ROZPRAWY INŻYNIERSKIE • ENGINEERING TRANSACTIONS • 25, 3, 447—461, 1977
Polska Akademia Nauk • Instytut Podstawowych Problemów Techniki

INTERACTION OF SHEAR WALL WITH ELASTIC FOUNDATION UNDER THE EXCITATION OF SH WAVES

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The study of the interaction between soil and structure was started by J.E. Luco (1969) who treated an infinite shear wall on a rigid foundation of semi-circular cross section under the excitation of a plane harmonic SH wave. M.D. Trifunac (1972) found that the motion of this structure is independent of the angle of incidence. In this paper, an elastic foundation is considered. Furthermore, a rigid mass on top of the shear wall is added. However, the boundary conditions at the inteface between foundation and shear wall are satisfied only in average over the width of the latter. The numerical results are compared to those of the rigid foundation case. There are considerable differences.

1. Introduction

The interaction of a structure and ground motion can be conveniently analyzed by assuming that the foundation of the structure is connected to the ground by a system of springs and dashpots [1]. These springs and dashpots model the elasticity and damping properties of the soil. Although the elastic property of the soil can be estimated from static tests, the determination of the damping characteristics must be based on dynamic experiments and analysis.

There are two parts of the damping that should be considered: one is the material damping due to the plasticity and porosity of the soil, the other is the damping through radiation of energy. When incident ground waves are scattered by the foundation considerable amount of energy is carred away the scattered wave. For this wave a detailed analysis of wave motion in the soil and the foundation is required. A general discussion of the scattering of elastic waves in solids is given in the monograph by Pao and Mow [2].

The scattering wave theory was first applied to the interaction of foundation and ground motion by Luco [3]. In the original paper, he considered a semi-circular cylindrical foundation embedded in an elastic half-space. The cylinder is assumed to be infinitely rigid. A shear wall is installed on the cylinder, and the foundation is excited by an incident SH-wave (horizontally polarized shear wave). A closed form solution is obtained for the system of shear wall, the rigid foundation and the elastic soil, from which the foundation motion and the base shear force are calculated. Luco's numerical result shows that the value of the specific damping coefficient is about 0.46 which means that the radiation damping is significantly larger than the ordinary material damping.

Luco [3] considered only the case of normal incidence where the direction of wave propagation is perpendicular to the plane surface. The case of an arbitrary angle of incidence was considered later by TRIFUNAC [4]. However, the foundation displacement was found to be independent of the incident angle for a rigid semi-circular foundation. Only when the geometry of the foundation is changed to a half-elliptical cylinder, is the displacement affected by the direction of the incident angle [5]. This raised the question on whether a rigid foundation, which leads to a considerable scattering of the waves away from the foundation, is a realistic model.

In this paper we replace the rigid foundation by a semi-circular elastic cylinder. In addition, we consider a mass added on top of the shear wall because the total mass above the foundation has a considerable influence on the motion of the latter. Introducing an elastic foundation prevents us from finding a closed form solution which satisfies exactly the boundary conditions at the interface between the shear wall and the elastic foundation. This exact condition that the traction and displacement should be continuous across the interface degenerates to some simple from when the foundation is assumed to be rigid. However, through the addition of a concentrated shear force at the base of the shear wall, we are able to find a closed form solution which satisfies the conditions of continuity of the tractions and displacements averaged over the width of the shear wall. Our results reduce to the ones in Ref. [3] when the shear modulus of the foundation approaches infinity.

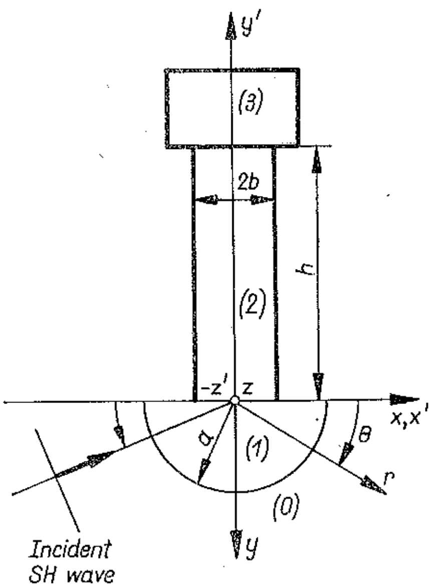


Fig. 1. Model of soil, elastic foundation and top mass.

2. THE MODEL OF ELASTIC FOUNDATION AND SHEAR WALL

Consider a system of soil, foundation, shear wall and a top weight as depicted in Fig. 1. The soil is represented by the elastic half-space with constant mass density ρ_0 and shear modulus μ_0. A semi-circular elastic cylinder of infinite lenght (in the direction of the z-axis) models the foundation. A circular-shaped foundation is assumed for the ease of analysis of wave motion. The foundation and shear wall have mass density ρ_1 and ρ_2, and shear modulus μ_1 and μ_2, respectively. The radius of the foundation is a; the thickness and height of the shear wall are $2b$ and h respectively, with $b<a$. The top weight is a rigid body with a total mass per unit length m_3. All physical quantities pertaining to the soil are identified by the index 0, those to the foundation and shear wall by 1 and 2 respectively, and those to the top weight by the index 3.

For convenience, we consider the soil-foundation, and the shear wall-concentrated mass as two subsystems. The motions of these two subsystems are coupled through a hitherto unspecified base shear, S, which is a concentrated force acting at the center of the semi-circular foundation. We shall discuss the interaction of the two subsystems in three parts, first the motion of the elastic foundation under the excitation of an incident SH wave and the base shear, next the motion of the shear wall with the added mass and the base shear, and finally the coupled motion of the entire system.

Only the steady state motion for the entire system is considered, and a time factor $\exp(-i\omega t)$ where ω is the circular frequency is assumed for all responses. The shear wave speed in each medium is given by $c=(\mu/\rho)^{1/2}$. For harmonic waves, $ck=\omega$ and $(2\pi/k)$ is the wave length. With an incident SH wave and a shear wall, the motion of the entire system is an anti-plane strain with displacements $(0, 0, w)$ along the (x, y, z) coordinate axes where w is independent of the z coordinate.

3. Waves in the half-space and foundation

Let $w^{(0)}(r, \theta; \omega)$ and $w^{(1)}(r, \theta, \omega)$ be the displacement amplitudes of the wave motion in the half-space and the foundation, respectively. The time factor $\exp(-i\omega t)$ is assumed for all responses and it is omitted in writing. Both displacements satisfy the Helmholtz equation

$$\nabla^2 w+k^2 w=\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}+k^2 w=0. \tag{3.1}$$

The shearing stresses in both media are given by

$$\sigma_{rz}=\mu\frac{\partial w}{\partial r}, \qquad \sigma_{\theta z}=\mu\frac{1}{r}\frac{\partial w}{\partial \theta}. \tag{3.2}$$

For the soil we should attach the superscript (0) to w and σ, and the subscript 0 to k and μ in the previous equations. Similarly, the index 1 should be attached if the equations are for the foundation.

For an obliquely incident SH wave at an angle γ with the free surface, the ground motion of the combined incident wave $w^{(i)}$ and the reflected wave $w^{(r)}$ can be represented by

$$w^{(i)}+w^{(r)}=w_0\, e^{ik_0(x\cos\gamma-y\sin\gamma)}+w_0\, e^{ik_0(x\cos\gamma+y\sin\gamma)}= \tag{3.3}$$

$$=2w_0\sum_{n=0}^{\infty}\varepsilon_n i^n J_n(k_0 r)\cos n\gamma\cos n\theta,$$

where J_n denotes the Bessel function of the first kind and $\varepsilon_n=1$ when $n=0$, and $\varepsilon_n=2$ when $n\geq 1$ (p. 116, Ref. 2). The maximum amplitude in the free half-space is $2w_0$.

The wave scattered by the foundation is

$$w^{(s)}=2w_0\sum_{n=0}^{\infty}A_n H_n(k_0 r)\cos n\theta \tag{3.4}$$

where H_n is the Bessel function of the third kind (Hankel function of the first kind) and A_n are unknown coefficients. Both Eqs. (3.3) and (3.4) satisfy the Helmholtz equation.

The resultant ground motion, $w^{(0)}=w^{(i)}+w^{(r)}+w^{(s)}$, is

$$w^{(0)}(r,\theta)=2w_0\sum_{n=0}^{\infty}[\varepsilon_n i^n J_n(k_0 r)\cos n\gamma+A_n H_n(k_0 r)]\cos n\theta. \tag{3.5}$$

The corresponding shearing stresses can easily be calculated from Eqs. (3.2) with $\mu=\mu_0$.

The waves inside the elastic foundation are expressed by

$$w^{(1)}(r,\theta)=(S/2\mu_1)\,Y_0(k_1 r)-2w_0\sum_{n=0}^{\infty}C_n J_n(k_1 r)\cos n\theta. \tag{3.6}$$

The corresponding stresses are

$$\sigma_{\theta z}{}^{(1)}=2\mu_1(w_0/r)\sum_{n=1}^{\infty}C_n n J_n(k_1 r)\sin n\theta, \tag{3.7}$$

$$\sigma_{rz}{}^{(1)}=(k_1 S/2)\,Y_0'(k_1 r)-2\mu_1 k_1 w_0\sum_{n=0}^{\infty}C_n J_n'(k_1 r)\cos n\theta, \tag{3.8}$$

where a prime indicates differentiating with respect to the argument. In the previous equations the series with the unknown coefficients C_n is the standard representation of standing waves, and the first term is added to account for the additional waves generated by the base shear S (force per unit length) at the location $r=0$. As a result of the concentrated force S, $w^{(1)}$ is singular at $r=0$. This is the reason for choosing the Bessel function of the second kind, Y_0, which is singular at $r=0$. By applying the asymptotic formula for $k_1 r\to 0$, $Y_0'(k_1 r)\to 2/(\pi k_1 r)$, we can show from Eq. (3.8) that

$$\lim_{r\to 0}\int_0^{\pi}\sigma_{rz}{}^{(1)}\,r\,d\theta=S. \tag{3.9}$$

4. Motion of the shear wall

The motion of the shear wall is usually assumed to be a function of the height (y' — coordinate) and time, which satisfies the one-dimensional wave equation

$$\rho(y')\frac{\partial^2 W(y',t)}{\partial t^2}=\mu(y')\frac{\partial^2 W(y',t)}{\partial y'^2}. \tag{4.1}$$

Let

$$W(y',t)=w^{(2)}(y')\exp(-i\omega t), \tag{4.2}$$
$$\rho(y')=(m_3/2b)\,\delta(y'-h)+\rho_2,\qquad \mu(y')=\mu_2.$$

We then obtain from Eq. (4.1) an equation for the amplitude $w^{(2)}(y')$ of the shear wall motion (in the direction of the z'-axis),

$$\frac{d^2 w^{(2)}}{dy'^2} + k_2^2 [1 + m_{32} h \delta(y'-h)] w^{(2)} = 0, \tag{4.3}$$

where $k_2 = \omega/c_2$ with $c_2^2 = \mu_2/\rho_2$, and $m_{32} = m_3/m_2$ with $m_2 = 2bh\rho_2$.

In the previous equations the delta function $\delta(y'-h)$ is introduced to account for the inertia of the concentrated mass m_3 at the top of the wall, and ρ_2 and μ_2 are constant. The boundary conditions for Eq. (4.3) are

$$2b\mu_2 (dw^{(2)}/dy') = 0 \quad \text{at } y' = h^+, \tag{4.4}$$

$$2b\mu_2 (dw^{(2)}/dy') = S \quad \text{at } y' = 0. \tag{4.5}$$

The location $y' = h^+$ means that y' is slightly larger than h, which is just above the concentrated mass m_3. Equation (4.4) means that the total shear force above m_3 vanishes; Eqs. (4.5) equates the total shear force in the wall at $y' = 0$ to the unspecified base shear S. Viewing toward the bottom face of the shear wall, S is positive when it acts along the $-z'$-axis (Fig. 1).

The solution for the equation of motion, Eq. (4.3) and the boundary condition equations (4.4) and (4.5) can be found by applying the Laplace transform [6]. The answer is

$$w^{(2)}(y) = \frac{S}{2b\mu_2 k_2} [\sin k_2 y + \Gamma(k_2 h) \cos k_2 y], \tag{4.6}$$

where

$$\Gamma(k_2 h) = \frac{-\cos k_2 h + m_{32} k_2 h \sin k_2 h}{\sin k_2 h + m_{32} k_2 h \cos k_2 h}, \tag{4.6'}$$

The shearing stress in the wall is

$$\sigma_{yz}^{(2)}(y) = \frac{S}{2b} [\cos k_2 y - \Gamma(k_2 h) \sin k_2 y]. \tag{4.7}$$

Note that if the shear wall is fixed at the location $y = 0$, we have $w^{(2)}(0) = 0$. This additional boundary condition is satisfied by $\Gamma(k_2 h) = 0$ or

$$k_2 h \tan k_2 h = 1/m_{32}. \tag{4.8}$$

This is the frequency equation ($\omega = c_2 k_2$) for the shear wall with a top mass m_3 in free vibration.

5. Boundary conditions and interaction of subsystems

So far we have found the general solutions for each and every subsystem with $w^{(0)}$ (Eq. (3.5)) for the soil, $w^{(1)}$ (Eq. (3.6)) for the foundation, and $w^{(2)}$ (Eq. (4.6)) for the shear wall. These equations contain the unknown coefficients A_n and C_n ($n = 0, 1, 2, \ldots$), and the unknown base shear S. These unknown constants are determined by the boundary conditions at the soil-foundation interface $r=a$, at the free surface $y=0$ and $|x|<b$, and at the base of the shear wall $y=0$ and $|x|\leqslant b$.

For the half-space (soil), the condition at the free surface is that $\sigma_{zy}^{(0)}=0$ at $y=0$. In polar coordinates it is

$$\sigma_{\theta z}^{(0)}=0 \quad \text{at} \quad \theta=0, \quad \pi \text{ and } r\geqslant a. \tag{5.1}$$

At the interface $r=a$ we have the continuity conditions

$$\sigma_{rz}^{(0)}=\sigma_{rz}^{(1)}, \quad \text{at } r=a, \quad 0\leqslant\theta\leqslant\pi, \tag{5.2}$$

$$w^{(0)}=w^{(1)}. \tag{5.3}$$

For the waves inside the foundation, in addition to the continuity conditions, Eqs. (5.2) and (5.3), some boundary conditions should be prescribed for $\sigma_{zy}^{(1)}$ and $w^{(1)}$ at $y=0$. In a more realistic case when the shear wall is connected elastically to the foundation in the region $-b<x<b$, the conditions are

$$\sigma_{zy}^{(1)}=0 \qquad \text{at } y=0,\ a<|x|<b, \tag{5.4}$$

$$\sigma_{zy}^{(1)}=\sigma_{zy}^{(2)} \quad \text{and} \quad w^{(1)}=w^{(2)} \qquad \text{at } y=0,\ |x|<b. \tag{5.5}$$

However, the conditions in Eq. (5.5) can not possibly be satisfied unless $w^{(2)}$ and $\sigma_{zy}^{(0)}$ are functions of both x and y coordinates. We have not assumed this because it is very difficult to find the two-dimensional solution for the motion of a shear wall. For the displacement functions given by Eqs. (3.6) and (4.6), we replace Eq. (5.5) by a single condition at $y=0$ and $|x|<b$:

$$\frac{1}{2b}\int_0^b [w^{(1)}(r,0)+w^{(1)}(r,\pi)]\,dr=w^{(2)}(0). \tag{5.6}$$

This condition equates the averaged value of the foundation displacement at the upper surface to the base motion of the shear wall. Since the condition of continuity of stresses across the same surface has already been satisfied approximately by assuming the common base shear in Eqs. (3.6) and (4.5), the boundary conditions, Eqs. (5.1)–(5.4), plus the approximate boundary condition equation (5.6) complete the specification of the interactions among the two subsystems.

Returning now to the general solutions, we find that the condition in Eq. (5.1) is identically satisfied by $w^{(0)}$ in Eq. (3.5); Eq. (5.4) is satisfied by $w^{(1)}$ in Eq. (3.7). Substituting Eqs. (3.5) and (3.6) and the corresponding expressions for σ_{rz} in Eqs. (5.2) and (5.3), we find that both conditions are satisfied by the following results:

For $n=1, 2, 3\ldots$

$$A_n=2i^n \cos n\gamma\,[J_n'(k_0 a)J_n(k_1 a)-\zeta_{10}J_n(k_0 a)J_n'(k_1 a)]/\Delta_n, \tag{5.7}$$

$$C_n=2i^n \cos n\gamma\,[J_n(k_0 a)H_n'(k_0 a)-H_n(k_0 a)J_n'(k_0 a)]/\Delta_n, \tag{5.8}$$

where

$$\zeta_{10}=\mu_1 k_1/(\mu_0 k_0)=\rho_1 c_1/(\rho_0 c_0)$$

and

$$\Delta_n = \zeta_{10} H_n(k_0 a) J_n'(k_1 a) - H_n'(k_0 a) J_n(k_1 a). \tag{5.9}$$

For $n=0$, with $H_0'(z) = -H_1(z)$,

$$A_0 H_0(k_0 a) + C_0 J_0(k_1 a) - (S/4\mu_1 w_0) Y_0(k_1 a) + J_0(k_0 a) = 0, \tag{5.10}$$

$$A_0 H_1(k_0 a) + \zeta_{10} C_0 J_1(k_1 a) - \zeta_{10}(S/4\mu_1 w_0) Y_1(k_1 a) + J_1(k_0 a) = 0. \tag{5.11}$$

The solutions in Eqs. (5.7) and (5.8) are the same as those for the elastic foundation without a superstructure [7]. Equations (5.10) and (5.11) contain three unknowns A_0, C_0 and S; and additional equation is provided by the last boundary condition Eqs. (5.6). Substituting Eqs. (3.6) and (4.6) in Eq. (5.6), we find

$$\sum_{n=1}^{\infty} C_{2n} \bar{J}_{2n} + C_0 \bar{J}_0 - \frac{S}{4w_0 \mu_2} \mu_{21} \bar{Y}_0 = \frac{-S}{2w_0 \mu_2} \cdot \frac{\Gamma(k_2 h)}{2k_2 b}, \tag{5.12}$$

where $\mu_{21} = \mu_2/\mu_1$ and

$$\bar{J}_0 = \frac{1}{b}\int_0^b J_0(k_1 r)\, dr = J_0(k_1 b) + \frac{\pi}{2}(J_1(k_1 b)\mathbf{H}_0(k_1 b) - J_0(k_1 b)\mathbf{H}_1(k_1 b)],$$

$$\bar{Y}_0 = \frac{1}{b}\int_0^b Y_0(k_1 r)\, dr = Y_0(k_1 b) + \frac{\pi}{2}[Y_1(k_1 b)\mathbf{H}_0(k_1 b) - Y_0(k_1 b)\mathbf{H}_1(k_1 b)], \tag{5.13}$$

$$\bar{J}_{2n} = \frac{1}{b}\int_0^b J_{2n}(k_1 r)\, dr = \frac{2}{k_1 b}\sum_{m=0}^{\infty} J_{n+2m+1}(k_1 b).$$

In the above, $\mathbf{H}_0$ and $\mathbf{H}_1$ are Struve functions [8].

In Eq. (5.12) all the integrals of Bessel functions $\bar{J}_0$, $\bar{Y}_0$ and $\bar{J}_{2n}$ can be evaluated numerically or calculated with the aid of tables (p. 493, Ref. /8/); the C_{2n} ($n \geq 1$) are given by Eq. (5.8). Hence Eq. (5.12) relates the two unknowns C_0 and $S/(2w_0\mu_2)$. Together with Eqs. (5.10) and (5.11) it yields the following solutions:

$$A_0 = [-\zeta_{10} Q_2 Q_3 + Q_1 Q_4]/D_1, \tag{5.14}$$

$$C_0 = [-H_0(k_0 a) Q_4 - H_1(k_0 a) Q_3]/D_1, \tag{5.15}$$

$$S/(2w_0 \mu_2) = -(C_0 \bar{J}_0 + \bar{K})/D_2, \tag{5.16}$$

where

$$\begin{aligned} D_1 &= \zeta_{10} Q_2 H_0(k_0 a) + Q_1 H_1(k_0 a), \\ D_2 &= \Gamma(k_2 h)/(2k_2 b) - \mu_{21} \bar{Y}_0/2, \\ \bar{K} &= \sum_{n=1}^{\infty} C_{2n} \bar{J}_{2n}, \\ Q_1 &= J_0(k_1 a) + \mu_{21} Y_0(k_1 a) \bar{J}_0/2D_2, \end{aligned} \tag{5.17}$$

$$Q_2 = -J_1(k_1 a) - \mu_{21} Y_1(k_1 a)\bar{J}_0/2D_2,$$

(5.18) [cont.]
$$Q_3 = J_0(k_0 a) + \mu_{21} Y_0(k_1 a)\bar{K}/2D_2,$$

$$Q_4 = -J_1(k_0 a) - \zeta_{10}\mu_{21} Y_1(k_1 a)\bar{K}/2D_2.$$

Substitution of Eqs. (5.7), (5.8), (5.14), (5.15) and (5.16) into Eqs. (3.5), (3.6) and (4.6) completes the answer.

6. Limiting case of a rigid foundation

If the foundation material is rigid, we set $\mu_1 \to \infty$ but keep ρ_1 finite. Since k_1 is of the order $1/\sqrt{\mu_1}$, it approaches zero as $\mu_1 \to \infty$.

For the small argument $z \to 0$, the limiting values of the Bessel functions in Eqs. (5.7)–(5.12) are given by the following formulae:

$$J_0(z) \to 1, \qquad Y_0(z) \to (2/\pi)\ln z,$$

$$J_n(z) \to \frac{1}{n!}\left(\frac{z}{2}\right)^n, \qquad Y_n(z) \to -\frac{1}{\pi}\left(\frac{2}{z}\right)^n (n-1)!.$$

Thus for $n = 1, 2, \ldots$, and when $k_1 a \to 0$, Eqs. (5.7) and (5.8) reduce to

(6.1)
$$A_n \to -2i^n \cos n\gamma J_n(k_0 a)/H_n(k_0 a),$$

(6.2)
$$C_n \to 0.$$

In Eq. (5.18), $\bar{J}_0 \to 1$, $\bar{J}_{2n} \to 0$, $\bar{K} \to 0$, and $\mu_{21}\bar{Y}_0 \to 0$ as $k_1 a \to 0$. Hence we find

(6.3)
$$Q_1 \to 1, \; Q_2 \to 0, \; Q_3 \to J_0(k_0 a), \qquad Q_4 \to -J_1(k_0 a),$$

(6.4)
$$D_2 \to \Gamma(k_2 h)/(2bk_2),$$

(6.5)
$$D_1 \to \left[-\frac{1}{2}k_0 a\rho_{10} + \frac{2b}{\pi a}\frac{\zeta_{20}}{\Gamma(k_2 h)}\right] H_0(k_0 a) + H_1(k_0 a).$$

In the calculation of limiting values for $\zeta_{10} Q_2$ in D_1, one should note that although $Q_2 \to 0$, the product $\zeta_{10} Q_2$ does not vanish. The result is

(6.6)
$$\zeta_{10} Q_2 \to -\frac{1}{2}k_0 a p_{10} + \frac{2b}{\pi a}\frac{\zeta_{20}}{\Gamma(k_2 h)}.$$

In Eq. (6.5) we introduced the notation $\rho_{10} = \rho_1/\rho_0$ and $\zeta_{20} = k_2\mu_2/(k_0\mu_0)$.

Substituting the above limiting values into Eqs. (5.14) and (5.15), we obtain

(6.7)
$$A_0 \to \left\{\left[\frac{1}{2}k_0 a p_{10} - \frac{2b}{\pi a}\frac{\zeta_{20}}{\Gamma(k_2 h)}\right] J_0(k_0 a) - J_1(k_0 a)\right\}/D_1,$$

(6.8)
$$C_0 \to 2i/(\pi k_0 a D_1),$$

where D_1 is given in Eq. (6.5). From Eq. (5.17) we find

(6.9)
$$\frac{S}{2w_0\mu_2} \to -C_0\frac{2bk_2}{\Gamma(k_2 h)}.$$

With the limiting values given by Eqs. (6.2) and (6.9), we obtain from Eqs. (3.6) and (4.6)

$$w^{(1)}(r, 0) \to -2w_0 C_0, \tag{6.10}$$

$$w^{(2)}(0) = \frac{S}{2b\mu_2 k_2} \Gamma(k_2 h) \to -2w_0 C_0. \tag{6.11}$$

Thus the displacements at both sides of the surface $y=0$ are equal, in the limit of a rigid foundation. For the case of an elastic foundation, only the *averaged value* of the foundation displacement equals $w^{(2)}(0)$.

When the top mass m_3 is removed, the $\Gamma(k_2 h)$ function in Eq. (4.6′) reduces to ($m_{32} \to 0$)

$$\Gamma(k_2 h) \to -\cot k_2 h. \tag{6.12}$$

After the substitution of Eq. (6.12) in Eq. (6.5), and then in Eq. (6.8), it can be shown that the movement of the rigid foundation, $w^{(1)}$ in Eq. (6.10), agrees with that derived by LUCO and by TRIFUNAC (Eq. 24 of Ref. 4).

7. NUMERICAL RESULTS

For rigid foundations and in the absence of a top mass, extensive numerical results were given in References 3 and 4. In this paper we shall concentrate on the effect of the elasticity of the foundation and the added top mass.

The important physical parameters of the entire system are the wave speed ratios $c_{10} = c_1/c_0$ and $c_{20} = c_2/c_0$, density ratios $\rho_{10} = \rho_1/\rho_0$ and $\rho_{20} = \rho_2/\rho_0$, mass ratio $m_{32} = m_3/m_2$ where $m_2 = 2bh\rho_2$, and length ratios h/a and b/a. The product ρc is often called the impedance of the material, thus $\zeta_{10} = \rho_1 c_1/(\rho_0 c_0)$ and $\zeta_{20} = \rho_2 c_2/ /(\rho_0 c_0)$ are the impedance ratios. The important parameters of the incident wave are its dimensionless wave number $k_0 a = \omega a/c_0$ and its angle of emergence, γ.

For comparison with the results known for a rigid foundation, we calculate the motion of the shear-wall footing based on Eq. (4.6) and the base shear force from Eq. (5.16). In addition, we calculate the motion of the added mass at the top of the shear wall also from Eq. (4.6). For convenience these formulae are repeated here:

$$\bar{w}_2 \equiv \frac{w^{(2)}(0)}{2w_0} = \frac{S}{2w_0 \mu_2} \cdot \frac{\Gamma(k_2 h)}{2bk_2}, \tag{7.1}$$

$$\bar{S} \equiv \frac{S}{2w_0 \mu_2} = -\frac{C_0 \bar{J}_0 + \bar{K}}{\Gamma(k_2 h)/(2bk_2) - \mu_{21} \bar{Y}_0/2}, \tag{7.2}$$

$$\bar{w}_3 \equiv \frac{w^{(2)}(-h)}{2w_0} = \frac{S}{2w_0 \mu_2} \cdot \frac{1}{2bk_2} [-\sin k_2 h + \Gamma(k_2 h) \cos k_2 h], \tag{7.3}$$

$$\Gamma(k_2 h) = \frac{-\cos k_2 h + m_{32} k_2 h \sin k_2 h}{\sin k_2 h + m_{32} k_2 h \cos k_2 h}. \tag{7.4}$$

In the previous equations the coefficient C_0 is given in Eq. (5.15), $\bar{J}_0$ and $\bar{Y}_0$ in Eq. (5.13) and $\bar{K}$ in Eq. (5.16). All displacements are normalized by the free field wave amplitude $2w_0$, and the base shear is made dimensionless by the factor $2w_0\,\mu_2$.

The effect of the top mass m_3 is characterized entirely by the function $\Gamma(k_2\,h)$. Elasticity of the foundation is contained in $\bar{J}_0$, $\bar{Y}_0$, $\bar{K}$, and C_0. The direction of the incident wave, angle γ in Figure 1, only affects C_0 and $\bar{K}$. The higher order coefficients C_{2n} $(n>0)$, which are absent in the case of a rigid foundation, are all contained in $\bar{K}$.

First we present the results for various angles of emergence. Figure 2 shows the base motion $|\bar{w}_2|$ of the shear wall on an elastic foundation with out top mass $\gamma=0°$ (reference curve, repeated in Figs. 4,5, and 6) and $\gamma=90°$ as a function of $k_0\,a$, the other parameters being $c_{10}=1.5$, $c_{20}=1.5$, $\rho_{10}=1$, $\rho_{20}=1$, $m_{32}=0$, $h/a=4$, $b/a=0.5$. Since $\bar{w}$ is complex valued, which means that the motion of the wall is out of phase with the incident wave, the absolute values are shown in Fig. 2, and the phase angle for $\gamma=0°$ in Fig. 3. At higher frequencies the response is quite dependent on the angle of incidence. The zeros of $|\bar{w}_2|$ coincide with the roots of $\Gamma(k_2\,h)=0$ (Eq. (4.4) with $m_{32}=0$). At these frequencies the super structure acts like a dynamic shock absorber for the foundation. The locations of these zeros are independent of the elasticity of the foundation.

In Fig. 4 the motion $|\bar{w}_2|$ of a rigid foundation is presented $(1/c_{10}=0)$. All the other parameters agree with those of the reference curve. The striking features are the smaller first peak and the shift of the other extremes to smaller frequencies.

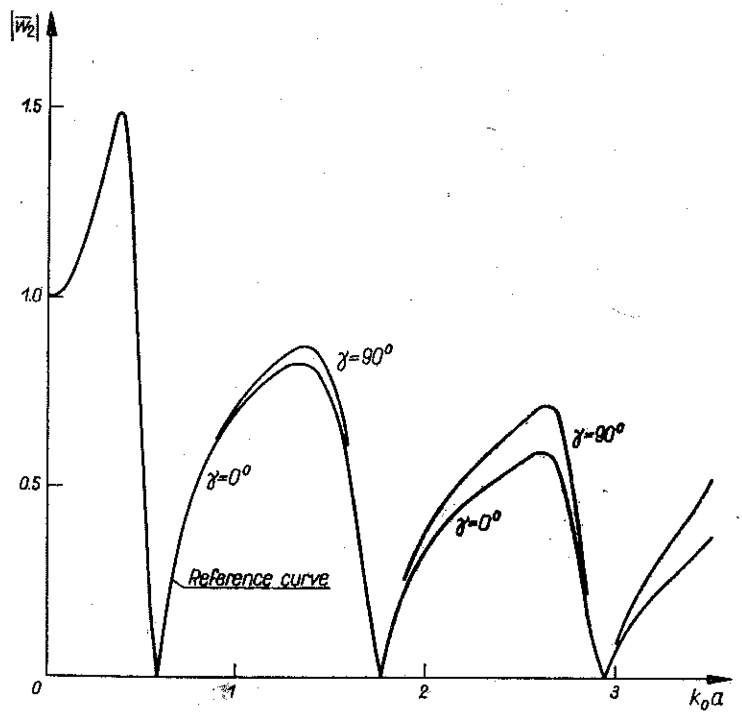


Fig. 2.

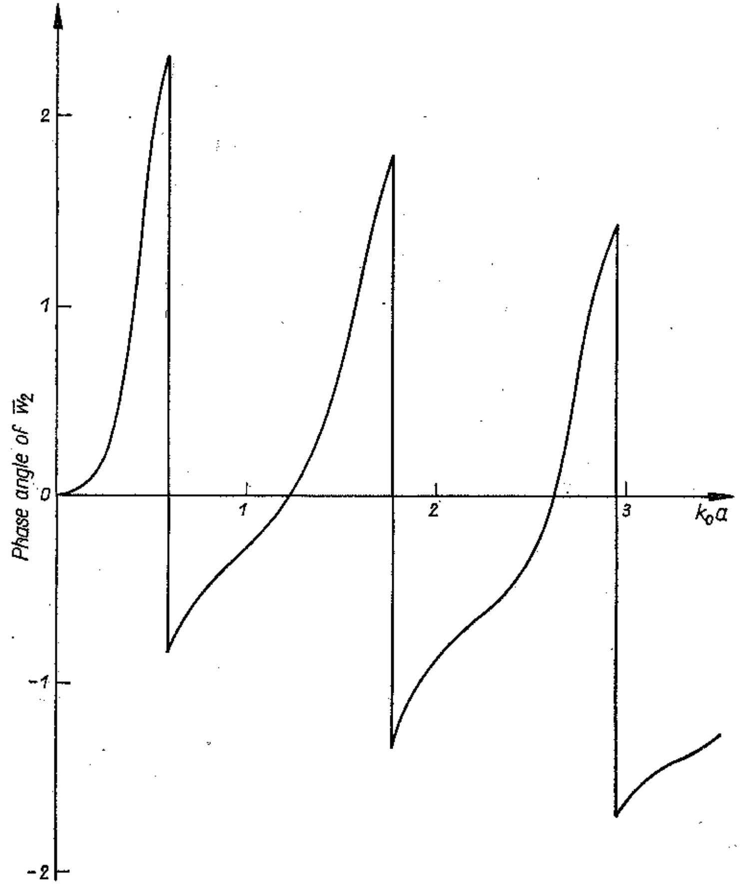

Phase angle of $\bar{w}_2$
2
1
0
-1
-2
1
2
3
k_0a

Fig. 3.

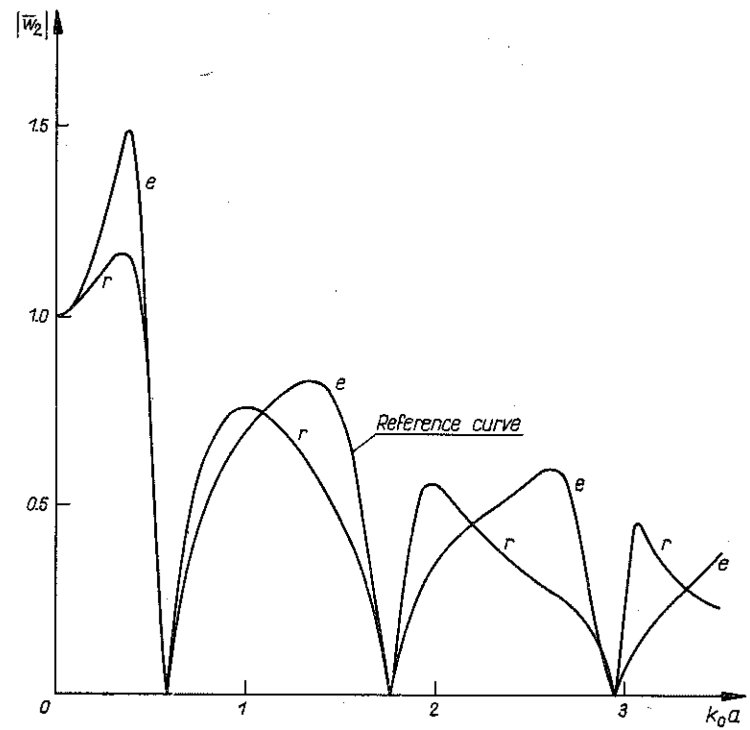
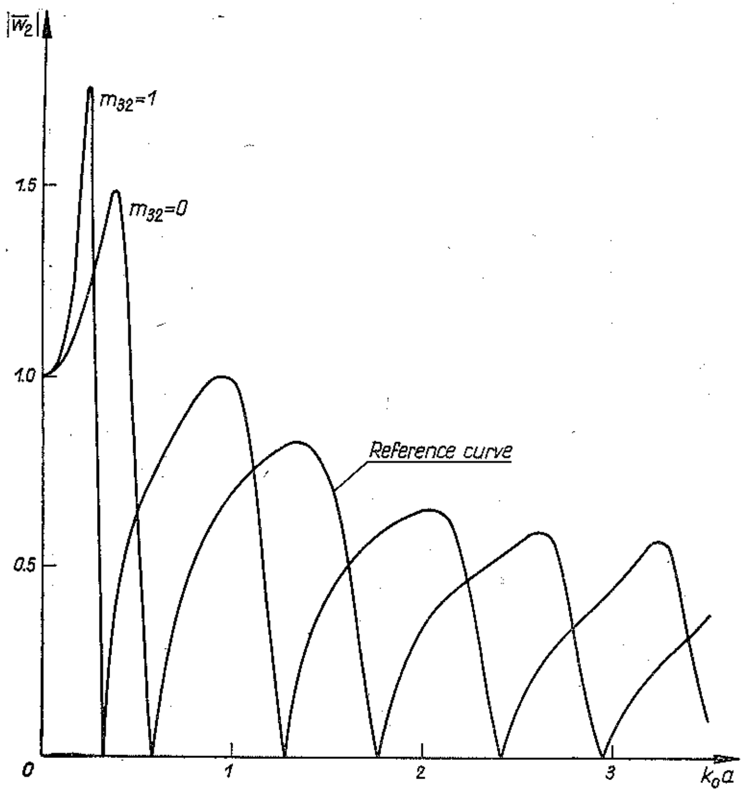
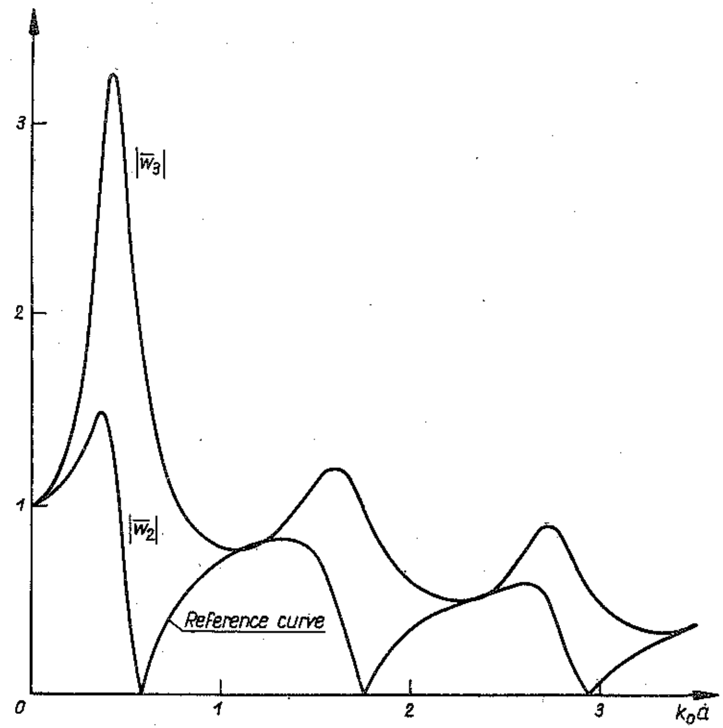

$|\bar{w}_2|$
1.5
1.0
0.5
0
1
2
3
k_0a
e
r
Reference curve

Fig .4.



$|\overline{w}_2|$
1.5
1.0
0.5
0
1
2
3
k_0a
$m_{32}=1$
$m_{32}=0$
Reference curve

Fig. 5.



3
2
1
0
1
2
3
k_0a
$|\overline{w}_3|$
$|\overline{w}_2|$
Reference curve

Fig. 6.

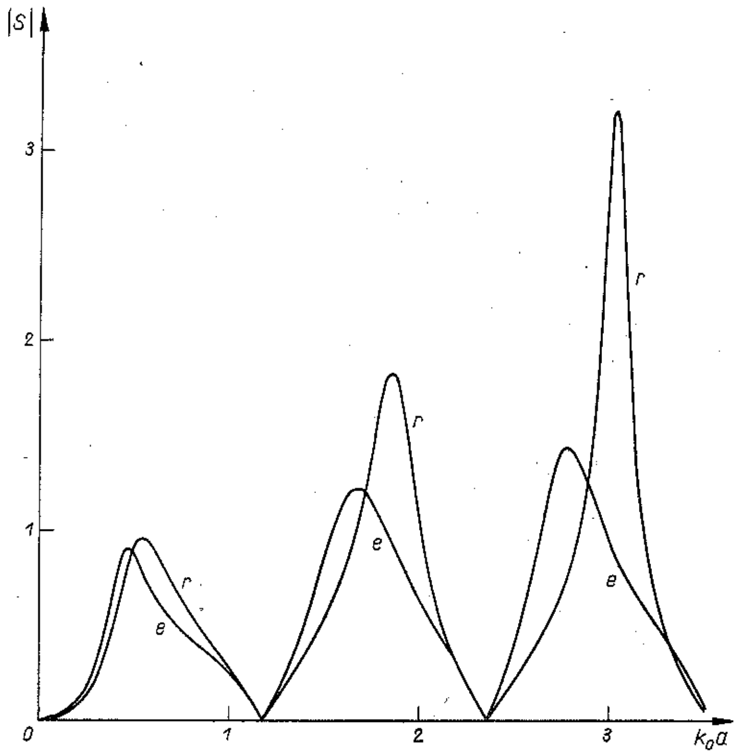


Fig. 7.

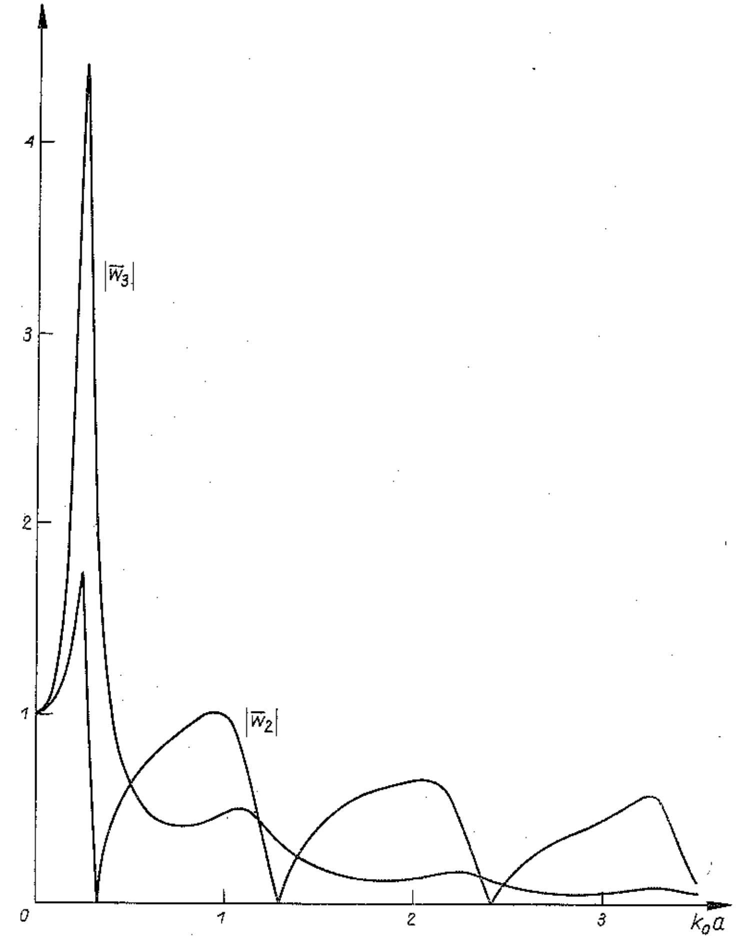


Fig. 8.

Next, we consider the effect of the concentrated mass on $|\bar{w}_2|$ as shown in Fig. 5. $m_{32}=1$, and the other parameters are the ones of the reference curve. Because of the added mass the resonance frequencies of the super structure are changed. This is evidenced by the shifting of the zeros along the $k_0\, a$ axis for different m_{32} ratios. The top mass raises the peak value for $|\bar{w}_2|$ considerably at low frequencies, but it has less influence at high frequencies.

Figure 6 shows the amplitude of the motion of the top, $|\bar{w}_3|$, and the base, $|\bar{w}_2|$ (reference curve), of the shear wall on an elastic foundation without top mass. The amplitude of the top is everywhere larger than or equal to the amplitude of the base. The points of osculation exist only in the absence of a top mass. The corresponding frequencies are determined by

$$\cos k_2 h = \pm 1 . \tag{7.5}$$

There is no force acting on the shear wall (cf. Fig. 8).

The amplitude of the top mass ($m_{32}=1$) is shown in Fig. 7. There is only one important peak but rather small displacements for higher frequencies. For better comparison the motion of the foot of the shear wall is repeated from Fig. 5 in a suitable scale.

We are interested also in the net shear force acting between the foundation and the foot of the shear wall. Figure 8 gives $|\bar{S}|$ as a function of $k_0\, a$ for the shear wall without top mass on an elastic foundation (parameters of the reference curve) and on a rigid foundation ($1/c_{10}=0$). At certain frequencies the time rate of change of momentum, and therefore the force, vanishes. They follow from $1/\Gamma(k_2\, h)=0$ or

$$k_2\, h \cot k_2\, h = -1/m_{32} . \tag{7.6}$$

Much higher peaks occur in the rigid footing case.

The novel feature in the present investigation is the elasticity of the foundation. As can be seen from the graphs, the influence of the angle of emergence is present but not essential unless rather high frequencies are considered. The amplification of the first peak of $|\bar{w}_2|$, and the shift of the other peaks to higher frequencies are important. The influence of the elasticity of the foundation on the shear force is also interesting. It is much smaller than the shear force acting between the shear wall and a rigid foundation.

The added mass on top of the shear wall results in a shift of the frequencies at which the motion of the foot of the shear wall vanishes and also in a shift of the frequenceis at which the shear force vanishes; however this effect exists, of course, also in the rigid footing case.

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Streszczenie

ODDZIAŁYWANIE POMIĘDZY ŚCIANAMI A PODŁOŻEM SPRĘŻYSTYM PRZY ZABURZENIACH FALAMI TYPU SH

Badania wzajemnego wpływu podłoża na konstrukcję zapoczątkowane zostało w pracy J. E. Luco (1969), w której rozważana była nieskończona ściana spoczywająca na sztywnym fundamencie o półokrągłym przekroju poddana działaniu płaskich fal harmonicznych typu SH. M.D. Trifunac (1972) udowodnił następnie że ruch takiej konstrukcji jest niezależny od kąta padania fali. W niniejszej pracy rozważane jest podłoże sprężyste oraz dodatkowa sztywna masa spoczywająca na ściance. Warunki brzegowe na powierzchni kontaktu podłoża ze ścianką spełnione są jedynie w sposób uśredniony po szerokości ścianki. Porównanie wyników numerycznych uzyskanych dla sprężystego i sztywnego podłoża wskazuje na znaczne różnice pomiędzy tymi rozwiązaniami.

Резюме

ВЗАИМОДЕЙСТВИЕ МЕЖДУ СТЕНКАМИ И УПРУГИМ ОСНОВАНИЕМ ПРИ ВОЗМУЩЕНИЯХ ВОЛНАМИ ТИПА SH

Исследования взаимного влияния основания на конструкции начались изучаться в работе Дж. Э. Луко (1969), в которой рассматривалась бесконечная стенка, находящаяся на жестком фундаменте с полукриглым сечением, подвергнута действию плоских гармонических волн типа SH. Н.Д. Трифунац (1972) доказал затем, что движение такой конструкции не зависит от угла падения волны. В настоящей работе рассматривается упругое основание и дополнительная жесткая масса находящаяся на стенке. Граничные условия на поверхности контакта основания со стенкой удовлетрвоения только усредненным образом по ширине стенки. Сравнение численных результатов, полученных для упругого и жесткого оснований указывает на значительные разницы между этими решениями.

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Received August 2, 1976.