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WAVE PROPAGATION IN A PRE-STRESSED REINFORCED COMPOSITE

HILMI DEMIRAY (ISTAMBUL)

In this study, a small time dependent displacement field is superimposed on an initial large static deformation, for a reinforced composite, and the linearized field equations and associated boundary conditions are obtained. Propagation of harmonic waves in such a composite medium is studied and various special cases are discussed. From the condition of propagation some criteria for the stability of equlibrium configuration are deducé.

1. Introduction

Finite or small deformation theories of reinforced composite materials have been studied by many researchers in mechanics (cf. Rivlin [1], Adkins [2], and many others). As might easily be seen, the finite deformation theory leads us to highly non-linear differential equations which could only be solved for some special cases. The method of solutions is similar to that of finite elasticity theory. The main problem of such elastic bodies subjected to a finite deformation is the question of stability of equilibrium. To our knowledge, this problem has not yet been investigated for the reinforced composite materials. Moreover, the propagation of elastic waves in reinforced composites, to which the continuum theory is applicable, is not yet a well studied subject. In this regard we refer the reader to studies by Demiray [3], Weitsman and Benverste [4], and Bose and Mall [5].

In this paper we study the propagation of small amplitude harmonic waves in a composite reinforced by a family of inextensible fibers and subjected to a large initial static deformation. For this purpose, we superimpose a small time dependent displacement field on a given large static deformation, and obtain the linearized field equations, governing this small displacement, and associated boundary conditions. These are given in Section 3. In the last section of the paper the propagation of harmonic waves in such a pre-stressed medium is studied and various special cases are discussed. From the propagation condition, we also obtained some criteria about the stability of initial equilibrium configuration.

2. Theoretical preliminaries

Consider an elastic body B_0 reinforced with a family of continuous and inextensible fibers which are the envelop of the unit vector field $\mathbf{A}(\mathbf{X})$. We assume that this vector field is at least piece-wise continuous in the domain of the body B_0.

Upon application of the external forces, the body occupies a new configuration denoted by B. Let $\mathbf{X}\,(X_K,\ K=1,2,3)\ \varepsilon B_0$ and $\mathbf{x}\,(x_k,\ k=1,2,3)\ \varepsilon \mathscr{B}$, respectively, be the material and space coordinates of a material point in the body. Thus the motion of the body is characterized by

$$\mathbf{x}=\mathbf{x}(\mathbf{X},t)\,. \tag{2.1}$$

Let $\mathbf{a}(\mathbf{x})$ be the image of the vector field $\mathbf{A}(\mathbf{x})$ in the deformed configuration. Then the inextensibility of material in the direction of fibers may be stated as (cf. SPENCER [7]):

$$C_{KL}A_K A_L=1\,,\qquad \text{or}\qquad c_{kl}a_k a_l=1\,, \tag{2.2}$$

where $C_{KL}=x_{k,K}x_{k,L}$, $\left(x_{k\,k}\equiv\dfrac{\partial x}{\partial X_K}\right)$, $c_{kl}=X_{K,k}X_K$, $\left(X_{K\,k}\equiv\dfrac{\partial X_K}{\partial x_k}\right)$ are respectively the Green and Cauchy deformation tensors. For this and other details of the subject, the reader is referred to ERINGEN [6].

The deformation so introduced must be the solution of Cauchy equations of equilibrium

$$T_{Kk,K}+\rho_0 f_k=0 \tag{2.3}$$

and associated boundary conditions

$$T_{Kk}N_K=T_k\,, \tag{2.4}$$

where ρ_0, f_k, T_{Kk}, T_k, and N_K are respectively the initial mass density, body force, Piola-Kirchhoff stress tensor, surface traction measured on undeformed body, and exterior unit normal vector of the material surface of the body.

The constitutive equations of a composite reinforced with a single family of continuous and inextensible fibers are given by (cf. SPENCER [7])

$$\begin{aligned}T_{Kk}=(I_3)^{-1/2}\,[SA_K a_k+PX_{K,k}+\Phi x_{k,K}+\Psi(I_1\,x_{k,K}-C_{KL}\,x_{k,L})+\\ +H(A_K\,C_{MN}\,A_N\,x_{k,M}+C_{KM}\,A_M\,A_N\,x_{k,N})]\,,\end{aligned} \tag{2.5}$$

where S is a scalar quantity known as "fibers reaction" force and must be determined through the field equations and boundary conditions. Other quantities appearing in (2.5) are defined by

$$\begin{aligned}&\Phi\equiv 2(I_3)^{-1/2}\frac{\partial\Sigma}{\partial I_1}\,,\qquad \Psi\equiv 2(I_3)^{-1/2}\frac{\partial\Sigma}{\partial I_2}\,,\\ &P\equiv 2(I_3)^{1/2}\frac{\partial\Sigma}{\partial I_3}\,,\qquad H\equiv 2(I_3)^{-1/2}\frac{\partial\Sigma}{\partial I_4}\,,\end{aligned} \tag{2.6}$$

with

$$\begin{aligned}&I_1\equiv\operatorname{tr}\mathbf{C}\,,\qquad I_2\equiv\frac{1}{2}\,[(\operatorname{tr}\mathbf{C})^2-\operatorname{tr}\mathbf{C}^2]\,,\\ &I_3\equiv\det\mathbf{C}\,,\qquad I_4\equiv\operatorname{tr}(\mathbf{A}\mathbf{A}^T\mathbf{C}^2)\,,\qquad I_5\equiv\operatorname{tr}(\mathbf{A}\mathbf{A}^T\mathbf{C})\equiv 1\,,\end{aligned} \tag{2.7}$$

where Σ is the strain energy density function of elastic composite medium.

Equations (2.3)–(2.7) are sufficient to determine the mechanical field completely. Assuming an initial large static deformation is given, the equations of motion and the constitutive equations for small deformations superimposed on this initial deformation are presented in the following section.

3. Small deformation superimposed on inital large static deformations

We now superimpose a small time dependent displacement field $\varepsilon \mathbf{u}(\mathbf{x}, t)$, where ε is a small parameter, on a given initial static deformation, and obtain the governing field equations and boundary conditions. Let B' be the final configuration of the body and $\mathbf{x}' = \mathbf{x} + \varepsilon \mathbf{u}(\mathbf{x}, t)$ denotes the space Cartesian coordinates of a material point at time t. Then the equations of motion for this final configuration, in the absence of body forces, and the boundary conditions are given by

$$T'_{kl,k} = \rho \ddot{x}'_2 , \tag{3.1}$$

$$T'_{kl} n_k = T'_2 , \tag{3.2}$$

where T'_{kl}, T, ρ, and n_k are respectively the Piola-Kirchhoff stress tensor and surface traction referred to area on B, mass density measured at configuration B, and the exterior unit normal vector to B.

We then write $T'_{kl} = t_{kl} + \varepsilon T_{kl}$, $T'_l = t_l + \varepsilon T_l$, where t_{kl} is the Euler stress tensor of configuration B, t_l is associated surface traction measured on B, T_{kl} and T are respectively the incremental stress tensor and surface traction of B' referred to area on B. Employing these definitions of T'_{kl} and T'_l in (3.1) and (3.2), and recalling the static nature of initial field, we obtain

$$T_{kl,k} = \rho \ddot{u}_l , \tag{3.3}$$

$$T_{kl} n_k = T_l . \tag{3.4}$$

In order to proceed further, one must know the explicit form of the tensor T_{kl}. At this stage it is convenient to work with Piola-Kirchhoff stress tensor T'_{Kl} referred to area on B_0. The relation between T'_{kl} and T^*_{Kl} may be shown to be

$$T'_{kl} = (I_3)^{-1/2} x_{k,K} T'_{Kl} . \tag{3.5}$$

If we denote the increment in T_{Kl} by $\varepsilon \check{T}_{Kl}$, i.e., $T'_{kl} = T_{Kl} + \varepsilon \check{T}_{Kl}$, one can show that

$$T_{kl} = (I_3)^{-1/2} x_{k,K} \check{T}_{Kl} . \tag{3.6}$$

The stress tensor T'_{kl} in the final configuration is expressed in terms of the strain energy function Σ' as

$$T'_{Kk} = (I'_3)^{1/2} S' A_K a'_k + \frac{\partial \Sigma'}{\partial x'_{k,K}} . \tag{3.7}$$

Defining $S'=S+\varepsilon s$ and noting that

$$
\begin{aligned}
&x'_{k,K}\cong x_{k,K}+\varepsilon u_{k,m}x_{m,K},\\
(3.8)\qquad &(I'_3)^{1/2}\cong (I_3)^{1/2}(1+\varepsilon u_{r,r}),\\
&a'_k\cong a_k+\varepsilon u_{k,n}a_n,
\end{aligned}
$$

Eq. (3.7) can be expanded into a power series of ε around $\varepsilon=0$. Performing this operation and then equating the coefficients of same powers of ε in the expansion, we obtain

$$(3.9)\qquad \check{T}_{Kk}=(I_3)^{1/2}\,[SA_K(u_{r,r}a_k+u_{k,n}a_n)+sA_Ka_k]+\frac{\partial^2\Sigma}{\partial x_{k,K}\partial x_{m,M}}x_{n,M}u_{m,n}\,.$$

In arriving at this result we kept only the zeroth and first order terms of ε in the expansion. Combined (3.7) and (3.9) follow the expression of T_{kl}

$$(3.10)\qquad T_{kl}=\check{s}a_ka_l+B_{klmn}u_{m,n}\,,\qquad \check{s}=s+u_{r,r}S\,,$$

where the fourth order tensor B_{klmn} is defined by

$$(3.11)\qquad B_{klmn}\equiv(I_3)^{-1/2}\frac{\partial^2\Sigma}{\partial x_{l,K}\partial x_{m,L}}x_{n,L}x_{k,K}+Sa_ka_n\delta_{lm}\,.$$

This quantity (if desired) may be expressed as follows

$$(3.12)\qquad B_{klmn}\equiv t_{kn}\delta_{lm}+C_{klmn}$$

with

$$(3.13)\qquad C_{klmn}=4(I_3)^{-1/2}\frac{\partial^2\Sigma}{\partial C_{KL}\partial C_{MN}}x_{k,K}x_{l,L}x_{M,m}x_{n,N}\,.$$

This tensor may be termed as the tensor of elasticity and has the following symmetry relations

$$C_{klmn}=C_{lknm}=C_{mnkl}\,.$$

If the matrix material is isotropic and homogeneous, the tensor of elasticity may be written in terms of the invariants given by equation (2.7), i.e.,

$$
\begin{aligned}
(3.14)\qquad C_{klmn}\equiv{}&P(2\delta_{kl}\delta_{mn}-\delta_{kn}\delta_{lm}-\delta_{km}\delta_{ln})+A_{11}c^{-1}_{kl}c^{-1}_{mn}+A_{22}B_{kl}B_{mn}+\\
&+A_{33}I_3^2\delta_{kl}\delta_{mn}+A_{44}D_{kl}D_{mn}+A_{12}(c^{-1}_{kl}B_{mn}+B_{kl}c^{-1}_{mn})+A_{13}I_3(c^{-1}_{kl}\delta_{mn}+\\
&+\delta_{kl}c^{-1}_{mn})+A_{23}I_3(B_{kl}\delta_{mn}+\delta_{kl}B_{mn})+A_{24}(B_{kl}D_{mn}+D_{kl}B_{mn})+\\
&+A_{34}I_3(D_{kl}\delta_{mn}+D_{mn}\delta_{kl})+\Psi(2c^{-1}_{mn}c^{-1}_{kl}-c^{-1}_{km}c^{-1}_{nl}-c^{-1}_{lm}c^{-1}_{nk})+\\
&+H(a_la_nc^{-1}_{mk}+c^{-1}_{kl}a_ma_n+a_ka_nc^{-1}_{ml}+c^{-1}_{nl}a_ma_k)\,,
\end{aligned}
$$

where the coefficients A_{ij} and other quantities appearing in Eq. (3.14) are defined by

$$A_{ij} \equiv 4(I_3)^{-1/2} \frac{\partial^2 \Sigma}{\partial I_i \partial I_j}, \quad i, j=1, 2, 3, \tag{3.15}$$

$$c_{ij}^{-1} \equiv X_{K,i} X_{K,j}, \quad B_{kl} \equiv (I_1 c_{kl}^{-1} - c_{km}^{-1} c_{ml}^{-1}), \quad D_{kl} \equiv a_k a_r c_{rl}^{-1} + a_l c_{kr}^{-1} a_r. \tag{3.16}$$

In general the tensor B_{klmn} is a function of the position vector $\mathbf{x}$, i.e., $B_{klmn} \equiv \equiv B_{klmn}(\mathbf{x})$. Thus, by use of Eq. (3.10) in (3.3) and (3.4), we obtain the equations of motion and the boundary conditions for the superposed deformation field

$$(\check{s} a_k a_l)_{,k} + (B_{klmn} u_{m,n})_{,k} = \rho \ddot{u}_l, \tag{3.17}$$

$$\check{s}(a \cdot n) a_l + B_{klmn} u_{m,n} n_k = T_l. \tag{3.18}$$

If the initial deformation is such that the tensors B_{klmn} and a_k are constants (this mostly corresponds to the case of uniform deformation), Eq. (3.17) is simplified to

$$\check{s}_{,k} a_k a_l + B_{klmn} u_{m,nk} = \rho \ddot{u}_l. \tag{3.19}$$

This equation is to be supplemented by inextensibility condition given by

$$u_{k,l} a_k a_l = 0. \tag{3.20}$$

These equations will be used for the investigation of small amplitude harmonic waves propagating in such a composite medium.

4. Propagation of harmonic waves

We consider an elastic composite medium reinforced with a family of inextensible fibers and subjected to a homogeneous large static deformation. Upon this initial large deformation we superimpose a time dependent displacement field $\varepsilon \mathbf{u}(\mathbf{x}, t)$ of which the governing equations are given by Eqs. (3.18), (3.19) and (3.20). We now seek a harmonic wave type of solution to the equations, and set

$$u_j = U_{0j} \exp[i(\mathbf{k} \cdot \mathbf{x} - \omega t)], \tag{4.1}$$

$$\hat{s} = \check{s}_0 \exp[i(\mathbf{k} \cdot x - \omega t)], \tag{4.2}$$

where $\mathbf{k}$, ω, U_{0j} and S_0 are respectively the wave vector whose magnitude is the wave number, i.e., $k = |\mathbf{k}|$, the frequency of wave, and the complex amplitudes of the wave.

Introducing (4.1) and (4.2) into equations (3.19) and (3.20) we obtain

$$\{Q_{mn} - [\rho c^2 - S(\mathbf{a} \cdot \mathbf{v})^2] \delta_{mn}] S_{0n} - i \frac{S_0}{k} a_l v_l a_m = 0, \tag{4.3}$$

$$U_{0m} a_m (a_l v_l) = 0, \tag{4.4}$$

where $c=\omega/k$ is the phase velocity, $\mathbf{v}=\mathbf{k}/|\mathbf{k}|$ is unit propagation vector, and the acoustical tensor Q_{mn} is defined by

$$Q_{mn}=B^{0}_{kmln}\,\nu_k\,\nu_l-S\,(a\cdot\mathbf{v})^2\,\delta_{mn}\,. \tag{4.5}$$

Examination of Eq. (4.4) shows that this equation is automatically satisfied if the following conditions hold true.

i) $\mathbf{a}\perp\mathbf{v}$ ($\mathbf{a}$ and $\mathbf{v}$ are perpendicular),

ii) $\mathbf{U}_0\perp\mathbf{a}$ ($\mathbf{U}_0$ and $\mathbf{a}$ are perdendicular).

Here we study the implications of these two cases separately. The case (i) will be studied first.

Case (i)

$$\mathbf{a}\cdot\mathbf{v}=0\,. \tag{4.6}$$

In this case there exists no wave propagating along the fibers direction, i.e., propagation direction is perpendicular to the orientation of fibers lying in the composite. Using this condition in Eq. (4.3) we obtain

$$(Q_{mn}-\rho c^2\,\delta_{mn})\,U_{0n}=0\,. \tag{4.7}$$

Since $\mathbf{Q}$ is a symmetric second order tensor, the characteristic equations obtained from Eq. (4.7) has in general three real roots. In order that these roots correspond to the speed of a real wave, these roots must be positive, as well. This requirement imposes certain restrictions on initial large deformations, that is to say, it gives a stability criteria for the equilibrium described by the initial deformation. From theory of matrices we know that these conditions are satisfied if

$$\operatorname{tr}\mathbf{Q}\geqslant 0\,,\qquad (\operatorname{tr}\mathbf{Q})^2\geqslant\operatorname{tr}\mathbf{Q}^2\,,\qquad \det\mathbf{Q}\geqslant 0 \tag{4.8}$$

holds for a given initial static deformation.

In general it is very difficult to make any progress for the solution unless we know the specific form of homogeneous deformation. For its convenience, we select the vectors $\mathbf{a}$ and $\mathbf{v}$ to be

$$\mathbf{a}=(1,0,0)\,,\qquad \mathbf{v}=(0,1,0)\,. \tag{4.9}$$

Furthermore, in order to be able to obtain a simple form for the tensor $\mathbf{Q}$, among other alternatives, we should have the principal direction of the deformation as our space coordinate system. Thus the homogeneous deformation may be given by

$$x_1=\lambda_1\,X_1\,,\qquad x_2=\lambda_2\,X_2\,,\qquad x_3=\lambda_3\,X_3 \tag{4.10}$$

where λ_i's are the stretch ratio in the direction of principal axis. Inextensibility of the composite under consideration, in the direction of a, (or x_1) imposes further restrictions on the deformation. It requires that $\lambda_1=1$; so that we are left with only two deformation parameters λ_z, λ_3 and one parameter S resulting from the constraint imposed on the elastic body.

If one uses this given deformation in equation (3.16), and then in (4.5), the nonvanishing components of Q_{ij} are found to be

$$
\begin{aligned}
&Q_{11}=P+\lambda_2^2\Phi+\lambda_2^2\lambda_3^2\Psi+\lambda_2^2 H\,,\\
&Q_{22}=P+\lambda_2^2\Phi+\lambda_3^2(1+\lambda_3^2)\Psi+\lambda_2^2A_{11}+\lambda_2^4(1+\lambda_3^2)^2A_{22}+\lambda_2^4\lambda_3^4A_{33}+\\
&\qquad\qquad +2\lambda_2^4(1+\lambda_3^2)A_{12}+2\lambda_2^4\lambda_3^2A_{13}+2\lambda_2^4\lambda_3^2(1+\lambda_3^2)A_{23}\,,\\
&Q_{33}=P+\lambda_2^2\Phi+\lambda_2^2\Psi\,.
\end{aligned}
\tag{4.11}
$$

Thus the speeds of longitudinal and transverse waves are given by

$$
\begin{aligned}
&C_L^2=[P+\lambda_2^2\Phi+\lambda_2^2(1+\lambda)_2^3\Psi+\lambda_2^2A_{11}+\lambda_2^4(1+\lambda_3^2)^2A_{22}+\lambda_2^4\lambda_3^4A_{33}+\\
&\qquad\qquad +2\lambda_2^4(1+\lambda_3^2)A_{12}+2\lambda_2^4\lambda_3^2A_{13}+2\lambda_2^4\lambda_2^3(1+\lambda_3^2)A_{23}]/\rho\geqslant 0\,,\\
&C_{T1}^2=(P+\lambda_2^2\Phi+\lambda_2^2\lambda_3^2\Psi+\lambda_2^2H)/\rho\geqslant 0\,,\\
&C_{T2}^2=(P+\lambda_2^2\Phi+\lambda_2^2\Psi)/\rho\geqslant 0\,.
\end{aligned}
\tag{4.12}
$$

In order that this equilibrium configuration, corresponding to this large initial deformation, be stable, the expressions given by Eq. (4.12) must be positive. These conditions impose certain restrictions on the material constants and given large deformation. It is interesting to note that these wave speeds are independent of fibers reaction force S. The conditions given by Eq. (4.12) should be independent of the magnitude of the deformation, that is they must even be valid at the natural state. Setting $\lambda_1=\lambda_2=\lambda_3=1$ in Eq. (4.12) we obtain the following restrictions

$$
\begin{aligned}
&(P_0+\Phi_0+2\Psi_0+A_{11}^0+2A_{22}^0+A_{33}^0+4A_{12}^0+2A_{13}^0+4A_{23}^0)/\rho_0\geqslant 0\,,\\
&(P_0+\Phi_0+\Psi_0+H_0)/\rho_0\geqslant 0\,,\\
&(P_0+\Phi_0+\Psi_0)/\rho_0\geqslant 0\,,
\end{aligned}
\tag{4.13}
$$

where P_0, Φ_0, Ψ_0 and A_{ij}^0 are the values of P, Φ, Ψ, and A_{ij} evaluated at the natural state. In fact, these quantities correspond to the real material constants of the composite body.

Thus far we have not used specific form for the strain energy function. The simplest and commonly used strain energy density is a linear function of strain invariants. This may be expressed by

$$
\Sigma=\alpha(I_1-3)+\beta(I_2-3)+\gamma(I_3-1)+\kappa(I_4-1)\,.
\tag{4.14}
$$

Here α, β, γ, and κ are elastic constants, which must be determined through experimental means. In this case we have

$$
P=2\gamma(I_3)^{1/2}\,,\qquad \Phi=2\alpha(I_3)^{-1/2}\,,\qquad \Psi=2\beta(I_3)^{-1/2}\,,\qquad H=2\kappa(I_3)^{-1/2}
\tag{4.15}
$$

and all the A_{ij} are zero. For this particular composites, the speeds of wave are given by

$$C_L^2=\left[2(\beta+\gamma)\lambda_2\lambda_3+2(\alpha+\beta)\frac{\lambda_2}{\lambda_3}\right]\Big/\rho>0,$$

(4.16) $$C_{T1}^2=\left[2(\beta+\gamma)\lambda_2\lambda_3+2(\alpha+\kappa)\frac{\lambda_2}{\lambda_3}\right]\Big/\rho>0,$$

$$C_{T2}^2=\left[2\gamma\lambda_z\lambda_3+2(\alpha+\beta)\frac{\lambda_2}{\lambda_3}\right]\Big/\rho>0.$$

These inequalities should also be valid when $\lambda_1=\lambda_2=\lambda_3=1$. Thus we get

(4.17) $$\alpha+\beta+\gamma+\kappa>0,\qquad \alpha+2\beta+\kappa>0,\qquad \alpha+\beta+\gamma>0.$$

This is some of the restrictions that the material constants must obey.

Case (ii)

(4.18) $$U_0\cdot a=0.$$

In this case, by multiplying Eq. (4.3) by a_m and making use of the condition (4.18), we get

(4.19) $$i\frac{\check{s}_0}{k}\cos\varphi=Q_{ij}U_{0j}a_i$$

with

$$\cos\varphi\equiv\mathbf{a}\cdot\boldsymbol{\nu}.$$

Resubstitution of (4.19) in (4.3) yields the following equation

(4.20) $$\{Q_{mn}^*=[\rho c^2-S(\mathbf{a}\cdot\boldsymbol{\nu})^2]\,\delta_{mn}\}\,U_{0n}=0,$$

where the tensor Q_{mn}^* is defined to be

(4.21) $$Q_{mn}^*\equiv Q_{mn}-Q_{im}a_i a_n.$$

It is easily seen that this matrix has the following property

(4.22) $$Q_{mn}^*a_n=Q_{mn}a_n-Q_{im}a_i=0.$$

Since the vector $\mathbf{a}\neq0$, the above relation is only possible if, and only if, the matrix is singular, i.e., $\det\mathbf{Q}^*=0$. As we see later, this fact greatly simplifies our calculations.

In order to obtain a nonzero solution to U_{0n} we must have the determinant of the coefficient matrix equal to zero. Using the singular property of the matrix $\mathbf{Q}^*$, the characteristic equation may be given as follows:

(4.22) $$(\rho c_0^2)^2-I_{Q^*}\rho c_0^2+II_{Q^*}=0.$$

Here we have defined

(4.23) $$\rho c^2=\rho c_0^2+S(\mathbf{a}\cdot\boldsymbol{\nu})^2,\qquad I_{Q^*}\equiv\operatorname{tr}\mathbf{Q}^*,\qquad II_{Q^*}\equiv\frac{1}{2}\,[(\operatorname{tr}\mathbf{Q}^*)^2-\operatorname{tr}\mathbf{Q}^{*2}].$$

The roots of bi-quadratic equation (4.22) are given by

$$(\rho c_0^2)_{\mp}^{+} \frac{1}{2} [I_{Q*} \mp (I_{Q*}^2 - 4II_{Q*})^{1/2}]. \tag{4.24}$$

We note that c_0 corresponds to the speed of wave when the fibers reaction equilibrium force S vanishes. If this configuration is stable we must have

$$I_{Q*} \geqslant 0, \quad I_{Q*} \geqslant 2(II_{Q*})^{1/2}. \tag{4.25}$$

From this, and expression $(4.23)_1$ we can deduce some interesting results: (i) If the configuration corresponding to $S=0$ is stable, i.e., $c_0^2>0$, the fibers with tensile reaction forces increase the speed of propagation, while the fibers with compressive reaction forces reduce the speed of wave. In the latter case there is a critical value for S, beyond which no real wave can propagate in the medium. From (Eq. $(4.23)_1$ this critical value of S is found to be

$$S_{cc} = -\frac{\rho c_0^2}{(\mathbf{a}\cdot\mathbf{v})^2} \quad \text{(compressive)}, \tag{4.26}$$

so that, when $S>S_{cr}$, the configuration is stable while for $S<S_{cr}$ it is unstable, i.e., there is no real wave. This result is no be expected from physical considerations. (ii) If the configuration corresponding to $S=0$ is unstable, i.e., $c_0^2<0$, the fibers with compressive reaction forces speed up the process of failure, while the fibers with tensile forces may provide additional supports for the system; and as a result of this, initially unstable system may turn out to be stable. The latter case may be used to strengthen the weak systems in the sense of stability. From Eq. $(4.32)_1$ this critical value of S is given to be

$$S_{cr} = -\frac{\rho c_0^2}{(\mathbf{a}\cdot\mathbf{v})^2} \quad \text{(tensile)}. \tag{4.27}$$

Here we note that c_0^2 is a negative quantity. Again, when $S>S_{cr}$, the system is stable but fot $S<S_{cr}$ it is unstable.

If desired, the components of matrix Q_{ij}^* can be expressed in terms of given homogeneous deformation. Since it has no practical value, we do not list them here. However, it should be noted that the forms of acoustic tensors and the expressions of wave speeds found in the present study and singular surfaces are the same (cf. DEMIRAY [3]). It is true that this coincidence is only valid when initial deformation is homogeneous.

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Streszczenie

ROZPRZESTRZENIANIE SIĘ FAL WE WSTĘPNIE SPRĘŻONYM ZBROJONYM KOMPOZYCIE

W niniejszej pracy bada się nałożenie pola infinitezymalnych, zależnych od czasu przemieszczeń na duże początkowe odkształcenia statyczne w zbrojnym kompozycie. Wyprowadzono zlinearyzowane równania pola i odpowiadające im warunki brzegowe. Zbadano rozprzestrzenianie się fal harmonicznych w takim ośrodku kompozytowym i przedyskutowano różne przypadki szczególne. Z warunku propagacji wyprowadzono pewne kryteria stabilności konfiguracji równowagi.

Резюме

РАСПРОСТРАНЕНИЕ ВОЛН В ПРЕДВАРИТЕЛЬНО НАПРЯЖЕННОМ АРМИРОВАННОМ КОМПОЗИТЕ

В настоящей работе исследуется наложение поля инфинитезимальных, зависящих от времени, перемещений на большие начальные статические деформации в армированном композите. Выведены линеаризованные уравнения поля и отвечающие им граничные условия. Исследовано распространение гармонических волн в такой композитной среде и обсуждены разные частные случаи. Из условия распространения выведены некоторые критерия стабильности конфигурации равновесия.

APPLIED MATHEMATICS DIVISION
MARMARA RESEARCH INSTITUTE

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