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LOCAL METHOD IN RAREFIED GAS AERODYNAMICS

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An approximate method for calculating aerodynamic characteristics of bodies in hypersonic rarefied gas flow is set forth. It is based on assuming that the momentum flux $\bar{p}$ at the body surface is to be determined by the local incidence angle θ_1 irrespective of the body form. The $\bar{p}(\theta_1)$-approximation contains a number of empirical coefficients depending on regime parameters. As a result the problem is split into two parts: first calculating form functions independent of the flow regime and, second finding regime coefficients independent of the body form. Treatment with experimental data has shown the local approach to be sufficiently exact for all Knudsen numbers. An extension of the method to finite Mach numbers is also proposed. Some advisable trends towards the further development of the local theory are discussed.

1. Momentum flux at the body surface

Let us consider a hypersonic rarefied gas flow of a velocity $\bar{U}_\infty$ past a convex body. We venture to suppose that the momentum flux $\bar{p}$ at the body surface is determined by local properties of the surface only. The main local characteristic is the local incidence angle $\theta_1 = \sphericalangle(\bar{n}, -\bar{U}_\infty)$, $\bar{n}$ being the outward normal. Let $\bar{p} = \bar{p}(\theta_1)$ [1]. Neglecting the base pressure one has $\bar{p}=0$ for $\pi/2<\theta_1\leqslant\pi$. At the forepart of the body S_+ we consider the $\bar{p}$-components in the flow coordinate system $\{p_x, p_y, p_z\}$. If $\{p_x, p_y^*\}$ are the $\bar{p}$-components at the $(\bar{n}, \bar{U}_\infty)$-plane, then $p_y = -p_y^* n_y \operatorname{cosec}\theta_1$, $p_z = -p_y^* n_z \operatorname{cosec}\theta_1$. The normal and tangential $\bar{p}$-components are connected with p_x, p_y^* by $p = p_x \cos\theta_1 + p_y^* \sin\theta_1$, $\tau = p_x \sin\theta_1 - p_y^* \cos\theta_1$.

In the framework of the local theory it is reasonable to approximate the function $\bar{p}(\theta_1)$ by formulae of an appropriate exactness level. For the purpose of simplicity, efficiency, and generality [2] we have the expansions

$$(1.1)\qquad p_x = \cos\theta_1 \sum_{k=0} \lambda_{2k}\cos 2k\theta_1, \qquad p_y^* = \sin\theta_1\cos^2\theta_1 \sum_{k=0} \mu_{2k}\cos 2k\theta_1 .$$

Then

$$(1.2)\qquad p = \cos^2\theta_1 \sum_{k=0} p_{2k}\cos 2k\theta_1, \qquad \tau = \sin\theta_1\cos\theta_1 \sum_{k=0} \tau_{2k}\cos 2k\theta_1 .$$

As a first approximation we take

$$(1.3)\qquad p_x = \cos\theta_1(\lambda_0 + \lambda_2\cos 2\theta_1), \qquad p_y^* = \mu_0 \sin\theta_1\cos^2\theta_1 .$$

2. Aerodynamic coefficients

With $\bar{p}$ referred to $\frac{1}{2}\rho_\infty U_\infty^2$ and the radius-vector $\bar{r}$ to a characteristic length R, the dimensionless aerodynamic force and moment coefficients can be written as

$$\bar{c}=\frac{1}{S_0}\iint_{S_+}\bar{p}dS, \quad \bar{c}_m=\frac{1}{S_0 l_0}\iint_{S_+}\bar{r}\times\bar{p}dS, \tag{2.1}$$

where $l_0=L/R$, L is the body length, S_0 — a dimensionless characteristic area; the moment is referred to the coordinate centre. Substituting the expressions (1.3) into Eqs. (2.1) we have for axisymmeteric bodies

$$\begin{gathered} c_x=\lambda_0 c_{x0}+\lambda_2 c_{x2}, \quad c_y=\mu_0 c_{y0}, \\ c_{mz}=\lambda_0 m_{x0}+\lambda_2 m_{x2}+\mu_0 m_{y0}, \end{gathered} \tag{2.2}$$

where

$$c_{xk}=\frac{1}{\pi}\iint_{S_+}\cos\theta_1\cos k\theta_1\,dS, \quad c_{y0}=\frac{-1}{\pi}\iint_{S_+}n_y\cos^2\theta_1\,dS, \tag{2.3}$$

$$m_{xk}=\frac{-1}{\pi l_0}\iint_{S_+}y\cos\theta_1\cos k\theta_1\,dS, \quad m_{y0}=\frac{-1}{\pi l_0}\iint_{S_+}xn_y\cos^2\theta_1\,dS. \tag{2.4}$$

It is essential that if the body form is given, the integrals (2.3), (2.4) are the same for all flow regimes. They will be called as form functions. On the other hand the coefficients λ_k, μ_k are the same for all bodies and depend on the flow regime only. They will be called regime coefficients.

Tables of the form functions for conical, cylindrical, and spherical elements at angles of attack α have been calculated in [3]. Using them one can simply find the form functions for any axisymmetric body composed of these elements.

3. Regime coefficients

The coefficients λ_k, μ_k are determined experimentally for a certain flow regime. Owing to the locality hypothesis they can be found not only by measuring $\bar{p}$ at a given body but also through experimental data for integral aerodynamic characteristics of any body.

Let us have, for example, experimental values c_x^e for a certain body in the interval $\alpha\in[0,\alpha_*]$. Then the coefficients λ_0, λ_2 can be found by minimizing the functional

$$J_x=\int_0^{\alpha_*}(\lambda_0 c_{x0}+\lambda_2 c_{x2}-c_x^e)^2\,d\alpha. \tag{3.1}$$

The flow regime is mainly characterized by the average effective Knudsen number Kn (or Reynolds number Re) based on the α — averaged characteristic length

$$L_*=\frac{1}{\alpha_*}\int_0^{\alpha_*}L(\alpha)\,d\alpha, \tag{3.2}$$

where $L=\sqrt{S_+}$, S_+ is the visible body surface area.

Table 1 contains the values λ_0, λ_2 obtained through experimental data for blunted cones [1] at Re=25÷175. The main value μ_0 in this range is 1.36.

At Kn$\sim 10^{-3}$ the experimental data [4] provide the following regime coefficient values: $\lambda_0=1.05$, $\lambda_2=0.92$, $\mu_0=1.40$.

Table 1

Re		25	50	75	100	125	150	175
M=4÷5	λ_0	2.03	1.71	1.59	1.53	1.48	1.44	1.41
	λ_2	0.24	0.30	0.39	0.40	0.46	0.52	0.56
M=7÷8	λ_0	2.08	1.81	1.69	1.62	1.58	1.55	1.52
	λ_2	0.14	0.20	0.24	0.30	0.34	0.38	0.44

In continuum regime (Kn=0) a treatment with computational data for perfect gas gives $\lambda_0=0.898$, $\lambda_2=0.913$, $\mu_0=1.81$.

In free molecule regime (Kn=∞), where the local theory becomes exact, the coefficients λ_k, μ_k are determined by gas-surface interaction laws [5]. For diffuse scattering with full accommodation one has

$$\lambda_0=2+\frac{4}{\pi}u_m, \qquad \lambda_2=\frac{8}{3\pi}u_m, \qquad \mu_0=\frac{8}{\pi}u_m, \tag{3.3}$$

where u_m is the mean relative reflection velocity connected to the temperature factor t_s by $t_s=4\kappa u_m^2\,[\pi(\kappa-1)]^{-1}$, $\kappa=c_p/c_v$. An analytic approximation of Kn-dependence for the regime coefficients has been proposed in [6].

4. Effect of finite Mach numbers

In the expressions (1.1) $\bar p(\theta_1)\to 0$ as $\theta_1\to\pi/2$, so for finite Mach numbers some corrections are needed. In the framework of the local theory it is reasonable to assume

$$\bar p=\left(\frac{s}{\sqrt{\pi}}\right)^3\int\int\int_{u_{in}<0}\bar p_\infty(\bar u_1)\,u_1^2\exp\left[-s^2(\bar u_1-\bar U_\infty)^2\right]d\bar u_1, \qquad s=\sqrt{\frac{\kappa}{2}}\,\mathrm{M}, \tag{4.1}$$

$\bar p_\infty$ being the momentum flux at $M=\infty$. The asymptotics of $\bar p$ for large M derived from the expressions (4.1) in [2] has a singularity at the shadow boundary. Neglecting shadow neighbourhood leads to convenient formulae with no additonal form functions and regime coefficients simply expressed through the limiting ones. But this provides appreciable errors especially in the case of slender bodies at small incidence. Asymptotic formulae valid up to the shadow boundary are complicated and require the calculation of additional form functions depending on M.

Non-uniformity of the asymptotics appears through the parameter $z=s\cos\theta_1$ which is large for $s\to\infty$ throughout S_+ except near $\theta_1=\pi/2$. Combining the expansions for large and small z so that the main terms are exact we obtain a simplified composed asymptotics which makes it possible to avoid the M-dependence of additional form functions. For example, for p_x we have, to within $O(s^{-1})$,

(4.2) $$p_x = \cos\theta_1 (\lambda_0 + \lambda_2 \cos 2\theta_1) + \frac{\lambda_0 - \lambda_2}{2s\sqrt{\pi}} \sin^2\theta_1 ,$$

so

(4.3) $$c_x = \lambda_0 \left(c_{x0} + \frac{c_{x*}}{2s\sqrt{\pi}} \right) + \lambda_2 \left(c_{x2} - \frac{c_{x*}}{2s\sqrt{\pi}} \right),$$

where

(4.4) $$c_{x*} = \frac{1}{\pi} \int\int_{S_+} \sin^2\theta_1 \, dS .$$

Tables of the additional form functions are included in [3] as well.

Let us evaluate the friction effect on the basis of experimental data [1] for sharp cones of half-angles $\beta = 5$ and $10°$ at angles of attact α from zero to $\alpha_* = 80°$. Together with Eq. (3.1) we minimize the functional

(4.5) $$J_{xs} = \int_0^{\alpha_*} \left[\lambda_0 \left(c_{x0} + \frac{c_{x*}}{2s\sqrt{\pi}} \right) + \lambda_2 \left(c_{x2} - \frac{c_{x*}}{2s\sqrt{\pi}} \right) - c_x^e \right]^2 d\alpha .$$

The values of λ_0, λ_2 obtained for four flow regimes are given in Table 2 which also contains $\Delta = \sqrt{J_x}$, $\Delta_s = \sqrt{J_{xs}}$ and the ratio Δ_s/Δ.

One can see that $\Delta_s < \Delta$ in all cases. It means that the formula (4.3) brings theoretical values of c_x nearer to experimental ones. The ratio Δ_s/Δ decreases with diminishing β. In other words, the smaller β the stronger the corrective effect of

Table 2

β	M	Re	without s			with s			Δ_s/Δ
			λ_0	λ_2	Δ	λ_0	λ_2	Δ_s	
	5	183	1.37	0.55	0.269	1.31	0.66	0.174	0.65
5°	5	66	1.56	0.47	0.225	1.49	0.63	0.099	0.44
	8	183	1.51	0.52	0.309	1.47	0.62	0.236	0.76
	8	83	1.68	0.48	0.336	1.63	0.60	0.203	0.60
	5	138	1.49	0.40	0.064	1.41	0.56	0.051	0.79
10°	5	50	1.72	0.31	0.057	1.62	0.54	0.041	0.72
	8	138	1.65	0.39	0.131	1.58	0.52	0.101	0.77
	8	62	1.95	0.27	0.083	1.86	0.46	0.035	0.42

friction. With increasing M this effect naturally weakens but for $\beta = 10°$ such a regularity is not distinctive. For $\beta = 15°$ the friction effect proves to be within errors of the theory and experiment. Hence for non-slender bodies the limiting formulae (1.3) are sufficient.

Having the regime coefficients λ_0, λ_2 one can calculate theoretical values of c_x by Eqs. (2.2) without taking into account friction effects and by Eq. (4.3) when these effects are included. Figure 1 shows both curves (——— Eqs. (2.2), —·— Eq. (4.3))

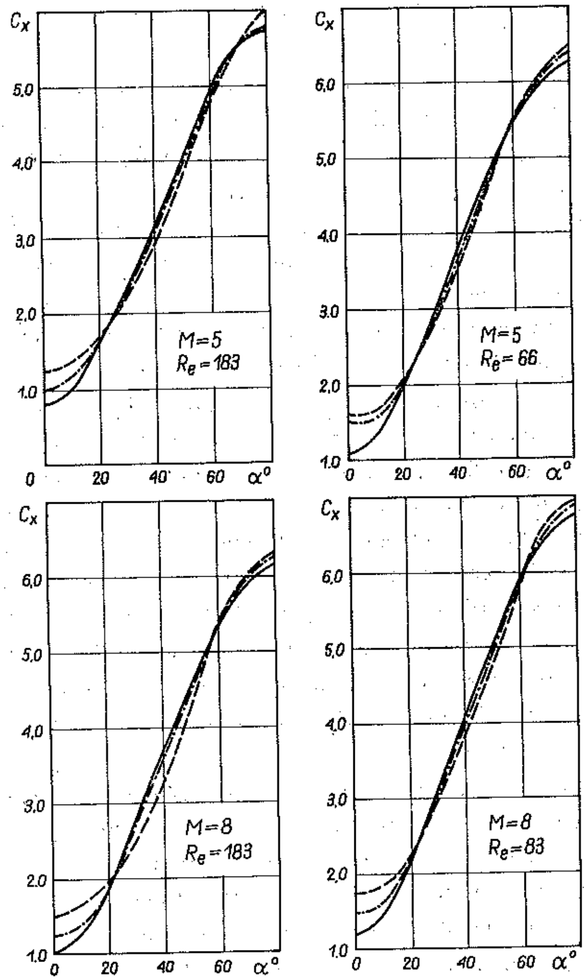

FIG. 1.

together with experimental (— — —) for each of the four regimes at $\beta = 5°$. It is clear that the friction effect is maximum at small angles of attack where the local incidence angle θ_1 is near $\pi/2$ throughout the cone surface.

5. CONCLUSIONS AND SUGGESTIONS

Calculation of aerodynamic characteristics of blunted cones and other bodies by the local interaction theory has shown that the latter is quite applicable to the transitional regime of rarefied gas dynamics as well. The results obtained enable us to hope that the local method will be useful for an engineering solution of problems of hypersonic rarefied gas flow past convex bodies.

The success of the local interaction theory makes it advisible to improve and to extend it in the directions where it has not yet exhausted all the possibilities. Let us point out some prospects of improvement and development of the method with regard to recent relevant papers.

1. Available evidence on Kn-dependence of the regime coefficients is not sufficiently complete and exact [6]. On the basis of experimental data for different regimes it is necessary, first of all, to specify λ_0 (Kn), λ_2 (Kn) and to find μ_0 (Kn).

2. The straggling of values λ_k (Kn), μ_k (Kn) is caused, to a considerable extent, by other regime parameters especially in a highly rarefied gas where an essential role is played by the gas-surface interaction laws [5] which determine the limiting values λ_k^∞, μ_k^∞. Continuing the study of M-dependence it is also necessary to separate effects of t_s and other parameters of interaction with surface. For large Kn the temperature factor becomes a principal argument of μ_k.

3. The macroparameters Kn, M, t_s are not usually sufficient for a proper correlation of experimental data in a highly rarefied gas. Microparameters appearing with the description of gas-surface interaction at the molecular level are unfortunately too numerous [5]. The regime coefficients λ_k, μ_k can play the role of similarity parameters at an intermediate description level. Their only shortcoming consisting in the lack of a direct physical interpretation is compensated by convenience of usage and by clearness of specifying the extension of the set of such parameters.

4. Aerodynamic calculation of canonical forms by the local interaction theory. Using simple analytic expressions to find qualitative regularities, in particular, to solve extremal problems. Using the relations between aerodynamic forces and moments [7]. Studying the non-affine similarity.

5. Improvement and estimation of $\bar{p}\,(\theta_1)$-approximations. Application of the regression analysis to choose the best approximation [8] and the factorial analysis to separate principal parameters. Determination of body and regime classes permitting the improvement of $\bar{p}\,(\theta_1)$-dependence in the framework of the local theory. The tables in [3] are intended to include the coefficients λ_4, μ_2.

6. Estimation and generalization of the very locality hypothesis. Account of curvature and non-local effects of edges and plane elements [9].

7. Extension of the local method by moving from the body surface into the whole flow field of a rarefied gas. We can complet the conservation equations by means of representation of the gasdynamic quantities through functions of a smaller number of arguments [10].

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STRESZCZENIE

METODA LOKALNA W AERODYNAMICE GAZU ROZRZEDZONEGO

W pracy pokazano przybliżoną metodę obliczania charakterystyk aerodynamicznych w naddźwiękowym przepływie gazu rozrzedzonego. Polega ona na założeniu, że strumień pędu $\bar{p}$ na powierzchni ciała jest określony przez lokalny kąt natarcia θ_1 niezależnie od kształtu ciała. Przybliżenie $\bar{p}(\theta_1)$ zawiera pewną ilość współczynników empirycznych zależnych od parametrów układu. W rezultacie problem został rozdzielony na dwa etapy: najpierw obliczono funkcje kształtu niezależne od charakteru przepływu, a następnie znaleziono współczynniki niezależne od kształtu ciała. Porównanie z danymi doświadczalnymi wykazało, że metoda lokala jest dostatecznie dokładna dla wszystkich liczb Knudsena. Zaproponowano również rozszerzenie metody na skończone liczby Macha. Przedyskutowano niektóre korzystne kierunki dalszego rozwoju teorii lokalnej.

Резюме

ЛОКАЛЬНЫЙ МЕТОД В АЭРОДИНАМИКЕ РАЗРЕЖЕННОГО ГАЗА

Излагается приближенный метод аэродинамического расчета тел в гиперзвуковом потоке разреженного газа, основанный на предположении, что поток импульса $\bar{p}$ на поверхности тела определяется местным углом падения θ_1 независимо от формы тела. Аппроксимация функции $\bar{p}(\theta_1)$ содержит несколько эмпирических коэффициентов, зависящих от параметров режима. В результате задача расщепляется на две: вычисление функций формы, не зависящих от режима обтекания и нахождение коэффициентов режима не зависящих от формы тела. Обработка экспериментальных данных показывает достаточно хорошую точность локального подхода для всех чисел Кнудсена. Предлагается также обобщение метода на конечные числа Маха. Обсуждаются целесообразные направления дальнейщего развития локальной теории.

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