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STRUCTURAL DAMPING IN SANDWICH SHELLS

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The basic equations governing the motion of sandwich shells are derived through the use of a variational procedure. Structural damping is investigated in the case of a cylindrical sandwich shell with dissimilar facings, executing axially symmetric free vibrations. The concept of the complex modulus is used in order to describe the viscoelasticity of the core. The various frequency parameters of the shell and their associated damping parameters are obtained and analyzed with regard to the ratios of different layers thicknesses and other geometrical and physical properties of the sandwich shell. Curves obtained from numerical results are presented.

Notation

$\check{}$	sign denoting complex quantity,
,	differentiation w.r.t. the variable standing after the comma,
$i=1, 2, 3$	subscript corersponding to upper face, core and lower face, respectively,
A, B	Lame's coefficients,
R_α, R_β	curvature radii in α-and β-directions, respectively,
$k_\alpha=R_\alpha^{-1}, k_\beta=R_\beta^{-1}$	curvatures,
$r=t_1/R$	core thickness to curvature radius ratio,
u, v, w	displacement components in α-, β-and z-directions, respectively,
$\bar{g}_\alpha, \bar{g}_\beta$	components of changes of slope in α-and β-directions, respectively,
h	total thickness of the sandwich construction,
t_i	thickness of the i-th layer,
$k=t_2/t_1$	core thickness ratio,
$m=t_3/t_1$	face thickness ratio,
E_α, E_β	storage moduli (for flexure and extension) in α- and β-directions, respectively,
ν_α, ν_β	Poisson's ratios in two orthogonal directions,
$E_{\alpha\beta}, E_{\alpha z}, E_{\beta z}$	storage shear moduli in $\alpha\beta$-, αz- and βz- planes, respectively,
$n=E_3/E_1$	modulus ratio of the faces,
$n_2=E_2/E_1$	modulus ratio of the core,
N_α, N_β	normal forces in α- and β-directions, respectively,
S	longitudinal shearing forces,
H	twisting moment,
G_α, G_β	bending moments,
Q_α, Q_β	transverse shearing forces,
σ_k	normal (direct) stress,
τ_{kl}	shearing stress,
e_α, e_β	normal strains in α- and β-directions, respectively,
$e_{\alpha\beta}, e_{\alpha z}, e_{\beta z}$	shearings strains in $\alpha\beta$- and αz- and βz-planes, respectively,
$s_1=E_1(1-\nu_1^2)$	stiffness coefficient of the upper face,

$B_1=E_1\,t_1\,(1-\nu_1^2)$ extensional stiffness of the upper face,
M_i mass of the i-th layer per unit area,
$M=M_1+M_2+M_3$ mass of the sandwich structure per unit area,
γ_i mass density of the i-th layer material,
$\vartheta_2=\gamma_2/\gamma_1$ mass density ratio of the core material,
$\vartheta_3=\gamma_3/\gamma_1$ mass density ratio of the face material,
$d=1+\vartheta_2\,k+\vartheta_3\,m$ mass ratio of the sandwich construction per unit area,
G_2 relaxed (static) elastic transverse modules of the core,
μ_q material loss factor associated with the q-th type of deformation,
$\Lambda=\sqrt{\lambda}$ wave-number parameter,
$\varepsilon_q=\mu_q/\mu_0$ loss factor ratio,
$\tau=r/\lambda$ curvature parameter,
$\gamma_{yz},\,\gamma_{xz}=G_2\,(1-\nu_1^2)/E_1$ transverse shear modulus ratio of the core,
$\psi_1=(1-\nu_1^2)\,G_2/\lambda E_1$ shear parameter,
ω frequency,
$\Omega=\gamma_1\,t_1^2\,\omega^2/s_1$ frequency parameter,
δ logarithmic decrement,
δ^* damping parameter,
$\{\cdot\}$ transposed matrix

1. Introduction

The paper is a study of the damped vibrations of sandwich shells and follows the previously published ones [1, 2]. This problem was considered by Bieniek and Freudenthal in [3] and Yu in [4], however, attention was focussed only on the sandwich shell of equal facing. In the present paper structural damping is investigated as the function of the face thickness ratio of the sandwich shell and its curvature. General equations of motion of a sandwich shell of asymmetrical structure are derived. The displacements at the midsurface of the core chosen as unknowns. This facilitates the damping analysis. The concept of the complex modulus is used and three damping parameters associated with the three vibration frequencies of the sandwich shell are obtained.

2. General equations of motion of the sandwich shell

The assumptions are as follows:

1) The material of the facings as well as that of the core is considered as homogeneous and orthotropic.

2) A straight line through the underformed core remains straight under deformation but not necessarily perpendicular to the midsurface of the core.

3) The Kirchhoff-Love's assumption holds true only for the facings.

4) There is no normal interaction between the layers parallel to the core midsurface.

5) The deformations of the shell are small as compared to its thickness h. The ratio between the thickness h of the shell and its curvature is small as compared to the unity.

6) No slip will occur at the contact surface between core and facings.

Shell characteristics and the forces acting on a shell element are presented in Fig. 1. The deformation relations of the shell used here are derived from [5, 6]. It is assumed that the coefficients of the first quadratic form of any layer parallel to the core midsurface do not differ from those of the core midsurface.

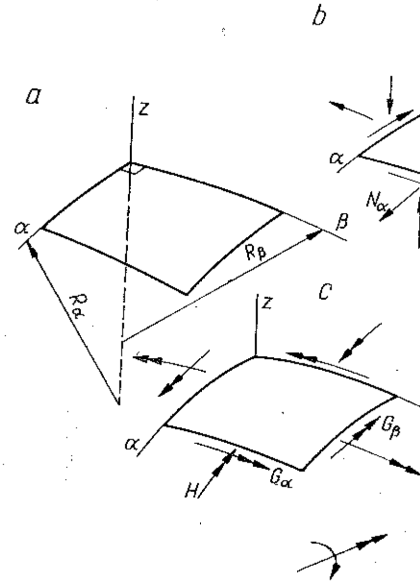


FIG. 1

For each face, the strain-displacement relations at its midsurface are

$$
(2.1)\qquad
\begin{aligned}
e_{\alpha i} &= A^{-1} u_{i,\alpha} + AB^{-1} v_i A_{,\beta} + R_\alpha^{-1} w,\\
e_{\beta i} &= B^{-1} v_{i,\beta} + AB^{-1} u_i B_{,\alpha} + R_\beta^{-1} w,\\
e_{\alpha\beta i} &= A^{-1} B (B^{-1} v_i)_{,\alpha} + AB^{-1} (u_i A^{-1})_{,\beta},
\end{aligned}
$$

while the curvature and twist changes are

$$
(2.2)\qquad
\begin{aligned}
h_{\alpha i} &= A^{-1} (R_\alpha^{-1} u_i - A^{-1} w_{,\alpha})_{,\alpha} + (R_\beta^{-1} v_i - B^{-1} w_{,\beta}) (AB)^{-1} A_{,\beta},\\
h_{\beta i} &= B^{-1} (R_\beta^{-1} v_i - B^{-1} w_{,\beta})_{,\beta} + (R_\alpha^{-1} u_i - A^{-1} w_{,\alpha}) (AB)^{-1} B_{,\alpha},\\
h_{\alpha\beta i} &= -AB^{-1} (A^{-2} w_{,\alpha})_{,\beta} - BA^{-1} (B^{-2} w_{,\beta})_{,\alpha} +\\
&\quad + A (BR_\alpha)^{-1} (A^{-1} u_i)_{,\beta} + B (AR_\beta)^{-1} (B^{-1} v_i)_{,\alpha},
\end{aligned}
$$

where the subscript $i=1, 3$ refers to the upper and lower faces, respectively.

A typical cross section of the sandwich shell is shown in Fig. 2. Simple geometric considerations thus give the following longitudinal displacements u_2 and v_2 at any layer of the core:

$$
(2.3)\qquad
\begin{aligned}
u_2 &= (1 + R_\alpha^{-1} z) u_2 - g_{\alpha 2} z,\\
v_2 &= (1 + R_\beta^{-1} z) v_2 - g_{\beta 2} z
\end{aligned}
$$

and the longitudinal displacements u_i, v_i $(i=1, 3)$ at the face midsurface are related with those at the core midsurface as follows:

$$u_i=u_2\mp\frac{t_2}{2}g_{\alpha 2}\mp\frac{t_i}{2}A^{-1}w_{,\alpha},$$

$$(2.4)\qquad v_i=v_2\mp\frac{t_2}{2}g_{\beta 2}\pm\frac{t_i}{2}B^{-1}w_{,\beta},$$

$$\hat{u}_2=u_2+\bar{g}_{\alpha 2}z,\quad \hat{v}_2=v_2+\bar{g}_{\beta 2}z,$$

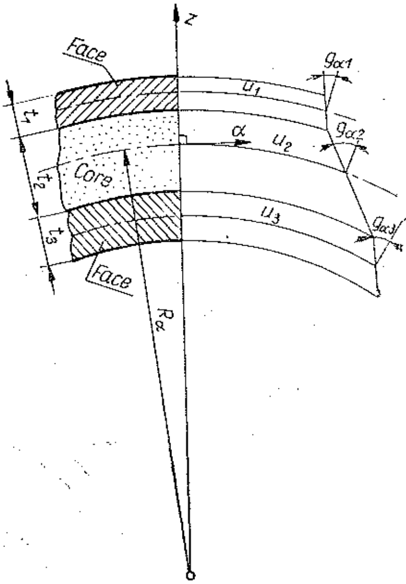


FIG. 2

where

$$(2.5)\quad \bar{g}_{\alpha i}=R_\alpha^{-1}u_i-g_{\alpha i},\qquad \bar{g}_{\beta i}=R_\beta^{-1}v_i-g_{\beta i},$$

the sign $-$ corresponds to $i=1$ and the sign $+$ corresponds to $i=3$; the quantities $\bar{g}_{\alpha i}$ and $\bar{g}_{\beta i}$ are, respectively, the changes of slope in α- and β-directions, while $g_{\alpha i}$ and $g_{\beta i}$ are the components of slope changes produced by deflection and shear deformation only.

The following relations exist between the quantities $\bar{g}_{\alpha i}$ and $\bar{g}_{\beta i}$:

$$\bar{g}_{\alpha i}=R_\alpha^{-1}u-g_{\alpha i},\qquad \bar{g}_{\beta i}=R_\beta^{-1}v-g_{\beta i};$$

$g_{\alpha i}$, $g_{\beta i}$ are the components of changes of slope in α- and β-directions. The effect of the displacements u_i and v_i in (2.5) is usually small and in the case of a simplified theory it can be neglected. Thus, we derive the following deformation relations at the core midsurface:

$$e_{\alpha 2}=A^{-1}u_{2,\alpha}+(AB)^{-1}v_2A_{,\beta}+R_\beta^{-1}w,$$

$$e_{\beta 2}=B^{-1}v_{2,\beta}+(AB)^{-1}v_2B_{,\alpha}+R_\beta^{-1}w,$$

$$e_{\alpha\beta 2}=A^{-1}B(v_2B^{-1})_{,\alpha}+AB^{-1}(A^{-1}u_2)_{,\beta},$$

$$(2.6)\qquad h_{\alpha 2}=A^{-1}\bar{g}_{\alpha 2}+(AB^{-1})A_{,\beta}\bar{g}_{\beta 2},$$

$$h_{\beta 2}=B^{-1}\bar{g}_{\beta 2}+(AB)^{-1}B_{,\alpha}\bar{g}_{\alpha 2},$$

$$h_{\alpha\beta 2}=-AB^{-1}(g_{\alpha 2}A^{-1})_{,\beta}-A^{-1}B(B^{-1}g_{\beta 2})_{,\alpha}+$$
$$+A(BR_\alpha)^{-1}(A^{-1}u_2)_{,\beta}+B(AR_\beta)^{-1}(B^{-1}v_2)_{\alpha}.$$

On the other hand, the core also undergoes transverse shear deformations which can be expressed as follows:

$$e_{\alpha z 2}=A^{-1}\, w_{,\alpha}-g_{\alpha 2}\,,\qquad e_{\beta z 2}=B^{-1}\, w_{,\beta}-g_{\beta 2}\,. \tag{2.7}$$

As in [1, 2], a variational procedure is used to obtain the basic equations which govern the behaviour of the sandwich shell. According to Hamilton's principle, we have

$$\delta\int_{t_0}^{t_1} L dt=0\,, \tag{2.8}$$

where t_0 and t_1 are arbitrarily chosen instants of time and the Lagrangian L is

$$L=V-K-T \tag{2.9}$$

with V — strain energy, T — external forces work and K — kinetic energy.

For the i-th layer of the sandwich shell, the first variation δV_i is

$$\delta V_i=\int_{\alpha_1}^{\alpha_2}\int_{\beta_1}^{\beta_2}[N_{\alpha i}\,\delta e_{\alpha i}+N_{\beta i}\,\delta e_{\beta i}+S_i\,\delta e_{\alpha\beta i}+G_{\alpha i}\,\delta h_{\alpha i}+G_{\beta i}\,\delta h_{\beta i}+ +H_i\,\delta h_{\alpha\beta i}+\delta_{2i}\,(Q_{\alpha i}\,\delta e_{\alpha z i}+Q_{\beta i}\,\delta e_{\beta z i})]\,AB\,d\alpha\,d\beta, \tag{2.10}$$

where the subscript $i=1, 2, 3$, refers to the upper face, the core and the lower face, respectively, while

$$\delta_{2i}=\begin{cases}0, & i\neq 2,\\ 1, & i=2.\end{cases}$$

The work of external forces is

$$\delta T=\int_{\alpha_1}^{\alpha_2}\int_{\beta_1}^{\beta_2} p_z\,\delta w\,AB d\alpha\,d\beta \tag{2.11}$$

since only the transverse load p_z is assumed to exist.

The kinetic energy in each layer is

$$K_i=\frac{\gamma i}{2}\int_{\alpha_1}^{\alpha_2}\int_{\beta_1}^{\beta_2}\int_{z_{li}}^{z_{ui}}[(u_{i t})^2+(v_{i t})^2+(w_{,t})^2]\,AB\,d\alpha\,d\beta\,dz, \tag{2.12}$$

where z_{ui} and z_{li} are the upper and lower bounds in the z — direction of the considered layer.

Replacing u_i and v_i by (2.3) into (2.12) and, next, intergrating the resulting expression with respect to z, we obtain

$$K=\sum_{i=1,2,3} K_i=\frac{1}{2}\int_{\alpha_1}^{\alpha_2}\int_{\beta_1}^{\beta_2}\Big[\sum_{i=1,2,3}\gamma_i\, t_i\,(u_{i,t}^2+v_{i,t}^2)+Mw_{,t}^2\Big]AB\,d\alpha\,d\beta+ +\frac{1}{24}\int_{\alpha_1}^{\alpha_2}\int_{\beta_1}^{\beta_2}\sum_{i=1,2,3}\gamma_i\, t_i^3\,(\bar{g}_{\alpha i,t}^2+\bar{g}_{i\alpha,t}^2)\,AB\,d\alpha\,d\beta, \tag{2.13}$$

where, for external layers, we have

$$g_{\alpha i}=A^{-1}\,w_{,\alpha},\quad g_{\beta i}=B^{-1}\,w_{,\beta},\quad i=1 \text{ or } 3 \tag{2.14}$$

because the effect of the shear deformations is taken into account only for the core. It should be noted that the second term of K is the effect of rotatory inertia.

The first variation of the kinetic energy is then

$$\int_{t_0}^{t_1}\delta K dt=\int_{t_0}^{t_1}\int_{\alpha_1}^{\alpha_2}\int_{\beta_1}^{\beta_2}\sum_{i=1,2,3}\gamma_i\,t_i\,(u_{i,t^2}\,\delta u_i+v_{i,t}\,2\delta v_i+w_{,t}\,2\delta w)\,AB\,d\alpha\,d\beta\,dt-$$
$$-\frac{1}{12}\int_{t_0}^{t_1}\int_{\alpha_1}^{\alpha_2}\int_{\beta_1}^{\beta_2}\Big[\sum_{i=1,2,3}\gamma_i\,t^3\,(\bar{g}_{\alpha i,t^2}\,\delta\bar{g}_{\alpha i}+\bar{g}_{\beta i,t^2}\,\delta\bar{g}_{\beta i})\Big]AB\,d\alpha\,d\beta\,dt \tag{2.15}$$

since the virtual displacements vanish at the end points of the arbitratry interval $t_0\leqslant t\leqslant t_1$.

Now, we introduce the expressions (2.1)–(2.7) into Eqs. (2.10), (2.11), (2.13). Next, the resulting expressions (in terms of the displacements related to the core midsurface) are introduced into Eq. (2.8). Since the variation in Eq. (2.8) for all arbitrary values of δu_2, δv_2, $\delta g_{\alpha 2}$, $\delta g_{\beta 2}$ and δw vanish, we have the following equations:

(2.14′)

1) $$N_{\alpha,a}+\bar{k}_\beta\,(N_\alpha-N_\beta)+S_{,b}+2\bar{k}_\alpha\,S+k_\alpha\,\bar{k}_\beta\,(G_\alpha-G_\beta)+$$
$$+k_\alpha\,G_{\alpha,a}+(k_\alpha\,H)_{,b}+2k_\alpha\,\bar{k}_\alpha\,H=\sum_{i=1,2,3}\gamma_i\,t_i\,u_{i,t^2}-\frac{t_2^3}{12}\,\gamma_2\,kg_{\alpha 2,t^2},$$

2) $$N_{\beta b}+\bar{k}_\alpha\,(N_\beta-N_\alpha)+S_{,a}+2\bar{k}_\beta\,S+k_\beta\,\bar{k}_a\,(G_\alpha-G_\beta)+$$
$$+k_\beta\,G_{\beta,b}+(k_\beta\,H)_{,a}+2k_\beta\,\bar{k}_\beta\,H=\sum_{i=1,2,3}\gamma_i\,t_i\,v_{i,t^2}-\frac{t_2^3}{12}\,\gamma_2\,k_\beta\,g_{\beta 2,t^2},$$

3) $$\tilde{G}_{\alpha,a}+\bar{k}_\beta\,(\tilde{G}_\alpha-\tilde{G}_\beta)+\tilde{H}_{,b}+2\bar{k}_\alpha\,\tilde{H}+\frac{t_2}{2}\,k_\alpha\,\bar{G}_{\alpha,a}+$$
$$+\frac{t_2}{2}\,k_\alpha\,\bar{k}_\beta\,(\bar{G}_\alpha-\bar{G}_\beta)+\frac{t_2}{2}\,(k_\alpha\,\tilde{H})_{,b}+t_2\,k_\alpha\,\bar{k}_\alpha\,\tilde{H}-Q_{\alpha 2}=$$
$$=\frac{t_2}{2}\Big[\sum_{i=1,2,3}\mp\gamma_i\,t_i\,u_{i,t^2}-\frac{t_2^2}{6}\,\gamma_2\,(k_\alpha\,u_2-g_{\alpha 2})_{,t^2}\Big],$$

4) $$\tilde{G}_{\beta,b}+\bar{k}_\alpha\,(\tilde{G}_\beta-\tilde{G}_\alpha)+\tilde{H}_{,a}+2\bar{k}_\beta\,\tilde{H}+\frac{t_2}{2}\,k_\beta\,\bar{G}_{\beta,b}+$$
$$+\frac{t_2}{2}\,\bar{k}_\alpha\,k_\beta\,(\bar{G}_\alpha-\bar{G}_\beta)+\frac{t_2}{2}\,(k_\alpha\,\tilde{H})_{,a}+t_2\,k_\alpha\,\bar{k}_\alpha\,\tilde{H}-Q_{\beta 2}=$$
$$=-\frac{t_2}{2}\Big[\sum_{i=1,2,3}\mp\gamma_i\,t_i\,v_{i,t^2}-\frac{t_2^2}{6}\,\gamma_2\,(k_\beta\,v_2-g_{\beta 2})_{,t^2}\Big],$$

(2.14′) [cont.]

$$5)\quad k_\alpha N_\alpha + k_\beta N_\beta + (\bar{k}_\alpha^2 + \bar{k}_{\alpha,b} - \bar{k}_\beta^2 - \bar{k}_{\beta,a})(\mathring{G}_\alpha - \mathring{G}_\beta) - \bar{k}_\beta (2\mathring{G}_\alpha - \mathring{G}_\beta)_{,a} + \bar{k}_\alpha (\mathring{G}_\alpha - 2\mathring{G}_\beta)_{,b} - (\mathring{G}_{,a})_{,a} - (\mathring{G}_{\beta,b})_{,b} - 2(\bar{k}_{\alpha,a} + 2\bar{k}_\alpha \bar{k}_\beta + \bar{k}_{\beta,b})\mathring{H} - 2(\bar{k}_\alpha \mathring{H}_{,a} + \bar{k}_\beta \mathring{H}_{,b} + \mathring{H}_{,ab}) - Q_{\alpha 2} - Q_{\beta 2,b} - \bar{k}_a Q_{a2} - - \bar{k}_\beta Q_{\beta 2} = -Mw_{,t^2} + \frac{1}{2} \sum_{i=1,2,3} \gamma_i t_i^2 \left\{ \bar{k}_\beta \left(\mp u_i + \frac{t_i}{6} g_{\alpha i} \right)_{,t^2} + \bar{k}_\alpha \left(\mp v_i + \frac{t_i}{6} g_{\beta i} \right)_{,t^2} + \left(\mp u_i + \frac{t_i}{6} g_{\alpha i} \right)_{,at^2} + \left(\mp v_i + \frac{t_i}{6} g_{\beta i} \right)_{,bt^2} \right\} + AB\, p_z .$$

where the curvilinear lengths and the geodesic curvatures are introduced

$$da = A d\alpha, \qquad db = B d\beta,$$

$$(2.16)\qquad k_\alpha = R_\alpha^{-1}, \qquad k_\beta = R_\beta^{-1},$$

$$\bar{k}_\alpha = (AB)^{-1} A_{,\beta}, \qquad \bar{k}_\beta = (AB)^{-1} B_{,\alpha} .$$

Moreover, for convenience the following has been denoted:

(2.17)

$$N_j = \sum_{i=1,2,3} N_{ji}, \qquad \bar{N}_j = N_{j1} - N_{j3},$$

$$S = \sum_{i=1,2,3} S_i, \qquad \bar{S} = S_1 - S_3,$$

$$G_j = \sum_{i=1,2,3} G_{ji}, \qquad \mathring{G}_j = G_{j1} + G_{j3} + \frac{1}{2}(N_{j1} t_1 - N_{j3} t_3)$$

$$\tilde{G}_j = G_{j2} + \frac{t_2}{2} \bar{N}_j, \qquad \bar{G}_j = G_{j1} - G_{j3},$$

$$H = \sum_{i=1,2,3} H_i, \qquad \bar{H} = H_1 - H_3,$$

$$\tilde{H} = H_2 + \frac{t_2}{2} \bar{S}, \qquad \mathring{H} = H_1 + H_3 + \frac{1}{2}(t_1 S_1 - t_3 S_3)$$

$$(j = \alpha, \beta).$$

The boundary conditions also yield from Eq. (2.8) but they are not reported here.

Equations (2.14′) are the equations of equilibrium of the considered sandwich shell. Tey may be expressed in terms of the shell displacements if the consitutive equations are considered.

We assume that the stress-strain relations for any individual layer of the sandwich shell are essentially the same as those for a single-layered homogeneous shell:

(2.18)

$$N_{\alpha i} = \frac{E_{\alpha i} t_i}{1 - \nu_{\alpha i} \nu_{\beta i}} \left[e_{\alpha i} + \frac{1}{2} \left(\nu_{\alpha i} + \frac{E_{\beta i}}{E_{\alpha i}} \nu_{\beta i} \right) e_{\beta i} \right],$$

$$N_{\beta i} = \frac{E_{\beta i} t_i}{1 - \nu_{\alpha i} \nu_{\beta i}} \left[e_{\beta i} + \frac{1}{2} \left(\nu_{\beta i} + \frac{E_{\alpha i}}{E_{\beta i}} \nu_{\alpha i} \right) e_{\alpha i} \right],$$

$$S_i = t_i E_{\alpha\beta i} e_{\alpha\beta i},$$

(2.18) [cont.]

$$G_{\alpha i}=\frac{E_{\alpha i}\,t_1^3}{12\,(1-\nu_{\alpha i}\,\nu_{\beta i})}\left[h_i+\frac{1}{2}\left(\nu_{\alpha i}+\frac{E_{\beta i}}{E_{\alpha i}}\nu_{\beta i}\right)h_{\beta i}\right],$$

$$G_{\beta i}=\frac{E_{\beta i}\,t_1^3}{12\,(1-\nu_{\alpha i}\,\nu_{\beta i})}\left[h_{\beta i}+\frac{1}{2}\left(\nu_{\beta i}+\frac{E_{\alpha i}}{E_{\beta i}}\nu_{\alpha i}\right)h_{\alpha i}\right],$$

$$H_i=\frac{t_1^3}{12}E_{\alpha\beta i}\,h_{\alpha\beta i},\qquad Q_{\alpha 2}=t_2\,E_{\alpha z2}\,e_{\alpha z2},$$

$$Q_{\beta 2}=t_2\,E_{\beta z2}\,e_{\beta z2}.$$

3. Governing equations of a sandwich cylindrical shell

The shell characteristics are thus (Fig. 3)

$$\begin{aligned}&a=x, && b=R\beta,\\ &R_\alpha=\infty, && R_\beta=R=\text{const},\\ &A=1, && B=R,\\ &\bar{k}_\alpha=\bar{k}_\beta=k_\alpha=0, && k_\beta=R^{-1}=\text{const}.\end{aligned}\tag{3.1}$$

The following equations of equilibrium result from Eqs. $(2.14')_4$:

$$1)\ N_{x,x}+R^{-1}S_{,\beta}=\sum_{i=1,2,3}\gamma_i\,t_i\,u_{i,t^2},$$

$$2)\ R^{-1}N_{\beta,\beta}+S_{,x}+R^{-2}G_{\beta,\beta}+R^{-1}H_{,x}=\sum_{i=1,2,3}\gamma_i\,t_i\,v_{i,t^2}-\frac{t_2^3}{12R}\gamma_2\,g_{\beta 2,t^2},$$

$$3)\ \tilde{G}_{x,x}+R^{-1}\tilde{H}_{,\beta}-Q_{x2}=-\frac{t_2}{2}\left[\sum_{i=1,2,3}\mp\gamma_i\,t_i\,u_{i,t^2}+\frac{t_2^2}{6}\gamma_2\,g_{x2,t^2}\right],$$

(3.2)

$$4)\ R^{-1}\tilde{G}_{\beta,\beta}+\tilde{H}_{,x}-Q_{\beta 2}=-\frac{t_2}{2}\left[\sum_{i=1,2,3}\mp\gamma_i\,t_i\,v_{i,t^2}-\frac{t_2^2}{6}\gamma_2\,(R^{-1}v_2-g_{\beta 2})_{,t^2}\right],$$

$$5)\ \mathring{G}_{x,x^2}+R^{-2}\mathring{G}_{\beta,\beta^2}+2R^{-1}\mathring{H}_{,x\beta}+Q_{x2,x}+R^{-1}Q_{\beta 2,\beta}-R^{-1}N_\beta=$$
$$=Mw_{,t^2}-\frac{1}{2}\sum_{i=1,2,3}\gamma_i\,t_i^2\left[\left(\mp u_i+\frac{t_i}{6}g_{xi}\right)_{,xt^2}+R^{-1}\left(\mp v_i+\frac{t_i}{6}g_{\beta i}\right)_{,\beta t^2}-p_z\right].$$

Let us consider the case where the faces and core materials are such that

$$E_{xi}=E_{\beta i}=E_i,\quad E_{x\beta i}=\frac{E_i}{2\,(1+\nu_i)},\quad \nu_{\alpha i}=\nu_{\beta i}=\nu_i\quad i=1,2,3,\quad E_{xz2}=E_{\beta z2}=G_2,\tag{3.3}$$

Moreover, for the sake of simplicity we assume

$$\nu_1 \approx \nu_2 \approx \nu_3 = \nu. \tag{3.4}$$

Now, we substitute the expressions (2.1), (2.2), (2.6) ,(2.7) into the relations (2.18), taking into account (2.5′), (3.1), (3.3) and (3.4). For convenience, we introduce the new variables

$$\bar{u}_2 = \frac{t_2}{2} g_{\alpha 2}, \qquad \bar{v}_2 = \frac{t_2}{2} g_{\beta 2}. \tag{3.5}$$

Next, we introduce the resulting constitutive equations into Eqs. (3.2); thus we obtain the following equation in a matrix form:

$$\rho\, [m_{rs}]\, \{\ddot{d}_r\} + [s_{rs}]\, \{d_r\} = \{L_r\}, \qquad r, s = 1, 2, 3, 4, 5, \tag{3.6}$$

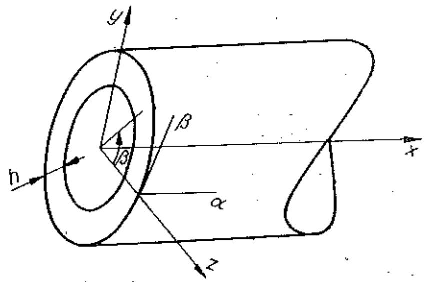


FIG. 3.

where $\{d_r\}$ is the dimensionless vector of displacements

$$\{d_r\} = t_1^{-1} \begin{Bmatrix} u_2 \\ v_2 \\ \bar{u}_2 \\ \bar{v}_2 \\ w \end{Bmatrix}, \tag{3.7}$$

$\{L_r\}$ is the dimensionless vector of loads

$$\{L_r\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ p/s_1 \end{Bmatrix}, \tag{3.8}$$

$[m_{rs}]$ is the dimensionless and symmetrical mass matrix, the elements of which are as follows:

$$m_{11} = -d, \quad m_{13} = 1 - \theta_3\, m, \quad m_{15} = \frac{t_1}{2}(1 - \theta_3\, m^2)\frac{\partial}{\partial x}, \quad m_{22} = -d,$$

$$m_{24} = 1 + \frac{\theta_2}{6} rk^2 - \theta_3\, m, \quad m_{25} = \frac{t_1}{2}(1 - \theta_3\, m^2)\frac{\partial}{R\partial\beta},$$

$$m_{33}=m_{44}=-\left(d-\frac{2}{3}\theta_2 k\right), \qquad m_{35}=-\frac{t_1}{2}(1+\theta_3 m^2)\frac{\partial}{\partial x},$$

$$m_{45}=-\frac{t_1}{2}(1+\theta_3 m^2)\frac{\partial}{R\partial\beta}, \qquad m_{55}=d-\frac{t_1^2}{3}(1+\theta_3 m^3)\nabla^2,$$

$$m_{12}=m_{14}=m_{23}=m_{34}=0$$

with $d=1+\theta_2 k+\theta_3 m$.

$[s_{rs}]$ is the dimensionless and symmetrical stiffness matrix, with

$$\begin{aligned}
&s_{11}=t_1^2(1+n_2 k+nm)\left(\frac{\partial^2}{\partial x^2}+\frac{1-\nu}{2}\frac{\partial^2}{R^2\partial\beta^2}\right),\\
&s_{12}=t_1^2(1+n_2 k+nm)\frac{1+\nu}{2}\frac{\partial^2}{R\partial\beta\,\partial x},\\
&s_{13}=t_1{}^2(nm-1)\left(\frac{\partial^2}{\partial x^2}+\frac{1-\nu}{2}\frac{\partial^2}{R^2\partial\beta^2}\right),\\
&s_{14}=\frac{1+\nu}{2}t_1^2(nm-1)\frac{\partial^2}{R\partial\beta\,\partial x},\\
&s_{15}=\left[t_1^2(1+n_2 k+nm)\frac{\nu}{R}+\frac{t_1}{2}(nm^2-1)\nabla^2\right]\frac{\partial}{\partial x},\\
&s_{22}=t_1^2(1+n_2 k+nm)\left(\frac{1-\nu}{2}\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{R^2\partial\beta^2}\right),\\
&s_{23}=\frac{1+\nu}{2}t_1^2\left(nm-1-\frac{n_2}{6}\frac{t_1}{R}k^2\right)\frac{\partial^2}{R\partial\beta\,\partial x},\\
&s_{24}=t_1^2\left(nm-1-\frac{t_1}{6R}n_2 k^2\right)\left(\frac{1-\nu}{2}\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{R^2\partial\beta^2}\right),\\
&s_{25}=t_1^2\left[(1+n_2 k+nm)R^{-1}+\frac{t_1}{2}(nm^2-1)\nabla^2\right]\frac{\partial}{R\partial\beta},\\
&s_{33}=t_1^2\left(1+\frac{k}{3}n_2+nm\right)\left(\frac{\partial^2}{\partial x^2}+\frac{1-\nu}{2}\frac{\partial^2}{R^2\partial\beta^2}\right)-4\frac{\gamma_{xz}}{k},\\
&s_{34}=L_{43}=\frac{1+\nu}{2}t_1^2\left(1+\frac{k}{3}n_2+nm\right)\frac{\partial^2}{R\partial\beta\,\partial x},\\
&s_{35}=t_1^2\left[(nm-1)\frac{\nu}{R}+\frac{2}{t_1}\gamma_{xz}+\frac{t_1}{2}(1+nm^2)\nabla^2\right]\frac{\partial}{\partial x},\\
&s_{44}=t_1^2\left[\left(1+\frac{k}{3}n_2+nm\right)\left(\frac{1-\nu}{2}\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{R^2\partial\beta^2}\right)-4\frac{\gamma_{yz}}{k},\right.\\
&s_{45}=t_1^2\left[(nm-1)R^{-1}+\frac{2\gamma_{yz}}{t_1}+\frac{t_1}{2}(nm^2+1)\nabla^2\right]\frac{\partial}{R\partial\beta},\\
&s_{55}=t_1^2(1+n_2 k+nm)R^{-2}+\frac{t_1^3}{R}(nm^2-1)\left(\nu\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{R^2\partial\beta^2}\right)+\\
&\qquad+\frac{t_1^4}{3}(nm^3+1)\nabla^4-kt_1^2\left(\gamma_{xz}\frac{\partial^2}{\partial x^2}+\gamma_{zy}\frac{\partial^2}{R^2\partial\beta^2}\right).
\end{aligned} \tag{3.10}$$

Equation (3.6) is the general equation governing the motion of a cylindrical sandwich shell in terms of the displacements (3.7). In the following section we shall study only the axially symmetrical free vibrations of cylindrical shells.

4. Axially symmetric free vibrations of damped sandwich shells

For reason of symmetry of the shell deformations, we thus have

$$v_2 = \bar{v}_2 = 0, \tag{4.1}$$

while the remaining variables u_2, $\bar{u}_2$ and a are only functions of the variable x. Since only free vibrations are considered, we set

$$p_z = 0.$$

Moreover, for sake of simplicity, we assume that:

(i) the contribution of $\frac{\partial^2 g_{xz}}{\partial t^2}$ to translatory motions u_2 and w, and inversely, those of $\frac{\partial^2}{u_2\,\partial t^2}$ and $\partial^2 w \partial t^2$ to rotatory motion are negligible;

(ii) the contribution of $\frac{\partial^2 u_2}{\partial t^2}$ to transverse motion w and that of $\frac{\partial^2 w}{\partial t^2}$ to longitudinal motion u_2 are also negligible.

Within the above-mentioned simplifications, the 5-dimensional equation (3.6) turns into a 3-dimensional and homogeneous one, namely:

$$\rho\,[m_{rs}]\,\{\ddot{d}_r\} + [s_{rs}]\,\{d_r\} = \{0\}, \qquad r, s = 1, 2, 3; \tag{4.2}$$

$$\{d_r\} = \begin{Bmatrix} u_2 \\ \bar{u}_2 \\ w \end{Bmatrix}; \tag{4.3}$$

$$\begin{gathered} m_{11} = -d; \quad m_{22} = -\left(d - \frac{2}{3}\theta_2 k\right), \quad m_{33} = d - \frac{t_1^2}{3}(1 + \theta_3 m^3)\frac{\partial^2}{\partial x^2}, \\ m_{12} = m_{13} = m_{23} = 0; \end{gathered} \tag{4.4}$$

$$\begin{aligned} s_{11} &= (1 + n_2 k + nm)\, t_1^2 \frac{\partial^2}{\partial x^2}, \qquad s_{12} = (nm - 1)\, t_1^2 \frac{\partial^2}{\partial x^2}, \\ s_{13} &= \left[(1 + n_2 k + nm)\, v \frac{t_1}{R} + \frac{1}{2}(nm^2 - 1)\, t_1^2 \frac{\partial^2}{\partial x^2}\right] t_1 \frac{\partial}{\partial x}, \\ s_{22} &= \left(1 + \frac{k}{3} n_2 + nm\right) t_1^2 \frac{\partial^2}{\partial x^2} - 4\frac{\gamma_{xz}}{k}, \\ s_{23} &= \left[(nm - 1)\frac{t_1}{R} + 2\gamma_{xz} + \frac{1}{2}(1 + nm^2)\, t_1^2 \frac{\partial^2}{\partial x^2}\right] t_1 \frac{\partial}{\partial x}, \\ s_{33} &= (1 + n_2 k + nm)\, t_1^2 R^{-2} + v t_1 R^{-1}(nm^2 - 1)\, t_1^2 \frac{\partial^2}{\partial x^2} + \\ &\quad + \frac{1}{3}(1 + nm^3)\, t_1^4 \frac{\partial^4}{\partial x^4} - k\,\gamma_{xz}\, t_1^2 \frac{\partial^2}{\partial x^2}. \end{aligned} \tag{4.5}$$

Let us now include the mechanical loss associated with the deformation of the core material through the use of complex material characteristies. Since only complex moduli are here considered, we have

$$\check{G}_2 = G_2 (1 + I \mu_0 \varepsilon_1), \qquad \check{E}_2 = E_2 (1 + I \mu_0 \varepsilon_2), \tag{4.6}$$

where

$$\varepsilon_1 = \frac{\mu_1}{\mu_0}; \qquad \varepsilon_z = \frac{\mu_z}{\mu_0} \tag{4.7}$$

with μ_0 — the arbitrarily chosen material loss factor, μ_1 — the material loss factor associated with the transverse shear modulus, μ_z — the material loss factor associated with Young's modulus.

In Eqs. (4.5) the following substitutions are made:

$$n_2 := \check{n}_2 = n_2 (1 + I \mu_0 \varepsilon_z), \qquad \gamma_{xz} := \check{\gamma}_{xz} = \gamma_{xz} (1 + I \mu_0 \varepsilon_1). \tag{4.8}$$

Let us consider the case of a composite shell simply supported at its free edges (i.e., at $x = 0, 1$).

According to the boundary conditions, the following forms of the shell — displacements may be employed:

$$u_2 = \sum_q U_q \cos \frac{q\pi x}{l} e^{I\check{\omega}_t}, \qquad \bar{u}_2 = \frac{t_2}{2} g_{x2} = \sum_q G_q \cos \frac{q\pi x}{l} e^{I\check{\omega}_t}, \qquad w = \sum_q \Lambda^{-1} W_i \sin \frac{q\pi x}{l} e^{I\check{\omega}_t}, \tag{4.9}$$

$$q = 1, 2, \ldots, q,$$

where

$$\omega = \check{\omega} + Is, \tag{4.10}$$

$$\Lambda = q\pi \frac{t_1}{l} \tag{4.11}$$

— wave number parameter.

Let the expressions (4.9) be introduced in Eqs. (4.2):

$$([\check{S}_{rs}] - \check{\Omega} [M_{rs}]) \{D_r\} = \{0\}, \tag{4.12}$$

where

$$\{D\}^T=\{u_q, G_q, w_q\}, \quad M_{11}=d\lambda^{-1}, \quad M_{22}=\left(d-\frac{2}{3}\vartheta_2 k\right)\lambda^{-1},$$

(4.13)
$$\begin{aligned}
&M_{33}=\left[d+\frac{\lambda}{3}(1+\vartheta_3 m^3)\right]\lambda^{-2},\\
&\check{S}_{11}=-(1+nm)-k\check{n}_2,\\
&\check{S}_{12}=S_{12}=(1-nm),\\
&\hat{S}_{13}=\nu\tau(1+nm)+0.5(1-nm^2)+\nu\tau k\check{n}_2,\\
&\check{S}_{22}=-(1+nm)-\frac{4}{k}\psi_1-\frac{k}{3}\check{n}_2,\\
&\check{S}_{23}=\nu\tau(nm-1)-0.5(1+nm^2)+2\check{\psi}_1,\\
&\check{S}_{33}=-\left\{\tau^2(1+nm)-\nu\tau(nm^2-1)+\frac{1}{3}(1+nm^3)+k\psi_1+k\tau^2\check{n}_2\right\},\\
&\lambda=A^2, \quad \tau=\frac{t_1}{R}\lambda^{-1}, \quad \check{\psi}_1=\frac{1-\nu^2}{\lambda}\frac{\check{G}_2}{E_1},
\end{aligned}$$

with

(4.14)
$$\check{\Omega}=\rho\check{\omega}^2.$$

It is seen from Eq. (4.13) that if nontrivial solutions for U_q, G_q and W_q are to exist, the determinant formed by the coefficients of these unknowns must vanish, e.g.,

(4.15)
$$\det([S_{rs}]-\check{\Omega}[M_{rs}])=0.$$

Thus we obtain the "frequency" equation

(4.16)
$$\check{a}_0\check{\Omega}^3-\check{a}_1\Omega^2+\check{a}_2\check{\Omega}-\check{a}_3=0,$$

where a_0, a_1, a_2, a_3 are complex coefficients which can be obtained on solving Eq. (4.15). Moreover, we have from Eqs. (4.14) and (4.10)

(4.17)
$$\check{\Omega}=\rho\check{\omega}^2\simeq\rho\omega^2\left(1+2I\frac{s}{\omega}\right),$$

where it has been assumed

(4.18)
$$\left(\frac{s}{\omega}\right)^2\ll 1.$$

Introducing the frequency parameter

(4.19)
$$\Omega=\rho\omega^2$$

and the logarithmic decrement

(4.20)
$$\delta=2\pi\frac{s}{\omega}$$

we obtain

$$\check{\Omega}=\Omega\left(1+I\frac{\delta}{\pi}\right) \tag{4.21}$$

or

$$\check{\Omega}=\Omega\left(1+I\mu_0\,\delta^*\right), \tag{4.22}$$

where

$$\delta^*=\frac{\delta}{\mu_0\,\pi} \tag{4.23}$$

is defined as the damping parameter.

In order to carry out the calculation of the shell frequencies and their associated damping parameters from Eq. (4.16), we set up from [7] a suitable procedure. First, we identify the shell frequencies as follows: the lowest frequency corresponds to the flexural mode, the highest frequency is predominantly of the thickness — shear type, while the remaining frequency is predominantly of the extensional type.

We denote the frequency parameter and its associated damping parameter, respectively, as corresponding to the flexural, extensional and thickness-shear types, by Ω_w and δ^*_w, Ω_u and δ^*_u Ω_g and δ^*_g. Let us note that all these quantities are positive values.

$$\check{\Omega}_w=\Omega_w\left(1+I\mu_0\,\delta^*_w\right), \qquad \check{\Omega}_u=\Omega_u\left(1+I\mu_0\,\delta^*_u\right), \qquad \hat{\Omega}_g=\Omega_g\left(1+I\mu_0\,\delta^*_h\right), \tag{4.24}$$

where, as it follows from the above remark

$$\Omega_w<\Omega_u<\Omega_g. \tag{4.25}$$

Viete's formulas give from Eq. (4.16)

$$\begin{aligned} \check{\Omega}_w+\check{\Omega}_u+\check{\Omega}_g&=\check{a}_1/a_0,\\ \check{\Omega}_w\,\check{\Omega}_u+\check{\Omega}_u\,\check{\Omega}_g+\check{\Omega}_g\,\check{\Omega}_w&=\check{a}_2/a_0,\\ \check{\Omega}_w\,\check{\Omega}_u\,\check{\Omega}_g&=\check{a}_3/a_0. \end{aligned} \tag{4.26}$$

By substituting Eqs. (4.24) and (4.17) into Eq. (4.26) and assuming

$$\mu_0^2\left(\delta^*_u\,\delta^*_g+\delta^*_g\,\delta^*_w+\delta^*_w\,\delta^*_u\right)\ll 1 \tag{4.27}$$

and, after equating in each equation the real part in the left-hand side member to that in the right-hand side member, we obtain

$$\begin{aligned} \Omega_w+\Omega_u+\Omega_g&=a_1/a_0,\\ \Omega_w\,\Omega_u+\Omega_u\,\Omega_g+\Omega_g\,\Omega_w&=a_2/a_0,\\ \Omega_w\,\Omega_u\,\Omega_g&=a_3/a_0. \end{aligned} \tag{4.28}$$

Likewise, a similar procedure for the imaginary part in each equation gives

$$\begin{aligned} &\delta^*_g+K_{23}\,\delta^*_u+K_{13}\,\delta^*_w=\eta_1\left(1+K_{23}+K_{13}\right),\\ &(1+K_{12})\,\delta^*_g+(1+K_{13})\,\delta^*_u+(K_{12}+K_{13})\,\delta^*_w=\eta_2\left(1+K_{13}+K_{12}\right),\\ &\delta^*_g+\delta^*_u+\delta^*_w=\eta_3, \end{aligned} \tag{4.29}$$

where

$$(4.30)\quad K_{12}=\frac{\Omega_w}{\Omega_u},\qquad K_{13}=\frac{\Omega_w}{\Omega_g},\qquad K_{23}=\frac{\Omega_u}{\Omega_g},\qquad \eta_j=a_{jp}/a_j,\quad j=1,2,3.$$

The set of Eqs. (4.28) yields the frequency of the sandwich shell

$$(4.31)\qquad a_0\,\Omega^3-a_1\,\Omega^2+a_2\,\Omega-a_3=0.$$

Solving Eqs. (4.29), we obtain

$$(4.32)\qquad \begin{aligned}
\delta_g^*&=[(1-K_{12})\,\eta_{1e}+(K_{23}-K_{13})\,(\eta_3\,K_{13}-\eta_{2e})]/(1-K_{13})\times\\
&\qquad\times(1+K_{13}-K_{12}-K_{23}),\\
\delta_u^*&=(\eta_{2e}-\eta_{1e}-\eta_3\,K_{12})/(1+K_{13}-K_{12}-K_{23},\\
\delta_w^*&=[\eta_{1e}\,(K_{12}-K_{13})+(\eta_3-\eta_{2e})\,(1-K_{23})](1-K_{13})\,(1+K_{13}-K_{23}-K_{12}),
\end{aligned}$$

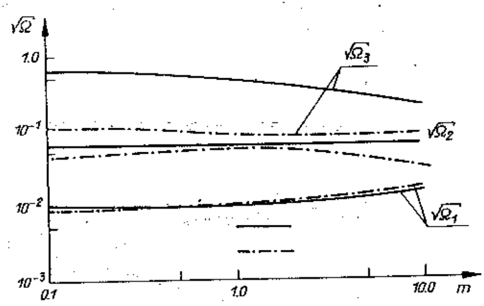
where

$$(4.23)\qquad \begin{aligned}
\eta_{1e}&=\eta_1\,(1+K_{23}+K_{13}),\\
\eta_{2e}&=\eta_2\,(1+K_{13}+K_{12}).
\end{aligned}$$

5. Numerical results

In the present section we present the results for the case of axially symmetric vibrations only.

For a shell of thin core and face thickness numerical results show that its three natural frequency parameters are separated far apart, as this can be seen in Fig. 4. Therefore, it is pertinent to point out that in this case the computation of the vibration characteristics may be performed by an approximate method as outlined below.



Rys. 4

We assume

$$(5.1)\quad K_{13}\ll K_{12}\ll 1,\ K_{13}\ll K_{23},\ K_{23}\ll 1.$$

Then, by applying Eq. (5.1) to Eq. (4.28) we get the approximate formulas for frequencies

$$(5.2)\qquad \Omega_w=-\frac{a_3}{a_2},\qquad \Omega_u=-\frac{a_2}{a_1},\qquad \Omega_g=-\frac{a_1}{a_0}.$$

Next, by considering Eqs. (4.32) and (5.1) we obtain

$$(5.3)\qquad \begin{aligned}
\delta_g^*&=\eta_1+K_{23}\,(\eta_3\,K_{13}-\eta_2),\\
\delta_u^*&=\eta_2-\eta_1-K_{12}\,\eta_3,\\
\delta_w^*&=\eta_1\,K_{12}+\eta_3-\eta_2,
\end{aligned}$$

— the approximate formulas of the various damping parameters. We will use mainly Eqs. (4.31) and (4.32) in the computation below. Geometrical parameters of the sandwich shell are chosen as follows

$$t_1/a = 1/50, \quad (t_1+t_2+t_3)/2R \leqslant 1/20,$$

the ratios $m=t_3/t_1$ and $k=t_2/t_1$ are varying within the ranges (0.05; 15.0) and (0.02; 10.0), respectively.

For the sake of simplicity we assume that the material of both facings is the same, e.g.,

$$n=E_3/E_1=1, \quad \theta_3=\delta_3/\gamma_1=1$$

and the core material characteristics are as follows: $n_2=E_2/E_1=0.0$ and 0.05 (for the case of "weak" core and "rigid" core, respectively),

$$\theta_2=\gamma_2/\gamma_1=0.1,$$

where γ_1, γ_2 are, respectively, the mass densities of the upper face and of the core;

$$\varepsilon_1=\varepsilon_2=1,$$

where $\mu_0=0.2$.

Two ratios of the core transverse shear modulus to the face Young's modulus will be considered:

$$G_2/E_1=1/100 \quad \text{and} \quad 1/10.$$

Therefore, since $\lambda=\pi^2(t_1/a)^2=0.04$, there two values of the shear parameter axe yielded:

$$\psi_1=\frac{1-\nu^2}{\lambda}\,\frac{G_2}{E_1}=2.5 \quad \text{and} \quad 25.0,$$

where $\lambda=0.3$.

Numerical results are presented in figures as follows. Figure 5 shows the effect of the face thickness on the various damping parameters δ^*_u, δ^*_g, δ^*_w in a sandwich shell, within the range of m from 0.1 to 10.0.

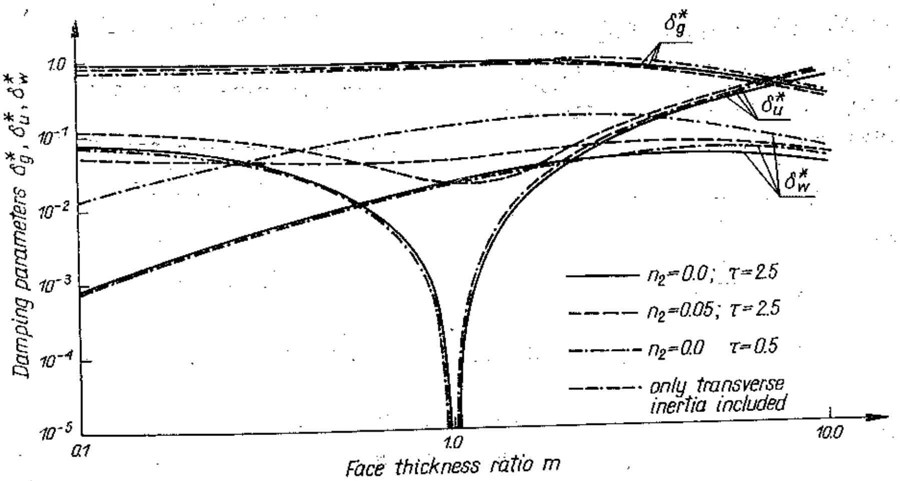


FIG. 5

I. The damping of thickness-shear type is the highest in magnitude among the three types of damping and differs scarcely from one, except for too large m. Within this limit, the increase in face thickness ratio m diminishes this type of damping but in rather slight manner and the dependence δ_g^* upon m may thus be plotted by a decreasing straight line.

II. Through the variation of the face thickness, the damping of extensional type, identified by δ_u^*, decreases at first to a very low in magnitude minimum at $m=1$ (e.g., when the plate has a symmetrical structure) and next increases for further increase in m. This minimum-peak has a very "local" character because the damping parameter δ_u^* suddenly decreases to very low in magnitude values only in the neighbourhood of $m=1.0$. It should be pointed out that this minimum-peak agrees closely with the results obtained by Yu from his theory of sandwich plates and shells of symmetrical structure [4]: Yu, deducing from his approximate formulas, asserted that in shells of symmetrical structure $\delta_u^*=0$, e.g., the shear damping of the core is seen to be totally ineffective for the extensional vibration of a sandwich plate or cylindrical shell.

III. The variation of the damping of the flexural type (identified by δ_w) versus the face thickness ratio m has the typical form mentioned in [2] for sandwich plates. For given λ, ψ_1, n_2 and k, an optimum structure m_{opt} may be reached providing a maximum damping of flexural type.

IV. The contribution of the modulus in flexure and in the extension of the core ($n_2 \neq 0$) is most effective for the damping of flexural type and for that of extensional type, while for the damping of thickness-shear type this contribution is all but ineffective, as illustrated in Fig. 5. The minimum-peak of the extensional type damping seems then to disappear completely.

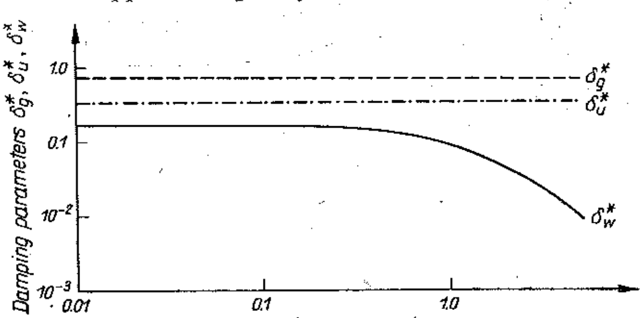


FIG. 6

V. Figure 5 shows also the effect induced by the shell curvature, identified by τ, on the various types of damping. The decrease in curvature will, in general, slightly diminish the damping of thickness-shear type and that of extensional type (except at $m=1$ where the minimum-peak strongly increases) but will improve noticeably the damping of flexural type. This type of damping would attain maximum values only if the curvature vanished, e.g., when the sandwich shell degenerates into a sandwich plate (see also Fig. 6). For a sandwich shell of asymmetrical structure the two types of damping, δ_u^* and δ_g^*, may be considered as independent of the curvature change.

VI. In general, (Fig. 7.), the thicker the core, the better the damping of extensional type (δ_u^*) and the damping of flexural type (δ_w^*). But the rule is opposite for the damping of thickness-shear type although the decrease is rather negligible when the core is not too thick. At small values of core thickness ratio k, the two types of

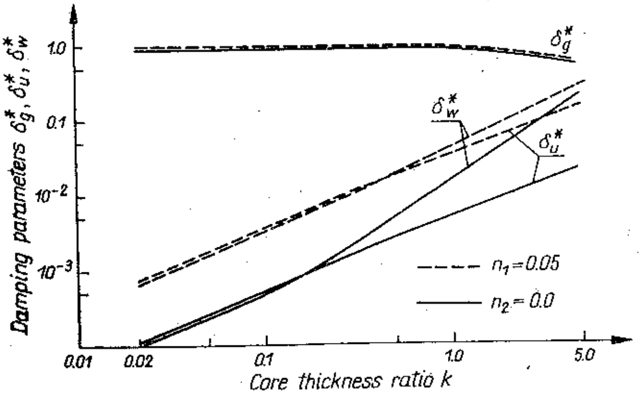


FIG. 7

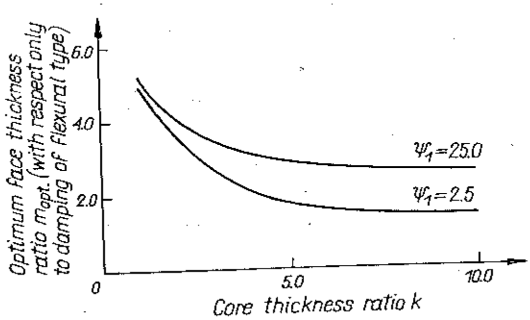


FIG. 8

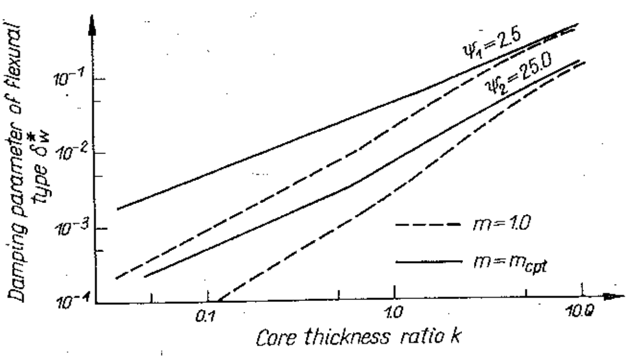


FIG. 9

damping δ_u^* and δ_w^* are all but the same. Only from a certain value of k does the difference between these two types of damping become stronger; the damping of flexural type then increases faster than that of extensional type.

VII. Figure 8 presents the relation between the core thickness k and the face thickness ratio m_{opt}, optimum with respect to damping of flexural type, at fixed shear parameters ψ_1 in the case of sandwich shells having a "weak" core [2]. This relation is plotted by "mean" curves which are similar to those obtained for sandwich plates with a "weak" core. In the considered cases it is obvious that in general, the optimum structure is the asymmetrical one.

VIII. The difference between the damping of flexural type optimum structure with that in symmetrical structure is shown in Fig. 9. It follows from the considered cases that only for a sandwich shell with a thick "weak" core may the symmetrical structure become an optimum one.

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Streszczenie

TŁUMIENIE STRUKTURALNE W POWŁOKACH WARSTWOWYCH

Stosując metodę wariacyjną wyprowadzono równania rządzące ruchem powłoki sandwiczowej. Tłumienie strukturalne zbadano szczegółowo w przypadku sandwiczowej powłoki walcowej o różnych warstwach i wykonującej osiowo symetryczne drgania swobodne. Do opisania lepkosprężystych własności wypełniacza wykorzystano koncepcję zespolonego modułu. Uzyskano i przedyskutowano różne parametry częstości powłoki i odpowiadające im parametry tłumienia w zależności od stosunków różnych grubości warstw oraz innych fizycznych i geometrycznych charakterystyk powłoki wypełniacza. Wyniki obliczeń numerycznych przedstawiono w postaci wykresów.

Резюме

СТРУКТУРНОЕ ЗАТУХАНИЕ В СЛОИСТЫХ ОБОЛОЧКАХ

Применяя вариационный метод, выведены уравнения описывающие движение слоистой оболочки. Структурное затухание исследовано подробно в случае слоистой цилиндрической оболочки с разными слоями и совершающей осесимметриченые свободные колебания. Для описания вязкоупругих свойств ядра использована концепция комплексного модуля. Получены и обсуждены разные параметры частоты оболочки и отвечающие им параметры затухания в зависимости от отношений разных толщин слоев, а также других физических и геометрических характеристик слоистой оболочки. Результаты численных расчетов представлены в виде графиков.

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