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INVESTIGATION OF THE RABOTNOV-SHESTERIKOV CREEP STABILITY UNDER GENERAL LOADING PROGRAMS

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The purpose of the present paper is to analyse the critical state of a creeping column, for an arbitrary loading process prescribed either by stresses or by strains. Creep stability in the conditions of pure relaxation is investigated. Some numerical calculations of the critical time for the particular processes are presented. The concept of the boundary process and of the process with safety factors constant in time is introduced. Finally, some remarks concerning the influence of the loading history on the critical time are given.

1. INTRODUCTION

The first theories of creep buckling considered quasi-static motion of imperfect columns (A. M. FREUDENTHAL, A. D. ROSS, A. R. RZHANITSYN, 1946). The theory of creep stability of perfect columns was initiated by F. R. SHANLEY [12] in 1952. His suggestion results in the replacement of E in Euler's formula by tangent modulus $d\sigma/d\varepsilon$ at $t=\text{const}$, calculated for given constant stress σ (slope of the isochronous curve). So the critical state is determined by a certain relation between time and stress (critical time for a given stress or critical stress for a given time). Another approach, proposed by G. GERARD [3] in 1956, uses the secant modulus (concept of constant critical strain); such a proposal for elastic-plastic buckling was suggested earlier by M. BROSZKO [1]. Those hypotheses, however, are not sufficiently justified and lead to upper bounds of exact solutions.

The most rigorous theory of creep stability of perfect columns — including dynamic criterion of stability — was published by YU. N. RABOTNOV and S. A. SHESTERIKOV [9] in 1957. For conservative behaviour of the force it leads to the replacement of E by another tangent modulus, namely $d\sigma/d\varepsilon$ at constant creep strain rate. The differences with respect to the SHANLEY theory are small for large strain-hardening of the material; however, they increase with decreasing strain-hardening and are particularly essential for steady creep — the SHANLEY theory gives here a certain finite critical time, whereas the RABOTNOV-SHESTERIKOV theory yields $t_k=0$. A detailed comparison of the basic theories of creep stability of perfect columns was given by N. J. HOFF [4] and W. E. JAHSMAN and F. A. FIELD [6, 7].

The fundamental concept of the RABOTNOV-SHESTERIKOV theory was developed in many papers. N. J. HOFF [5] and Y. YAMAMOTO [15] investigated the components of the vibrating motion of the column. S. A. SHESTERIKOV [13, 14] considered the influence of the initial conditions and the change of the tangent modulus in time. A. R. RZHANITSYN [10] and J. N. DISTÉFANO and J. L. SACKMAN [2] derived the stability criterion for the materials the rheological behaviour of which is described by an integral equation. O. N. SAVINOV [11] and W. E. JAHSMAN [8] investigated the stability of an imperfect column. M. ŻYCZKOWSKI and R. WOJDANOWSKA-ZAJĄC [18] used the RABOTNOV-SHESTERIKOV theory to determine the optimal shape of a column subject to creep buckling.

Although RABOTNOV and SHESTERIKOV derived their criterion of stability for arbitrary behaviour of the force in time, it was used almost exclusively for the case of a constant force (described by a Heaviside function). The purpose of the present paper is to analyse the critical state of the column for an arbitrary process, prescribed either by stresses or by strains. Creep buckling in the conditions of pure relaxation is investigated. The concept of the boundary process and of the processes with constant safety factors is introduced. Finally, some remarks concerning the influence of the loading history on the critical time are given.

2. THE RABOTNOV-SHESTERIKOV CRITERION OF STABILITY

RABOTNOV and SHESTERIKOV [9] investigate the stability of a column, the material of which is subject to the non-linear creep law

$$\Phi(\sigma, p, \dot{p}) = 0, \tag{2.1}$$

where $p = \varepsilon_c = \varepsilon - \sigma/E$ denotes the inelastic (creep) strain. Small variations of stress and strain, superposed on the basic (pure compression) state, are governed by the relation

$$\lambda^* \,\delta\sigma + \mu^* \,\delta p + \nu^* \,\delta\dot{p} = 0 \tag{2.2}$$

in which $\lambda^* = \partial\Phi/\partial\sigma$, $\mu^* = \partial\Phi/\partial p$, $\nu^* = \partial\Phi/\partial\dot{p}$. The Bernoulli assumption $\delta\varepsilon = z\delta\kappa = \kappa z$ (since the initial curvature $\kappa_0 = 0$) and the integration of (2.2) multiplied by z over the cross-sectional area gives the moment-curvature relation

$$(E\lambda^* - \mu^*) M - \nu^* \dot{M} + EI(\mu^* \kappa + \nu^* \dot{\kappa}) = 0. \tag{2.3}$$

RABOTNOV and SHESTERIKOV investigated the vibrations of a simply supported column and derived the only condition of stability

$$\frac{E\lambda^* P}{P_E - P} + \mu^* - \frac{\nu^* \dot{P}}{P_E - P} > 0, \tag{2.4}$$

where P_E stands for the Euler force. This criterion corresponds to the vanishing frequency of vibrations, so it coincides with the purely static approach.

The derivatives λ^*, μ^* and ν^* are to be evaluated from (2.1) integrated for the case of pure compression. Here we confine ourselves to the particular form of (2.1), namely

$$\Phi = \dot{p}p^{\alpha} - k\sigma^{n} = 0 \tag{2.5}$$

containing three material constants, k, n, and α (E being the fourth constant). Calculating the corresponding derivatives and substituting into (2.4), we obtain the stability condition

$$-Ekn\sigma^n+\alpha\dot{p}p^{\alpha-1}(\sigma_E-\sigma)-p^\alpha\dot{\sigma}>0, \tag{2.6}$$

where σ_E denotes the Euler stress. This form is valid for $\sigma_E>\sigma$, but in the opposite case the straight column cannot be stable at all. The compressive stresses and strains are here assumed to be positive; our analysis will be restricted to compressive stresses and strains only. Now, we are going to the detailed investigation of the stability condition (2.6).

3. Stress as the "control function"

The loading program, or the history of loading, may be prescribed by means of various quantities ("control functions", "exertion factors" [19]), but two such quantities are the most typical: stress and strain. The loading program prescribed in stresses is considered as classical, whereas the program in strains is easier to be realized at most testing machines.

If the program is prescribed in stresses, the stability condition in its general form (2.6) may be effectively expressed without difficulties. Integrating (2.5), we obtain at first

$$p^{\alpha+1}=k(\alpha+1)\int_0^t \sigma^n(\xi)\,d\xi \tag{3.1}$$

(since $p=0$ for $t=0$), and hence we find the general formula for the strain

$$\varepsilon=\frac{\sigma}{E}+\sqrt[\alpha+1]{k(\alpha+1)\int_0^t\sigma^n(\xi)\,d\xi}\,. \tag{3.2}$$

The symbol ξ stands here for the variable of integration (time during the loading history).

Substitution of (3.1) or (3.2) into (2.6) gives the general form of the stability condition expressed in stresses. To present it in the most compact manner we introduce the dimensionless quantities: stress s and time τ, defined as follows

$$s=\frac{\sigma}{\sigma_E}=\frac{\lambda^2}{\pi^2 E}\sigma,\quad \tau=k\left(\frac{\lambda^2}{\pi^2}\right)^{\alpha+1}\sigma_E^n t=k\,E^n\left(\frac{\pi^2}{\lambda^2}\right)^{n-\alpha-1}t, \tag{3.3}$$

where $\lambda=l/i$ denotes the slenderness ratio of the column. These substitutions are very convenient, since they eliminate two material constants, E and k, and the slenderness ratio λ from the equations. Using (3.3) we express the condition (2.6) in the form of the following integro-differential inequality

$$\dot{s}<\frac{s^n}{(\alpha+1)\int_0^\tau s^n(\xi)\,d\xi}\left[\alpha(1-s)-n\sqrt[\alpha+1]{(\alpha+1)\int_0^\tau s^n(\xi)\,d\xi}\right]. \tag{3.4}$$

The dot denotes here differentiation with respect to the dimensionless time τ, and ξ is now dimensionless variable of integration. The fundamental question arises, whether the loading represented by a Heaviside function $P=P_0 H(t)$ or $s=s_0 H(\tau)$ is admissible in view of the condition (3.4); such a loading program is assumed by almost all investigators. It turns out, however, that starting from $\tau=0$ and $s=0$ and analysing the symbol 0/0 in (3.4) we find $\dot{s}<\infty$, so the infinite derivative in the Heaviside function may be considered as lying at the boundary of the admissible region up to $s=1$. Other initial conditions do not admit infinite value of the stress rate: so other Heaviside functions would cause the instability of the column. The behaviour of (3.4) in the vicinity of $s=0$ and $\tau=0$ will be discussed later (Sect. 5, the cases $\psi_\sigma=1$ and $\psi_\varepsilon=1$).

As an example of application of the general formula (3.4) let us consider the following loading program:

$$\sigma=\bar{C}t^m \quad \text{or} \quad s=C\tau^m, \tag{3.5}$$

where m denotes an arbitrary nonnegative constant. The inequality (3.4) yields here

$$\frac{\alpha+1}{mn+1}\,Cm\,\tau^m<\alpha\,(1-C\tau^m)-n\left(\frac{\alpha+1}{mn+1}\right)^{\frac{1}{\alpha+1}} C^{\frac{n}{\alpha+1}}\,\tau^{\frac{mn+1}{\alpha+1}}. \tag{3.6}$$

Replacing the sign of inequality by that of equality we obtain the equation determining the critical time in RABOTNOV-SHESTERIKOV sense for the process under consideration. In the simplest case $m=0$, $s=C=\text{const}$, $\sigma=\bar{C}=\text{const}$, the obtained equation

$$\alpha\,(1-C)-n\,(\alpha+1)^{\frac{1}{\alpha+1}}\,C^{\frac{n}{\alpha+1}}\,\tau^{\frac{1}{\alpha+1}}=0 \tag{3.7}$$

may easily be solved with respect to τ, namely

$$\tau_{cr}=\frac{[\alpha\,(1-C)]^{\alpha+1}}{(\alpha+1)\,n^{\alpha+1}\,C^n}, \tag{3.8}$$

$$t_{cr}=\left(\frac{\lambda^2}{\pi^2}\right)^{n-\alpha-1}\frac{\left[\frac{\alpha}{n}(1-\bar{C})\right]^{\alpha+1}}{(\alpha+1)\,k\,(\bar{C}E)^n}=\frac{\left[\frac{\alpha}{n}\left(\frac{\pi^2}{\lambda^2}-\frac{\sigma}{E}\right)\right]^{\alpha+1}}{(\alpha+1)\,k\sigma^n}. \tag{3.9}$$

This formula for the stress constant in time may be considered as classical.

The second important case of (3.5), namely $m=1$ (constant-stress rate), leads to the equation

$$n\left(\frac{\alpha+1}{n+1}\right)^{\frac{1}{\alpha+1}} C^{\frac{n}{\alpha+1}}\,\tau^{\frac{n+1}{\alpha+1}}+\frac{n\alpha+2\alpha+1}{n+1}\,C\tau-\alpha=0 \tag{3.10}$$

which, in general, cannot be solved analytically with respect to the critical time $\tau=\tau_{cr}$. Some numerical results are gathered in Table 1. The material constants for a polyvinyl chloride S-PVC are assumed as follows: $n=3.918$, $\alpha=1.598$.

On the other hand, the condition (3.6), treated as an equation, may be solved analytically with respect to τ for some other values of the exponent m. For example, if $m=1/(2\alpha+2-n)$, this equation may be reduced to a quadratic one. Such an exponent may appear in practical applications: for a polyethylene with $n=11$, $\alpha=6$, we have $m=1/3$. The final formula for the critical time, however, is in this case rather lengthy and will not be quoted here.

Table 1. Critical times for the programs prescribed in stresses

$s=C\tau^m$			
$m=0$		$m=1$	
C	τ_{cr}	C	τ_{cr}
0.1	235.8	0.01	22.9
0.3	1.66	0.1	3.28
0.5	0.0935	1.0	0.443
0.7	0.00664	10.0	0.0556
0.9	0.000143	100.0	0.00645

As another example of application of (3.4) we determine critical stress rates $\dot{s}_{cr}$ after the constant-stress program within a certain interval $0<\tau<\tau_0$. Substituting $s=\text{const}=s_0$ into (3.4), we obtain

$$\dot{s}_{cr}=\frac{1}{(\alpha+1)\tau_0}\left[\alpha(1-s_0)-n(\alpha+1)^{\frac{1}{\alpha+1}}\tau_0^{\frac{1}{\alpha+1}}s_0^{\frac{n}{\alpha+1}}\right]. \tag{3.11}$$

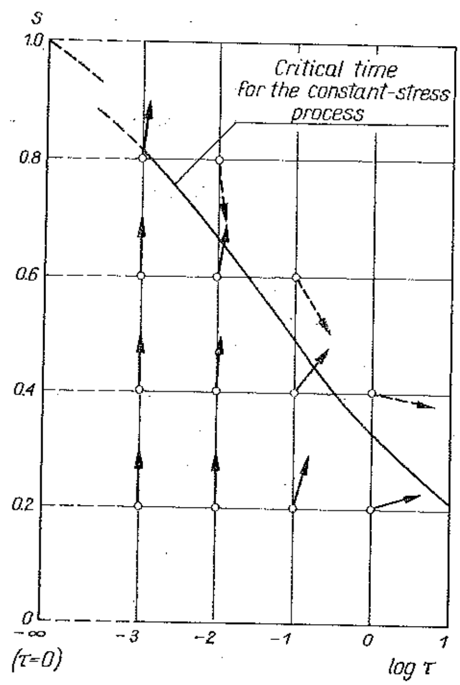

FIG. 1. Critical stress rates after the constant-stress process.

Infinitely short process with the stress rate (3.11), following the constant-stress program will cause instability of the column. For $\tau_0 = \tau_{cr}$ determined by (3.8) we obtain, of course, $\dot{s}_{cr} = 0$; for larger values of τ_0 the critical stress rates are negative — this result is purely theoretical, since the stability will be lost earlier, during the constant-stress period.

Numerical results of (3.11) for $n=3.918$, $\alpha=1.598$ (for S-PVC) are gathered in Table 2 and shown in vectorial form in Fig. 1.

Table 2. Critical stress rates after a constant-stress process

s_0	τ_0			
	0.001	0.01	0.1	1.0
0.8	14.089	(−14.126)		
0.6	175.450	7.478	(−1.694)	
0.4	330.761	27.614	1.436	(−0.178)
0.2	478.612	45.940	4.128	0.300

4. Strain as the "control function"

The considered creep law (2.5) cannot be reverted with respect to σ, it means that for arbitrary function $\varepsilon = \varepsilon(t)$ we cannot express the operator $\bar{\Omega}$ determining $\sigma = \bar{\Omega}[\varepsilon(t)]$ by means of elementary functions and operators. However, this operator may be defined and realized numerically, by numerical integration of (2.5). Introducing dimensionless quantities (3.3) and a new, dimensionless strain

$$e = \frac{\lambda^2}{\pi^2}\varepsilon, \tag{4.1}$$

we rewrite (2.5) in the form

$$ds = de - \frac{s^n}{(e-s)^\alpha}d\tau. \tag{4.2}$$

Integration of (4.2) determines the operator Ω in the expression $s = \Omega[e(\tau)]$ for any given function $e = e(\tau)$.

The start of the integration presents here some difficulties, since for $\tau = 0$ we have always $s = e$ and the second term at the right-hand side of (4.2) contains a singularity. The expansion of the solution into a generalized power series [16, 17], seems to be the most effective in this case. Suppose that the given function $e = e(\tau)$ may be expanded into the series

$$e = A_1\tau^{m_1} + A_2\tau^{m_2} + \ldots, \tag{4.3}$$

where m_i are nonnegative constants. Making use of (4.2) we determine at first the difference $e - s$. Assuming

$$e - s = B_1\tau^{q_1} + \ldots, \tag{4.4}$$

and substituting (4.4) and (4.3) into (4.2) we obtain a set of equations determining B_i and q_i. For the first term we have

$$B_1 q_1 \tau^{q_1-1} = \frac{A_1^n \tau^{nm_1}}{B_1^\alpha \tau^{\alpha q_1}}, \tag{4.5}$$

and hence both q_1 and B_1 may be determined

$$q_1 = \frac{nm_1+1}{\alpha+1}, \quad B_1 = \left(\frac{A_1^n}{q_1}\right)^{\frac{1}{\alpha+1}} = \left(\frac{\alpha+1}{nm_1+1} A_1^n\right)^{\frac{1}{\alpha+1}}. \tag{4.6}$$

Going back to (4.4) we may now derive the series for the dimensionless stress s. Three cases may be distinguished:

a) if $q_1 < m_2$, then the first two terms are

$$s = A_1 \tau^{m_1} - B_1 \tau^{q_1} + \ldots; \tag{4.7}$$

b) if $q_1 = m_2$, then

$$s = A_1 \tau^{m_1} + (A_2 - B_1) \tau^{m_2} + \ldots; \tag{4.8}$$

c) finally, if $q_1 > m_2$, then the first two terms are identical with (4.3)

$$s = A_1 \tau^{m_1} + A_2 \tau^{m_2} + \ldots \tag{4.9}$$

and the difference appears in the third term or even later. So numerical integration of (4.2) together with the expansions (4.7)–(4.9) determines the operator $\Omega[e]$ in a quite effective manner.

Substituting now into (2.6) the dimensionless quantities (3.3) and (4.1), eliminating $\dot{s}$ from (2.5) and expressing s in the form $s = \Omega[e]$, we obtain after some simple rearrangements the following condition of stability expressed in strains

$$\dot{e} < \frac{\{\Omega[e]\}^n}{\{e - \Omega[e]\}^{\alpha+1}} \{\alpha + (n-\alpha-1)\Omega[e] - (n-1)e\}. \tag{4.10}$$

The analysis of particular cases of (4.10) is much more difficult than that of (3.4). However, some simple conclusions may be drawn for the case of pure relaxation, $\dot{e} = 0$. The critical time is then determined from

$$\alpha + (n-\alpha-1)\Omega[e] - (n-1)e = 0. \tag{4.11}$$

The dimensionless stress $s = \Omega[e]$ may vary within the interval $0 < s < 1$. If $s = 1$, then the instability will occur immediately; as matter of fact (4.11) yields then $e = 1, \varepsilon = \varepsilon_E = \pi^2/\lambda^2$. On the other hand, if — in the limiting case — $s = 0$ determines the critical state, then the corresponding time will be infinitely large; this will take place for $e = \alpha/(n-1)$. So we have determined the limits of ε, inside which the instability under pure relaxation may occur

$$\frac{\alpha}{n-1} \frac{\pi^2}{\lambda^2} < \varepsilon < \frac{\pi^2}{\lambda^2}. \tag{4.12}$$

For lower ε the column is always stable, for higher ones — unstable from the beginning. For the material described in Sect. 3, we have $0.5476 < e < 1$. Numerical results for e=const and $e = C\tau$ are gathered in Table 3.

Table 3. Critical times for the programs prescribed in strains

$e = C\tau^m$			
$m=0$		$m=1$	
C	τ_{cr}	C	τ_{cr}
0.55	>1000	0.01	22.2
0.65	4.64	0.1	5.04
0.75	0.238	1.0	0.588
0.85	0.0177	10.0	0.0641
0.95	0.000389	100.0	0.00697

To the authors' knowledge, creep buckling in the case of pure relaxation has not yet been observed in experiments; however, it is predicted by the theory within the limits (4.12).

5. Loading programs corresponding to constant safety factors against creep buckling

Replacing in (3.4) or in (4.10) the signs of inequality by those of equality, we obtain the integro-differential equations of the boundary process: for every τ the column is then at the boundary of stability. These equations seem to be without practical meaning, since the instability will then occur at once, from the beginning; however, such an approach makes it possible to determine the processes with safety factors constant in time, and such processes may be interesting from the engineering point of view.

We introduce two safety factors: ψ_σ for stresses and ψ_ε for strains. As a process with constant safety factor for stresses we understand the process described by (3.4) as an equality with s replaced by $\psi_\sigma s$. After some simple rearrangements we write its equation in the form

$$(5.1)\quad (\alpha+1)\psi_\sigma \dot{s} \int_0^\tau s^n(\xi)\,d\xi = s^n \left[\alpha(1-\psi_\sigma s) - n\psi_\sigma^{\frac{n}{\alpha+1}} \sqrt[\alpha+1]{(\alpha+1)\int_0^\tau s^n(\xi)\,d\xi}\right].$$

The corresponding function $\varepsilon = \varepsilon(t)$ or $e = e(\tau)$ may be found from (3.2).

If all the physical relations had been linear, then the constant safety factor for stresses would have also imposed constant safety for strains. This is not the case, and the process with constant safety factor for strains will be defined separately. We understand here the process described by (4.10) as an equality with e replaced by $\psi_\varepsilon e$

$$(5.2)\quad \psi_\varepsilon \dot{e} \{\psi_\varepsilon e - \Omega[\psi_\varepsilon e]\}^{\alpha+1} = \{\Omega[\psi_\varepsilon e]\}^n \{\alpha + (n-\alpha-1)\Omega[\psi_\varepsilon e] - (n-1)\psi_\varepsilon e\}.$$

The operator Ω in the equation $\bar{s}=\Omega[\psi_\varepsilon e]$ is consequently defined by (4.2) with substituted $\psi_\varepsilon e$ instead of e

$$d\bar{s}=\psi_\varepsilon de-\frac{\bar{s}^n}{(\psi_\varepsilon e-\bar{s})^\alpha}d\tau; \tag{5.3}$$

however, this equation for $\bar{s}$ is only of auxiliary character, and the stresses s corresponding to (5.2) should be calculated separately from the original equation (4.2).

The Eqs. (5.1), (5.2) and (5.3) were integrated numerically. The only difficulty was connected with the singularity for $\tau=0$; so in the vicinity of $\tau=0$ generalized power series were used.

Consider, at first, the process with constant safety factor ψ_σ. For $\tau=0$ the infinitely large stress rate is admissible up to $s=1/\psi_\sigma$, so we assume the expansion of (5.1) in the form

$$s=\frac{1}{\psi_\sigma}+D_1\tau^m+\ldots. \tag{5.4}$$

Substituting (5.4) into (5.1) and taking just the first (basic) terms into consideration we arrive at the equation

$$(\alpha+1)\psi_\sigma^{1-n}mD_1\tau^m=-\alpha\psi_\sigma^{1-n}D_1\tau^m-n\psi_\sigma^{-n}(\alpha+1)^{\frac{1}{\alpha+1}}\tau^{\frac{1}{\alpha+1}} \tag{5.5}$$

and hence

$$m=\frac{1}{\alpha+1},\qquad D_1=-\frac{n}{\psi_\sigma(\alpha+1)^{\frac{\alpha}{\alpha+1}}}, \tag{5.6}$$

$$s=\frac{1}{\psi_\sigma}\left[1-\frac{n}{(\alpha+1)^{\frac{\alpha}{\alpha+1}}}\tau^{\frac{1}{\alpha+1}}+\ldots\right]. \tag{5.7}$$

The corresponding expansion for strains may be found from (3.2). Performing appropriate operations, we obtain

$$e=\frac{1}{\psi_\sigma}-\frac{n\psi_\sigma^{\frac{n-\alpha-1}{\alpha+1}}-(\alpha+1)}{\psi_\sigma^{\frac{n}{\alpha+1}}(\alpha+1)^{\frac{\alpha}{\alpha+1}}}\tau^{\frac{1}{\alpha+1}}+\ldots. \tag{5.8}$$

It may be seen that the safety factor ψ_σ reduces the stresses proportionally (as it was assumed), whereas the strains are not subject to proportional reduction.

The expansion of (5.2) and (5.3) is somewhat more complicated. We assume the expansions for the strains and for the auxiliary (non-reduced) stresses $\bar{s}$ in the form similar to (5.8), namely

$$e=\frac{1}{\psi_\varepsilon}+A\tau^{\frac{1}{\alpha+1}}+\ldots,\qquad \bar{s}=1+B\tau^{\frac{1}{\alpha+1}}+\ldots. \tag{5.9}$$

Substituting these expressions into (5.2) and (5.3) we prove that the exponents are suitably chosen and obtain the following non-linear equations

$$(5.10)\qquad \begin{aligned} &(A\psi_\varepsilon - B)^{\alpha+1} = \alpha+1,\\ &A\psi_\varepsilon(A\psi_\varepsilon - B)^{\alpha+1} = (\alpha+1)\,[(n-\alpha-1)B-(n-1)A\psi_\varepsilon].\end{aligned}$$

They may easily be reduced to linear ones, and finally

$$(5.11)\qquad A = -\frac{n-\alpha-1}{\psi_\varepsilon(\alpha+1)^{\frac{\alpha}{\alpha+1}}},\qquad B = -\frac{n}{(\alpha+1)^{\frac{\alpha}{\alpha+1}}},$$

$$(5.12)\qquad e = \frac{1}{\psi_\varepsilon}\left[1-\frac{n-\alpha-1}{(\alpha+1)^{\frac{\alpha}{\alpha+1}}}\tau^{\frac{1}{\alpha+1}}+\dots\right].$$

The corresponding expansion for stresses may be found from (4.6). Since in the considered case $q_1 = 1/(\alpha+1) = m_2$, we apply the formula (4.8) and obtain

$$(5.13)\qquad s = \frac{1}{\psi_\varepsilon} - \frac{(n-\alpha-1)\psi_\varepsilon^{\frac{n-\alpha-1}{\alpha+1}}+(\alpha+1)}{\psi_\varepsilon^{\frac{n}{\alpha+1}}(\alpha+1)^{\frac{\alpha}{\alpha+1}}}\tau^{\frac{1}{\alpha+1}}+\dots.$$

In this case the strains are reduced proportionally whereas the stresses are not.

For $\psi_\sigma = \psi_\varepsilon = 1$ the expansions (5.7) and (5.13) as well as (5.8) and (5.12) coincide. For larger safety factors they may differ considerably; if the process is controlled by stresses, then (5.1), (5.7) and (5.8) should be used, and if by strains — the equations (5.2), (5.12) and (5.13) are justified.

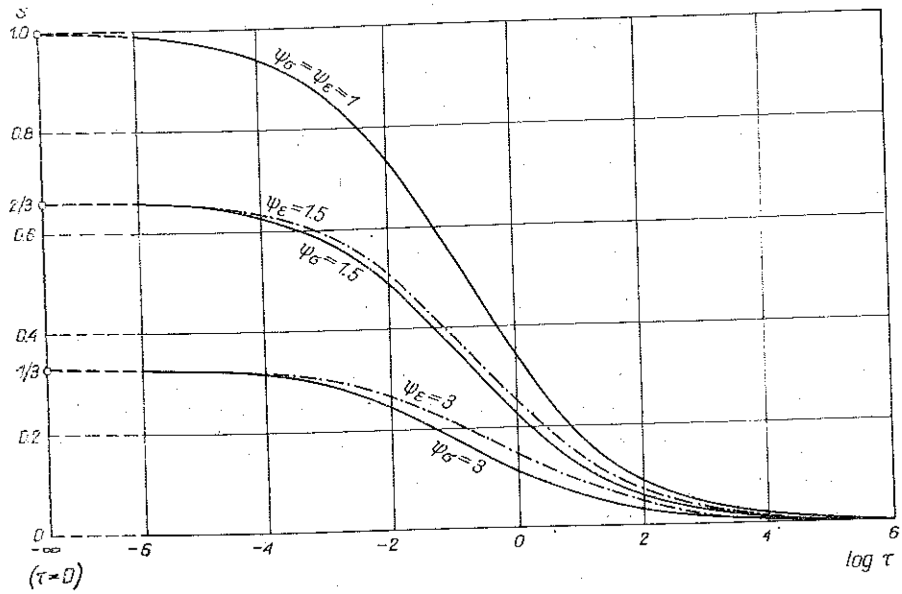

FIG. 2. Processes with constant safety factors (stresses versus time).

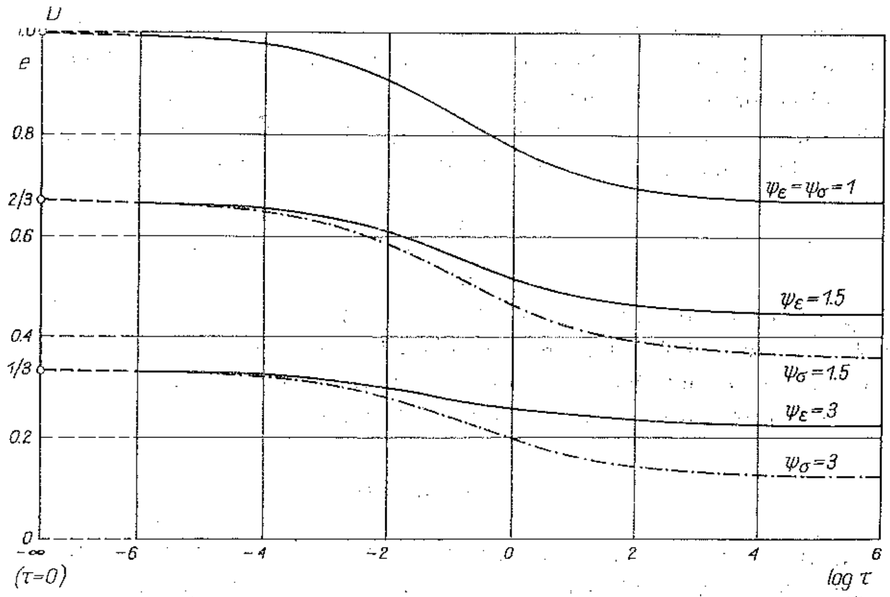

FIG. 3. Processes with constant safety factors (strains versus time).

The results of numerical integration are shown in Figs. 2 and 3 for $\psi_\sigma=1$, 1.5, and 3, $\psi_\varepsilon=1$, 1.5, and 3. For the processes with $\psi_\sigma>1$ or $\psi_\varepsilon>1$ the critical time is, of course, infinitely large.

6. Influence of the loading history on the critical time

The loading history, for example described by $s=s(\tau)$, has an essential influence on the critical time even if final stress s^* is assumed the same. To show the differences we compare the critical times for four various loading programs: $s=\text{const}=s^*$, $s=C_1\tau$, $e=\text{const}=C_2$ and $e=C_3\tau$. The constants C_1, C_2 and C_3 were chosen in

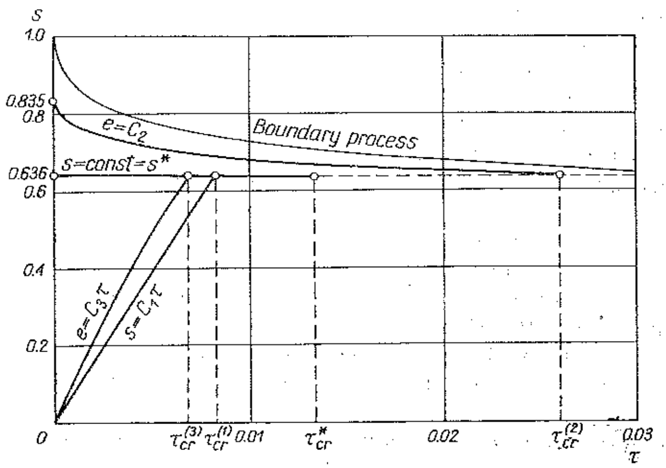

FIG. 4. Critical times for various loading histories and the same final stress.

such a way as to obtain the same stress, $s=s^*$, at the critical time. The results are shown in Fig. 4 for $s^*=0.636$ and the material constants as above; the boundary process (5.1) for $\psi_\sigma=1$ is shown as well.

The diagrams in Fig. 4 lead to interesting conclusions. One may suspect that for the loading varying in time, the critical time might depend on the stress impulse

$$S=\int_0^{\tau_{cr}} s(\tau)\,d\tau. \tag{6.1}$$

Such a statement is quite false, since the stress impulses for the processes shown in Fig. 4 are entirely different from each other. On the contrary, the stress rate $\dot{s}$ is much more important than the impulse: a negative stress rate has a stabilizing effect, and a positive stress rate — destabilizing one.

7. Final remarks

The general analysis, given in the paper, is based on the strain-hardening creep law (2.5). However, the results are very sensitive to the form of the creep law and any application of the theory must be preceded by careful examination of the actual behaviour of the material. Further, the considerations are based on the classical Rabotnov-Shesterikov assumption of the constancy of the derivatives λ^*, μ^* and ν^* during the perturbed motion; Shesterikov [14] proved that such an assumption may lead only to a certain underestimation of the critical time, so the obtained results remain on the safe side.

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Streszczenie

BADANIE STATECZNOŚCI PEŁZANIA RABOTNOWA-SZESTERIKOWA PRZY OGÓLNYCH PROGRAMACH OBCIĄŻENIA

Celem niniejszej pracy jest analiza stanu krytycznego pełzającego pręta, w ogólnym przypadku procesu obciążania, danego w naprężeniach lub odkształceniach. Przeanalizowano wyboczenie pełzające w przypadku relaksacji. Przytoczono również liczbowe wartości czasów krytycznych dla pewnych procesów obciążania. Wprowadzone zostało pojęcie procesu granicznego i procesu o stałym w czasie współczynniku bezpieczeństwa. W zakończeniu są przedstawione spostrzeżenia dotyczące wpływu historii obciążenia na czas krytyczny.

Резюме

ИССЛЕДОВАНИЕ УСТОЙЧИВОСТИ ПРИ ПОЛЗУЧЕСТИ РАБОТНОВА-ШЕСТЕРИКОВА ПРИ ОБЩИХ ПРОГРАММАХ НАГРУЖЕНИЯ

Целью настоящей работы является анализ критического состояния сжатого стержня в условиях ползучести, для общего процесса нагружения, заданного в напряжениях или деформациях. Рассматривается устойчивость в условиях релаксации. Приведены численные значения критического времени для некоторых процессов нагружения. Вводится понятие граничного процесса и процесса о постоянном во времени коэффициенте безопасности. В заключении поданы замечания касающиеся влияния истории нагружения на критическое время.

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