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HYDROMAGNETIC UNSTEADY MIXED CONVECTION AND MASS TRANSFER FLOW PAST A VERTICAL POROUS PLATE IMMERSED IN A POROUS MEDIUM WITH THE HALL EFFECT

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The present work is concerned with unsteady mixed convection and mass transfer flow with Hall effect of an electrically conducting incompressible viscous fluid through a porous medium bounded by an infinite vertical plate subjected to suction/injection velocity in the presence of a constant magnetic field. The magnetic field is applied transversely to the direction of the flow. The resulting problem has been solved analytically and the solutions are found for velocity, temperature, concentration of the species, skin-friction, surface heat flux and mass flux. The effects of material parameters on the flow characteristics are expressed and illustrated/discussed by graphs and table.

Key words: Hall effect, porous medium, mixed convection, mass transfer.

Notations

$\mathbf{H}$	applied magnetic field,
$\mathbf{J}$	current density vector,
$\mathbf{E}$	electric field,
e	charge of electron,
n_e	number density of electrons,
P_e	electron pressure,
$B_0 = \mu_e H_0$	magnetic field induction,
K	permeability of porous medium,
$m = \omega_e T_e$	Hall parameter,
κ	thermal conductivity,
T_∞	temperature of the fluid far away from the plate,
C_∞	concentration of the species far away from the plate,
$\mathbf{B}$	induced magnetic field,
$\mathbf{V}$	velocity vector,
g	acceleration due to gravity,
C_p	specific heat of the fluid at constant pressure,
M	magnetic field parameter,
Pr	Prandtl number,
Sc	Schmidt number,

G	Grashof number,
Gc	modified Grashof number,
θ	non-dimensional temperature,
C	non-dimensional concentration,
D	the chemical molecular diffusivity,
t	non-dimensional time,
T	reference temperature,
u	x-component of flow velocity,
v	y-component of flow velocity,
w	z-component of flow velocity,
x, y, z	Cartesian co-ordinate system,
V_0	injection velocity,
ω_e	cyclotron frequency,
τe	electron collision time,
μ	coefficient of viscosity of the fluid,
μe	magnetic permeability,
ρ	density of the fluid,
σ	electrical conductivity,
ν	kinematic viscosity,
β	free convection term,
β^*	volumetric coefficient,
η	modified y-coordinate,
Ω	frequency parameter,
ω	angular frequency,
Ψ	velocity function,
γ	phase angle for temperature field,
τ_1	skin friction along x-axis,
τ_2	skin friction along z-axis,
r	real part,
i	imaginary part.

1. Introduction

The phenomenon of heat and mass transfer has been the object of extensive research due to its applications in science and technology. Such phenomenon is observed in buoyancy – induced motions in the atmosphere, in bodies of water, quasi-solid bodies such as earth and so on. Some of the convective heat and mass transfer processes with phase change include the evaporation of a liquid at the interface between a gas and a liquid or the sublimation at a gas-solid interface. They can be described using the method for convective heat and mass transfer. Separation process in chemical engineering such as drying of solid materials, distillation, extraction and absorption, are all affected by the process of mass transfer. They also play a role in the production of materials in order to obtain the desired properties of a substance. Chemical reactions, including the combustion process, are often decisively determined by mass transfer. As examples of these types of processes, the evaporation, condensations, distillation, rectification and absorption of a fluid should all be mentioned (Baehr [1]).

The requirements of modern technology have stimulated interest in fluid flow studies which involve the interaction of several phenomena. One of them is related to the effect of free convection flow through a porous medium which plays an important role in agriculture, engineering, petroleum industries and heat transfer. The convection problem in a porous medium has also important applications in geothermal reservoirs and geothermal energy extractions. In order to utilize the geothermal energy to a maximum, one should have a complete and precise knowledge of the amount of perturbations needed to generate the convection currents in geothermal fluids. A comprehensive review of the studies of convective heat transfer mechanism through porous media has been made by NIELD and BEJAN [2]. Free convection flow past a vertical plate has been studied extensively by OSTRACH [3–6], RILEY *et al.* [7], DEY *et al.* [8], KAWOSE *et al.* [9], WEISS *et al.* [10] and PANTOKRATORAS [11] in numerous ways to include various physical aspects. GALLAHAN *et al.* [12], SOUNDALGEKAR *et al.* [13, 14], KHAIR *et al.* [15], LIN *et al.* [16, 17] and RAPTIS [18] have also studied the combined effect of thermal and mass diffusion along the vertical plate in numerous ways. The problem of magnetohydrodynamic viscous flow through porous medium past a vertical plate has been studied by TAKHAR *et al.* [19], ALCHAR *et al.* [20], ALDOSS *et al.* [21], SINGH *et al.* [22], SATTAR *et al.* [23], K.A. HELMY [24], with different physical conditions.

Important progress has been made during the last few decades in the development of magnetohydrodynamics due to its importance in engineering applications. The interest in these new problems stems from their importance in liquid metals, electrolytes and ionized gases. The thermal physics of MHD processes and MHD mass transfer are of interest in power engineering and metallurgy. The boundary zone between hydraulics and thermal physics is the area of many cross galvano and thermomagnetic effects. These phenomena are important in the study of semiconductor materials. In magnetohydrodynamics, serious attention has been given only to the transverse galvanomagnetic effect, i.e. the Hall effect: crossed phenomena also occur in the interaction of heat and mass transfer and hydraulics and mass transfer processes. The mechanism of conduction in ionized gases (low density) in presence of strong magnetic field is different from that in a metallic substance. The electric current in ionized gas is usually carried by electrons which undergo successive collisions with other charged or neutral particles. In case of ionized gas, the current is not proportional to the applied potential except when the electric field is very weak. When the electric field is strong, the conductivity parallel to the electric field is reduced and current is induced in the direction normal to both the electric and magnetic fields. This phenomenon is known as the Hall effect. The effect can be taken into account within the range of magnetohydrodynamical approximation.

The Hall effect on the fluid with variable concentration has a lot of applications in MHD power generations, several astrophysical and meteorological studies as well as in flow of plasma through MHD power generators. From the point of view of applications, model studies on the Hall effect on the free and forced convection flows have been made by several investigators. Some of them are Datta *et al.* [25], Acharya *et al.* [26–27] and Biswal [28]. However, the authors [25–28] studied the Hall effect on convection and mass transfer flow past a porous plate only, while [28] considered the effect of Hall on free convection flow of a visco-elastic fluid.

The problem investigated here is the study of the Hall effect on the combined heat and mass transfer unsteady flow, which occur due to buoyancy forces caused by thermal diffusion (temperature differences) and mass diffusion (concentration differences) of comparable magnitude, past a vertical porous plate which is immersed in porous medium with a constant magnetic field applied perpendicular to the plate. The plate is kept at the oscillating temperature and concentration.

2. Formulation

Consider the unsteady flow of a viscous incompressible and electrically conducting fluid past an infinite vertical porous plate in presence of transverse magnetic field. The x-axis is chosen along the plate in the upward direction while the y-axis is chosen normal to it and pointing away from the plate surface. All the properties of the fluid are assumed to be constant, except the body force term causing the buoyancy effect. The effect of Hall current gives rise to a force in the z-direction which induces a cross-flow in that direction. Thus the flow becomes three-dimensional. The physical configuration considered here is shown in the figure A. The equation governing the flow of fluid together with Maxwell's electromagnetic equations are as follows:

Continuity equation

$$\nabla.\mathbf{V} = 0, \tag{2.1}$$

Momentum equation

$$\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}.\nabla)\mathbf{V} = -\frac{1}{\rho}\nabla P+\nu\nabla^2\mathbf{V}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)+\frac{1}{\rho}(\mathbf{J}\times\mathbf{B}), \tag{2.2}$$

Energy equation

$$\frac{\partial T}{\partial t} + (\mathbf{V}.\nabla)T = \kappa\nabla^2 T, \tag{2.3}$$

Generlized Ohm's Law

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}) - \frac{\sigma}{en_e}(\mathbf{J} \times \mathbf{B} - \nabla P_e), \tag{2.4}$$

Maxwell's equation

$$\nabla \times \mathbf{H} = \mathbf{J}, \qquad \nabla \times \mathbf{E} = 0, \qquad \nabla.\mathbf{B} = 0. \tag{2.5}$$

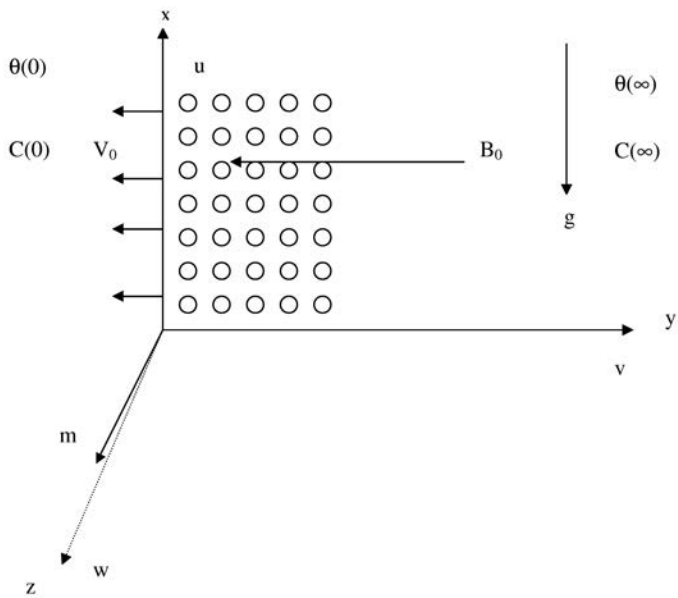

Fig. A. Physical model of the problem.

By assuming a very small magnetic Reynolds number, the induced magnetic field is neglected [29] in comparison to the applied magnetic field, so that $\mathbf{B} = (0, B_0, 0)$. Since no applied or polarization voltage is imposed on the flow field, the electric field vector $\mathbf{E} = 0$. This then corresponds to the case when no energy is added or extracted from the fluid by the electric field. The equation of conservation of electric charge $\nabla.\mathbf{J} = 0$ gives $J_y =$ constant, where $\mathbf{J} = (J_x, J_y, J_z)$. As the plate is non-conducting, $J_y = 0$ at the plate and hence vanishes everywhere. Considering the magnetic field strength to be very large, the corresponding generalized Ohm's law in the absence of electric field takes the following form:

$$\mathbf{J} + \frac{\omega_e \tau_e}{B_0}(\mathbf{J} \times \mathbf{B}) = \sigma\left(\mathbf{V} \times \mathbf{B} + \frac{\nabla P_e}{en_e}\right). \tag{2.6}$$

For weakly ionized gases, the electron pressure gradient and ion slip effects (arising out of imperfect coupling between ions and neutrals) are neglected. Then Eq. (2.6) reduces to

$$J_x = \frac{\sigma B_0}{1+m^2}(mu - w),$$
$$J_z = \frac{\sigma B_0}{1+m^2}(u + mw). \tag{2.7}$$

The equations of motion, energy and concentration governing the flow under the usual Boussinesq approximation are:

Momentum equations

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2(u+mw)}{\rho(1+m^2)} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\nu u}{K}, \tag{2.8}$$

$$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y} = \nu\frac{\partial^2 w}{\partial y^2} + \sigma\frac{B_0^2(mu - w)}{\rho(1+m^2)} - \frac{\nu w}{K}, \tag{2.9}$$

Energy equation

$$\frac{\partial}{\partial t}(T - T_\infty) + v\frac{\partial(T - T_\infty)}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2(T - T_\infty)}{\partial y^2}, \tag{2.10}$$

Concentration equation

$$\frac{\partial}{\partial t}(C - C_\infty) + v\frac{\partial(C - C_\infty)}{\partial y} = \frac{D\partial^2(C - C_\infty)}{\partial y^2}. \tag{2.11}$$

In Eq. (2.10), the viscous dissipation and Ohmic dissipation are neglected and in Eq. (2.11), the term due to chemical reaction is assumed to be absent. Now using $v = -V_0$ in Eqs. (2.8) and (2.9), $T(y,t) - T_\infty = \theta(y,t)$ in Eq. (2.10) and $C(y,t) - C_\infty = C^*(y,t)$ in Eq. (2.11), subjected to the initial boundary conditions

$$\begin{array}{llllll} t\leq 0: & & u(y,t)=w(y,t)=0, & \theta=0, & C^*=0 & \text{for all } y, \\ t>0: & \left\{\begin{array}{l} \\ \\ \end{array}\right. & u(0,t)=w(0,t)=0, & \theta(0,t)=ae^{i\omega t}, & C^*(0,t)=be^{i\omega t}, & \text{at } y=0, \\ & & u(\infty,t)=w(\infty,t)=0, & \theta(\infty,t)=0, & C^*(\infty,t)=0 & \text{as } y\to\infty, \end{array} \tag{2.12}$$

and using non-dimensional parameters

$$\eta=\frac{V_0 y}{\nu},\qquad t'=\frac{V_0^2 t}{4\nu},\qquad u'=\frac{u}{V_0},\qquad w'=\frac{w}{V_0},$$
$$\theta'=\frac{\theta}{a},\qquad C'=\frac{C^*}{b},\qquad \mathrm{G}=\frac{4g\beta\nu a}{V_0^3},\qquad \mathrm{Gc}=\frac{4g\beta^*\nu b}{V_0^3},\tag{2.13}$$
$$M=\frac{4\beta_0^2\sigma\nu}{\rho V_0^3},\qquad \mathrm{Pr}=\frac{\nu\rho C_p}{\kappa},\qquad K'=\frac{V_0^2 K}{4\nu^2},\qquad \mathrm{Sc}=\frac{\nu}{D},$$

Eqs. (2.8) to (2.11) are transformed to their corresponding non-dimensional form (dropping the dashes) as

$$\frac{\partial u}{\partial t}-4\frac{\partial u}{\partial \eta}=4\frac{\partial^2 u}{\partial \eta^2}-\frac{M}{1+m^2}(mw+u)+\mathrm{G}\theta+\mathrm{Gc}C-\frac{u}{K},\tag{2.14}$$

$$\frac{\partial w}{\partial t}-4\frac{\partial w}{\partial \eta}=4\frac{\partial^2 w}{\partial \eta^2}+\frac{M}{1+m^2}(mu-w)-\frac{w}{K},\tag{2.15}$$

$$\frac{\partial \theta}{\partial t}-4\frac{\partial \theta}{\partial \eta}=\frac{4}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial \eta^2},\tag{2.16}$$

$$\frac{\partial C}{\partial t}-4\frac{\partial C}{\partial \eta}=\frac{4}{\mathrm{Sc}}\frac{\partial^2 C}{\partial \eta^2}.\tag{2.17}$$

The modified boundary conditions become

$$\begin{array}{lllll} t\le 0: & u(\eta,t)=w(\eta,t)=0, & \theta=0, & C=0 & \forall\eta,\\ t>0: \left\{\begin{array}{l} \\ \\ \end{array}\right. & \begin{array}{l} u(0,t)=w(0,t)=0,\\ u(\infty,t)=w(\infty,t)=0,\end{array} & \begin{array}{l}\theta(0,t)=e^{i\omega t},\\ \theta(\infty,t)=0,\end{array} & \begin{array}{l} C(0,t)=e^{i\omega t},\\ C(\infty,t)=0\end{array} & \begin{array}{l}\text{at } \eta=0,\\ \text{as } \eta\to\infty,\end{array}\end{array}\tag{2.18}$$

3. Solution

Equations (2.14) and (2.15) can be combined using the complex variable

$$\Psi=u+iw\tag{3.1}$$

giving

$$\frac{\partial^2\Psi}{\partial\eta^2}+\frac{\partial\Psi}{\partial\eta}-\frac{1}{4}\frac{\partial\Psi}{\partial t}-\frac{1}{4}\left[\frac{M}{1+m^2}(1-im)+\frac{1}{K}\right]\Psi=-\frac{1}{4}\mathrm{G}\theta-\frac{1}{4}\mathrm{Gc}C.\tag{3.2}$$

Introducing the non-dimensional parameter $\Omega = \dfrac{4\nu\omega}{V_0^2}$ and using Eq. (3.1), the boundary conditions in (2.18) are transformed to

$$\begin{aligned} &\Psi(0,t) = \Psi(\infty,t) = 0 \qquad \text{and} \qquad C(0,t) = e^{i\Omega t}, \\ &\theta(0,t) = e^{i\Omega t}, \qquad \theta(\infty,t) = 0, \qquad C(\infty,t) = 0. \end{aligned} \tag{3.3}$$

Putting $\theta(\eta,t) = e^{i\Omega t} f(\eta)$ in Eq. (2.16), we get

$$f''(\eta) + \Pr f'(\eta) - \frac{i\Omega \Pr}{4} f(\eta) = 0, \tag{3.4}$$

which has to be solved under the boundary condition

$$f(0) = 1, \qquad f(\infty) = 0. \tag{3.5}$$

Hence

$$f(\eta) = e^{-\frac{\eta}{2}\left[\Pr + \sqrt{\Pr^2 + i\Omega \Pr}\right]}$$

$$\Rightarrow \theta(\eta,t) = e^{i\Omega t - \frac{\eta}{2}\left[\Pr + \sqrt{\Pr^2 + i\Omega \Pr}\right]}. \tag{3.6}$$

Separating real and imaginary parts, the real part is given by

$$\theta_r(\eta,t) = \cos\left\{\Omega t - \frac{\eta}{2} R_1 \sin\frac{\alpha}{2}\right\} e^{-\frac{\eta}{2}\left(\Pr + R_1 \cos\frac{\alpha}{2}\right)} \tag{3.7}$$

where

$$\begin{aligned} R_1 &= \Pr^{1/2}(\Pr^2 + \Omega^2)^{1/4}, \\ \alpha &= \tan^{-1}\left(\frac{\Omega}{\Pr}\right). \end{aligned} \tag{3.8}$$

Putting $C(\eta,t) = e^{i\Omega t} g(\eta)$ in Eq. (2.17), we get

$$g''(\eta) + \mathrm{Sc}\, g'(\eta) - \frac{i\Omega \mathrm{Sc}}{4} g(\eta) = 0, \tag{3.9}$$

which has to be solved under the boundary condition

$$g(0) = 1, \qquad g(\infty) = 0. \tag{3.10}$$

Hence

$$g(\eta) = e^{\frac{1}{2}\eta\left[-\mathrm{Sc} - \sqrt{\mathrm{Sc}^2 + i\Omega \mathrm{Sc}}\right]},$$

so that

$$C(\eta,t)=e^{i\Omega t-\frac{\eta}{2}\left[\mathrm{Sc}+\sqrt{\mathrm{Sc}^2+i\Omega\mathrm{Sc}}\right]}. \tag{3.11}$$

Separating real and imaginary parts, the real part is given by

$$C_r(\eta,t)=\cos\left\{\Omega t-\frac{\eta}{2}R_2\sin\frac{\beta}{2}\right\}e^{-\frac{\eta}{2}\left(\mathrm{Sc}+R_2\cos\frac{\beta}{2}\right)}, \tag{3.12}$$

where

$$\begin{aligned} R_2&=\mathrm{Sc}^{1/2}(\mathrm{Sc}^2+\Omega^2)^{1/4},\\ \beta&=\tan^{-1}\left(\frac{\Omega}{\mathrm{Sc}}\right). \end{aligned} \tag{3.13}$$

In order to solve Eq. (3.2), substituting $\Psi=e^{i\Omega t}f(\eta)$ and using boundary conditions

$$F(0)=0,\qquad F(\infty)=0. \tag{3.14}$$

and then separating real and imaginary parts, we obtain

$$\begin{aligned} u=\Big[&\{A_{18}\eta_{12}\cos(A_{21}\eta/2)+A_{19}\sin(A_{21}\eta/2)\}e^{-A_{20}\eta/2}\\ &-\{A_{22}\cos(\Omega t-A_3\eta/2)e^{-A_2\eta/2}+A_{23}\sin(\Omega t-A_3\eta/2)e^{-A_3\eta/2}\}A_{26}\\ &-\{A_{24}\cos(\Omega t-A_{10}\eta/2)e^{-A_9\eta/2}+A_{25}\sin(\Omega t-A_{10}\eta/2)e^{-A_{10}\eta/2}\}A_{27}\Big]\cos\Omega t\\ -\Big[&\{A_{19}\cos(A_{21}\eta/2)-A_{18}\sin(A_{21}\eta/2)\}e^{-A_{20}\eta/2}\\ &-\{A_{22}\sin(\Omega t-A_3\eta/2)e^{-A_3\eta/2}-A_{23}\cos(\Omega t-A_3\eta/2)e^{-A_2\eta/2}\}A_{26}\\ &-\{A_{24}\sin(\Omega t-A_{10}\eta/2)e^{-A_{10}\eta/2}\\ &-A_{25}\cos(\Omega t-A_{10}\eta/2)e^{-A_9\eta/2}\}A_{27}\Big]\sin\Omega t, \end{aligned} \tag{3.15}$$

$$\begin{aligned} w=\Big[&\{A_{18}\cos(A_{21}\eta/2)+A_{19}\sin(A_{21}\eta/2)\}e^{-A_{20}\eta/2}\\ &-\{A_{22}\cos(\Omega t-A_3\eta/2)e^{-A_2\eta/2}+A_{23}\sin(\Omega t-A_3\eta/2)e^{-A_3\eta/2}\}A_{26}\\ &-\{A_{24}\cos(\Omega t-A_{10}\eta/2)e^{-A_9\eta/2}+A_{25}\sin(\Omega t-A_{10}\eta/2)e^{-A_{10}\eta/2}\}A_{27}\Big]\sin\Omega t\\ +\Big[&\{A_{19}\cos(A_{21}\eta/2)-A_{18}\sin(A_{21}\eta/2)\}e^{-A_{20}\eta/2}\\ &-\{A_{22}\sin(\Omega t-A_3\eta/2)e^{-A_3\eta/2}-A_{23}\cos(\Omega t-A_3\eta/2)e^{-A_2\eta/2}\}A_{26}\\ &-\{A_{24}\sin(\Omega t-A_{10}\eta/2)e^{-A_{10}\eta/2}\\ &-A_{25}\cos(\Omega t-A_{10}\eta/2)e^{-A_9\eta/2}\}A_{27}\Big]\sin\Omega t. \end{aligned} \tag{3.16}$$

The shearing stress at the wall along the x-axis is given by

$$\tau_1 = \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0}, \tag{3.17}$$

and the shearing stress at the wall along the z-axis is given by

$$\tau_2 = \left(\frac{\partial w}{\partial \eta}\right)_{\eta=0}, \tag{3.18}$$

The surface heat flux is given by

$$Q(t) = \frac{1}{2}\left[\mathrm{Pr}\cos\Omega t + R_1\cos\left(\Omega t + \frac{\alpha}{2}\right)\right], \tag{3.19}$$

and the mass flux is given by

$$C(t) = \frac{1}{2}\left[\mathrm{Sc}\cos\Omega t + R_2\cos\left(\Omega t + \frac{\beta}{2}\right)\right], \tag{3.20}$$

where

$$R_1 = \mathrm{Pr}^{1/2}(\mathrm{Pr}^2 + \Omega^2)^{1/4}, \qquad R_2 = \mathrm{Sc}^{1/2}(\mathrm{Sc}^2 + \Omega^2)^{1/4},$$

$$R_3 = \left[\left(\frac{1}{K} + 1 + \frac{M}{1+m^2}\right)^2 + \left(\Omega - \frac{Mm}{1+m^2}\right)^2\right]^{1/4},$$

$$\alpha = \tan^{-1}\left(\frac{\Omega}{\mathrm{Pr}}\right), \qquad \beta = \tan^{-1}\left(\frac{\Omega}{\mathrm{Sc}}\right), \qquad \gamma = \tan^{-1}\frac{\left(\Omega - \frac{Mm}{1+m^2}\right)}{A_1},$$

$$A_1 = \frac{1}{K} + 1 + \frac{M}{1+m^2}, \qquad A_2 = \mathrm{Pr} + R_1\cos\alpha/2, \qquad A_3 = R_1\sin\alpha/2,$$

$$A_4 = \frac{1}{K} + \frac{M}{1+m^2}, \qquad A_5 = \Omega - \frac{Mm}{1+m^2}, \qquad A_6 = (A_2^2 - A_3^2 - 2A_2 - A_4),$$

$$A_7 = (2A_2A_3 - 2A_3 - A_5), \qquad A_8 = A_6^2 + A_7^2, \qquad A_9 = \mathrm{Sc} + R_2\cos\beta/2,$$

$$A_{10} = R_2\sin\beta/2, \qquad A_{11} = (A_9^2 - A_{10}^2 - 2A_9 - A_4),$$

$$A_{12} = (2A_9A_{10} - 2A_{10} - A_5), \qquad A_{13} = (\mathrm{G}A_6\cos\Omega t + \mathrm{G}A_7\sin + \Omega t)A_{26},$$

$$A_{14} = (\mathrm{G}A_6\sin\Omega t - \mathrm{G}A_7\cos\Omega t)A_{26},$$

$$A_{15} = (\mathrm{Gc}A_{11}\cos\Omega t + \mathrm{Gc}A_{12}\sin\Omega t)A_{27},$$

$$A_{16} = (\mathrm{Gc}A_{11}\sin\Omega t - \mathrm{Gc}A_{12}\cos\Omega t)A_{27}, \qquad A_{17} = A_{11}^2 + A_{12}^2,$$

$$A_{18} = A_{13} + A_{15}, \qquad A_{19} = A_{14} + A_{16}, \qquad A_{20} = 1 + R_3 \cos \gamma/2,$$

$$A_{21} = R_3 \sin \gamma/2, \qquad A_{22} = \mathrm{G}A_6, \qquad A_{23} = \mathrm{G}A_7, \qquad A_{24} = \mathrm{Gc}A_{11},$$

$$A_{25} = \mathrm{Gc}A_{12}, \qquad A_{26} = 1/A_8, \qquad A_{27} = 1/A_{17}.$$

4. Results and discussion

A study of the velocity field, variations of temperature and concentration, shearing stresses, surface heat flux and mass flux in hydromagnetic mixed convective flow past an infinite vertical plate through porous medium with Hall effect, has been carried out in the preceding sections. Approximate solutions are obtained for various flow variables. In order to get insight into the physical situation of the problem, we have computed the numerical values of the velocity, temperature, concentration, shearing stress, surface heat flux and mass flux for different values of m (Hall parameter), M (Magnetic parameter), Sc (Schmidt number), Pr (Prandtl number) and Ω (Frequency parameter). The values of G (Grashof number for heat transfer) are taken equal to 5.0 (G $>$ 0, cooled Newtonian fluid) and -5.0 (G $<$ 0 heated Newtonian fluid). The values of modified Grashoff number (Gc, for mass transfer) Ωt and permeability (K) are taken equal to 2.0, $\pi/2$ and 1, respectively. The obtained numerical results are illustrated and tabulated in Figs. 1 to 10 and Table 1. The velocity components, temperature and concentration versus η are shown in Figs. 1 to 6, but shearing stress versus m are shown in Fig. 7 to 10.

Table 1. Variations of $C(t)$ and $Q(t)$ for different values of Ω, Sc and Pr.

Ω	$C(t)$			$Q(t)$		
	Sc = 0.22	Sc = 0.30	Sc = 0.78	Pr = 0.025	Pr = 0.71	Pr = 7.0
0	0.22	0.30	0.78	0.025	0.71	7.0
0.2	0.08	0.12	0.38	0.012	0.34	3.74
0.4	−0.17	−0.21	−0.41	−0.011	−0.38	−3.00
0.6	−0.27	−0.35	−0.81	−0.024	−0.74	−6.95
0.8	−0.08	−0.12	0.39	−0.013	−0.35	−4.42
1.0	0.21	0.26	0.44	0.011	0.42	2.22

Figures 1 and 2 depict the velocity component u for a cooled Newtonian fluid (G $>$ 0) and for a heated Newtonian fluid (G $<$ 0), respectively. It is drawn for Pr $=$ 0.71 (Prandtl number for air at 20°C) and Pr $=$ 7.0 (Prandtl number for water at 20°C), taking different values of m, M, Ω and Sc. It is observed that an

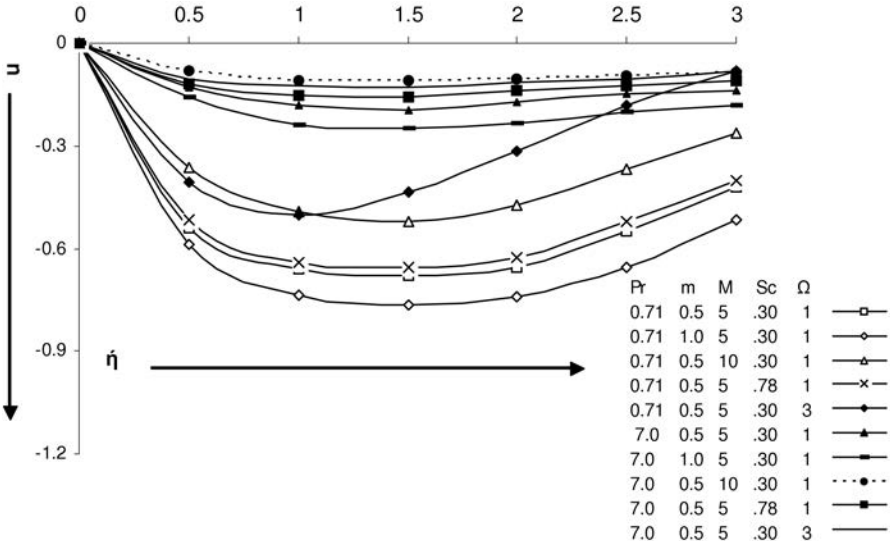

FIG. 1. Variation of velocity component u for Gc = 2.0, G = 5.0, $\Omega t = \pi/2$.

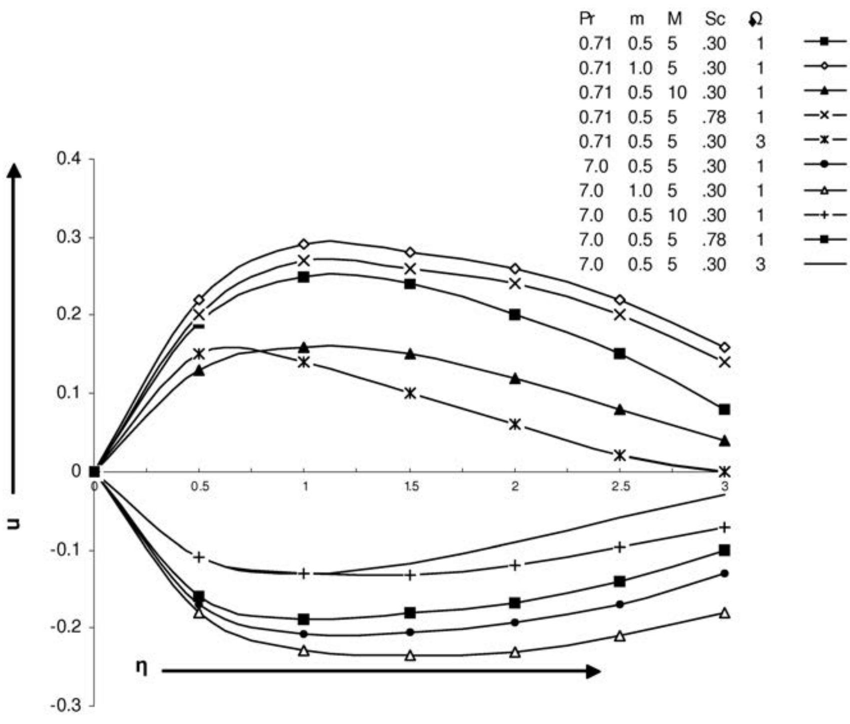

FIG. 2. Variation of velocity component u for Gc = 2.0, G = −5.0, $\Omega t = \pi/2$.

increase in the Hall parameter leads to decrease in the velocity for both air and water in a cooled Newtonian fluid. For a heated Newtonian fluid, the velocity increases with increasing Hall parameter (m) for air, but in the case of water, a reverse effect is observed. It is noticed that an increase in the magnetic parameter (M) leads to a rise in the velocity for both air and water for an externally cooled ($\mathrm{Gr} > 0$) plate. In the case of externally heated plate ($\mathrm{Gr} < 0$) and $\mathrm{Pr} = 0.71$ (air), we have observed that an increase in the magnetic parameter decreases the velocity, while a reverse effect is noticed in water ($\mathrm{Pr} = 7.0$). The velocity is greater for Ammonia ($\mathrm{Sc} = 0.78$, at temperature 25°C and 1 atmosphere) than that of Helium ($\mathrm{Sc} = 0.30$, at temperature 25°C and 1 atmosphere) for $\mathrm{Pr} = 0.71$ or 7.0 and G $>$ or $<$ 0. We also observe that an increase in the frequency parameter (Ω) gives rise to the velocity for air/water and $\mathrm{G} > 0$. For the heated plate in the air, it is found that an increase in Ω leads to a fall in the values of the velocity, while a reverse effect is observed for water. Further, it is noticed that the velocity distribution increases/decreases gradually near the plate ($0 < \eta \leq 1$) and then decreases/increases slowly far away from the plate ($\eta \gg 1$). A comparative study of the curves reveals that the values of the velocity increase/decrease at each point with variations in m or M or Sc or Ω or Pr. It is concluded that the maximum/minimum of the velocity occurs in the vicinity of the plate and the rise and fall in the values of the velocity are more dominant in the case of air ($\mathrm{Pr} = 0.71$) than those of water ($\mathrm{Pr} = 7.0$). The velocity profiles remain negative for $\mathrm{Gr} > 0$ (cooled Newtonian fluid) and positive for $\mathrm{G} < 0$ (heated Newtonian fluid) in the case of $\mathrm{Pr} = 0.71$ near the plate and fade far away from the plate. However, in the case when $\mathrm{Pr} = 7.0$, the velocity profiles remain negative for cooled/heated fluid. In all the situations, the velocity profiles remain always in phase.

The velocity component w has been shown in Figs. 3 and 4 for cooled Newtonian fluid ($\mathrm{G} > 0$) and heated Newtonian fluid ($\mathrm{G} < 0$), respectively. An increase in the Hall parameter leads to an increase in the velocity for both air and water in a cooled Newtonian fluid. For a heated Newtonian fluid, the velocity decreases with increasing Hall parameter for air, while reverse effect is observed in the case of water. It is observed that an increase in the magnetic parameter (M) leads to a rise in the velocity for both air and water for externally cooled ($\mathrm{G} > 0$) plate. For an increasing M, there is a fall in the velocity for air but a rise in the case of water for externally heated plate ($\mathrm{G} < 0$). The velocity is greater for Ammonia than that for Helium with $\mathrm{Pr} = 0.71$ or 7.0 and $\mathrm{G} > 0$ or $\mathrm{G} < 0$. Increasing Ω, the velocity decreases for air and water and $\mathrm{G} > 0$. When $\mathrm{G} < 0$ and Ω is increasing, w increases for $\mathrm{Pr} = 0.71$, while it decreases for $\mathrm{Pr} = 7.0$. The maximum/minimum of w occurs away from the plate ($\eta > 1$) and becomes almost constant as we move farther from the plate. The w remains positive in a Newtonian cooled fluid for both $\mathrm{Pr} = 0.71$ or 7.0. For $\mathrm{G} < 0$ (heated

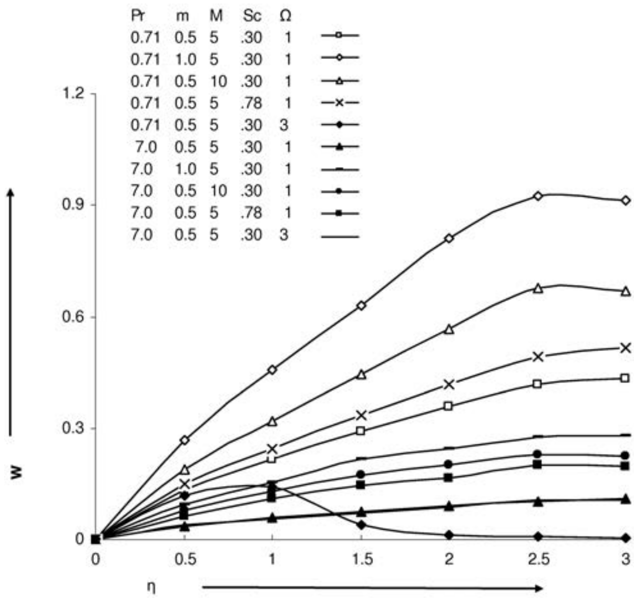

FIG. 3. Variation of velocity component w for Gc = 2.0, G = 5.0, $\Omega t = \pi/2$.

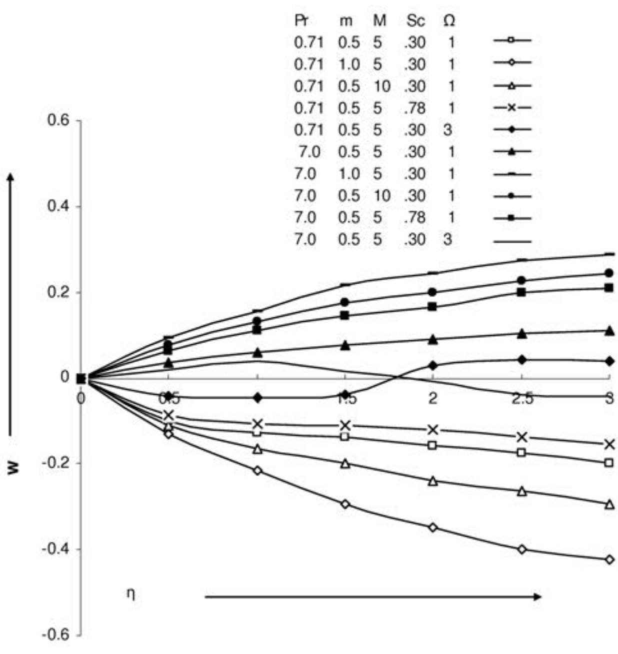

FIG. 4. Variation of velocity component w for Gc = 2.0, G = −5.0, $\Omega t = \pi/2$.

Newtonian fluid), the w remains negative for Pr = 0.71, while it is positive for Pr = 7.0. However, for increasing Ω, its value oscillates between negative and positive values for G < 0.

The variation of temperature θ_r has been shown in Fig. 5 for Pr = 0.71 (air) and Pr = 7.0 (water) and for different values of Ω. It is noticed that an increase in Ω leads to a rise in the temperature for both air and water. It is observed that maximum of θ_r occurs more quickly in water than that in air, in the neighbourhood of the plate and as the distance from the plate increases it decays faster in water than in air. An increase in Ω gives a rise in θ_r at each point. Figure 6 depicts the variation of concentration Cr for Helium (Sc = 0.30) and Ammonia (Sc = 0.78) for different values of Ω. An increase in Ω leads to a rise in Cr for both Helium and Ammonia. A comparative study of the curves reveals that the values of Cr increase/decrease at each point with variation in Ω and the same pattern is found as that of θ_r. It is further observed that the values of Cr are higher in Ammonia (heavier particles) than in Helium (lighter particles) near the plate ($0 < \eta < 1$). The shearing stress τ_1 is presented in Figs. 7 and 8 for cooled Newtonian fluid (G > 0) and heated Newtonian fluid (G < 0), respectively. τ_1 is drawn for Pr = 0.71 and Pr = 7.0, taking different values of M and Sc as a function of m. It is observed that for increasing M that the τ_1 first increases, reaches a maximum (at $m = 0.2$) and then becomes constant for G > 0 both in air and water. For extremely heated plate, the values of τ_1 first decrease, reach a minimum (at $m = 0.2$) and then become constant for large values of M in air, but a reverse effect is observed for water. We have

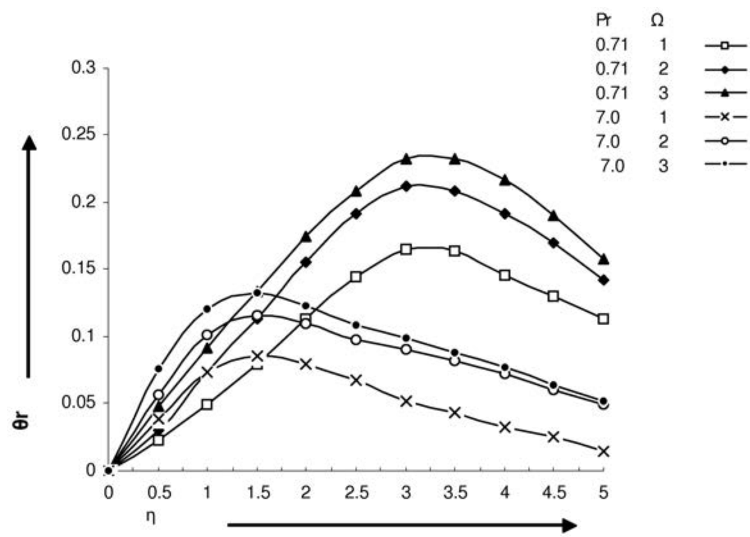

FIG. 5. Variation of temperature θ_r.

noticed that the values of τ_1 are greater for Helium than Ammonia, for both air and water and G > 0 or G < 0. Further, it is found that the values of τ_1 are smaller in air than in water for cooled Newtonian fluid, while a reverse effect is observed for a heated Newtonian fluid. Also, the values of τ_1 increase/decrease for small values of m and then remain constant for $m \geq 0.2$.

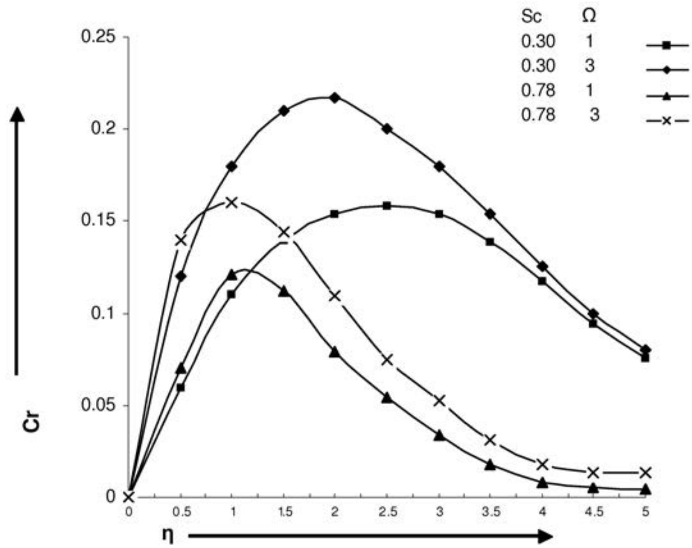

FIG. 6. Variation of concentration Cr.

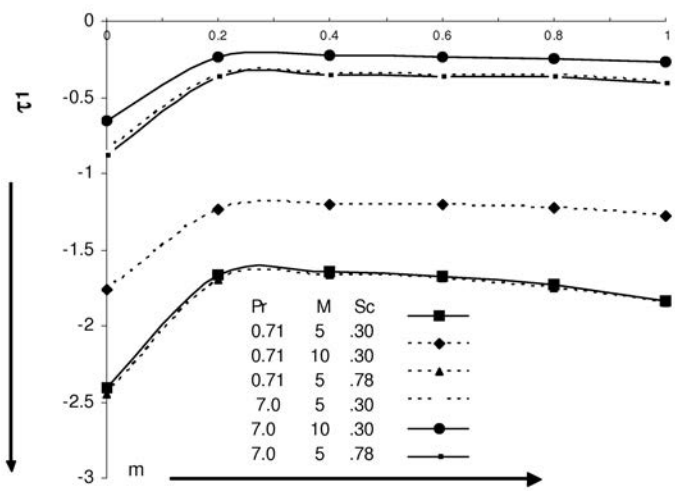

FIG. 7. Variation of shearing stress τ_1 for Gc = 2.0, G = 5.0, $\Omega t = \pi/2$.

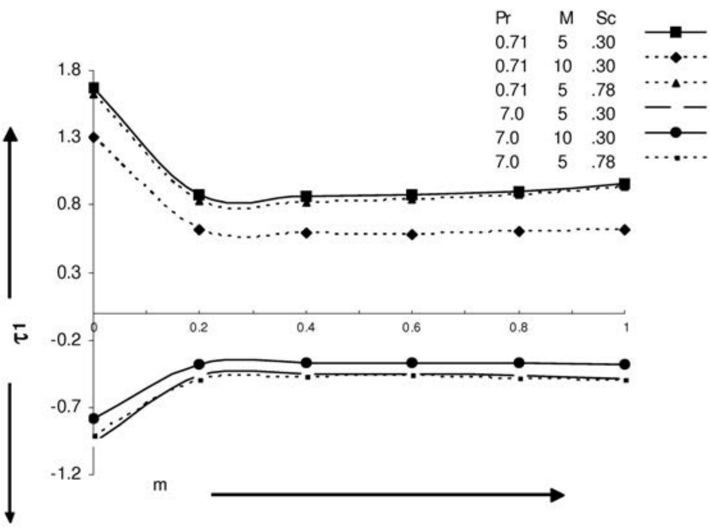

FIG. 8. Variation of shearing stress τ_1 for Gc = 2.0, G = −5.0, $\Omega t = \pi/2$.

Figures 9 and 10 depict the variation of the shearing stress τ_2 for different values of M, Sc and Pr versus the Hall parameter (m). It is observed that an increase in M leads to a fall in τ_2 for both air and water and G > 0. However, the values of the skin friction are greater in air than those of water for small Hall parameter, but for large parameter a reverse effect is observed. For G < 0 and

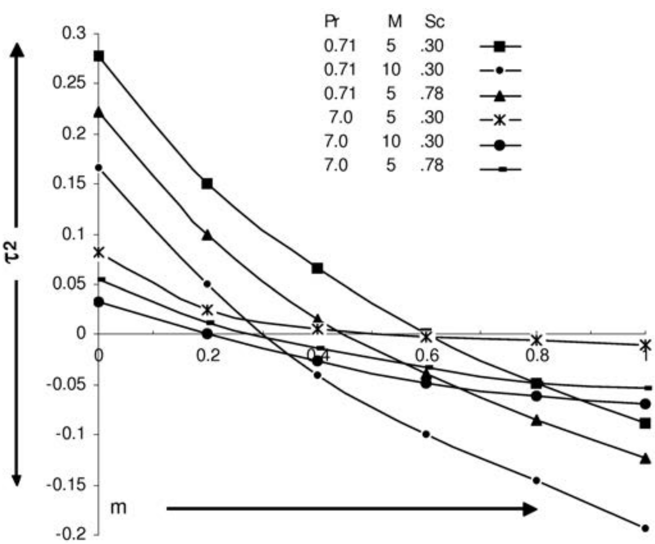

FIG. 9. Variation of shearing stress τ_2 for Gc = 2.0, G = 5.0, $\Omega t = \pi/2$.

Pr = 0.71 it is found that for increasing M, the values of τ_2 rise, while a reverse effect is observed for water. The effect of small induced magnetic field is greater for Pr = 7.0 than that for Pr = 0.71. We have found that the values of τ_2 are greater in Helium than in Ammonia for air or water and G > 0 or G < 0. For large induced magnetic field ($m \geq 1$) the skin friction becomes almost constant in all situations.

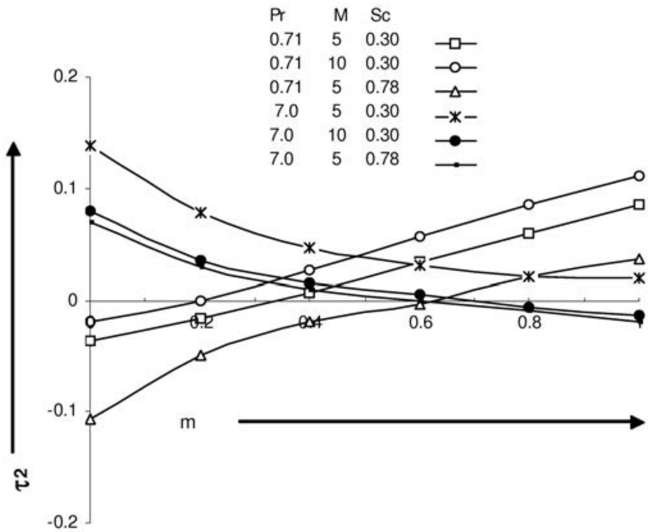

FIG. 10. Variation of shearing stress τ_2 for Gc = 2.0, G = −5.0, $\Omega t = \pi/2$.

One would also certainly like to know the quantity of heat exchange between the body and the fluid. The heat flux across the surface Q (t) is given in the table for different values of Pr = 0.025 (Mercury), 0.71 (air) and 7.0 (water). The variation of Q (t) is reported for various values of Ω (frequency). It is noted that for increasing Prandtl number, the flux increases for small values of the frequency. However, it oscillates i.e. it increases/decreases with the increasing values of Ω or Pr. The mass flux across the surface C (t) is also given in the table for different values of Sc = 0.22 (Hydrogen), 0.30 (Helium) and 0.78 (Ammonia). The variation of C (t) is reported for various values of Ω (frequency). It is noticed that for increasing Sc, the flux increases for small values of the frequency, however it oscillates, i.e. it increases/decreases with the increasing values of Ω or Sc.

5. Conclusions

In this work the problem of unsteady mixed convection and mass transfer flow with Hall effect of a viscous, electrically conducting fluid through a porous medium, bounded by an infinite vertical plate under the action of a uniform transverse magnetic field is investigated. The resulting governing equations are solved by a perturbation scheme. The results are presented for variations of major parameters, including the magnetic field parameter, the Prandtl number, the Grashof number, the Schmidt number and Hall parameter. A systematic study of the effects of the various parameters of flow, heat and mass flux characteristics is carried out. Some of the important findings, obtained from the graphs and table are listed here with:

1. An increase in magnetic parameter (M) or Hall parameter (m) leads to a rise in the velocity for both air and water for a cooled Newtonian fluid.
2. The velocity is higher for Ammonia than that for Helium with $\mathrm{Pr} = 0.71$ or 7.0 and $\mathrm{G} > 0$ or $\mathrm{G} < 0$.
3. For a cooled Newtonian fluid, an increase in frequency parameter (Ω) gives a rise in the velocity for both air and water.
4. The values of concentration (Cr) are higher in Ammonia than that in Helium near the plate.
5. An increase in Ω leads to a rise in the temperature and concentration for air/water and Helium/Ammonia, respectively.
6. The values of shearing stresses are greater in Helium than in Ammonia for air or water and $\mathrm{G} > 0$ or $\mathrm{G} < 0$.
7. The mass flux across the surface oscillates with increasing Ω or Sc.
8. The heat flux across the surface oscillates with increasing Ω or Pr.

It is hoped that the present investigation of the study of physics of flow over a vertical surface can be utilized, as the basis for many scientific and engineering applications, for studying more complex problems involving the flow of electrically conducting fluids. The findings may be useful for the study of movement of oil or gas and water through the reservoir of oil or gas field, in migration of underground water and in the filtration and water purification processes. The results of the problem are also of great interest in geophysics in the study of interaction of the geomagnetic field with the fluid in the geothermal region.

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