9)}\quad \begin{aligned} \sigma_\varphi &= \frac{pR}{2h}\left[1\pm\delta\frac{3}{2}\frac{1}{\sqrt[4]{3(1-\nu^2)}}\sqrt{\frac{R}{h}}\right], \\ \sigma_\vartheta &= \frac{pR}{2h}\left\{1+\delta\left[\left(\frac{1}{2}\sqrt{3(1-\nu^2)}\pm\frac{3}{2}\frac{\nu}{\sqrt[4]{3(1-\nu^2)}}\right)\sqrt{\frac{R}{h}} - \frac{1}{2}\operatorname{ctg}\frac{\alpha_0}{2}\right]\right\}. \end{aligned}$$

The sign + corresponds to the external surface and – to the internal surfaces of the shell. The factor of concentration of stress can be defined as

$$k_1 = \frac{\sigma_z}{\sigma_z \text{ ideal sphere}},$$

where

$$\sigma_z^2 = \sigma_\varphi^2 - \sigma_\vartheta \sigma_\varphi + \sigma_\vartheta^2, \qquad \sigma_z \text{ ideal sphere} = \frac{pR}{2h}.$$

At the external surface we have

$$k_1 = 1 + \delta\left((0.25\sqrt[4]{3(1-\nu^2)} + 0.75\frac{1+\nu}{\sqrt[4]{3(1-\nu^2)}}\right)\sqrt{\frac{R}{h}}, \tag{2.10}$$

for $\nu = 0.3$ we have

$$k_1 = 1 + 1.08\delta\sqrt{\frac{R}{h}}, \qquad \delta > 0.$$

The factor k_1 of concentration of stress at the internal surface of the shell for $\nu = 0.3$ is

$$k_1 = \sqrt{1 + 0.884|\delta|\sqrt{\frac{R}{h}} + 1.8\,\delta^2\frac{R}{h}}.$$

The further conclusion resulting from the solution is as follows: The membrane forces in the shell can be calculated as for the ideal spherical shell, if the widths of the segments are not too small.

The case of a spherical shell with angle discontinuity near the pole — i.e., when the angle α_{zi} is small — should be solved separately using, for example, the shallow shells equations. However, such calculations give somewhat smaller stresses in the junction than those for large α_{zi}.

We notice that the concentration of stress produced by the symmetrical discontinuity of the surface of the spherical shell is relatively large. Already for $\delta = 2.5°$ and $h/R = 0.01$ the increment of the reduced stress is 50 per cent by comparison with the reduced stress produced by the membrane state in the ideal shell.

3. Effect of the angle discontinuity in an arbitrary shell

The calculations performed for the previous case of the spherical shell demonstrated that the discontinuity of the middle surface of the shell produces a local and relatively narrow area of bending stresses appearing along the line of the discontinuity. The width of this area is of order of several

$$l = \sqrt{R_2 h}/\sqrt[4]{12(1-\nu^2)},$$

where l is the characteristic length of the shell.

We may expect that the arbitrary shell of positive Gauss curvature behaves in a similar way, since in such a shell all bending effects are very quickly stifled [2].

Let us consider an arbitrary thin shell built of segments. The middle surface of the shell is discontinuous and δ denotes the angle of discontinuity. Let us introduce the system of non-dimensional coordinates x, y on the middle surface of the shell with the y-axis following the line of discontinuity. The curvature of the shell in the area close to this line can be characterised by the principal radii of curvature R_1 and R_2. We assume that the direction of the line of discontinuity follows the direction of the larger of the principal curvatures $R_2 < R_1$, Fig. 5.

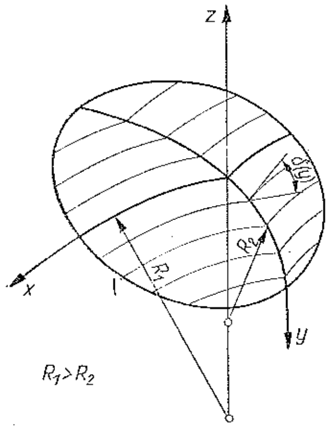


Fig. 5

The angle $\delta = \delta(y)$ describes the discontinuity of the middle surface and is positive when the shell is bent inwards. We assume that the function $\delta(y)$ vanishes with increasing distance from the origin of the system of coordinates. This means that this geometrical disturbance is of local character. On the basis of the results of the case considered previously, we can introduce the following assumptions and simplifications:

a) we disregard the effect of the discontinuity of the middle surface on the membrane stresses in the shell — i.e., we calculate these stresses as for an ideal shell;

b) we assume that the effect of angle discontinuity consists in the action of an additional fictitious load $P(y)$ resulting from the action of the membrane forces in the junction of the segments;

c) we assume that the real state of stress in the shell can be presented by superposition of the membrane and bending states of the stresses;

d) the shell in the area close to the discontinuity can be considered as shallow, which enables us to assume that the first quadratic form of the surface considered differs slightly from the first quadratic form of the plane or of the cylindrical surface.

Considering the conditions of equilibrium of the region close to the line of the discontinuity, we find that the load $P(y)$ can be defined as:

$$P(y) = N_{xx_0}\,\delta(y),$$

where N_{xx_0} is the membrane force in the ideal shell acting in the direction perpendicular to the line y. The function $\delta(y)$ depends on the geometry of the discontinuity of the surface and can be assumed arbitrarily. The fictitious load $P(y)$, symmetric in respect to the x-axis, can be presented by the following surface, Fourier integral

$$P(y) = \int_0^\infty \int_0^\infty Q(\beta) \cos\alpha x \cos\beta y \, d\alpha \, d\beta, \tag{3.1}$$

where $Q(\beta)$ is a certain function of the parameter β defining the distribution of the load $P(y)$. On the basis of the equations of shallow shells, we find similar integral expressions for the normal deflection and the stress function:

$$w=\frac{l^4}{D}\int_0^\infty\int_0^\infty \frac{Q(\beta)}{M}(\alpha^2+\beta^2)^2\cos\alpha x\cos\beta y\,d\alpha\,d\beta, \tag{3.2}$$

$$\Phi=l^2R_2\int_0^\infty\int_0^\infty \frac{Q(\beta)}{M}(\alpha^2+\lambda\beta^2)\cos\alpha x\cos\beta y\,d\alpha\,d\beta, \tag{3.3}$$

where l is the characteristic length of the shell and M denotes the denominator of the integral which is

$$M=(\alpha^2+\beta^2)^4+(\alpha^2+\lambda\beta^2)^2.$$

The internal forces and moments in the shell can be calculated on the basis of the above solutions. For example, for the bending moment M_{xx}, we find the following integral:

$$M_{xx}=l^2\int_0^\infty\int_0^\infty \frac{Q(\beta)}{M}(\alpha^2+\nu\beta^2)(\alpha^2+\beta^2)^2\cos\alpha x\cos\beta y\,d\alpha\,d\beta. \tag{3.4}$$

The above expressions can easily be once integrated by means of the calculus of residues. For example,

$$M_{xx}=-\operatorname{im}\frac{\pi l^2}{4}\int_0^\infty Q(\beta)\sum_{k=1}^{2}\frac{e^{i\alpha_k}(\alpha_k^2+\nu\beta^2)}{\alpha_k\left(\alpha_k^2+\beta^2+\frac{1}{2}\,i\,(-1)^{k+1}\right)}\cos\beta y\,d\beta,$$

where α_k are the roots of the characteristic equation $M=0$ with positive imaginary parts. The above integral is complex, slowly convergent and cannot be evaluated analytically. However, it can be calculated numerically. In order to speed up computation, we can apply the method proposed in [2]. This method consists in calculating the integral in two stages: numerically from 0 to a certain value B and, analytically from B to ∞.

If similarly we dissolve the bending moment

$$M_{xx}=\int_0^\infty m(\beta)\,d\beta=\int_0^B m(\beta)\,d\beta+\int_B^\infty m(\beta)\,d\beta=M_{xx}^0+M_{xx}^B,$$

we find for M_{xx}^B the simplified formula presenting the expressions for the roots and the integrand $m(\beta)$ by means of power series in respect to $1/\beta$.

Taking into account two terms only, we find

$$M_{xx}^B=\frac{\pi l^2}{4}\int_B^\infty\left[(1+\nu)\,C_1(k_1x)-(1-\nu)\frac{S_1(k_1x)}{2k_1}\beta\right]\frac{Q(\beta)}{\beta}e^{-\beta x}\cos\beta y\,d\beta,$$

where

$$C_1(k_1x)=\cos h(k_1x)\cos(k_1x),$$

$$S_1(k_1x)=\sin h(k_1x)\cos(k_1x)+\cos h(k_1x)\sin(k_1x),$$

$$k_1=\sqrt{\frac{1}{8}(1-\lambda)}.$$

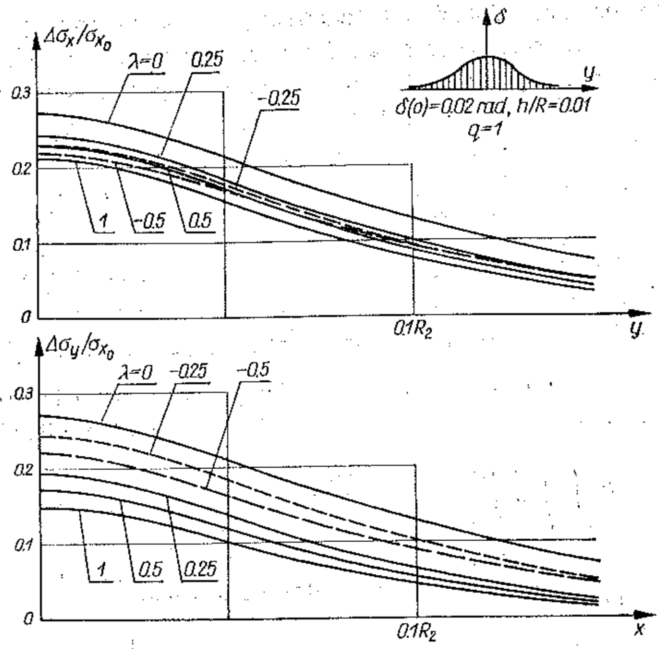
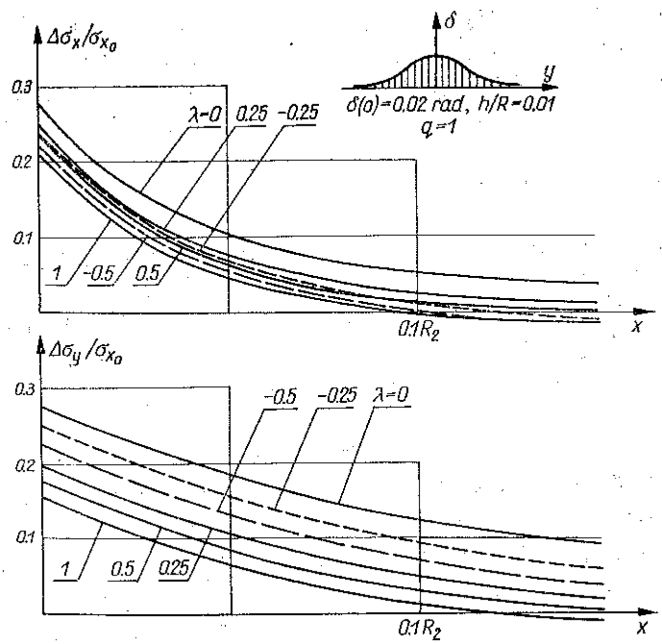

δ
y
δ(o)=0.02 rad, h/R=0.01
q=1
Δσx/σx0
λ=0
0.25
-0.25
1
-0.5
0.5
0.1R2
y
Δσy/σx0
λ=0
-0.25
-0.5
1
0.5
0.25
0.1R2
x

Fig. 6


δ
y
δ(o)=0.02 rad, h/R=0.01
q=1
Δσx/σx0
λ=0
0.25
-0.25
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-0.5
0.5
0.1R2
x
Δσy/σx0
-0.5
-0.25
λ=0
1
0.5
0.25
0.1R2
x

Fig. 7

The above integral is much more simple than (4.4) and can usually be calculated analytically if $Q(\beta)$ is not too complex a function. The results of the computations of the stresses in the shell produced by the angle discontinuity are presented in Figs. 6 – 8. In Fig. 6 and 7, the distribution of the additional stresses appearing on the external surface for the vanishing discontinuity defined by the function

$$\delta(y) = \pm 0.02 \frac{1}{1+y^2/q^2} \text{ [rad]}$$

is given for the different values of the parameter $\lambda = R_2/R_1$, $\lambda = 1, 0.5, 0.25, 0, -0.25, -0.5$; q is here a certain arbitrary parameter which is taken now as 1. We observe that the additional stresses resulting from the effect of discontinuity of the middle

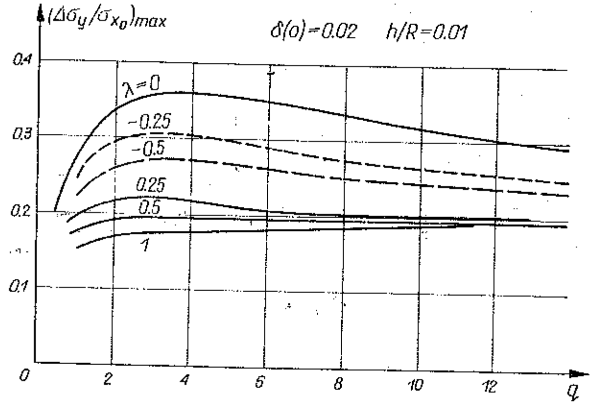

Fig. 8

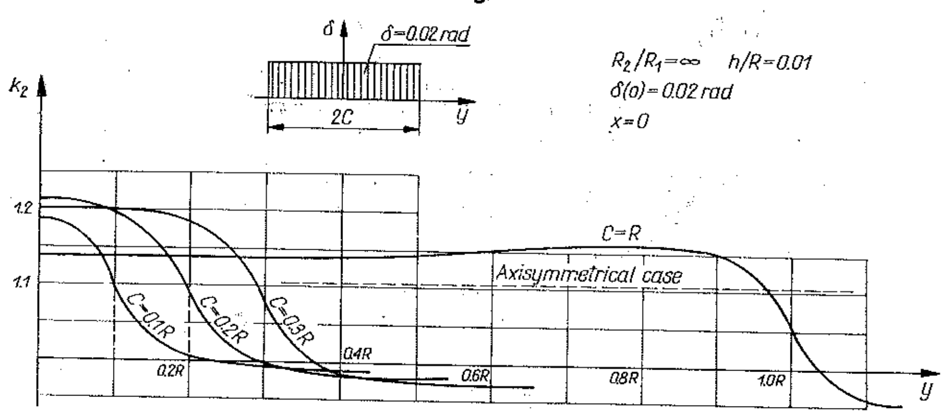

Fig. 9

surface increase as the parameter q decreases. They reach the maximum value for the cylindrical shell — i.e., for $\lambda=0$. The damping of the local stress also decreases with decrease of λ. Figure 8 presents maximum increment of the stress σ_{yy} at the external surface as a function of the parameter q. We observe that with increase of this parameter the increment of the stress reaches the value which was previously obtained for the axially symmetrical case. For the cylindrical shell, the maximum value is for $q=3-5$. Figure 9 presents the concentration factor for the case of the constant angle discontinuity along a certain segment of the circumference of the cylindrical surface. The solution presented can be applied to cases in which the discontinuity of the shell surface follows the direction of the larger of the principal

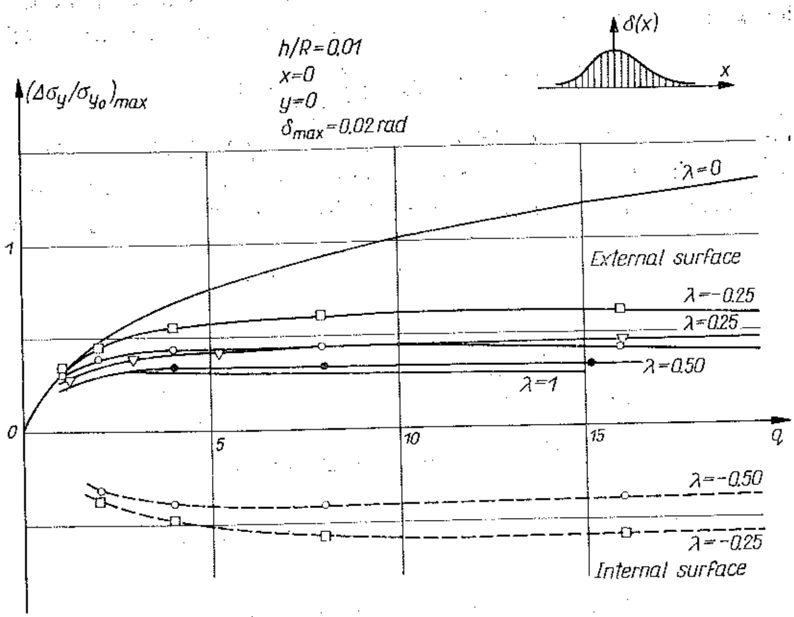

Fig. 10

curvatures of the shell. The cases in which the discontinuities follow the directions of smaller curvature can be solved in the similar way by rotation of the system of coordinates. We obtain similar integral expressions for the internal forces and moments. The results presented in Fig. 10 are derived for the discontinuity along the generator for various values of the parameter λ. The largest values of the increment of the stress σ_{yy} are obtained for $\lambda=0$. The behaviour of the increment $\Delta\sigma_{xx}$ is similar. But the numerical values are about $\Delta\sigma_{xx}\approx 0.4\Delta\sigma_{yy}$.

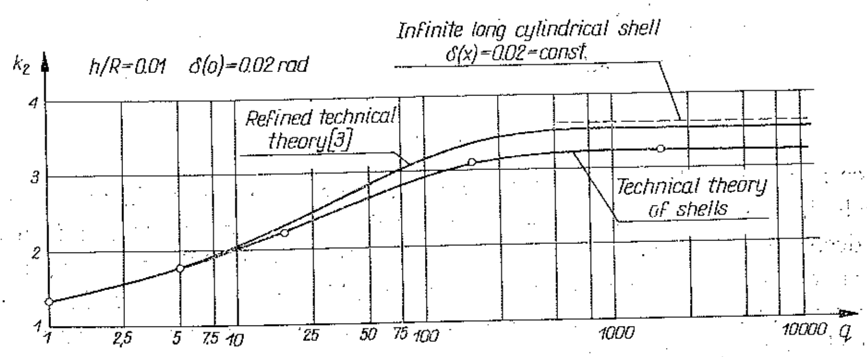

Fig. 11

However, the case of large q and $\lambda=0$ should be here excluded, because in this case the problem considered becomes not local and the shell under the fictitious load is not in equilibrium. Thus, the solution applied becomes singular. This means that the case of a cylindrical shell with the discontinuity along the generator must be solved separately, on the basis of a more exact assumption. Resolving the forces obtained from the membrane and bending states, the complete state of loads should be taken into account (together with the boundary forces). Also, for $\lambda=0$, the solution should satisfy the condition of periodicity in the circumferential direction. This can be achieved if we use the Fourier series as the presentation of the displacement w and the stress function Φ. The results given in Fig. 11 are so obtained. We observe that the stress concentration factor defined as

$$k_2 = \frac{\sigma_z}{\sigma_z \text{ ideal cylinder}}$$

increases with increase of the parameter q tending to a value which corresponds to the infinitely long shell with constant angle discontinuity:

$$k_2 = 1 \pm 2.57\delta\frac{R}{h} + 2.2\delta^2\frac{R^2}{h^2}(1-\nu+\nu^2).$$

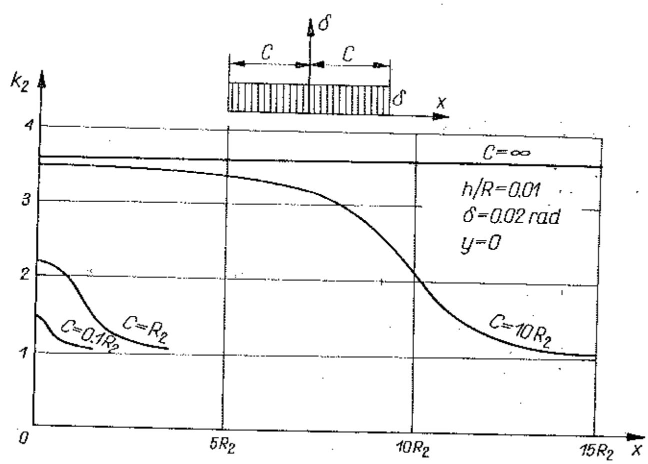

Fig. 12

The sign + corresponds to the external surface of the shell, – the internal surface. The dotted line in Fig. 11 presents the values obtained by means of shallow shell equations. The full line — by means of what is called "equations of the improved technical theory of shells". Figure 12 presents the factor k for the case $\delta(y)=\text{const}$ along a certain segment of the generator.

4. Conclusions

The inaccuracy of the shell structure consisting in the angle discontinuity of the middle surface produces local bending in the shell and is the reason for the significant stress concentration in the area close to the line of discontinuity. The maximum value of the concentration factor for the spherical shell is given by the formulae (2.10).

It should be emphasized that most containers for gas or liquid are built of rectangular elements. In this case, it may happen that three elements meet each other at one point of the shell surface. On the basis of the theorem of superposition of the solutions, we find that at this point the stress concentration is about 1.5 times greater than that for the single discontinuity.

The approximate formula for the concentration factor for the cylindrical shell with discontinuity along the circumference takes the form:

$$k_2 = 1 + 1.1\delta_{max}\sqrt{\frac{R}{h}}.$$

Very large concentrations of stresses in the cylindrical shell produce angle discontinuity in the generator direction. In this case, the factor k_2 is proportional to the ratio R/h but in all cases considered previously it was proportional $\sqrt{R/h}$ only.

The solutions presented above are based on the linear theory of shells of small deflections. Note that the effect of the change in the geometry of the shell with the load leads to partial elimination of inaccuracies. In reality, the increment of the stress will be smaller than that resulting from the linear theory.

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Streszczenie

KONCENTRACJA NAPRĘŻENIA W POWŁOKACH Z NIECIĄGŁOŚCIAMI POWIERZCHNI

Tematem pracy jest określenie koncentracji naprężeń wywołanej w powłoce przez nieciągłości kątowe (załomy) na powierzchni środkowej. Postawione zadanie rozwiązano najpierw dla powłok kulistych zbudowanych z segmentów toroidalnych. Następnie zbadano wpływ nieciągłości w powłokach dowolnych dla różnych funkcji opisujących rozkład nieciągłości.

Резюме

КОНЦЕНТРАЦИЯ НАПРЯЖЕНИЙ В ОБОЛОЧКАХ С РАЗРЫВАМИ ПОВЕРХНОСТИ

Темой работы является определение концентрации напряжений, вызванной в оболочке угловыми разрывами (изломами) на срединной поверхности. Представленная проблема решена сначала для сферических оболочек построенных из тороидальных сегментов. Затем исследовано влияние разрыва в произвольных оболочках для разных типов функции описывающей распределение разрыва.

POLITECHNIKA WARSZAWSKA

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