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BENDING OF FRAGMENTED ARCHITRAVES RESTORED WITH BOLTED TITANIUM BARS: A NUMERICAL ANALYSIS

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The mechanical behaviour of restored structural members of ancient monuments is studied in the present paper with the aid of the Finite Element Method. The study is motivated by the needs of the conservation project in progress on the Parthenon Temple of the Acropolis of Athens; the results however could be valuable for various stone monuments under conservation. Centrally fractured prismatic marble architraves (epistyles) of rectangular cross-section restored with either threaded or smooth titanium bars are modelled. The architraves are resting on marble blocks simulating the capitals (abacuses) of the columns of the temple. They are subjected to bending under uniformly distributed loading along their span, following the results of earlier studies, concerning the influence of the loading mode on the overall behaviour of restored structural members. The method used for determination of the reinforcement required is the one introduced recently by the scientists working for the restoration of the Acropolis monuments. All the loads that could be applied on the member after it is replaced in its initial position in the monument were taken into account, including the own weight of the member, the weights of the members that will rest on it after the restoration, as well as possible dynamic (earthquake) loads. Emphasis is laid on the influence of the threads of the bolted bars in comparison to the results for the unbolted ones, in an effort to quantify the maximum anchoring length required in order to minimize the intervention on the authentic stones. The distribution of the stress and strain fields all over the architrave-abacus-reinforcing bar system is investigated and conclusions are drawn concerning the extreme stresses and the points where they are developed.

1. Introduction

Restoring and conserving an ancient monument is a complicated, multidisciplinary scientific task. Many problems are to be considered and solved before final decisions are made. These problems vary from elementary ones (for example the strength and deformability of the materials used) to rather complex ones (such as preservation of the structural system, determination of the minimum possible intervention, reversibility of the interventions and of course, their durability). Archaeologists, architects, materials scientists, civil and chemical engineers collaborate in order to meet the final target, i.e. the extension of the life of the monument. The decisions made are usually a compromise between various, and often contradictory, points of view.

A typical example of a complicated restoration program is that of the Parthenon Temple of the Acropolis of Athens. Parthenon, the masterpiece of Fidias and Kallikratis, was built in the 5th century BC as a temple of goddess Athena. It is the most famous surviving building of ancient Greece and it has been praised as the finest achievement of Greek architecture. The Parthenon is an enduring symbol of Ancient Greece and of Athenian democracy and it is regarded as one of the world's greatest cultural monuments.

The temple has been damaged by fire during the invasion of the Herulians, but the exact date of the fire and subsequent repairs are debated (suggestions range from 150 B.C. to 267 A.D.). Later Parthenon was converted to a Christian church of the Virgin Mary (around 600 A.D.). At the time of the Latin Empire it became, for about 250 years, a Roman Catholic Church. In 1456, Athens fell to the Ottomans and the Parthenon was converted into a mosque. Besides the successive conversions, European visitors in the 17th century testified that the building was largely intact. In 1687, the Parthenon suffered its greatest blow when the Venetians attacked Athens, and the Ottomans fortified the Acropolis and used the building as a gunpowder magazine. A Venetian mortar exploded in the magazine and the building was partly destroyed. The internal structures were demolished, the roof collapsed, and some of the pillars, particularly those on the southern side, were decapitated. After this, much of the building fell into disuse and a smaller mosque was erected.

The condition of the monument was aggravated by older restoration projects (between 1840 and 1930) and many structural problems appeared (intrusion of moisture, appearance of funguses, corrosion and inflation of the iron joints and fracture of marble structural members). The Athens' 1981 earthquake made the situation worse and from this moment on, an ambitious scientific restoration project was approved by the Hellenic State, which is in progress until today. Series of problems have been confronted in the frame of this project concerning especially protection of the authentic material and minimization of the interventions on the original structural elements of the Temple. Various innovative solutions were proposed and applied and nowadays these solutions are considered as guidelines for the scientists working on similar projects.

In the frame of the above project, a pioneer method was developed for the restoration of fractured structural elements in an effort to confront the problem of the structural stability of the monument [1]. The method is based on the use of titanium bars in combination with suitable cement mortar and permitted to reduce the interventions on the authentic material in comparison to older approaches. It takes into account all the loads that could be applied on the member after it is replaced in its initial position in the monument, namely the weight of the member and these of any other members that will rest on it after the restoration is completed as well as possible dynamic loads [2, 3]. For the theoretical development of the method it is assumed that the stresses do not exceed the linearity limit, the strains developed in the marble body and in the reinforcing bar are compatible with each other, the marble behaves as a transversely isotropic material, the bending loads act normally to the bedding planes and the cross-sections remain plane and normal to the longitudinal, neutral axis of the beam.

Recently the method was assessed experimentally and numerically [4, 5]. It was concluded that the reproduction of actual bending conditions in the laboratory does not give reliable results since the simulation of bending under uniform load by a laboratory multi-point bending test does not lead to accurate conclusions unless the number of loading points is increased dramatically, something unrealizable for practical reasons. Thus the engineer who designs basing on an experimentally evaluated and calibrated model, should increase the safety factor accordingly, in order to take into account the fact that the stress field of the test may be weaker compared to the actually developed one. It was, also, indicated that some of the assumptions adopted are not always satisfied. Indeed, the axial strains do not vary linearly along the height of the architrave but rather they follow a sigmoid distribution. In addition, the neutral axis is translated towards the bottom side of the architrave, rendering the calculations carried out considering the centroidal longitudinal axis as the neutral one, a rather rough approximation of the real conditions.

It became therefore clear that the method for the calculation of the reinforcement required should be studied further before definite conclusions concerning its applicability are drawn. Towards this direction, the problem of joining together fractured structural elements is studied here using a more sophisticated numerical model, in an effort to enlighten the influence of some critical points related to the shape of the reinforcing bars.

2. The material: Dionysos marble

The material used by the ancient Greeks for erection of the monuments of the Parthenon Temple of the Acropolis of Athens was the Pentelic marble which is an extremely durable white marble quarried from Mount Pentelicus in Attica. However, the ancient quarries are nowadays exhausted and the needs of the restoration and conservation programs are covered almost exclusively by Dionysos marble, since it was found to be the most compatible candidate substitute of the authentic Pentelic marble [6, 7].

From the physical point of view, Dionysos marble is composed by 98% of calcite, 0.5% of muscovite, 0.3% of sericite, 0.2% of quartz and 0.1% of chlorite.

Its specific density is 2730 kg/m^3, its apparent density is 2717 kg/m^3 and its absorption coefficient by weight is around 0.11% [8]. The coefficient of thermal expansion is $9 \times 10^6/°$C between 15°C and 100°C. Its very low porosity varies between 0.3% in the virgin state to 0.7% after the action of various natural weathering and corrosive agents (superficial porosity). Its grain size varies around 0.43×10^{-3} m and the crystals have a polygonic shape of almost uniform size. The dimensions of the largest crystals vary between 900 μm × 650 μm and 950 μm × 874 μm. It is of white colour with a few thin parallel ash-green veins, following the schistosity of marble and containing locally silver areas due to the existence of chlorite and muscovite [9].

Concerning its mechanical behaviour, the Dionysos marble is an anisotropic material, characterized by three different anisotropy directions (parallel to the layers, along the width of the web and along the thickness of the web) and thus it appears to be orthotropic. The values of its mechanical properties reported in literature vary within very broad limits. It is mentioned characteristically that the values of its tensile strength vary between 2.4 MPa and 19.5 MPa, while the respective experimental values of Young's modulus range between 23 GPa and 90 GPa [6, 7, 10]. However, a long series of direct tension and uniaxial compression tests with cylindrical specimens [11] indicated that the material can be considered as transversely isotropic, since the mechanical properties along the first two of the above anisotropy directions are very similar to each other and therefore, the Dionysos marble can be described with the aid of five elastic constants: two elastic moduli, in the plane of transverse isotropy and normal to it, two Poisson's ratios characterizing the lateral strain response in the plane of transverse isotropy to a tensile stress acting parallel and normal to it, and the shear modulus in the planes normal to the plane of isotropy. It was also concluded that Dionysos marble appears to be slightly non-linear, both in the tension and in the compression regime, and slightly bimodular, i.e. the elastic modulus in tension is slightly different from the respective one in compression [12]. The values of the elastic moduli, Poisson's ratios and the values of tensile strength, σ_t, are recapitulated in Table 1, while in Fig. 1 the stress-strain curve is shown for the whole tension-compression regime.

Table 1. Mechanical properties of Dionysos marble in direct tension (loading rate equal to 10^{-6} m/min).

	E [GPa]	ν	σ_f [MPa]
Strong Direction	84.5	0.26	10.8
Intermediate Direction	79.5	0.26	9.5
Weak Direction	50.0	0.11	5.3

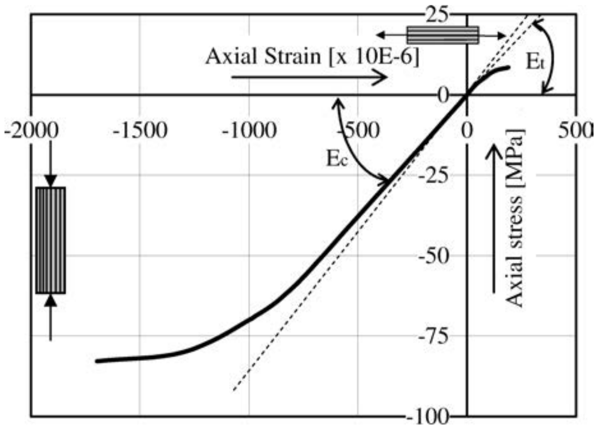

FIG. 1. The stress-strain curve of Dionysos marble along the strong anisotropy direction.

From the above tests it was also concluded that the size effect is very pronounced for Dionysos marble. For example, the Ultimate Compressive Strength of cylindrical specimens of height-to-diameter ratio equal to 2 is strongly dependent on the size of the specimens used for the laboratory tests and, as it is seen in Fig. 2, the dependence appears to be not monotone: a clear maximum exists for specimens of diameter equal to about 125 mm. The size effect appears to be independent of the lubrication conditions between the end platens and the bases of the specimens, although the absolute strength values in case of lubricated specimens are slightly higher. Similar conclusions were drawn also for the tensile strength of Dionysos marble (as obtained from diametral compression tests) as well as for its modulus of elasticity [13].

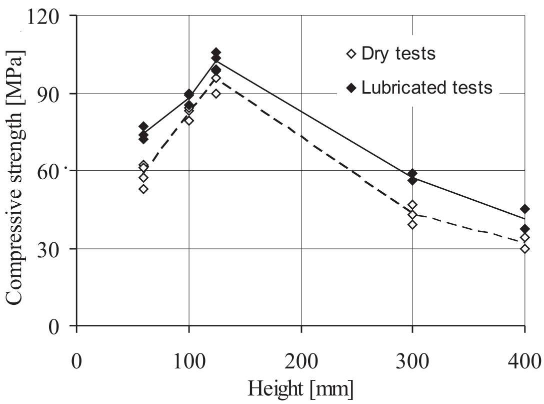

FIG. 2. The size effect for Dionysos marble.

3. Numerical modelling

3.1. Bending of restored marble architraves using cylindrical titanium bars

In actual conditions the architraves of a monument are loaded by an almost uniformly distributed load over their total length, since they carry their own weight and the weight of the superimposed structural elements. However, the realization of a bending test under homogeneous load in the laboratory is an extremely difficult experimental task and therefore, multi-point bending tests are carried out instead. In an effort to check the degree of approximation of the real conditions by the laboratory multi-point bending tests, a numerical analysis was carried out recently [14] using the Finite Element Method and the commercially available software ANSYS 9.0. As a first step and for CPU-time economy, intact marble architraves were considered in that study. Four loading types were simulated:

- Uniformly distributed load along the total length of the architrave (Fig. 3a),
- Uniformly distributed load along the span (Fig. 3b),
- Eight-point bending along the span (Fig. 3c), and
- Eight-point bending along the total length of the member (Fig. 3d).

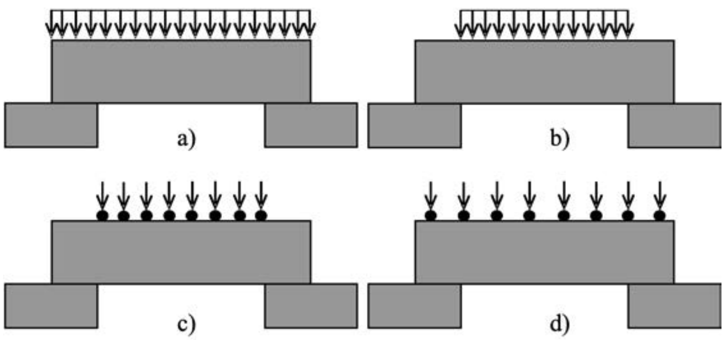

Fig. 3. The four loading cases studied numerically.

Among the most important conclusions of that study was the fact that the points most prone to failure were those in the vicinity of the corners of the supporting abacuses rather than those at the mid-span of the beam. The above conclusion is strongly supported by a thorough in-situ investigation of the architraves of the Parthenon Temple, which have never been removed from their original place from the antiquity until the present days: more than half of the fractures and the cracks observed (excluding those caused by interventions) have their origin very close to the edges of the abacuses [15].

It was also concluded that the strain is not linearly distributed along the height of the architraves but it exhibits a sigmoid variation, in accordance with earlier experimental results [16] and theoretical predictions [17]. The neutral axis of the bent architrave appeared to be displaced downwards. Finally it was indicated that the most intensive stress field developed at the central cross-section of the architrave was the one corresponding to the uniformly distributed load along the span (case b) rather than the one developed in the case of the eight-point bending. It was thus pointed out that the simulation of bending under uniform load with multi-point bending tests leads to underestimation of the stress field developed, which may be catastrophic if the design of the restoration is based on the results of the tests. Therefore an appropriate increase of the safety factors used by the design engineers appears to be absolutely necessary.

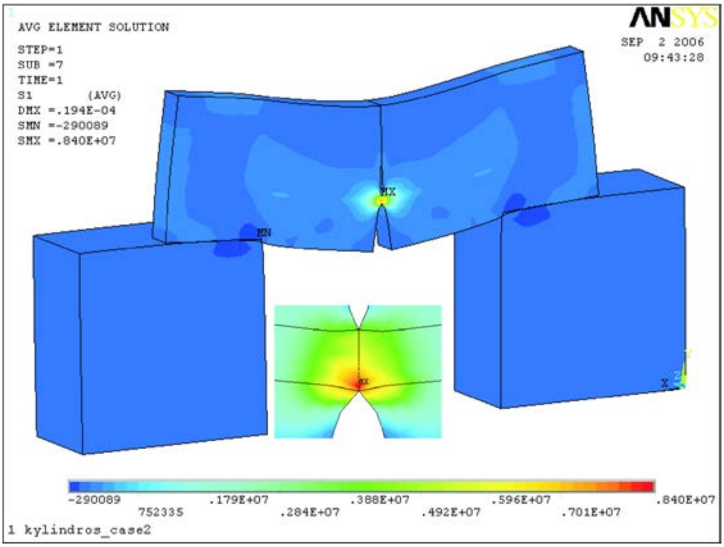

Fig. 4. The distribution of the 1st (maximum) principal stress in the restored architrave. The embedded figure shows a detail of the central section around the titanium bar.

As a second step, the problem of joining together the fragmented marble architraves using titanium bars was also explored numerically in the previous study, taking into account the conclusions drawn from the analysis concerning the intact architraves. The architrave was assumed to consist of two equal parts joined together by a single cylindrical titanium bar fully bonded with the marble. In other words, in this simplified analysis the influence of the threads of the titanium bars was ignored. The geometry of the model matched exactly that of the most damaged architrave of the north colonnade of the Parthenon Temple, namely the fifth external one, in a scale of 1:3. The architrave was subjected to bending and the load was assumed to be uniformly distributed along the span of the architrave (case b), since it corresponds to the worst case concerning the magnitude of the stress field developed.

An overall view of the distribution of the first (maximum) principal stress in the case of the restored architrave is shown in Fig. 4. As it was expected, the situation is completely different compared to that of the intact member. The points most prone to fail were the ones in the immediate vicinity of the reinforcing bar rather than at the abacuses' corners. In addition it was indicated that in the vicinity of the bar, both beyond and below it, the contact of the two constituent marble parts of the architrave tends to be lost (Fig. 4, embedded figure).

3.2. Bending of restored marble architraves using bolted (threaded) titanium bars; The numerical model

Up to now, the bar-marble interface was not studied extensively since attention was paid to the qualitative and comparative description of the stress and strain fields all over the restored architrave. Therefore the reinforcing bar was considered to be cylindrical in perfect contact with the marble. However in praxis, the bars used are bolted in order to optimize the load transfer mechanism avoiding the pull-out failure. It appears therefore absolutely necessary to study thoroughly the interaction between marble and the reinforcing bar as well as the stress and strain concentrations, inevitably generated at the corners of the threads of the bolted titanium bars. In this direction the same problem was modelled by considering the exact geometrical characteristics of the titanium bars, used in the restoration of the Parthenon Temple.

The architrave was considered again to be centrally fractured and restored with a single titanium bar, which now is assumed to be bolted all over its length, as it is shown schematically in Fig. 5. The geometry of the beam matched again exactly that of the fifth external architrave of the north colonnade of the Parthenon Temple in the scale of 1:3 (length $L = 1.43$ m, thickness $w = 0.18$ m and height $h = 0.45$ m). However, for reduction of the "running time" and taking advantage of various symmetry planes, only one quarter of the configuration was modelled. The diameter of the reinforcing bar was calculated according to the approach introduced by Ioannidou and Paschalides [2] and Mentzini [3] and it was placed at a distance $h_t = 0.305$ m from the upper side of the architrave. The anchoring length at each one of the two equal parts of the architrave was $L = 0.4$ m and the total thread number corresponding to the anchoring length was 200. The thin layer of cement used in practice to increase the adhesion was ignored again. The geometrical characteristics of the titanium bar are presented in Table 2.

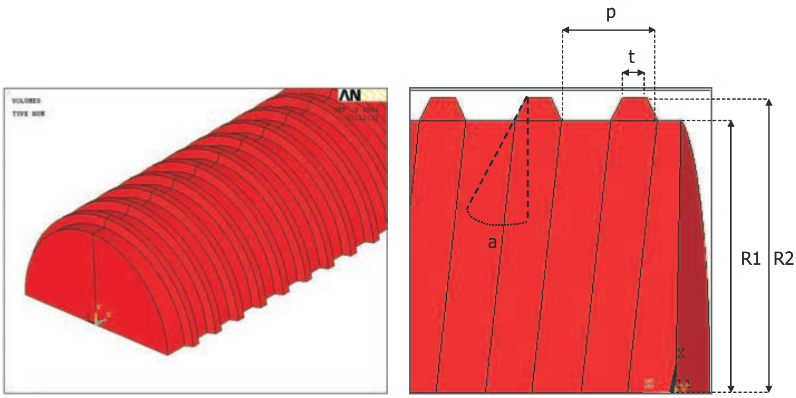

FIG. 5. Schematic representation of the geometrical characteristics of the bolted bar.

Table 2. Geometrical characteristics of the bolted titanium bar.

R1	R2	p	t	a
5.85 mm	6.35 mm	2 mm	0.5 mm	25^0

Assuming that the bedding planes of marble are parallel to the longitudinal axis of the architrave and the bending loads act normally to these planes, the transversely isotropic nature of Dionysos marble could be ignored. Its slight non-linearity and its bimodularity were ignored, also. According to these assumptions, the values of the mechanical properties of Dionysos marble used were those of the strong anisotropy direction of Table 1. The density of Dionysos marble was set equal to $\rho_m = 2.78$ g/cm^3 and the coefficient of static friction between the marble architrave and the marble abacuses was set equal to $\mu = 0.7$. The mechanical properties of the titanium bar were: Young's modulus $E_t = 105$ GPa, Poisson's ratio $v_t = 0.32$, density $\rho_t = 4.51$ g/cm^3. The coefficient of static friction between marble and titanium was assumed equal to 0.4.

The numerical model was discretized by creating a uniform and fine mesh in the vicinity of the titanium reinforcement, since this part of the model was, for the specific problem, the region of highest interest. In order to increase the flexibility of the model and to reduce the CPU running time , a marble "*cylinder*" was constructed around the titanium bar with radius R1+R2, where R1 and R2 are the minimum and maximum radii of the bolted bar, respectively. The regular mesh on the cylinder and the bolted bar was attained by using the Mapped Meshing Technique (Fig. 6b). According to this technique, a volume must have the shape of a brick or a wedge or a prism or even a tetrahedron, and must have equal numbers of element divisions specified on opposite sides. The two volumes (cylinder and bar) were constructed in an appropriate manner, but without changing their primary geometrical characteristics, in order to take the shape imposed by the mapped mesh. For the remaining part of the model, a coarser mesh was created without employing the mapped meshing technique, but using suitable divisions and spacing ratios on the unmeshed lines (Fig. 6a). The element used for the meshing was the SOLID186, a higher order 3-D structural solid element defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions (Fig. 7). It has quadratic displacement behaviour and is well suited for modelling irregular meshes. The final model consisted of 96784 such elements (Fig. 6a).

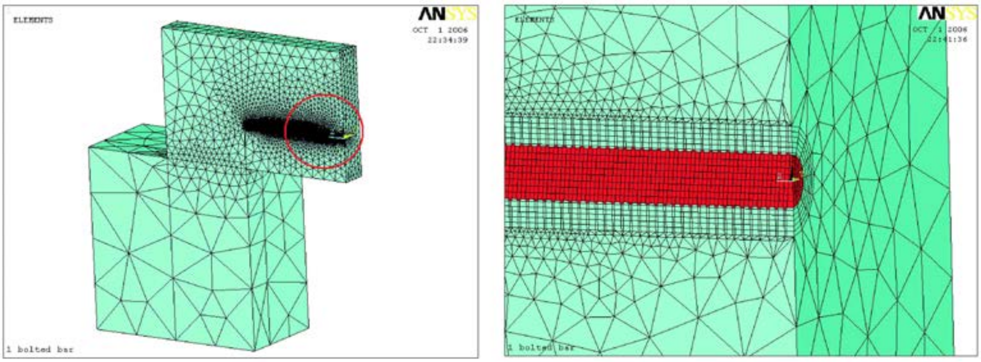

Fig. 6. An overall view of the mesh of the model (a), and a detailed view of the mesh in the vicinity of the titanium reinforcement (b).

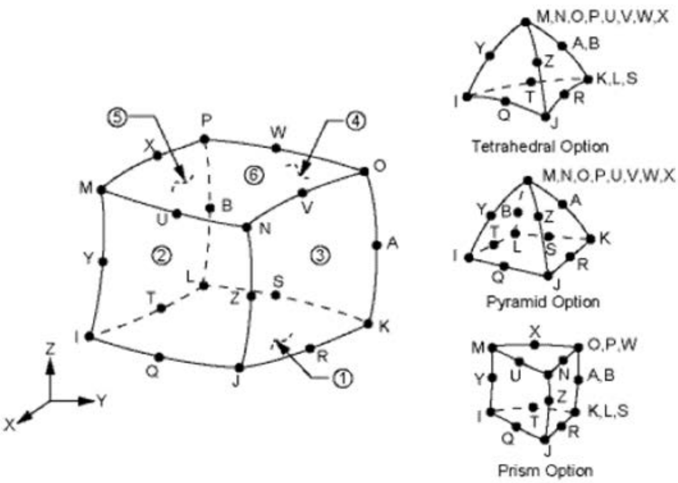

Fig. 7. The element SOLID186 used for the meshing of the model.

The next step of the numerical analysis was identification of the contact options of the problem according to which two surfaces, a "contact" one and a "target" one are to be defined. For modeling rigid-flexible contact, the rigid surface must be represented by the "target" surface. For the specific problem, three couples of 2D contact elements were created (Fig. 8). The first couple represented the contact between the architrave and the supporting abacus, which is a rigid-rigid contact since the contact surfaces are made of the same material (marble). In this context, the contact surface of the architrave was considered as the "target" and that of the abacus as the "contact". The second couple represented the contact between the architrave and the titanium bar, since for the specific problem the titanium bar was not considered to be fully bonded with the marble as in the previous model. Due to the fact that titanium is a rigid material, its contact surface was considered as the "target" and the respective one of the architrave as the "contact". Concerning the third couple, a new rigidly clamped area was created in the position of the central cross-section area of the architrave, in order to be in simple contact with the respective architrave's area. This new area was defined as the "target" surface and that of the architrave as the "contact" one. The elements used for the analysis were "TARGE170" and "CONTA174".

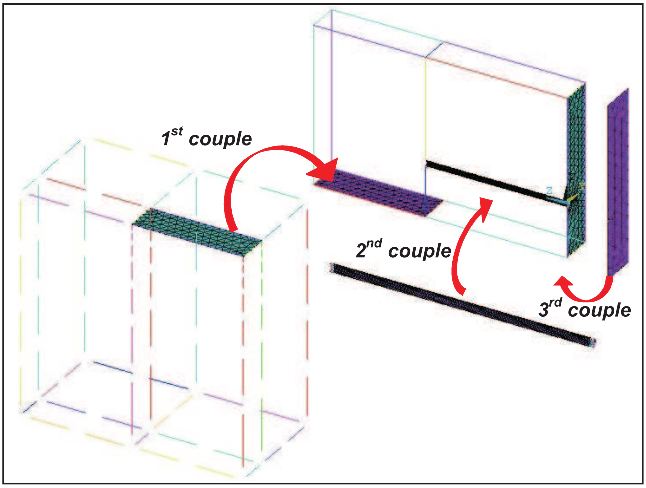

Fig. 8. Identification of the contact elements created for the model.

The boundary conditions were imposed in such a way that both the static determinacy of the problem as well as its symmetry were ensured since, as it has been already mentioned, only a quarter of the configuration was modelled (advantage was taken of the vertical plane of symmetry). In this context, the lower bases of the supporting abacuses were considered as rigidly clamped, while the central cross-section area of the titanium bar was allowed to be free only along the vertical direction. On the other hand, the central cross-section area of the architrave was restricted only to the right horizontal direction by creating the rigidly clamped target surface described previously (third couple of contact elements).

The load was assumed to be uniformly distributed along the span of the architrave. Its magnitude was equal to 65 kN, i.e. the maximum load expected for the particular architrave, after the completion of the restoration of the Parthenon Temple.

4. Results and discussion

A detailed view of the distribution of the first (maximum principal stress) in the restored architrave and the reinforcing bar in the immediate vicinity of the central cross-section is shown in Fig. 9. From a qualitative point of view, the conclusions drawn are of similar nature compared to those in the case of a restored member with one cylindrical (unbolted) titanium bar: the major part of the architrave is relieved and only at the central cross-section, around the titanium bar, the stresses aproach the fracture stress of marble under direct tension (∼6–8 MPa) and reach the values of the order of about 8 MPa, remaining however far from the respective limit of titanium (∼300 MPa). Comparing the two models, i.e. the bolted and unbolted reinforcing bars (Figs. 9 and 4, respectively), it is noted that the use of bolted bars leads to much higher stresses than those generated by the cylindrical titanium bars.

The distribution of the other two principal stresses is plotted in Figs. 10 and 11. The change of sign of these stresses as one moves towards the interior of the architrave is noteworthy. In addition, similarity of the distribution of the 3rd principal stress with the cone-type fracture surface commonly observed during pure pull-out tests is striking.

In Fig. 12, the variation of the von Mises equivalent stress in the (ductile) titanium bar is plotted. As it should be expected, the maximum value is observed in the immediate vicinity of the central section and it approaches the value of about 20 MPa. As one moves towards the interior of the architrave, the equivalent stress in the titanium bar decreases rapidly and after the 20-th thread (out of 200), its value is only one tenth of the respective maximum one. The above conclusions become more clear in Fig. 13 in which the maximum value of the equivalent stress appearing at the peaks and the roots of the threads of a bolted bar are plotted versus the order of the thread (the thread at the central cross-section is considered as the first one). It is seen from this figure that the stress field is higher at the roots of the thread rather than at the respective peak. However, the most striking conclusion is that the stress on both the peak and a)

b)

c)

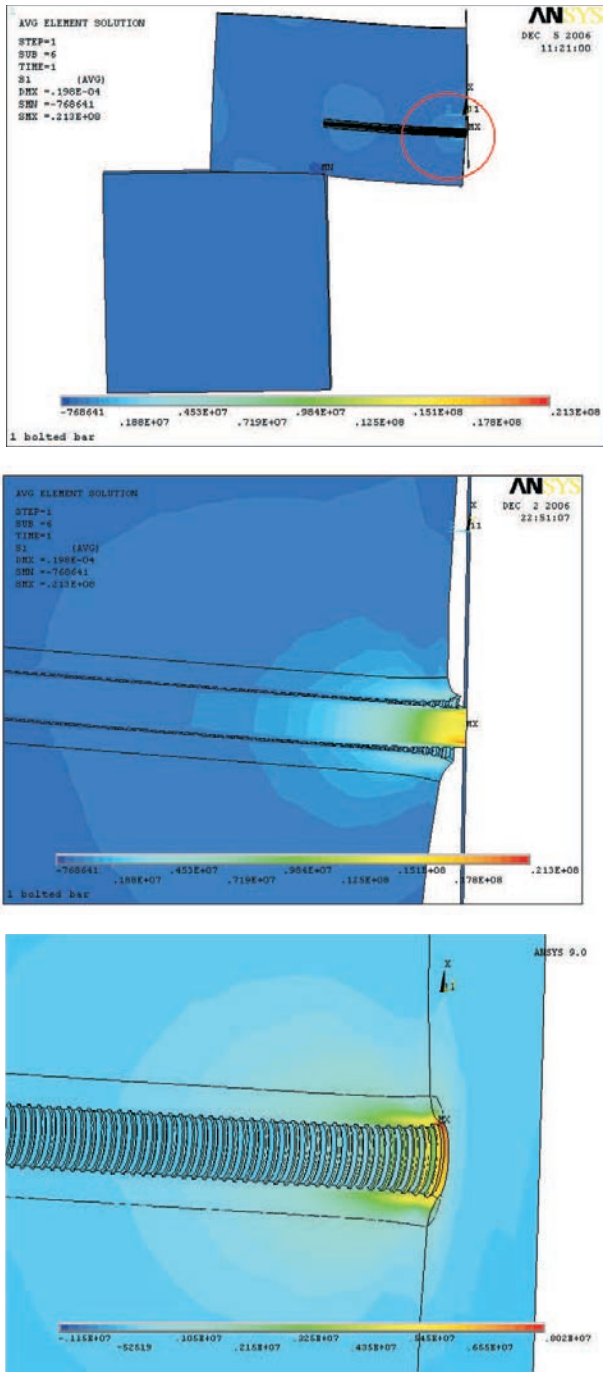

FIG. 9. The distribution of the 1st (maximum) principal stress in the restored architrave (a) and a detailed view of the central section of the titanium bar (b) and of the marble (c).

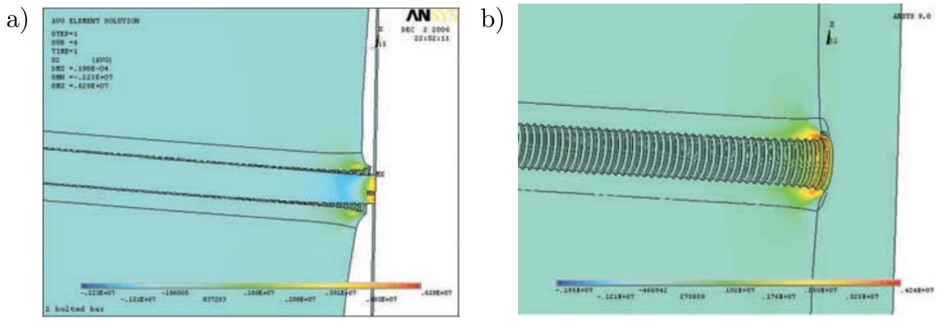

Fig. 10. A detailed view of the distribution of the 2-nd principal stress around the central cross-section area of the titanium bar (a) and of the marble (b).

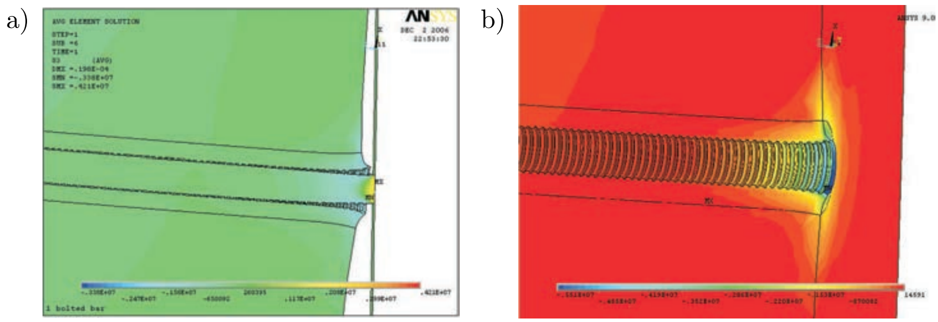

Fig. 11. A detailed view of the distribution of the 3-nd principal stress around the central cross-section area of the titanium bar (a) and of the marble (b).

the root of the threads is almost zero after the 50-th thread. From this point on, it increases very slightly as one approaches the end of the bar, but the maximum value reached does not exceed in any case the value of 1.5 MPa. Based on these observations, one should reconsider the empirical formulae yielding the anchoring length of the reinforcing bars, which is not in accordance with the minimum intervention principle.

In Fig. 14 the distribution of the maximum shear stress (Fig. 14a) and that of the maximum principal stress (Fig. 14b) are plotted in juxtaposition, around the area of the reinforcing bar. The non-monotonous variation of both stresses along the radial and the axial directions is clear from these figures, explaining the cone type fracture of marble during the pull-out tests [18].

In order to study the differences between the two models (bolted bar and cylindrical bar), a series of critical diagrams were plotted. As the first step, variation of the normal axial strain along the central vertical line of the cross-

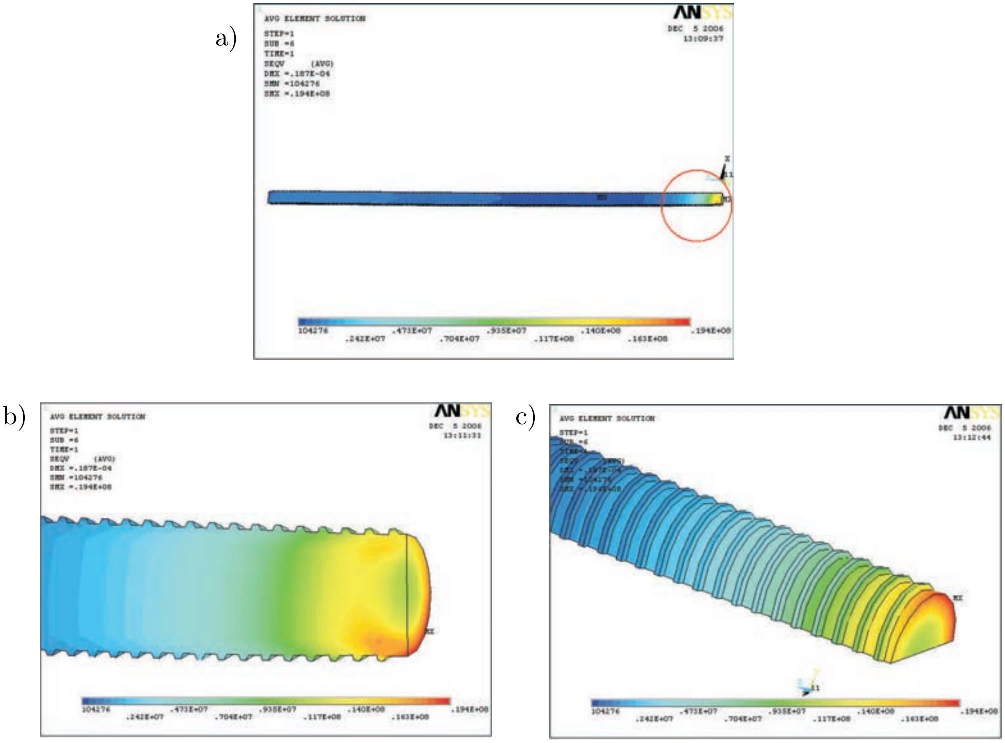

FIG. 12. The distribution of the equivalent stress in the reinforcing bar (a) and two detailed views in the vicinity of the central cross-section (b, c).

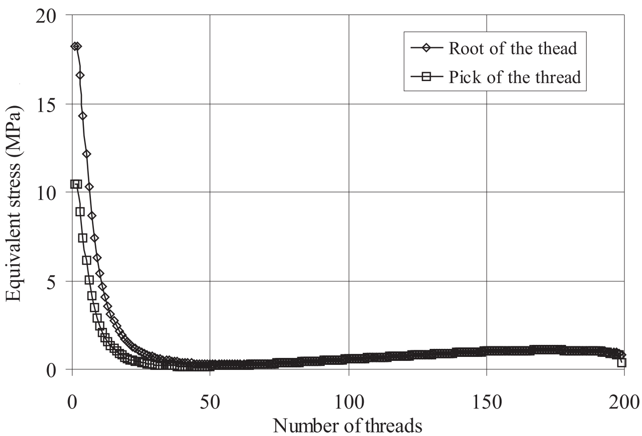

FIG. 13. The maximum equivalent stress at the peaks and the roots of the threads versus the order of the threads of the titanium bar.

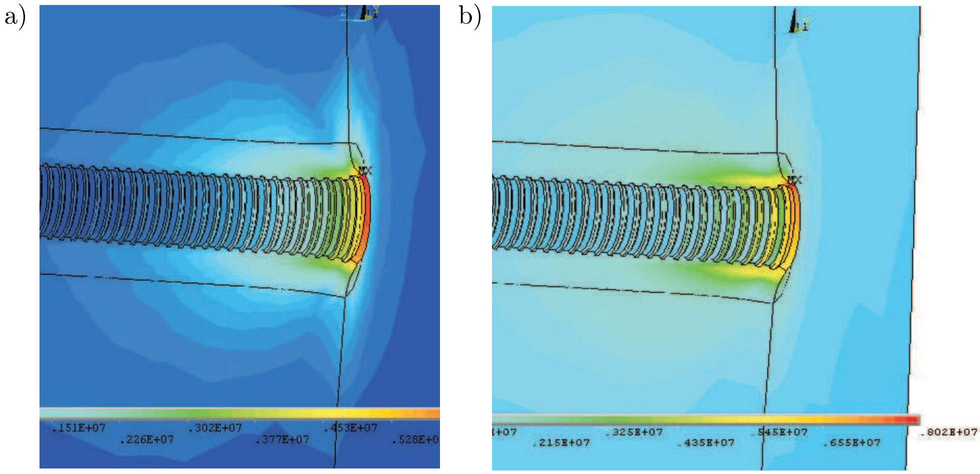

FIG. 14. The distribution of the maximum shear stress (a) and the maximum principal stress (b) in the marble in the central cross-section.

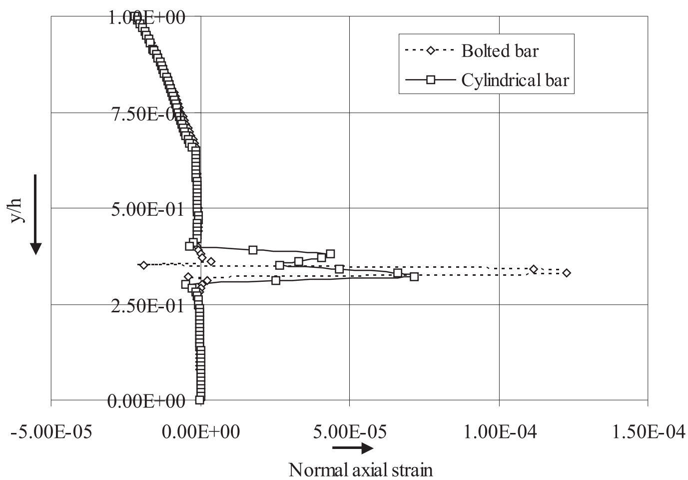

FIG. 15. The variation of the normal axial strain along the height of the architrave.

section of the architrave is plotted in Fig. 15. The vertical axis is reduced over the height of the beam, h, and is directed downwards. Point (0.00, 1.00) corresponds at the upper base of the architrave while point (0.00, 0.00) at the lower one. It is seen that in major part of the section, the values are of the same sign and order of magnitude for both models. For the upper one third of the height of the architrave, negative strain values are observed (corresponding to the compression exerted mutually from each part of the member on the other), which become zero at the regions just above and below the titanium bars, since in these regions contact of the two parts of the architrave is lost, as it was already concluded from Fig. 9. In the immediate vicinity of the reinforcing bar, the strain reaches high tensile values of magnitude about 120 μstrain in case when the bar is bolted. This value is almost double the respective maximum one developed in the case of the model with an unbolted bar ($\sim$70 μstrain). In any case, since the maximum fracture strain of Dionysos marble is about 200 μstrain [19], it is concluded that even in the region of the titanium bar the member is safe, at least for the specific load used in this analysis.

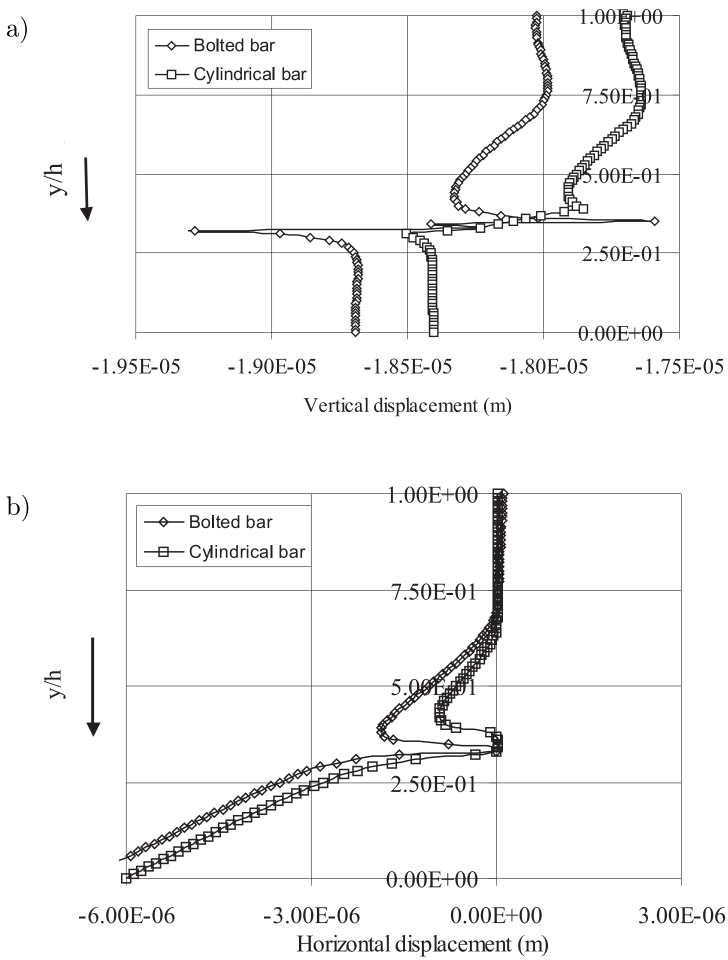

FIG. 16. The variation of the vertical (a) and the horizontal (b) displacements along the height of the architrave at its central cross-section.

Figure 16 describes variation of the components of the displacement vector along the same as previously line. As it is seen in Fig. 16a, where variation of the vertical displacement is plotted, behaviour of the two models is qualitatively similar. Indeed, in both cases, an almost constant (non-zero) vertical displacement is observed all along the height of the member, although it appears to increase slightly in the portion below the reinforcing bar. However, from a quantitative point of view the model with bolted bar exhibits deflections higher at any point of the architrave's height. In addition, the discontinuity observed close to the titanium bar is dramatically higher for this model: from -1.76×10^{-5} m the displacement reaches a value of -1.93×10^{-5}, namely it exhibits an abrupt change of almost 10%.

Similar conclusions are drawn for the variation of the horizontal displacement (Fig. 16b). The horizontal displacement is zero along the portions of the parts of architrave for which the contact is not lost (as it is expected, also, for symmetry reasons) and reaches considerably higher values, of the order of 6 μm, in the portions of the section where the contact of the two constituent parts is lost (both below and above the reinforcing titanium bar). Again the absolute values are higher in the case of the architrave restored using a bolted bar. Although such a displacement appears to be negligibly small, it must be considered seriously by experts working for the restoration projects, since it is the origin of a series of problems due to the penetration of moisture, bacteria etc in the body of the structural member.

Variation of the normal components of the stress field along the central horizontal line at the height of the axis of the reinforcing bar is plotted in Fig. 17(a,b). In this figure the horizontal axis is reduced over the half-length of the architrave and therefore point (0.00, 0.00) corresponds to the leftmost point of the member (resting on the capitals), while point (0.00, 1.00) corresponds to the mid-span of the architrave. It is observed that for the bolted bar, the axial stress (Fig. 17a) starts to increase abruptly as one approaches the central section and reaches a value of about 14 MPa, almost double as compared to the respective stress of the unbolted bar ($\sim$7 MPa). As one moves away from the central section, the axial stresses tend to decrease rapidly for both models and some perturbations are observed only in the region of the end of the titanium bar. The conclusions for the transverse normal stresses are of different nature. Compressive stresses appear at the region of the architrave resting on the abacuses which become tensile as one moves towards the central section (about 3.2 MPa for the unbolted bar and about 2.1 MPa for the bolted one). However, it should be mentioned that the use of the bolted bar creates high compressive values very close to the central section, just before the appearance of the tensile stresses at that section.

Finally, some interesting conclusions are drawn by plotting variation of the axial strain along the bottom central line of the member (Fig. 18). Initially the

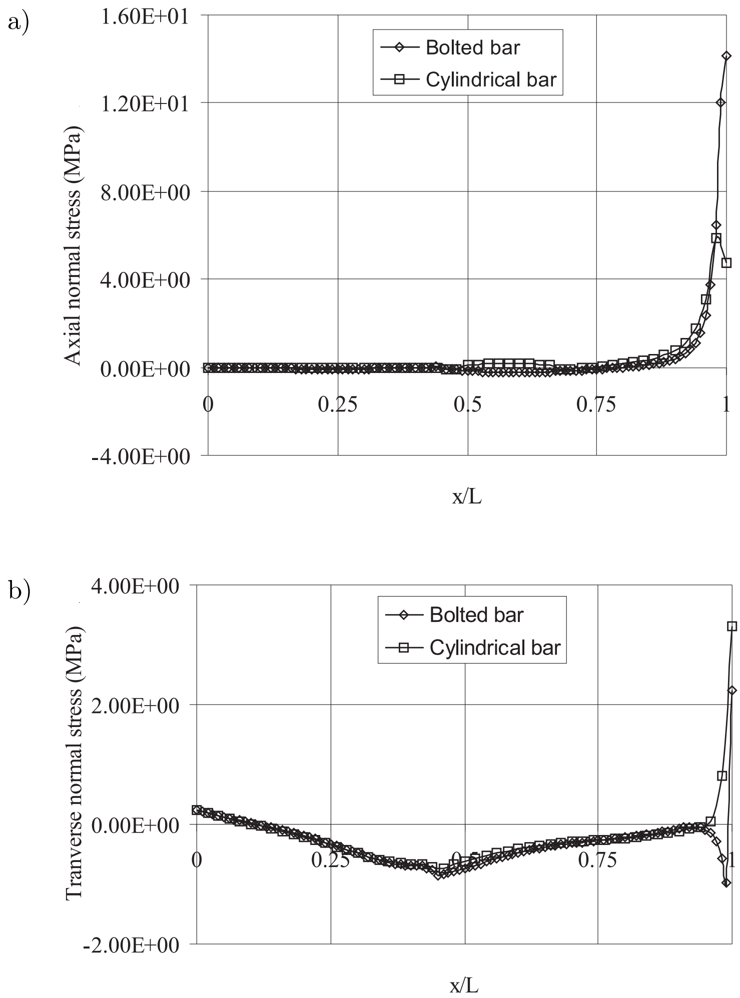

Fig. 17. The variation of the normal axial (a) and the normal transverse (b) stresses along the axial line passing from the centre of the reinforcing bar.

axial strain is almost zero for both models since part of the architrave, initially resting on the abacuses, tends to lift up losing the contact with the abacus in case when the load is applied only along the span of the member [4, 5]. However, as ones moves towards the corners of the supporting abacuses, the axial strain takes negative (compressive) values of the order of about -35 μstrain. From that point on, the strain starts to increase in an almost parabolic form until it becomes zero at the mid-span of the beam, where the contact between the two constituent parts is lost. Similarity of the variation of the strain along the bottom central line for the two models indicates that the stress and strain fields at the bottom base of the member are almost independent of the geometrical details of the titanium bar used.

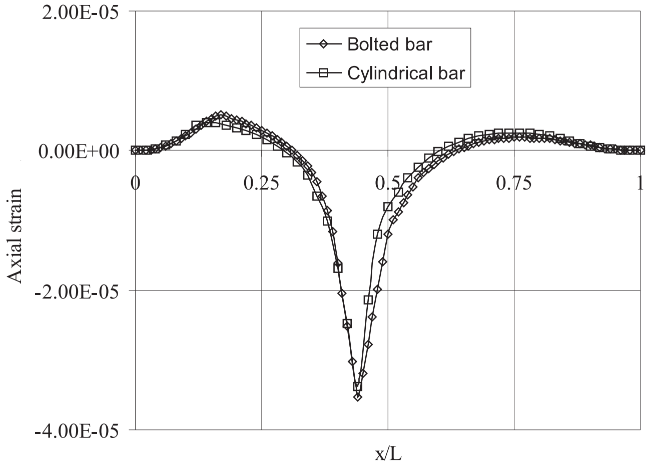

FIG. 18. The axial strain along the bottom central axial line.

5. Conclusions

The mechanical behaviour of fractured marble architraves restored with either cylindrical or bolted titanium bars was studied with the aid of the Finite Element Method. The findings of the analysis can be summarized as follows.

The simulation of bending under uniform load with multi-point laboratory bending tests leads to underestimate the stress field actually developed. In the vicinity of the reinforcing bar the contact between the two constituent marble parts of the architrave is lost.

The axial strain along the height of the architraves is not linear but of sigmoid variation. Taking into account, also, that the neutral axis of the bent architrave is displaced downwards (obviously due to the fact that the length – to – height ratio of typical architraves is lower than 4) it is indicated that higher order bending theories should be employed, compared to the simplified technical Bernoulli-Euler bending theory. Preliminary results using a modified Timoshenko bending theory indicate that the number of reinforcing bars should be slightly increased.

The existence of singularities at the corners and the roots of the threads yields more severe stress and strain fields in comparison to the restoration with unbolted bars. The equivalent stress in the titanium bar is more severe at the roots of the threads rather than at their respective peaks.

However, what is perhaps more important from a practical point of view, is the fact that the intensity of the stress field along the reinforcing bar decreases rapidly and is almost zero after the 50-th (out of 200) thread, indicating that the formula for the anchoring length of the reinforcing bars should be reconsidered in the direction of minimizing the intervention on the authentic stones.

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