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DYNAMICS OF THE COMPLEX SYSTEM WITH ELASTIC AND VISCO-ELASTIC INERTIAL INTERLAYERS

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In this paper is given the dynamic analysis of the free and forced vibration problems of a complex system with elastic and visco-elastic inertial interlayers. The analytical method of solving the free and forced vibrations problem of the system is presented in the paper [2]. The external layer of the complex system is treated as the plate made from elastic materials, coupled by visco-elastic inertial interlayers. The plate is described by the Kirchhoff–Love model. The visco-elastic, inertial interlayer possesses the characteristics of a continuous inertial Winkler foundation and has been described by the Voigt-Kelvin model. Small transverse displacements of the complex system are excited by the stationary and non-stationary dynamical loadings. The phenomenon of free and forced vibrations problems has been described using a non-homogeneous system of conjugate, partial differential equations. After separation of variables in the homogeneous system, the boundary value problem has been solved and two sequences have been obtained: the sequences of frequencies and the sequences of free vibrations modes. Then, the property of orthogonality of complex free vibrations has been presented. The free vibrations problem has been solved for some arbitrarily assumed initial conditions. The forced vibrations problem has been considered for different modes of dynamical loading [3].

The solution of the ecological safety problem and protection from exposure to dust, depend much on the equipment and techniques used in quarrying the brown coal. Thus, dynamics of loading the open cast colliery dump trucks which have a load-carrying capacity of hundreds of tons, mass of tens of tons and dimensions of tens of meters, is a very important problem. The numerical results of free and forced vibrations problems of the complex system with the elastic and visco-elastic inertial interlayer, for various parameters and different modes of dynamical loading, are given in this paper.

Key words: vibrations, two-layer system, damping, numerical results.

1. Introduction

The problems of vibrations with damping of complex structures play an important role in various engineering structures. Some mechanical and building constructional systems consisting of strings, beams, shafts, plates and shells, can be connected by elastic or visco-elastic constraints, working in complex conditions of stationary and non-stationary loading. In dynamics of various technical objects important influence on their character operation is exerted by unavoidable vibrations of certain structural elements.

A typical example of the above-mentioned constructional elements can be layers, which are made of soft elastic or visco-elastic materials. On this subject, the mathematical method was presented by Cabańska–Płaczkiewicz [3] taking into account not only stationary loading of complex systems using the methods based on the Kirchhoff–Love hypothesis [8], but also non-stationary loading of complex systems based on the Timoshenko model [26]. Among numerous precise models applied to the investigation of plates made of modern materials, the Reisner model [21] was used.

Wide bibliography concerning the classical, rheological models, were presented by Nashif, Jones, Henderson [12], Nowacki [14], Rymarz [23] and the operator methods were given by Osiowski [18].

In the paper by Jemielita [7], the criteria of choice of the shear coefficient in plates of medium thickness have been considered. Vibrations of elastic compound systems subjected to inertial moving load was presented by Bogacz [1], Oniszczuk [15, 16] using the Renaudot formula [22] and Szcześniak [24, 25].

The problem of non-axisymmetric deformation of flexible rotational shells was solved by Pankratova, Nikolaev, Świtoński [19] using the classical Kirchhoff-Love model and the improved Timoshenko model. The dynamic problem of elastic homogeneous bodies was presented by Taranto, Mc Graw [6], Kurnik, Tylikowski [10, 28], Mindlin, Schacknow [1], Pankratova, Mukoed [20] and Wang [29]. The interlayer is a one- or two-directional viscoelastic Winkler [30] layer, but it can also be a multiparametric viscoelastic layer presented by Woźniak [31].

In the above-mentioned complex cases, especially where viscosity and discrete elements occur, it is recommended to adopt the method of solving the dynamic problem of a system in the domain of functions of complex variable, following the papers by Tse, Morse, Hinkle [27], Nizioł, Snamina [13] and Cabańska–Płaczkiewicz [2–3]. The property of orthogonality of free vibrations of complex types was first described by Cremer, Heckel, Ungar [5] and Cabański [4] or discrete systems with damping, and for discrete – continuous systems with damping – by Nashif, Johnes and Henderson [12], and for continuous systems with damping – by Nowacki [14].

The aim of this paper is a dynamics analysis of a complex system with elastic and visco-elastic interlayers for various geometrical, physical and mechanical parameters, and for different modes of dynamical loading.

2. Problems of control of vibrations in ecologically-dangerous technical systems

2.1. Statement of the problem

Interdisciplinary range of problems connected with development and use of brown coal, is extremely urgent for Central and Western Europe. The practical significance of these problems is determined by many factors. One of them that should be noticed is that 58% of world productions of brown coal is concentrated in the given region. Life of lignite reserves is in excess of 240 years. Poland is one of the world leaders in this field and takes the 4-th place after Germany, Russia and USA. Therefore, great attention of engineering universities in Poland is given to various research aspects and directions within the given range of problems. A problem of ecological safety and protection of mine staff and inhabitants of the surrounding areas from exposure to dust, takes an important place among them.

For example, the capacity of the body of a dump truck is 337 m^3, its width is 8.53 m, length is 15.54 m, depth is 3.34 m and mass is tons. The analysis shows that the volume of dust ejection during loading of coal depends on the efficiency of control of body vibrations by the shock-absorbing system of a dump truck. At the same time, the standard models and methods of analysis and control of mechanical vibrations, are based on a combination of the control and vibration theories.

Thus, the analysis of dynamic response and of non-linear body vibrations is usually made with the help of models where the body is schematically represented as a load resting on a spring with one or several degrees of freedom. Such an approach does not take into account certain essential properties of dynamics of loading. These are, for instance, irregularity of loading the body, randomness of distribution of a shock dynamic loading over the surface of a body, dynamics of interaction of a body and a dump truck during the shock load. Therefore, there is a practical necessity for making a common formulation of the problem of analysing and controlling the vibrations of open cast colliery dump trucks as an interdisciplinary problem of vibration theory, control theory and visco-elastic theory. The last one takes into account real processes of a dynamic response during the action of a non-uniform shock loading on a real structure of a dump truck body on the whole.

It should be remembered that the dynamics of operation of material system is always influenced by the residual vibrations.

Typical examples concern the important dynamic problems of certain objects working in the open extraction of coal in Poland, e. g. deep-immersion at subsoil motors, truck and the special spring balances on which are resting many tons of coal mass. The dynamical problems occur also in the railroad engineering and the seaports, e.g. in the shock occurring by cargoes of a cargo ship.

In the design of such objects, an important practical meaning has the optimal choice of the main factors of the vibrational processes, providing the optimal compromise of controversial requirements to the dynamic elements system: shock-absorber-weight.

Let us consider stationary and non-stationary dynamical loading of complex system with damping (Fig. 1). The complex system is made from the elastic plate which is coupled by a visco-elastic inertial interlayer resting on a stiff foundation. The elastic plate is described as the Kirchhoff–Love model [8], and is simply supported at their edges. The interlayer connecting the plate with the rigid foundation [30, 31] will be replaced in further considerations by the so-called simplified foundation, which is modelled as the homogeneous foundation. Besides it is assumed that this simplified foundation consist of a close-packed set of homogeneous pillars appearing within the plate contour.

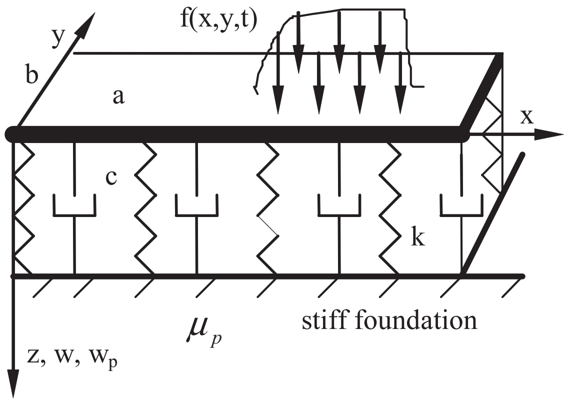

Fig. 1. Dynamic model of an elastic plate coupled with a visco-elastic inertial interlayer, under the stationary and non-stationary dynamical loading.

For this reason, the strains in the directions of co-ordinate axes x and y are equal to zero. Each of the pillars with unit area of cross-section, has length equal to thickness of this foundation and is made of visco-elastic material described by the Voigt–Kelvin model [12, 14, 17].

Due to the small angle of slope of the deflection surface of the plate, the shearing forces acting on lateral faces of these pillars are also very small and hence they can be neglected.

On grounds of the above assumed simplification it follows that these pillars are subjected to uniaxial strain but to the three-dimensional state of stress.

It is obvious that the displacement of the simplified foundation is identified with the displacement of the corresponding pillar which is placed at the point of co-ordinates x, y. In this situation these displacements are apparently described by a one-dimensional differential equation; nevertheless it should be observed that these displacements are not only dependent on the variable z, but also on the variables x, y.

In practical application, the combined system (Fig. 1) is treated as a platform with the stationary and non-stationary dynamical, concentrated or distributed loadings by the moving mass of coal [2, 3].

2.2. Mathematical problem

The phenomenon of small transverse vibrations of the elastic plate coupled with a visco-elastic inertial interlayer is described by the following non-homogeneous system of conjugate partial differential equations [2]:

$$\tag{2.1} \begin{gathered} D\Delta^2 w + \mu \frac{\partial^2 w}{\partial t^2} - \left(1 + c\frac{\partial}{\partial t}\right) k \frac{\partial w_p}{\partial z} \Big|_{z=0} = f(x,y,t), \\ \left(1 + c\frac{\partial}{\partial t}\right) k \frac{\partial^2 w_p}{\partial z^2} - \mu_p \frac{\partial^2 w_p}{\partial t^2} = 0 \end{gathered}$$

together with the corresponding homogeneous boundary conditions for the plate

$$\tag{2.2a} \begin{gathered} w\,\big|_{x=0} = 0, \qquad w\,\big|_{x=a} = 0, \qquad w\,\big|_{y=0} = 0, \qquad w\,\big|_{y=b} = 0, \\ \frac{\partial^2 w}{\partial x^2}\bigg|_{x=0} = 0, \qquad \frac{\partial^2 w}{\partial x^2}\bigg|_{x=a} = 0, \qquad \frac{\partial^2 w}{\partial y^2}\bigg|_{y=0} = 0, \qquad \frac{\partial^2 w}{\partial y^2}\bigg|_{y=b} = 0 \end{gathered}$$

and for the inertial foundation

$$\tag{2.2b} w_p\,\big|_{z=h_p} = 0,$$

as well as with the continuity condition of displacements of the plate and the simplified foundation

$$\tag{2.2c} w = w_p\,\big|_{z=0} \,.$$

In Eq. (2.1) are introduced the following notations:

$$\tag{2.3} D = \frac{Eh^3}{12\,(1-\nu_o^2)}\,, \qquad k = \frac{E_p(1-\nu_p)}{(1-2\nu_p)(1+\nu_p)}, \qquad \mu = \rho h\,, \qquad \mu_p = \rho_p\,.$$

Here $f(x,y,t)$ is the dynamical load of the complex system; $w = w(x,y,t)$, $w_p = w_p(x,y,z,t)$ are the transverse deflections of the plate and the visco-elastic inertial interlayer; E, E_p are the Young modulus of materials of the plate and the interlayer; c is the damping coefficient of the interlayer (retardation time); ρ, ρ_p are the mass densities of materials of the plate and the interlayer; h, h_p are the thicknesses of the plate and the interlayer; a, b are the dimensions of the complex system; ν_o, ν_p is the Poisson coefficient of the plate and the interlayer; x, y are the co-ordinate axes.

An analytical method of solving the problem of boundary-value problem as well as free and forced vibrations of mechanical system (Fig. 1), will be based on separation of variables.

Substituting the following dependences:

$$w = WT, \qquad w_p = W_p T \tag{2.4}$$

into the system of differential equations (2.1), one obtains the ordinary differential equation

$$\overset{o}{T} - i\nu T = 0 \tag{2.5}$$

and the system of partial differential equations

$$\begin{aligned} &D\Delta^2 W - \nu^2 \mu W - (1 + i\nu c)k \left.\frac{dW_p}{dz}\right|_{z=0} = 0, \\ &(1 + ic\nu)R + \nu^2 \mu_p W_p = 0, \end{aligned} \tag{2.6}$$

where $T = T(t)$ denotes the modal function; $W = W(x, y)$ and $W_p = W_p(x, y, z)$ stand for complex modes of vibration of the plate and the layer; ν is the complex eigenfrequency of vibrations.

Thanks to the relations (2.4), the boundary conditions (2.2) take the following form:

$$\begin{aligned} &W\big|_{x=0} = 0, \qquad W\big|_{x=a} = 0, \qquad W\big|_{y=0} = 0, \qquad W\big|_{y=b} = 0, \\ &\left.\frac{\partial^2 W}{\partial x^2}\right|_{x=0} = 0, \qquad \left.\frac{\partial^2 W}{\partial x^2}\right|_{x=a} = 0, \qquad \left.\frac{\partial^2 W}{\partial y^2}\right|_{y=0} = 0, \qquad \left.\frac{\partial^2 W}{\partial y^2}\right|_{y=b} = 0, \\ &W_p\big|_{z=h_p} = 0, \qquad W = W_p\big|_{z=0}. \end{aligned} \tag{2.7}$$

Further analytical procedures of solving this problem are presented in the papers [1, 2, 3, 24, 25].

2.3. Different modes of dynamical loading

In the first case, small transverse vibrations of the complex system with damping are excited by the following stationary, concentrated dynamical loading

$$f(x,y,t) = P\delta(x - x_o)\delta(y - y_o)\sin(\omega_o t) \tag{2.8}$$

at the point x_o, y_o and varying in time t.

In the second case, small transverse vibrations of the complex system with damping are excited by the following non-stationary concentrated dynamical loading

$$f(x,y,t) = b(t) - m\frac{d^2\overline{w}(x^*, y_o, t)}{dt^2}\delta(x - x^*)\delta(y - y_o), \tag{2.9}$$

or by the following non-stationary concentrated dynamical loading:

$$f(x,y,t) = b(t) - \frac{m}{d}\frac{d^2\overline{w}(x^*, y_o, t)}{dt^2}[H(x - x^* - d) - H(x - x^*)]. \tag{2.10}$$

Here m is the mass of coal; d is the length on which the moving mass is distributed; $\delta(\ldots)$ is the Dirac delta function; $H(\ldots)$ is the Heaviside function; $x^* = v^*t$, v^* is the constant speed; $y_o = 0.5b$; $\overline{w}(x^*, y_o, t)$ denote the transverse displacements of the plate in its first approximation at the points of location of the moving mass of coal, i.e. the trajectory of the moving mass of coal; P is the amplitude of harmonic force; $f(x,y,t)$ is the dynamical loading of the complex plate; v_n are complex frequencies of free vibrations; x_o, y_o are the co-ordinate coal for time $t = 0$; $b(t)$ is the constant loading in the direction of axis z.

3. Results and discussions

Calculations are carried out for the following data:
$E = 10^{10}$ Pa, $E_p = \gamma^* 10^8$ Pa, $\nu_o = 0.3$, $\nu_p = 0.2$, $\rho = 5^* 10^3$ Ns^2 m^{-4}, $\rho_p = 7^* 10^3$ Ns^2 m^{-4}, $h = 0.5$ m, $h_p = \varepsilon^* 3.34$ m, $a = 15.54$ m, $b = 8.53$ m, $c = \wp^* 0.00007$ Nsm^{-2}, $b(t) = 0$, $v^* = \varsigma^* 10$ ms^{-1}, $c_g = 0.00001$, $\omega_n = \mathrm{Re}[\nu_n]$, $P = 4^* 10^2$ kN, $m = 4^* 10^4$ kg.

In order to solve the boundary value problem, the following boundary conditions are used for the Kirchhoff–Love model.

Let us consider the free and forced vibrations problem of the complex system (see Fig. 1).

In order to find the Fourier coefficient $\Phi_{n_1 n_2}$, the following initial conditions are assumed:

$$w_o = A_{s1}\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right), \qquad \overset{o}{w}_o = 0, \qquad A_s = 0.0016\, m \tag{3.1}$$

where w_o is the initial displacement; $\overset{o}{w}_o$ is the initial velocity.

Some results of this problem given in Fig. 2 present absolute values of a complex determinant $|\Delta|$ and eigenfrequencies of free vibrations. The diagrams of the values of complex eigenfrequencies $\nu_{n_1n_2} = i\eta_{n_1n_2} \pm \omega_{n_1n_2}$ of free vibrations for various parameters $\wp = 1$, $g = 1$, $\gamma = 1; 0.01; 0.0001$, $\varepsilon = 1; 0.6; 0.3$ are shown in Fig. 2.

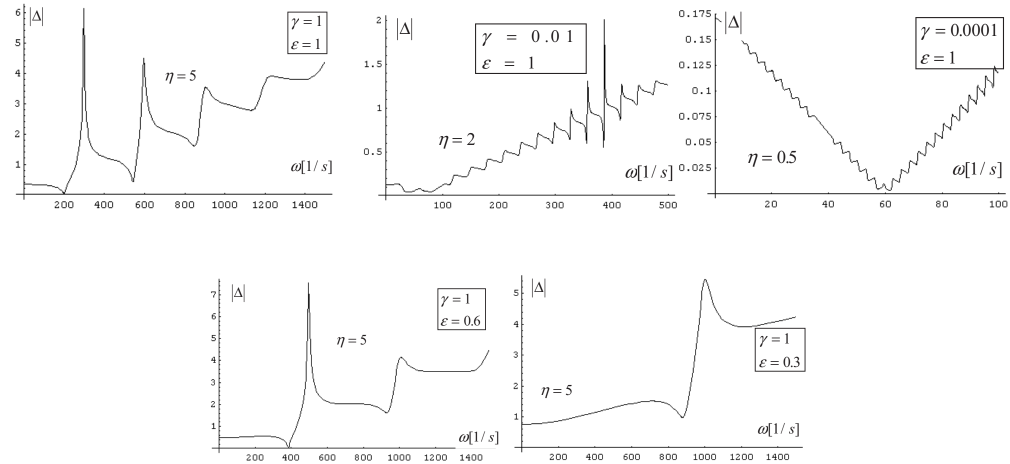

FIG. 2. Moduli of a complex determinant $|\Delta|$ and values of complex eigenfrequencies $\nu_{n_1n_2} = i\eta_{n_1n_2} \pm \omega_{n_1n_2}$ of free vibrations for $x = 0.6a$, $y = 0.6b$, $z = 0$ and $n_1 = 1$, $n_2 = 1$.

The space diagrams in Fig. 3 shows complex modes of free vibration of the visco-elastic inertial interlayer for $n_1 = 1$, $n_2 = 1$ and $n_1 = 1$, $n_2 = 2$. The space diagrams of $W(x)$ show the real ReW and the imaginary ImW parts of complex modes of free vibrations of the interlayer, in the ranges $0 < x < a$ and $0 < z < h$ and $y = 0.5b$. For $z = 0$, the diagrams show the real ReW_1 and the imaginary ImW_1 parts of complex modes of free vibrations of the plate.

The diagrams in Fig. 4 show free vibrations of the complex system with elastic and visco-elastic interlayers in time t in two cases; the first case where damping coefficient $\wp = 1$ occurs, the second case where damping coefficient does not occur, $\wp = 0$.

Calculations of dynamic displacements for the two cases where damping coefficient occurs $\wp = 1$, are compared with dynamic displacements in which the damping coefficient does not occur, $\wp = 0$ – Fig. 4. Amplitudes of free vibrations for damping coefficient $\wp = 1$ in a visco-elastic interlayer have the value approximately by 62% smaller than the amplitudes of free vibrations for the damping coefficient $\wp = 0$ in the elastic interlayer of the complex system.

The effects of various geometrical physical and mechanical parameters are shown in Figs. 5–12. In the first case, small transverse vibrations of the complex system with a viscoelastic inertial interlayer are excited by the stationary dynamical force (2.7) acting at the point $x = 0.6a$, $y - 0.6b$, $z = 0$ and varying in time t.

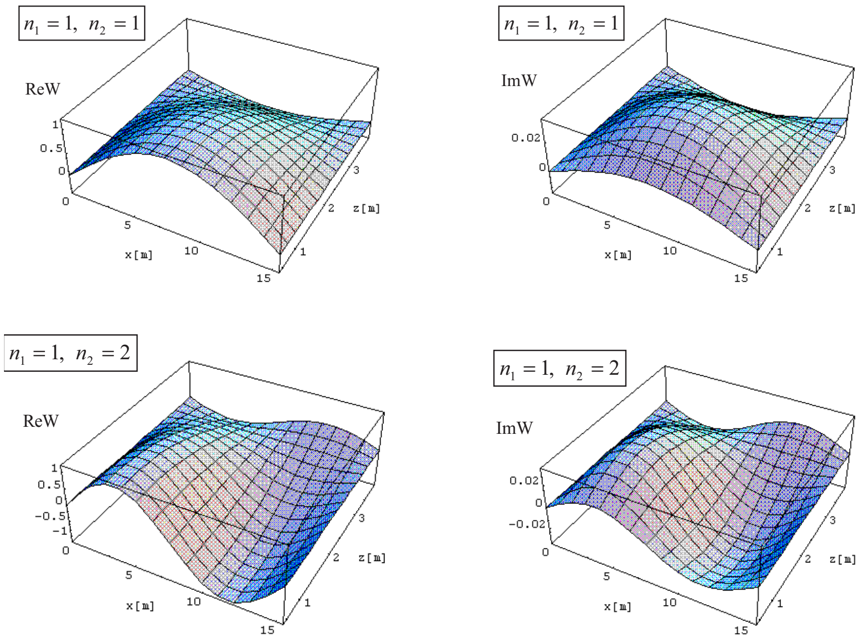

FIG. 3. Complex modes of free vibrations of the complex system with damping for $n_1 = 1$, $n_2 = 1$ and $n_1 = 1$, $n_2 = 2$; the elastic plate for $z = 0$ and the visco-elastic inertial interlayer for $0 < z < h$.

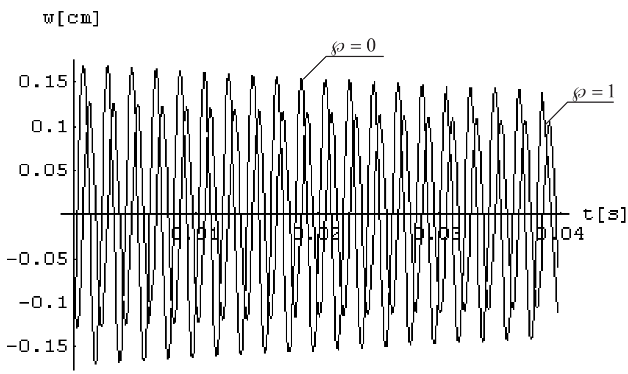

FIG. 4. Free vibrations of the complex system with the elastic $\wp = 0$ and visco-elastic $\wp = 1$ inertial interlayers for $\gamma = 1$, $\varepsilon = 1$.

The effect of stationary dynamical force in the complex system with elastic and visco-elastic interlayer is presented in Figs. 5–9 for various damping coefficients of the interlayer: $\wp = 0$ (Figs. 5a–9a) and $\wp = 1$ (Figs. 5b–9b).

In the case when the complex system is loaded by stationary concentrated force and for damping coefficient of the visco-elastic interlayer $\wp = 1$ (Figs. 5b–9b), the amplitudes of forced vibrations achieve a value approximately 55–63% smaller than the amplitudes of forced vibrations for damping coefficient of the elastic interlayer $\wp = 0$ (Figs. 5a–9a).

The effects of stationary dynamical force in the complex system with visco-elastic inertial interlayer $\wp = 1$ are presented in Fig. 5b for the theoretical A and experimental A^* investigations.

In the case when the complex system is loaded by a stationary dynamical force, for the experimental amplitudes A^* of forced vibrations we obtain a value approximately 7% smaller than the amplitudes of forced vibrations for the analytical amplitudes A.

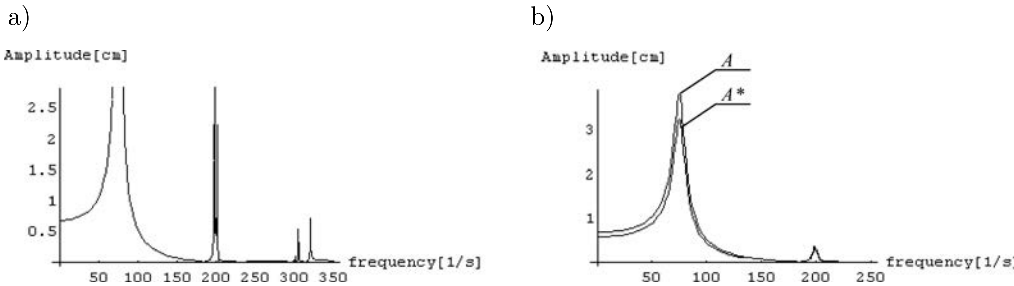

FIG. 5. Forced vibrations of the complex system for the stationary force and $\gamma = 1$, $\varepsilon = 1$; a) $\wp = 0$, b) $\wp = 1$.

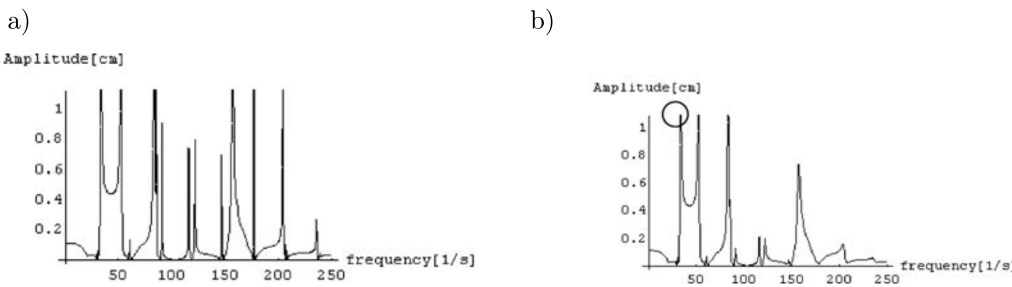

FIG. 6. Forced vibrations of the complex system for the stationary force and $\gamma = 0.01$, $\varepsilon = 1$; a) $\wp = 0$, b) $\wp = 1$.

After analysing the results presented in Figs. 5b–9b where damping coefficient in the interlayer occurs, we state that the visco-elastic inertial interlayer can be a vibration damper for the elastic plate which is loaded by the stationary dynamical force acting at the point $x = 0.6a$, $y = 0.6b$ and varying in time t.

In the case when the damping coefficient is equal to zero, presented in Figs. 5a–9a, resonance in the complex plate with an elastic inertial interlayer occurs, because real frequency $\pm\omega_{n_1 n_2}$ of free vibrations is coinciding with real frequency ω_o of forced vibrations. In the case when damping coefficient is different from zero, presented in Figs. 5b–9b, no resonance in the complex plate with a visco-elastic inertial interlayer, because complex eigenfrequency $\nu_{n_1 n_2} = i\eta_{n_1 n_2} \pm \omega_{n_1 n_2}$ of free vibrations (Fig. 2) is do not coinciding with real frequency ω_o of forced vibrations.

The effect of stationary dynamical force in the complex system with a visco-elastic interlayer is presented in Figs. 5–7 for the various Young moduli of the interlayer $\gamma = 1$ (Fig. 5), $\gamma = 0.01$ (Fig. 6) and $\gamma = 0.0001$ (Fig. 7).

a) b)

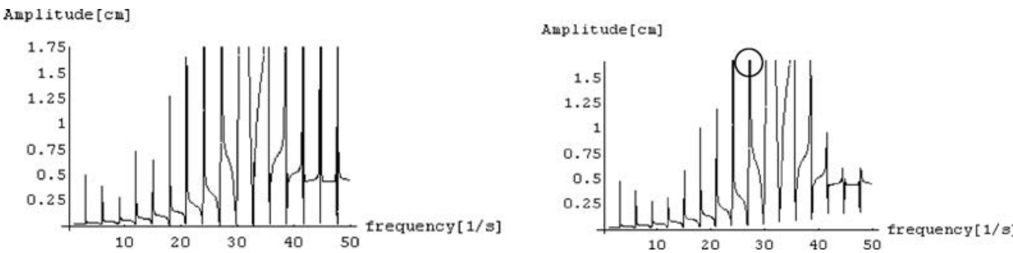

FIG. 7. Forced vibrations of the complex system for the stationary force and $\gamma = 0.0001$, $\varepsilon = 1$; a) $\wp = 0$, b) $\wp = 1$.

In the case when the complex system with damping is loaded by the stationary dynamical force, for the Young modulus of the visco-elastic interlayer $\gamma = 1$ (Fig. 5), the amplitudes of forced vibrations achieve a value approximately 65% smaller than the amplitudes of forced vibrations for the Young modulus of the visco-elastic interlayer $\gamma = 0.01$ (Fig. 6). For the Young modulus of the visco-elastic interlayer $\gamma = 0.01$ (Fig. 6), the amplitudes of forced vibrations achieve a value approximately 60% smaller than the amplitudes of forced vibrations for the Young modulus of the visco-elastic interlayer $\gamma = 0.0001$ (Fig. 7).

The effect of stationary dynamical force in the complex system with a visco-elastic interlayer is presented in Figs. 5, 8, 9 for various thicknesses of the interlayer $\varepsilon = 1$ (Fig. 5), $\varepsilon = 0.6$ (Fig. 8) and $\varepsilon = 0.3$ (Fig. 9).

a) b)

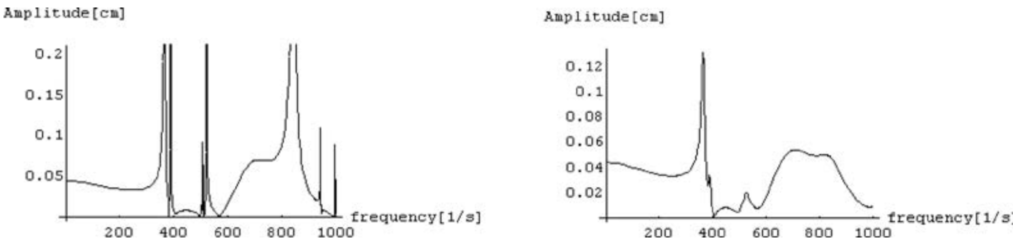

FIG. 8. Forced vibrations of the complex system for the stationary force and $\gamma = 1$, $\varepsilon = 0.6$; a) $\wp = 0$, b) $\wp = 1$.

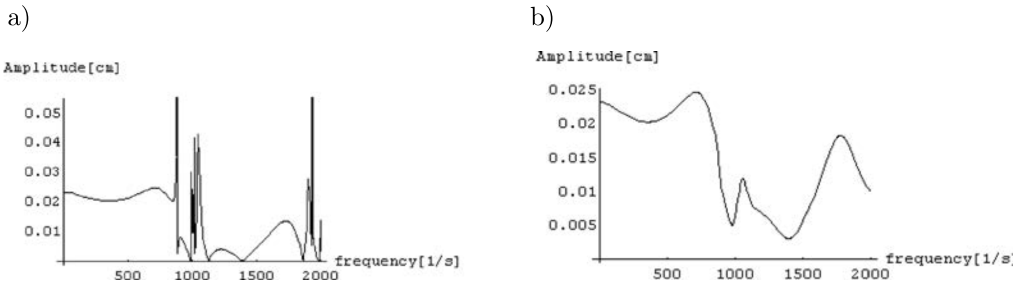

FIG. 9. Forced vibrations of the complex system for the stationary force and $\gamma = 0$, $\varepsilon = 0.3$; a) $\wp = 0$, b) $\wp = 1$.

In the case when the complex system with damping is loaded by the stationary dynamical force, for a small thickness of the visco-elastic interlayer $\varepsilon = 0.3$ (Fig. 9) the amplitudes of forced vibrations achieve a value approximately 79% smaller than amplitudes of forced vibrations for a large thickness of the visco-elastic interlayer $\varepsilon = 0.6$ (Fig. 8). For a small thickness of the visco-elastic interlayer $\varepsilon = 0.6$ (Fig. 8), the amplitudes of forced vibrations achieve a value approximately 95% smaller than the amplitudes of forced vibrations for a large thickness of the visco-elastic interlayer $\varepsilon = 1$ (Fig. 5).

In the second case, small transverse vibrations of the complex system with a visco-elastic inertial interlayer are excited by the dynamical non-stationary loading expressed by the equations (2.8) or (2.9). The mass is moving with the speed ς for $y = 0.5b$.

The effect of moving mass in the complex system with a visco-elastic inertial interlayer is presented in Fig. 10 for various speeds $\varsigma = \{1, 2, 3, 5, 6\}$ and various damping coefficients $\wp = \{0, 0.6, 1\}$.

In the case when the complex system is loaded by a moving mass and when the damping coefficient of the visco-elastic interlayer $\wp = 1$, the amplitudes of forced vibrations achieve a value approximately 20–30% smaller than the amplitudes of forced vibrations for damping coefficient of the visco-elastic interlayer $\wp = 0.6$. In the case when the complex system is loaded by a moving mass and when the damping coefficient of the visco-elastic interlayer $\wp = 0.6$, the amplitudes of forced vibrations achieve a value approximately 50–60% smaller than the amplitudes of forced vibrations for damping coefficient of the elastic interlayer $\wp = 0$ (Fig. 10).

In the case when the complex system with damping is loaded by the mass moving with speed $\varsigma = 1$, the amplitudes of forced vibrations achieve a value approximately 10% smaller than the amplitudes of forced vibrations for the speed $\varsigma = 2$. In the case when the complex system with damping is loaded by the moving mass with the speed $\varsigma = 2$, the amplitudes of forced vibrations

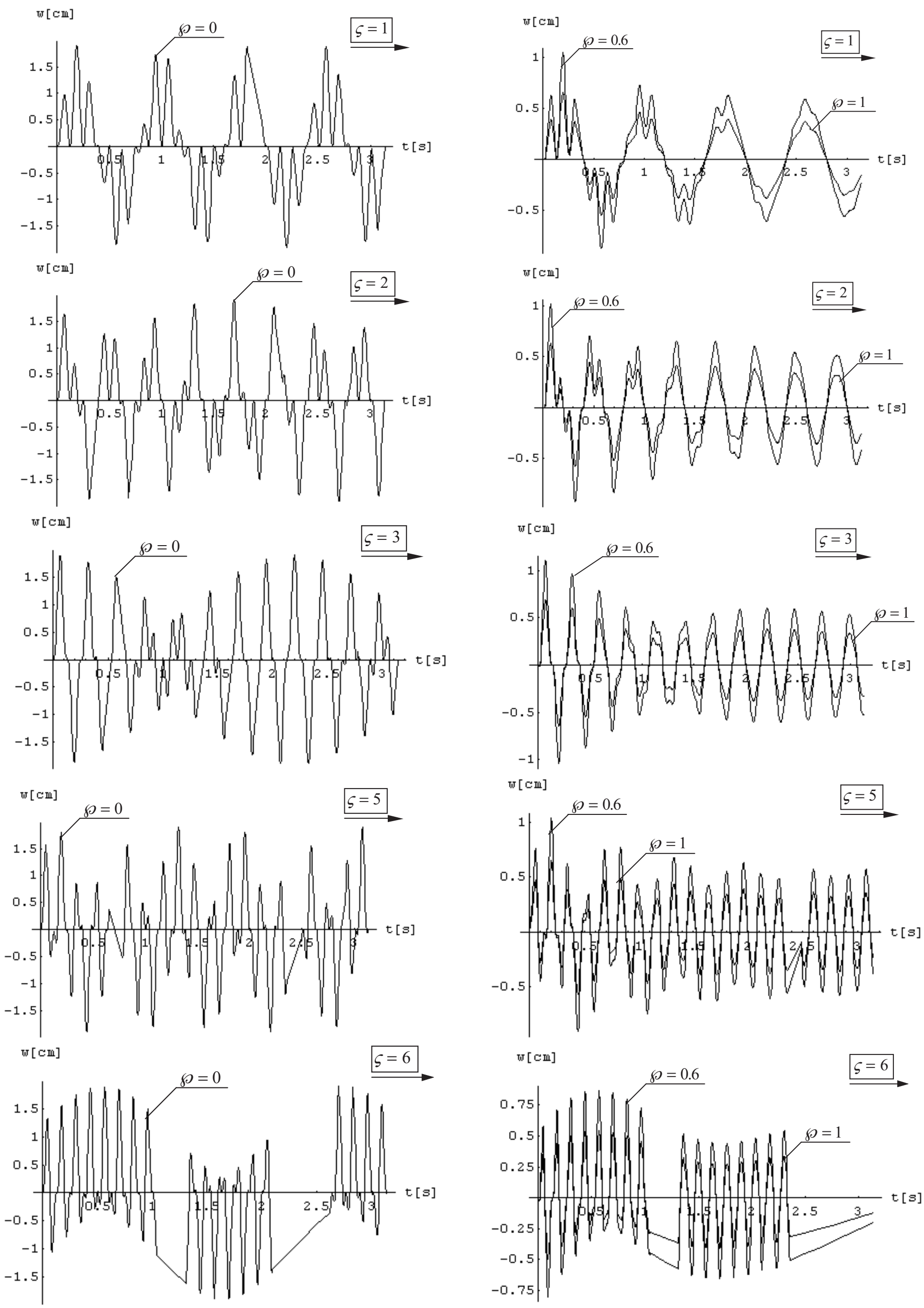

FIG. 10. Forced vibrations of the elastic and visco-elastic complex system for the moving mass with the speeds $\varsigma = \{1, 2, 3, 5, 6\}$ for $\gamma = 1$, $\varepsilon = 1$, $\wp = \{0, 0.6, 1\}$, $g = 1$.

achieve a value approximately 8% smaller than amplitudes of forced vibrations for the speed $\varsigma = 3$. At the critical speed $\varsigma_{\text{crit}} = 5$, the amplitudes of forced vibrations achieve a value approximately 35% larger than amplitudes obtained for the speed $\varsigma = 6$ (Fig. 10).

The effect of a moving inertial mass in the complex system with elastic $\wp = 0$ and visco-elastic $\wp = \{0.6, 1\}$ inertial interlayer, for the speed $\varsigma = 1$, is presented in Figs. 11–12.

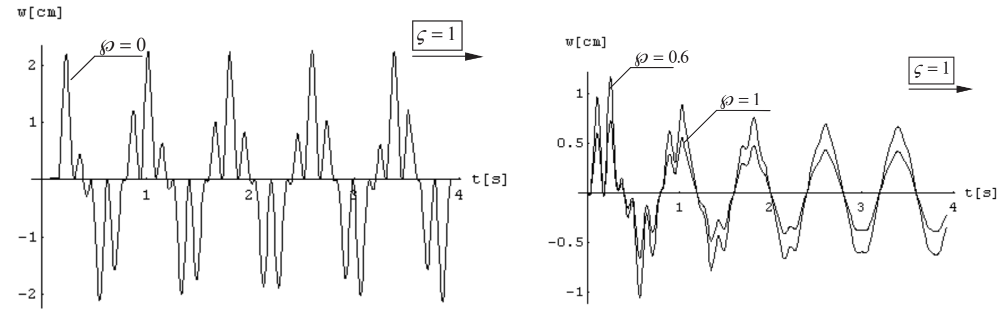

FIG. 11. Forced vibrations of the elastic and visco-elastic complex system for the mass moving with the speeds $\varsigma = 1$ and $\gamma = 0.01$, $\varepsilon = 1$, $\wp = \{0, 0.6, 1\}$, $g = 1$.

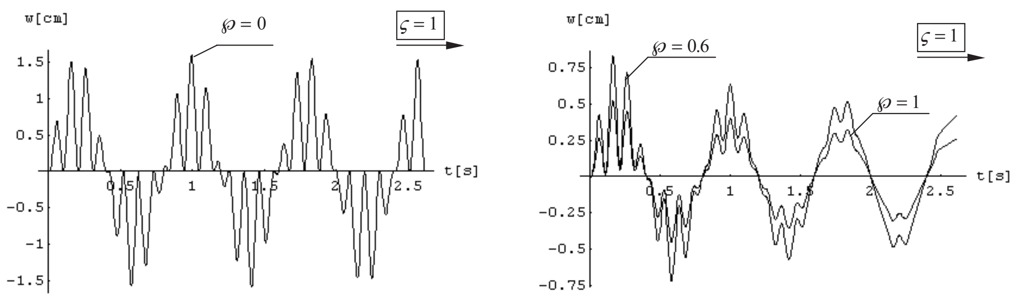

FIG. 12. Forced vibrations of the elastic and visco-elastic complex system for the mass moving with the speeds $\varsigma = 1$ and $\gamma = 1$, $\varepsilon = 0.03$, $\wp = \{0, 0.6, 1\}$, $g = 1$.

In the case when the complex system is loaded by the moving mass for damping coefficients of the visco-elastic interlayer $\wp = \{0.6, 1\}$, the amplitudes of forced vibrations achieve a value approximately 60–90% smaller than amplitudes of forced vibrations for damping coefficient of the elastic interlayer $\wp = 0$ (Figs. 11–12).

The effect of moving mass in the complex system with the elastic and visco-elastic interlayer is presented in Figs. 11, 12 for the various Young moduli of the interlayer $\gamma = 1$ (Fig. 12) and $\gamma = 0.01$ (Fig. 11).

In the case when the complex system with damping is loaded by a moving mass, for the Young modulus of the visco-elastic interlayer $\gamma = 0.01$ (Fig. 11), the amplitudes of forced vibrations achieve a value approximately 12% larger than amplitudes of forced vibrations for the Young modulus of the visco-elastic interlayer $\gamma = 1$ (Fig. 12).

The effect of moving mass in the complex system with the elastic and visco-elastic interlayer is presented in Figs. 11, 12 for various thicknesses of the interlayer $\varepsilon = 1$ (Fig. 11) and $\varepsilon = 0.3$ (Fig. 12). In the case when the complex system with damping is loaded by a moving mass, for a small thickness of the visco-elastic interlayer $\varepsilon = 0.3$ (Fig. 12), the amplitudes of forced vibrations achieve a value approximately 33% smaller than amplitudes of forced vibrations for a large thickness of the visco-elastic interlayer $\varepsilon = 1$ (Fig. 11).

After analysing the results presented in Figs. 11–12 when damping coefficient $\wp = \{0.6, 1\}$ in the interlayer occurs, we conclude that the visco-elastic inertial interlayer can be the vibration damper for the elastic plate which is dynamically loaded by the moving mass and varying in time t. In the case when damping coefficient is equal to zero $\wp = 0$ and in the case when damping coefficient is different from zero $\wp = \{0.6, 1\}$ presented in Figs. 11–12 no resonance occurs in the complex plate with the elastic and visco-elastic inertial interlayers.

4. Conclusions

- In the case of stationary harmonic loading acting on the compound system, i.e. the platform, the analysis of displacements and the investigations of resonance can be considered in the routine way. The choice of mechanical parameters of the compound system proceeds according to the principles of the classical theory of linear vibrations.
- The problem of vibrations of a compound system excited by the non-stationary inertial, moving load have specific attributes. Therefore the analysis of these vibrations to exceed the general theory of the linear vibrations in the mechanical systems with the time-dependent parameters. It is well-known that a very important quantity of this compound system is the relation between the velocity of moving mass and the so-called critical velocity, that is dependent on the ratio of moving mass to the stationary mass of the mechanical system. In turned out that in the case when the velocity of moving mass small in comparison with the critical high-velocity. Hence it follows that there is no danger to exceed the acceptable displacements in the mechanical system.

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