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MOTION AND HEAT TRANSFER IN TURBULENT VORTEX PAIRS

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A two-dimensional problem of turbulent vortex pairs originated by an instantaneous dynamical impulse (cylindrical puffs) or an instantaneous heat release (cylindrical thermals) is considered. The equations of vorticity and heat transfer for a turbulent motion of incompressible fluid are transformed into a non-dimensional form which includes the requirements to reach the similarity regime. The system of equations is solved numerically for various values of spatially constant eddy viscosity and heat diffusion coefficients. The obtained similarity distributions of vorticity, velocity and temperature for cylindrical puffs and thermals are discussed and compared with available experimental data. The results of computations for line puffs point at the existence of a critical value of the non-dimensional eddy viscosity coefficient. Below this value there exists a loss of momentum and heat to a wake at the rear of the vortex pair.

1. Introduction

A vortex ring may arise in a viscous fluid under the influence of an instantaneous dynamic impulse (for example, by pushing a finite amount of fluid with an initial velocity) or by releasing instantaneously a finite amount of buoyancy (for example, as a result of rapid heat release from a compact isolated source). The first type of a vortex ring is sometimes called a puff and the second type, a thermal. In a general case both cases mentioned above may coexist.

A two-dimensional analog of the vortex ring, a vortex pair, is the main factor that determines the development of a round jet in a cross flow at a sufficient distance from the source. This was suggested first by Scorer [1] and substantiated further in the paper [2]. Another practical application of the vortex pair theory is connected with trailing vortices behind an aircraft.

Experimental researches of turbulent vortex pairs, i.e. cylindrical thermals and puffs, were carried out by Richards [3, 4]. Tsang [5, 6] added new ideas to Richard's work in the part concerning cylindrical thermals. Scorer and Richards also deduced theoretically some basic features of the behaviour of turbulent vortex pairs in the similarity regime.

An attempt to solve numerically the motion and heat transfer equations, transformed in a special way for the case of cylindrical thermals in the similarity regime, was undertaken by Lilly [7]. In his computations the thermals were released from the lower boundary which was not penetrable. An analogous problem for cylindrical puffs, but in a fully infinite medium, was formulated by Lugovtzov [8], however, it was not solved.

The present paper based on the main principles put forward by Lilly and Lugovtzov deals with a general formulation of the problem and the numerical solution of governing equations for cylindrical puffs and thermals developing in a fully infinite medium in the similarity regime.

2. Formulation of the problem

We adopt the following basic assumptions: (i) a fluid is regarded as incompressible; maximum local temperature (density) differences are small in comparison with some characteristic temperature (density); (ii) molecular transfer of momentum and heat is small in comparison with their eddy transfer; (iii) eddy kinematic viscosity and eddy heat diffusion coefficients , ν_t and a_t are independent of spatial coordinates; as regards these coefficients $\nu_t \sim a_t \sim L(t)\, V(t)$ is assumed, where L and V are characteristic length and velocity scales, which are functions of time alone.

Taking into account these assumptions we have the following equations of vorticity and heat transfer in two-dimensional Cartesian coordinates:

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = -\beta g\frac{\partial T}{\partial y} + \nu_t(t)\,\nabla^2 \zeta\,, \tag{2.1}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a_t(t)\,\nabla^2 T\,, \tag{2.2}$$

while

$$u = \frac{\partial \psi}{\partial y}\,, \qquad v = -\frac{\partial \psi}{\partial x}\,, \tag{2.3}$$

$$\nabla^2 \psi = -\zeta\,, \tag{2.4}$$

where u and v are the velocity component in the x and y directions, ψ the stream function, ζ the vorticity, T the temperature deviation from the undisturbed environment temperature, g the acceleration of gravity, β the thermal expansion coefficient; $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

The boundary conditions for the problem under consideration are (at symmetry of the flow relative to the x-axis)

$$\begin{aligned} \psi = \zeta = \frac{\partial T}{\partial y} = 0 \quad &\text{at} \quad y = 0\,, \\ \psi = \zeta = T = 0 \quad &\text{at} \quad x^2 + y^2 \to \infty\,. \end{aligned} \tag{2.5}$$

Equations (2.1) (on condition of the sufficiently quick decay of vorticity at infinity) and (2.2) yield, after integration over the infinite plane, the integral conditions of the vortex pair momentum and heat conservation, respectively

$$\frac{1}{2}\int_{-\infty}^{\infty}\!\!\int \zeta y\,dx\,dy = I + Qt\,, \qquad \int_{-\infty}^{\infty}\!\!\int T\,dx\,dy = Q_T\,, \tag{2.6}$$

where $Q_T=2Q/\beta g$ and I, Q, Q_T are the initial momentum, buoyancy and heat content of the vortex pair, respectively.

We now introduce the dimensionless variables

$$x'=\frac{x}{L(t)},\quad y'=\frac{y}{L(t)},\quad u'=\frac{u}{V(t)},\quad v'=\frac{v}{V(t)},\quad \psi'=\frac{\psi}{L(t)V(t)},$$

$$\zeta'=\zeta\frac{L(t)}{V(t)},\quad T'=\frac{T}{\theta(t)}$$

as well as

$$\nu_t'=\frac{\nu_t}{L(t)V(t)},\quad a_t'=\frac{a_t}{L(t)V(t)},$$

where θ is a characteristic temperature scale.

Having changed the variables in Eq. (2.6) and denoted

$$\frac{1}{2}\int\int_{-\infty}^{\infty}\zeta y dx dy=\chi_1,\quad \int\int_{-\infty}^{\infty}T dx dy=\chi_2 \tag{2.7}$$

(dropping for convenience the primes here and further in the text) we obtain the following equations for the scale:

$$L^2V=\frac{1}{\chi_1}(I+Qt),\quad L^2\theta=\frac{Q_T}{\chi_2}. \tag{2.8}$$

As it appears from the dimensional analysis the problem under consideration has similarity solutions in the following two scaes:

1) $I\neq 0$, $Q=0$ i.e. for a cylindrical puff (in this case Eq. (2.1) becomes independent of Eq. (2.2), therefore here $Q_T=0$ is not obligatory and the equality $Q=0$ is fulfilled owing to the condition $\beta g L\theta/V^2\approx 0$);

2) $I=0$, $Q\neq 0$ i.e. for a cylindrical thermal.

For the similarity or shape-preserving regime it may be assumed $V=\kappa\frac{dL}{dt}$, where κ is an arbitrary constant. In the same case χ_1 and χ_2 in Eq. (2.7) are assumed to be constant with arbitrary chosen values.

Equations (2.8) are then solved with the initial condition $L(t_0)=L_0$, where L_0 is an initial length of the vortex pair. Then we obtain for a cylindrical puff choosing $t_0=\kappa\chi_1L_0^3/3I$

$$L=\left(\frac{3It}{\kappa\chi_1}\right)^{1/3},\quad V=\left(\frac{\kappa}{3}\right)^{2/3}\left(\frac{I}{\chi_1}\right)^{1/3}t^{-2/3},\quad \theta=\left(\frac{\kappa\chi_1}{3}\right)^{2/3}\frac{Q_T}{\chi_2}(It)^{-2/3} \tag{2.9}$$

and for a cylindrical thermal, if $t_0=\left(\frac{2\kappa\chi_1L_0^3}{3Q}\right)^{1/2}$,

$$L=\left(\frac{3Qt^2}{2\kappa\chi_1}\right)^{1/3},\quad V=\left(\frac{2\kappa}{3}\right)^{2/3}\left(\frac{Q}{\chi_1}\right)^{1/3}t^{-1/3},\quad \theta=\left(\frac{\kappa\chi_1}{3}\right)^{2/3}\frac{\beta g}{\chi_2}\left(\frac{Q}{3}\right)^{1/3}t^{-4/3}. \tag{2.10}$$

Taking into account the relations (2.9) or (2.10), Eqs. (2.1) and (2.2) can be transformed into a dimensionless form

$$\frac{\partial \zeta}{\partial \tau} - A\zeta + (\kappa u - x)\frac{\partial \zeta}{\partial x} + (\kappa v - y)\frac{\partial \zeta}{\partial y} = -B\frac{\partial T}{\partial y} + \nu_t \nabla^2 \zeta, \tag{2.11}$$

$$\frac{\partial T}{\partial \tau} - 2T + (\kappa u - x)\frac{\partial T}{\partial x} + (\kappa v - y)\frac{\partial T}{\partial y} = a_t \nabla^2 T, \tag{2.12}$$

where the dimensionless time $\tau = C \ln\left(\frac{t}{t_0}\right)$ and $A=3$, $B=0$, $C=1/3$ for a puff, $A=3/2$, $B=3$, $C=2/3$ for a thermal.

Equations (2.3) and (2.4) as well as the boundary conditions (2.5) remain unchanged after the coordinate transformation. The normalization conditions are given by the relations (2.7).

3. Numerical solution principles

For reasons of symmetry computations were carried out in half-plane, whereas the rectangle $x_2 \leqslant x \leqslant x_1$, $0 \leqslant y \leqslant y_1$, was chosen as the computation region. In order to estimate roughly the dimensions of this rectangle and its displacement relative to the origin of coordinates, some linearized equations were considered beforehand. These equations were obtained from Eqs. (2.11) and (2.12) by a formal change $u = u_0$, $v = v_0 = 0$ where u_0, v_0 are the components of some medium velocity of the vortex pair.

Upon considering Eqs. (2.3), (2.4), (2.5) and (2.7), the similarity solutions of such equations for a cylindrical puff are

$$\psi = \frac{1}{\pi\kappa\chi_1}\left[1 - e^{-\frac{(x-x_0)^2+y^2}{2\kappa\nu_t}}\right]\frac{y}{(x-x_0)^2+y^2}, \tag{3.1}$$

$$\zeta = \frac{1}{\pi\kappa^2 \chi_1 \nu_t^2}\, y e^{-\frac{(x-x_0)^2+y^2}{2\kappa\nu_t}}, \tag{3.2}$$

$$T = \frac{1}{2\pi\kappa\chi_2 a_t} e^{-\frac{(x-x_0)^2+y^2}{2\kappa a_t}}, \tag{3.3}$$

where $x_0 = u_0$. The same solutions are also correct for a cylindrical thermal at $\nu_t = a_t$, i.e. $Pr_t = 1{,}0$ (where Pr_t is the eddy Prandtl number).

Assuming $u = u_0$ in some point (x_0, y_0), the abcissa of which equals to

$$y_0 = \frac{\int_{-\infty}^{\infty}\int_{0}^{\infty} \zeta y\,dx\,dy}{\int_{-\infty}^{\infty}\int_{0}^{\infty} \zeta\,dx\,dy} = \frac{\chi_1}{\int_{-\infty}^{\infty}\int_{0}^{\infty} \zeta\,dx\,dy}. \tag{3.4}$$

we find from Eqs. (3.1) and (3.2) the approximate value of the displacement of the computation region centre relative to the origin of coordinates

$$x_0 = \frac{2}{\pi^2 \chi_1 \nu_t}\left\{\left[e^{-\frac{\pi}{4}}\left(\frac{\pi}{4} + 1\right) - 1\right]\right\} = \frac{0.034}{\chi_1 \nu_t}. \tag{3.5}$$

Equations (3.1) to (3.3) may be regarded as first approximations of the desired similarity solutions at sufficiently great values of ν_t and a_t. Besides, they satisfy Eqs. (2.11) and (2.12) when $u \ll x$, $v \ll y$. For the values of ν_t and a_t of physical interest, i.e. of the order $10^{-1}-10^{-2}$ the exponential components of the functions (3.1) to (3.3) tend to zero sufficiently quickly and, consequently, the flow may be considered potential already at a comparatively small distance from the point (x_0, y_0) assumed to be the vortex pair centre. Thus, if the outer boundaries of the computation region are in the potential flow, the distributions of ψ, ζ and T on these boundaries may be given according to the functions (3.1), (3.2) and (3.3). For ζ and T these boundary conditions correspond to referring the zero values form infinity to the outer boundaries of the rectangle. The performed calculations indicated also that giving the zero value of the stream function instead of

$$\psi = \frac{y}{\kappa\pi\chi_1\,[(x-x_0)^2+y^2]} \tag{3.6}$$

on the outer boundaries of the computation region did not affect the nature of the obtained numerical solutions in the active mixing zone (the zone where vorticity is substantially different from zero).

As to the dimensions of the rectangle, the assumed values of $x_2-x_1=3.6$—4.2 and $y_1=0.8-2.1$ together with the approximately found value of x_0 from Eq. (3.5) ensured the potential character of the flow on the outer boundaries of the computation region at all adopted values of ν_t, excluding only the smallest ones in the case of cylindrical puffs.

A detailed analysis of the effect of referring the theoretical boundary conditions (2.5) from infinity to the outer boundaries of the finite computation region was also carried out by LILLY [7].

Equations (2.11) and (2.12) together with Eqs. (2.3) and (2.4) were solved numerically by an explicit finite-difference scheme. A first-order approximation of the basic equations was used for time and a second-order approximation for space variables. The convective terms in Eq. (2.11) were approximated in a special way which ensured the conservation of the mean kinetic energy if $\nu_t \to 0$ and prevented the fictitious growth of vorticity [9]. A detailed description of this scheme is given by ARAKAWA [10]. The amount of iterations necessary to reach the steady state was substantially reduced by means of the multiplicative correction for the stream function obtained on every time layer from the condition (2.7).

The initial distributions of ψ and T were given according to the functions (3.1) and (3.3).

All computations were performed on a 42×21 or 61×14 mesh with a coordinate step $\Delta x=\Delta y=0.06-0.10$ and a time step $\Delta\tau=0.05-0.10$. Calculations were carried out for the range of $\nu_t=0.015-0.10$ at $Pr_t=0.5$, 0.75, 1.0 and 1.25. Besides, $\kappa=\chi_1=\chi_2=1$ was assumed. In most cases a convergence of numerical solution for ζ up to there decimal figures was obtained after 500–600 iterations. It was only at the smallest values of ν_t for cylindrical puffs that the convergence was getting worse substantially.

4. Results of computations

Figures 1–3 show the similarity distributions of ζ, u, v and T for cylindrical puffs at $Pr_t = 1$ and $\nu_t = 0.08$, 0.04 and 0.02, respectively. Fiugre 4 represents the distributions of T at $\nu_t = 0.04$ and $Pr_t = 0.5$ and 1.25.

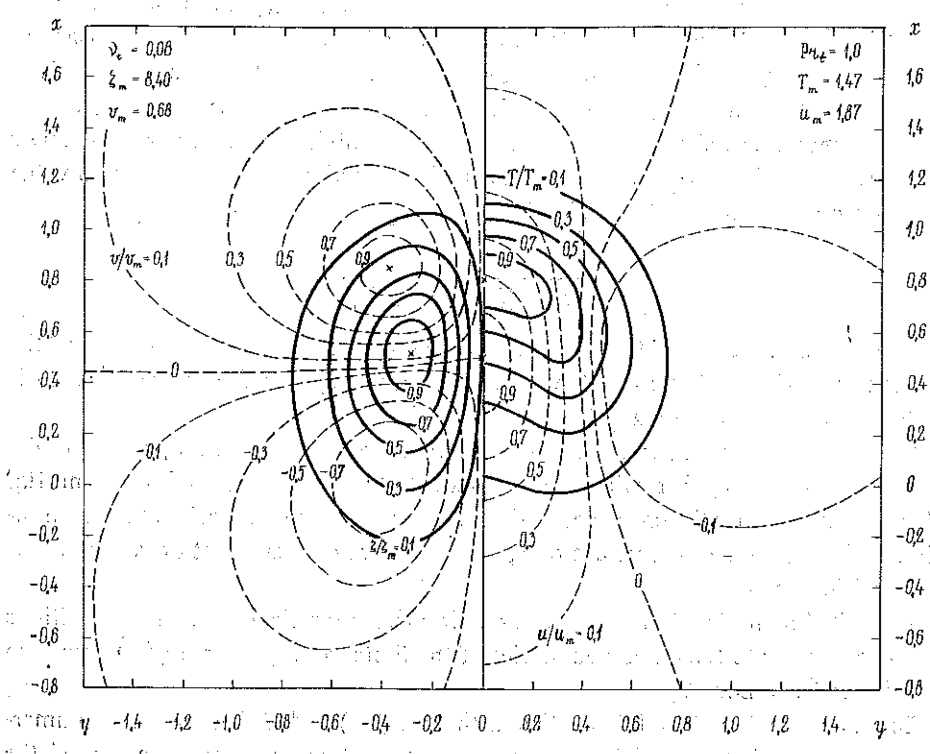
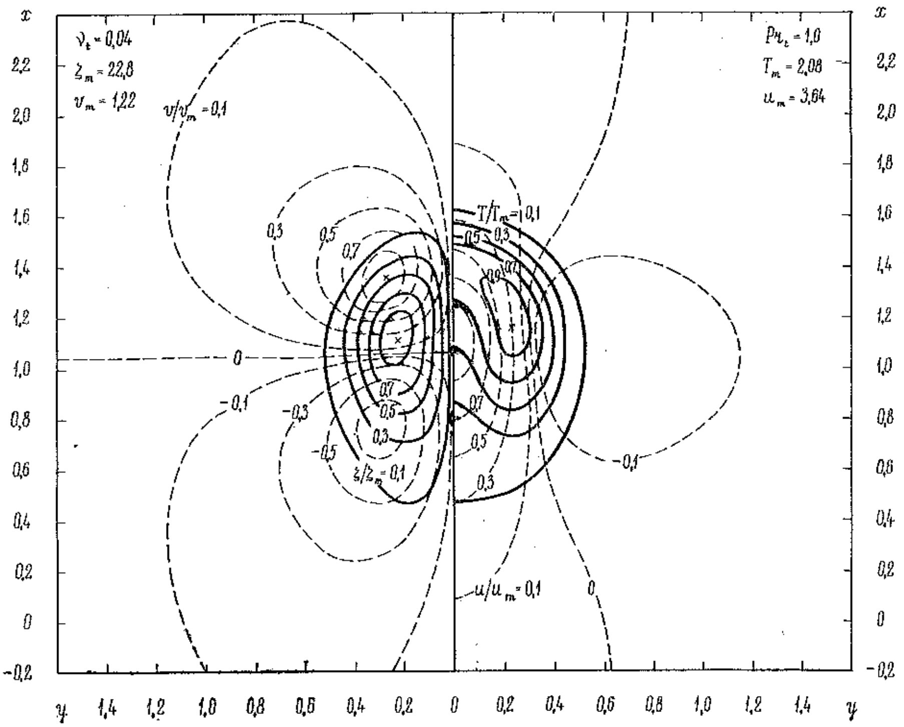


Fig. 1.

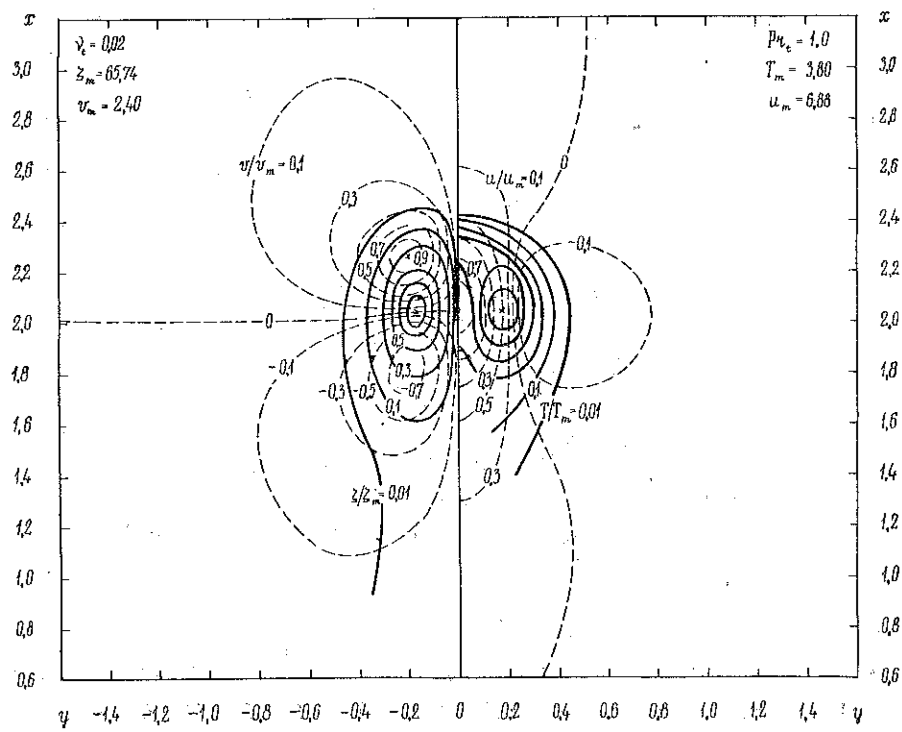
As it was to be expected, with decrease of ν_t the respective distributions start to differ more and more from the symmetrical distributions (3.1)–(3.3). In particular, for ζ this difference consists in the fact that the zone, where ζ has small, though substantially different from zero values, is stretched to the side opposite to the direction of vortex pair momentum. Already at $\nu_t = 0.02$ the character of vorticity field actually indicates the formation of a hydrodynamic wake behind the puff. As ν_t is being diminished the deformation of the temperature field takes first the form of an isotherm concavity at the rear of the thermally active zone. Then, beginning with a certain value of ν_t, a closed zone arises in the periphery; there T exceeds its maximum value at the axis of symmetry cf the puff. With ν_t decreasing further, this zone becomes more and more pronounced. The analogous effect on the temperature distribution has an increase of Pr_t, whereas at small values of Pr_t there is a tendency to form a heat wake behind the main part of the thermally active zone.

It should be pointed out that the temperature distributions of a vortex pair are equivalent to the distributions of passive admixture concentration, since



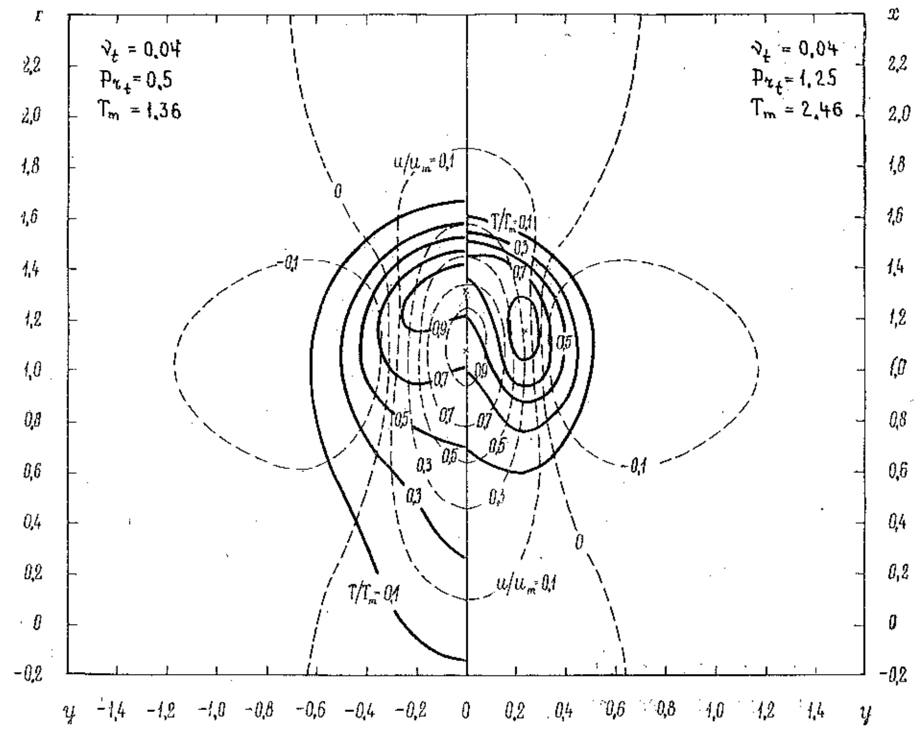
$\nu_t = 0{,}04$
$z_m = 22{,}8$
$v_m = 1{,}22$
$Pr_t = 1{,}0$
$T_m = 2{,}08$
$u_m = 3{,}64$
$v/v_m = 0{,}1$
$T/T_m = 0{,}1$
$z/z_m = 0{,}1$
$u/u_m = 0{,}1$

FIG. 2.



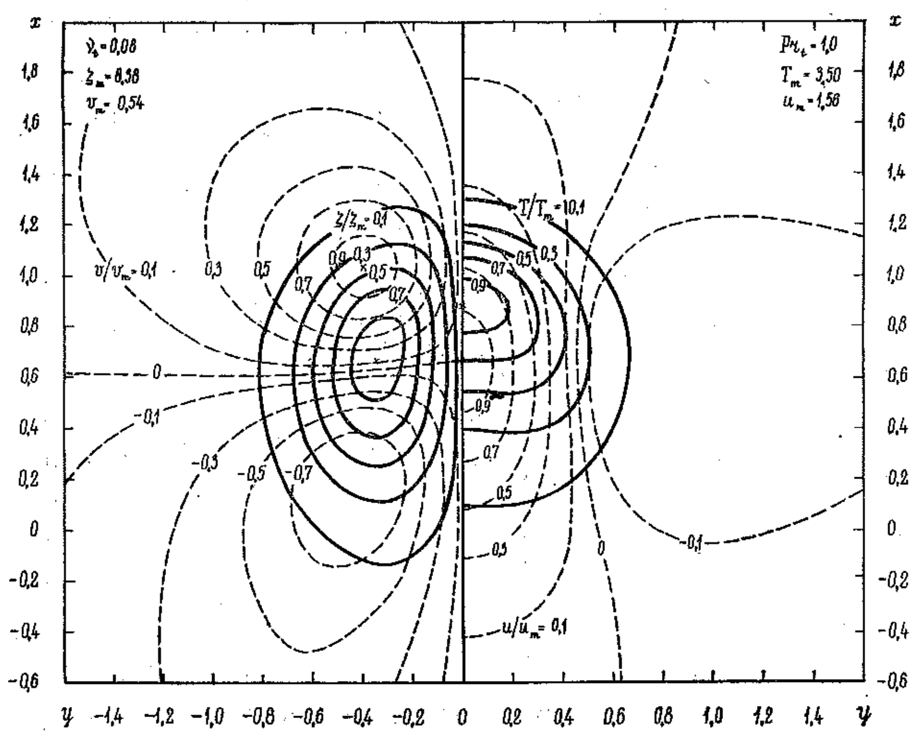
$\nu_t = 0{,}02$
$z_m = 65{,}74$
$v_m = 2{,}40$
$Pr_t = 1{,}0$
$T_m = 3{,}80$
$u_m = 6{,}88$
$v/v_m = 0{,}1$
$u/u_m = 0{,}1$
$T/T_m = 0{,}01$
$z/z_m = 0{,}01$

FIG. 3.



ϑ_t = 0,04
Pr_t = 0,5
T_m = 1,36
ϑ_t = 0,04
Pr_t = 1,25
T_m = 2,46
u/u_m = 0,1
T/T_m = 0,1
T/T_m = 0,1
u/u_m = 0,1
x
y

FIG. 4.



ϑ_t = 0,08
ζ_m = 8,38
v_m = 0,54
Pr_t = 1,0
T_m = 3,50
u_m = 1,56
v/v_m = 0,1
ζ/ζ_m = 0,1
T/T_m = 0,1
u/u_m = 0,1
x
y

FIG. 5.

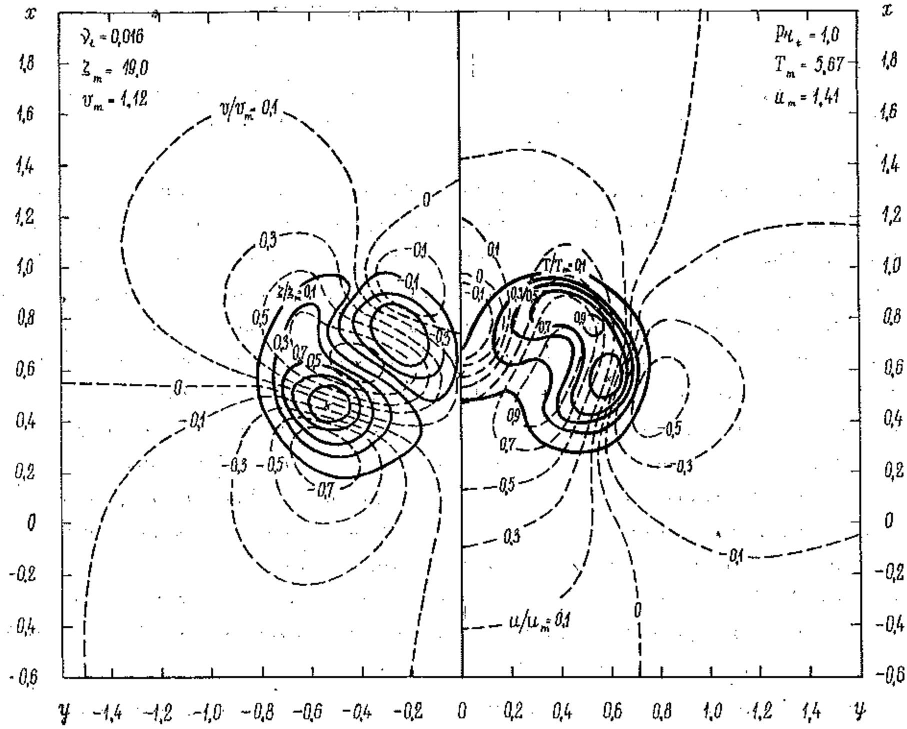

$\vartheta_t = 0{,}016$
$z_m = 19{,}0$
$v_m = 1{,}12$
$Pr_t = 1{,}0$
$T_m = 5{,}67$
$u_m = 1{,}41$
$v/v_m = 0{,}1$
$z/z_m = 0{,}1$
$T/T_m = 0{,}1$
$u/u_m = 0{,}1$

FIG. 6.

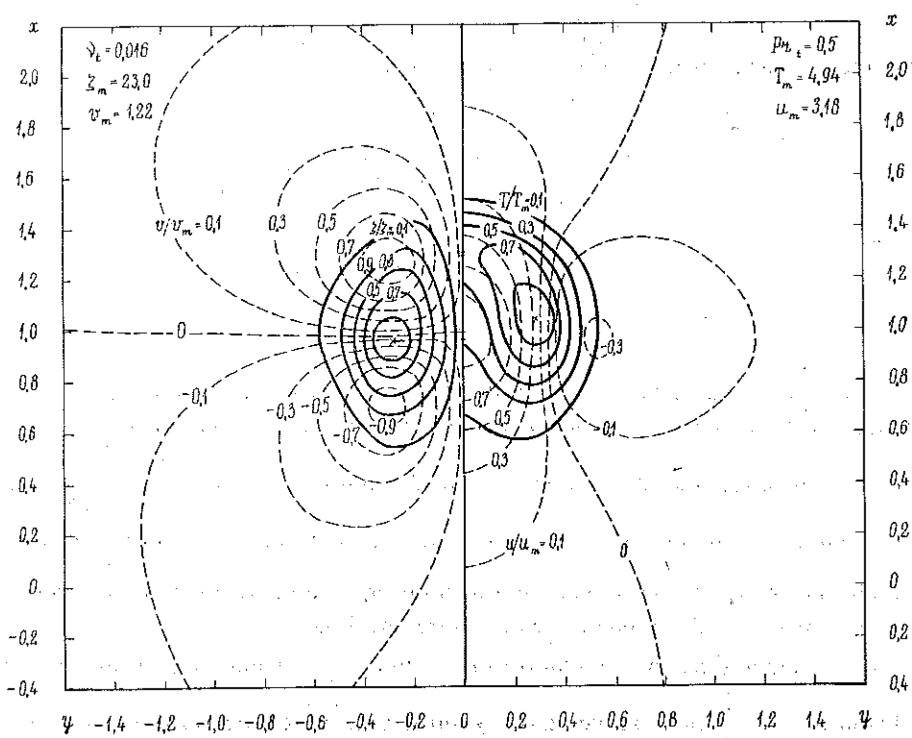

$\vartheta_t = 0{,}016$
$z_m = 23{,}0$
$v_m = 1{,}22$
$Pr_t = 0{,}5$
$T_m = 4{,}94$
$u_m = 3{,}18$
$v/v_m = 0{,}1$
$z/z_m = 0{,}1$
$T/T_m = 0{,}1$
$u/u_m = 0{,}1$

FIG. 7.

for an incompressible fluid the processes of heat transfer and admixture transfer are described by means of absolutely identical equations with the same boundary conditions.

The distributions of ζ, u, v and T for a cylindrical thermal at $\nu_t=0.08$ and $Pr_t=1.0$ are shown in Fig. 5. Their quantitative resemblance to the corresponding distributions for a cylindrical puff at the same value (see Fig. 1) is remarkable. However, with a decrease of ν_t the differences in behaviour of thermals and puffs become more noticeable, being also determined by the value of Pr_t. These differences can be seen in Fig. 6 and 7 where the distributions of ζ, u, v and T are shown for thermals at at values $\nu_t=0.016$ and $Pr_t=1.0$ and 0.5, respectively. The absence of a hydrodynamic wake is an outstanding feature of the vorticity field. On the other side, at $Pr_t=1.0$ and 0.75 a zone arises in front of the thermal, where ζ has an opposite sign, i.e. an oppositely directed circulation takes place. This, in turn, creates concavity of isotherms also in front of the thermally active zone. This fact, and also the existence of peripheral temperature maxima, indicates the tendency to a complete separation of the vortex pair at small values of ν_t. It is also worth noting that the obtained distributions are in good agreement with those in LILLY's [7] paper in spite of slightly different boundary conditions.

The formation of an oppositely directed circulation in cylindrical thermals and some other particularities of vortex pairs can be explained if we consider the equation for the circulation Γ, obtained by integrating Eq. (2.11) over the xy-half-plane.

For the similarity regime $\left(\frac{\partial \Gamma}{\partial \tau}\to 0\right)$ we then have

$$\Gamma=\frac{1}{A-2}\int_{-\infty}^{\infty}\left(\nu_t\frac{\partial \zeta}{\partial y}-BT\right)_{y=0}dx, \tag{4.1}$$

where

$$\Gamma=\int_{-\infty}^{\infty}\int_{0}^{\infty}\zeta dxdy.$$

It is evident that a cylindrical puff ($B=0$) in half-plane can have a one-way circulation only since Γ is simply connected with the distribution of $\partial\zeta/\partial y$ at the axis of symmetry, the latter being everywhere positive. At $\nu_t\to\infty$ $\left.\frac{\partial \zeta}{\partial y}\right|_{y=0}\to 0$ and $\Gamma\to 0$, but as ν_t diminishes $\left.\frac{\partial \zeta}{\partial y}\right|_{y=0}$ increases quicker and Γ generally grows. As a result, the growth of the convective heat transfer in comparison with diffusion leads to the formation of peripheral temperature maxima.

On the other side, the circulation in a cylindrical thermal is determined by the distributions of $\frac{\partial \zeta}{\partial y}$ and T on the axis of symmetry, characterized by the first and second terms in Eq. (4.1), respectively. As ν_t dimishes both terms first increase but the second term increases quicker, thus Γ grows. However, the growth of Γ causes delay in the growth of the second term in Eq. (4.1) owing to the more intensive removing of heat from the axis of symmetry by convection (it is clear that the rate of growth of this term depends also on Pr_t, i.e. on the rate of heat diffusion). Thus, with the further decrease of ν_t the circulation, having reached some maximum value, starts to diminish. But such a decrease of Γ is possible only when zones arise, where ζ has an opposite sign. This conclusion is confirmed by the computations.

The vector fields of relative velocity $\mathbf{w}\{u, v\}-\mathbf{r}\{x, y\}$ with the help of which streamlines of relative motion can be depicted, give a visual notion about the interaction between the vortex pair and the surrouding medium. For a cylindrical puff at considerable values of ν_t (Fig. 8a) all streamlines of relative motion are directed

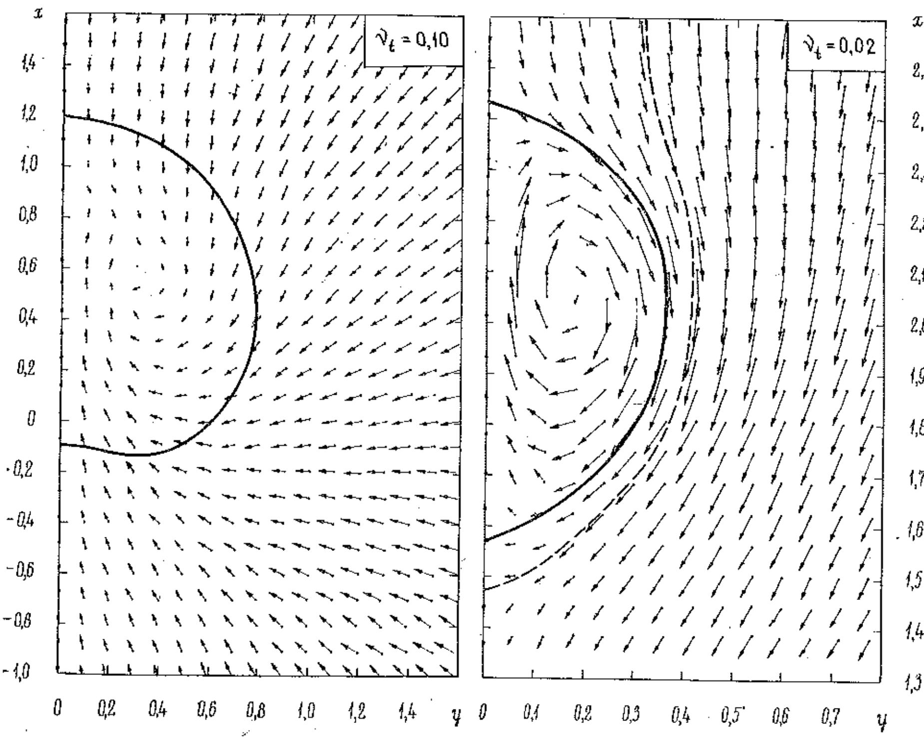


FIG. 8.

into the thermally active zone, and the entrainment velocity is distributed rather uniformly along its boundaries. Such a flow pattern is maintained roughly at $\nu_t > 0.04$. But as ν_t dimishes the relative part of fluid entrained into the active zone through its front boundary increases, while through the rear boundary a substantial entrainment takes place only near the axis of symmetry. In the range of $0.02 < \nu_t < 0.04$ the flow pattern changes (Fig. 8b). There appears a zero streamline (as shown in the dashed line) dividing the flow into two regions as in the case of Hill's vortex. As opposed to the latter, however, this streamline is not closed. The vorticity and heat penetrate from the inner region into the outer by diffusion, forming at the rear of the puff hydrodynamic and heat wakes. Thus, at values of ν_t lower than a certain critical value the momentum and heat content of a cylindrical puff are no longer conserved. This fact, not considered by the accepted theoretical model, is to all appearances the reason for the deterioration of numerical solution convergence at small values of ν_t. The possibility of wake formation is confirmed by experimental data of MAXWORTHY [11] on vortex rings.

Unlike cylindrical puffs the vector fields of relative velocity for cylindrical thermals (Fig. 7) do not point in the whole calculation range of ν_t and Pr_t to the formation of a wake.

It may be stressed that in numerical calculations the ordinate of point where ψ has its maximum value coincides sufficiently well with the value of x_0 calculated by the formula (2.5) for cylindrical puffs at sufficiently great values of ν_t. However, for cylindrical thermals, as well as puffs at small values of ν_t, the formula (2.5) does not yields satisfactory results.

5. COMPARISON WITH EXPERIMENTAL DATA

Comparison with the experimental data of RICHARDS [3, 4] and TSANG [5, 6] shows a good qualitative agreement concerning flow patterns, velocity distributions and character of entrainment, as well as general contours of the zone, where the heat (or passive admixture) is concentrated. However, the separation of thermals,

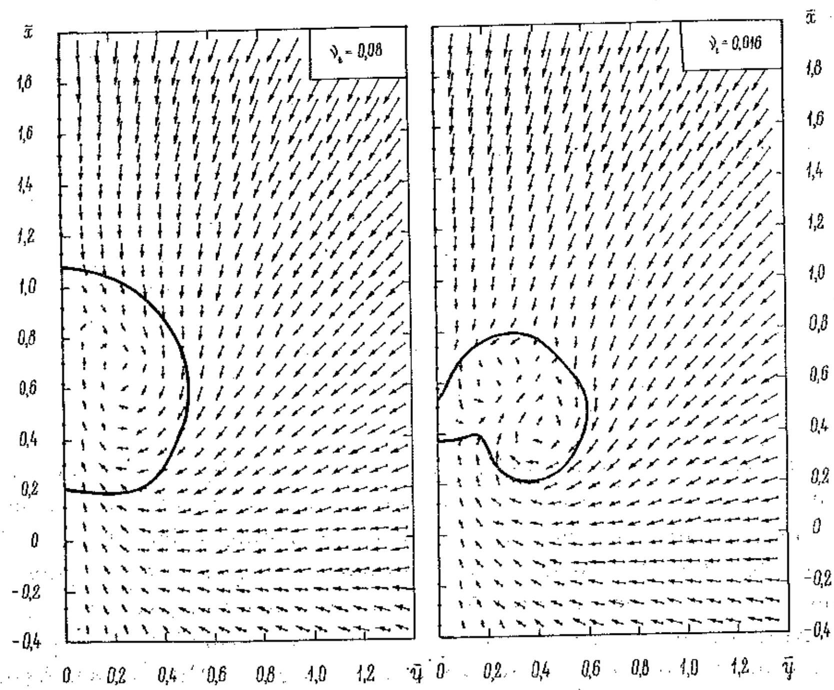


FIG. 9.

characteristic of small values of ν_t, was not observed. While comparing the computations with experiments in quantitative characteristics it must be kept in mind that the real values of Pr_t for vortex pairs are still unknown. We might suppose it to be between 0.7–1.0, i.e. the same as for the turbulent jets and wakes. Besides, it is not clear to what relative concentration of admixture in the experiments the visible outlines of puffs or thermals correspond.

In the case of the similarity regime we may write in the dimensionless variables (at $\kappa=\chi_1=1$)

$$a^3=Kn^2, \tag{5.1}$$

where $n=a/b$, a is the maximum ordinate of the front boundary, b the maximum half-width of the vortes pair, K a proportionality coefficient. In Richards' experiments [3, 4] on cylindrical thermals and puffs the value of K appeared to be practically constant with the mean value of 0.33, though the expansion rate n changed in a rather wide range, as the result of difference in conditions of the vortex pair formation. In Tsang's experiments [5, 6] on cylindrical thermals a comparatively small change of n was obtained owing to a good ordering of the vortex pair formation while the mean value of K was rather close to Richards' results.

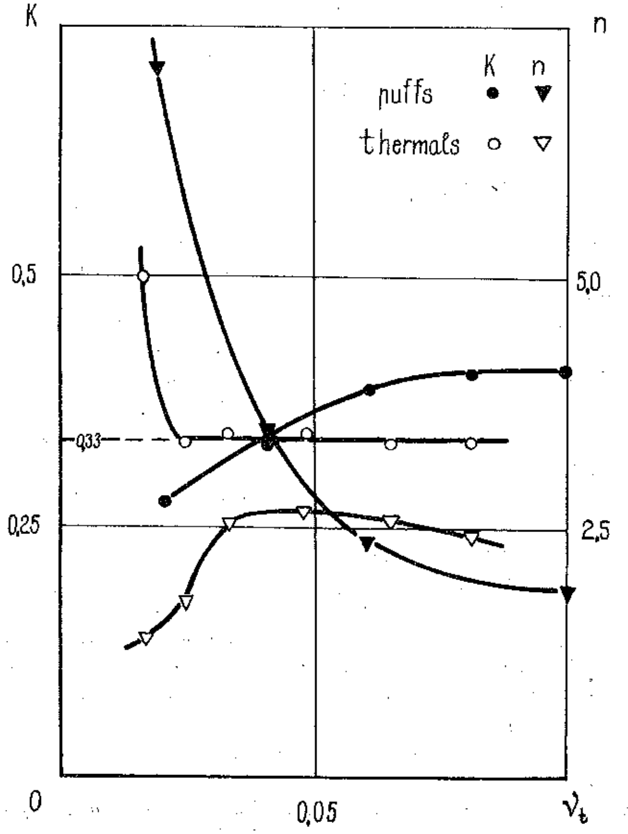


FIG. 10.

Figure 9 represents graphs which show how K and n depend on ν_t at $Pr_t=1$. The boundary of the vortex pair is defined as an isotherm corresponding to the temperature 0.3 T_m, where T_m is the maximum value of temperature. As the graph shows, for cylindrical thermals the value of K is practically constant being close to 0.33. For cylindrical puffs K varies more noticeably, however, in the range of $n=$ $=2.6$–5.7 observed in Richards' experiments the mean value of K is also close to 0.33. Analogous dependencies of K and n on ν_t take place also by defining in a different way of the vortex pair boundaries and at different values of Pr_t, however, the numerical values of these coefficients are somewhat different.

Of some interest are also the results of comparing the calculated temperature distributions in vortes pair with the temperature or admixture concentration distributions found in the experiments with a jet in a cross stream [12–14]. In particular, for non-buoyant jets it seems to be possible to determine qualitative and quantitative dependencies between the eddy diffusivity coefficient ν_t of the vortes pair and the ratio of the initial jet's dynamic pressure to the dynamic pressure of the cross flow q, the greater values of ν_t corresponding to the smaller values of q and vice versa. The result of these comparisons confirm a possibility of principle to create a method of calculating the parameters of a jet in a cross flow, in particular, a smoke plume in a windy environment, basing on the turbulent vortex pair model.

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STRESZCZENIE

RUCH I PRZENOSZENIE CIEPŁA W TURBULENTNYCH PARACH WIRÓW

Rozważany jest dwu-wymiarowy problem turbulentnych par wirów wywołanych nagłym impulsem dynamicznym (podmuchy cylindryczne), bądź przerwaniem ogrzewania (termiki cylindryczne). Równania wirowości i przenoszenia ciepła dla burzliwego przepływu nieściśliwej cieczy przekształcono do postaci bezwymiarowej, zawierającej żądanie osiągnięcia stanu samopodobieństwa. Układ równań rozwiązano numerycznie dla różnych wartości przestrzennej stałej lepkości wirowej i współczynników dyfuzji ciepła. Przedyskutowano otrzymane rozkłady podobieństwa wirowości, prędkości i temperatury dla cylindrycznych podmuchów i termik, i porównano je z dostępnymi danymi doświadczalnymi. Wyniki obliczeń dla podmuchów liniowych wykazują istnienie krytycznej wartości bezwymiarowego współczynnika lepkości wirowej. Poniżej tej wartości występuje zanik pędu i ciepła dla śladu na tyle par wirów.

Резюме

ДВИЖЕНИЕ И ПЕРЕНОС ТЕПЛА В ТУРБУЛЕНТНЫХ ВИХРЕВЫХ ПАРАХ

Рассматривается плоско-параллельная задача о развитии турбулентной вихревой пары, возникающей за счет мгновенного динамического импульса (цилиндрический клуб) или мгновенного тепловыделения (цилиндрический термик). Уравнения переноса завихренности и тепла для турбулентного движения преобразованы к безразмерному виду, который включает в себя требования достижения автомодельного режима. Система уравнений решена численно при различных значениях не зависящих от пространственных координат коэффициентов вихревой вязкости и температуропроводности. Обсуждаются и сравниваются с имеющимися экспериментальными данными полученные автомодельные распределения вихря, скорости и температуры. Результаты расчетов для цилиндрических клубов указывают на существование критического значения безразмерного коэффициента вихревой вязкости, ниже которого имеет место потеря импульса и тепла на образование следа за вихревой парой.

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