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CRUSHING OF THIN-WALLED STRAIN RATE SENSITIVE STRUCTURES

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A simplifying analysis is presented for evaluating the dynamic crushing strength of thin-walled structures subjected to compressive loading. Three major deformation mechanisms are distinguished and corresponding approximate expressions for the mean curvature rates are derived. Depending on the assumed form of the constitutive equation describing the strain rate properties of the material two alternative formulae are derived for the dynamic correction factor of compressed thin-gauge structures. Indications are given as to how crushing tests should be run in order to validate the theory.

1. Introduction

Most experiments concerning the plastic behaviour of metals at a high rate of loading are interpreted on the assumption that the state of strain is uniform throughout the specimen. Indeed in quasi-static tests (with strain rates not exceeding 10^{-1}) performed on the standard Instron machine the material in circular or sheet metal specimens undergoes uniform extension and the relation between the cross-head velocity V and the resulting strain rate $\dot{\varepsilon}$ is linear:

$$\dot{\varepsilon}=\frac{V}{l_0}, \tag{1.1}$$

where l_0 is the specimen gauge length. In experiments with higher strain rates accomplished on the Hopkinson pressure bar or MTS hydraulic tests machines the strain distribution ceases to be uniform due to longitudinal and lateral (radial) inertia effects. Still Eq. (1.1) can serve as a useful approximation (cf. [1]).

By contrast, in crushed sheet metal specimens the deformation field is highly nonuniform and plastic deformations are confined to localized regions while the remainder of the material remains elastic. It has been observed that the deformation field in compressed thin-walled structures consists predominantly of bending about stationary and moving hinge lines with very little stretching of the sheet metal [2, 3]. As a result of this, a complicated pattern of folds and wrinkles is formed where strain and strain rates usually reach very high magnitudes.

The purpose of the paper is to derive approximate expressions relating the impact velocity and the strain rate attained in the process of crushing of thin-walled members. On that basis estimate is given of the ratio between the static and dynamic strength of the compressed structures. In order to keep the analysis as simple as possible, a rigid-viscoplastic material idealization is assumed; all inertia effects and elastic deformations are disregarded. Thus we introduce the hypothesis that the difference between the strength of statically and dynamically crushed members is due solely to the strain rate sensitivity of the material. Neglecting inertia forces is justified when the deformation fields in static and dynamic problems are identical. This is often the case, especially in the compressed thin-walled columns of various cross-section. However, the present analysis is aimed at more general shell structures in which the local buckling and post-buckling mechanism can be distinguished.

2. Computation of average curvature rates

At a first glance the crushing process of thin-walled structures appears to be very complicated, (Fig. 1). However, upon closer examination one can distinguish few characteristic deformation mechanisms. The major deformation mechanism, observed in almost all experiments, is bending about stationary hinge lines. In a rigid-perferctly plastic material the curvature at the plastic hinge is infinite but remains always finite if strain-hardening and strain rate sensitivity are taken into account.

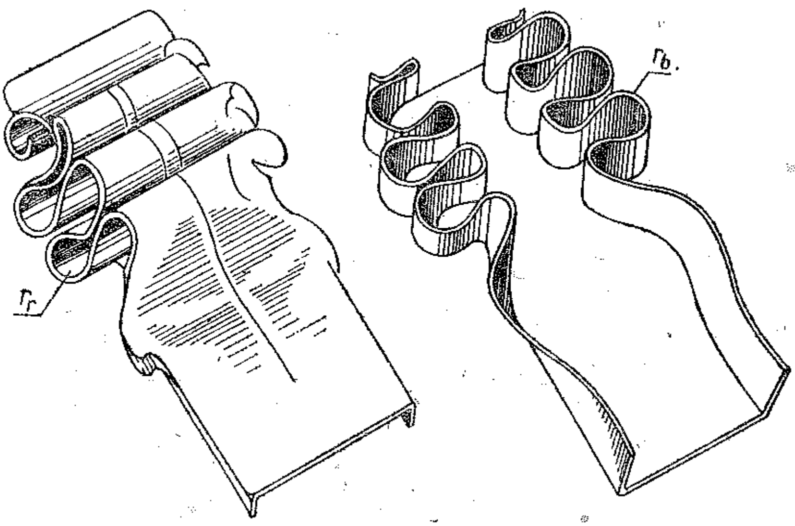

Fig. 1. Photographs of partially crushed thin-walled metal columns.

Consider a small, initially straight ($K_1=0$) element of a metal strip which is bent to the radius ($K_2=1/r_b$). The average curvature rate is then

$$\dot{K}_{av}=\frac{K_1-K_2}{\Delta t}, \tag{2.1}$$

where Δt is the duration time of the deformation process $\Delta t = h/V$, i.e. the time of formation of a single complete fold (Fig. 2). Here we denote by h the half-length of the local buckling wave. Hence

$$\dot{K}_{av} = \frac{V}{hr_b}. \tag{2.2}$$

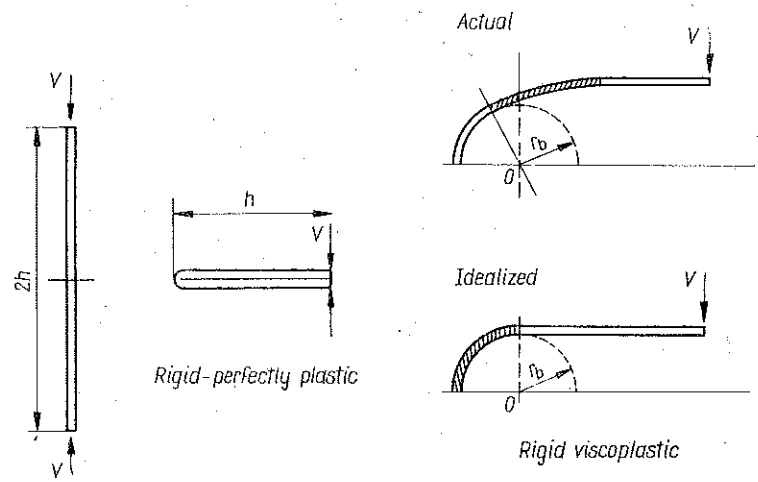

FIG. 2. Bending mechanisms in rigid-perfectly plastic and rigid-viscoplastic members.

A quasi-static analysis of compressed thin-walled members indicate that only a part of energy is dissipated by bending about stationary hinge lines. Depending upon the degree of symmetry and end conditions of the structure, some other deformation mechanisms can be activated. If the hinge lines are forced to travel, the corresponding deformation mode consists of the so-called rolling and corner type of deformation [3]. For example, in the crushing problem of closed-section prismatic columns the relative contribution of bending, rolling and corner deformation to the strength of the structure is respectively 44%, 44% and 12% [4]. It is thus necessary to derive expressions for the average strain rates also in these two deformation mechanisms.

Consider first the rolling deformation mode which is essentially bending and rebending of a metal strip at a hinge moving with the velocity V_r. For a rigid-perfectly plastic material all deformation is confined to two points A and B where the curvature suffers a jump from $K_1=0$ to $K_2=1/r_r$, (Fig. 3). The curvature rate is infinite at A and B. In a rigid-viscoplastic material the process of bending is spread over a finite region $(\xi+\eta)$ leading to a finite value of the curvature rate. Consider a small arc element ds moving with a constant velocity V_r. This element is initially straight $(K_1=0)$ but after passing through $x=\xi$ is subjected to bending and leaves the viscoplastic zone at $x=-\eta$ with a curvature $K=1/r_r$. The same element is subsequently rebent between C and D. Noting that $ds = V_r\, dt$, the rate of curvature can be expressed by

$$\dot{K} = \frac{dK}{dt} = V_r \frac{dK}{ds} = V_r \frac{d^2\theta}{ds^2}. \tag{2.3}$$

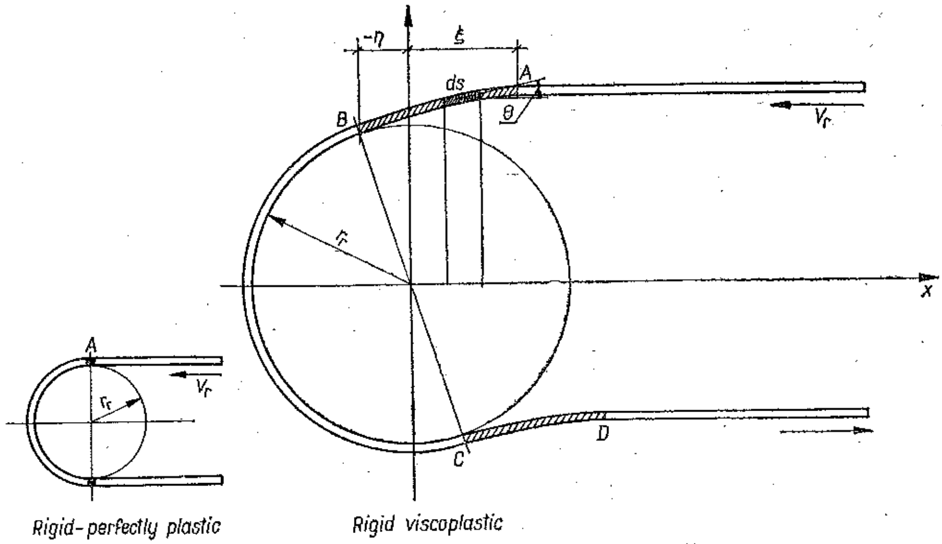

FIG. 3. Rolling mechanism in rigid-plastic and rigid-viscoplastic metal strip.

Introducing an approximation $dx = ds$, valid for small angles θ, Eq. (2.3) can be rewritten as

$$\dot{K} = V_r \frac{dK}{dx} = V_r \frac{d^2\theta}{dx^2}. \tag{2.4}$$

Now, the average strain rate over the deformation region $(\xi+\eta)$ can be easily computed:

$$\dot{K}_{av} = \frac{1}{\xi+\eta} \int_{-\eta}^{\xi} \dot{K}\, dx = \frac{V_r}{(\xi+\eta)\, r_r}. \tag{2.5}$$

It is easy to interpret Eq. (2.5) in a similar way as before. The average rate of curvature equals to the change in curvature $1/r_r$ of the arc element ds divided by the time required for ds to pass through the deformation zone $(\xi+\eta)/V_r$. However, the formula (2.5) is of no direct use since the distance $(\xi+\eta)$ is unknown and depends on the velocity V_r which in turn is some function of the crushing velocity V.

A useful approximation, valid at least for columns of a circular or rectangular cross-section, is that the ratio $V_r/(\xi+\eta)$ is constant and equal to

$$\frac{V_r}{\xi+\eta} = \frac{V}{h}. \tag{2.6}$$

This follows from the kinematic considerations and observation that initially V_r is large and so is the rolling radius and length of the viscoplastic zone [5]. At later stages of the formation of the local fold the rolling velocity V_r diminishes and at the same time the deformation zone shrinks.

With Eq. (2.6), the formula (2.5) for the average strain rate in the rolling process takes an analogous form as in the case of the bending process Eq. (2.2) with the exception that the radius r is not generally the same

$$\dot{K}_{av} = \frac{V}{hr_r}. \tag{2.7}$$

The corner type of deformation was first identified in [6] and described in detail by one of the present authors (W.A.) for a rigid-perfectly plastic material idealization [4]. As the hinge line with one lobe moves through the shell material, the curvature of the side panels changes from positive $1/r_b$ to negative $-1/r_b$ (Fig. 4). The deformation time is now $2h/V$ and the average rate of curvature is the same as before, (Eq. (2.2)).

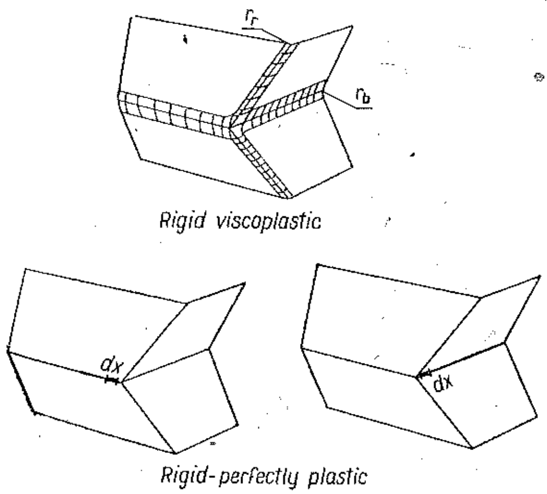

Fig. 4. Corner deformation in crushing of rigid-perfectly plastic and rigid-viscoplastic sheet metal structures.

3. Average strain rate

The present analysis is very crude, it ignores local extension of the shell and many secondary effects. Despite this, the derived formulae for $\dot{K}_{av}$ are believed to give at least the order of magnitude of the expected curvature rate in the crushing process of shell structures. More exact analysis, as for example that presented in Ref. [5], can be performed at the expense of more complex calculations. However, the increase in accuracy within the present rigid-viscoplastic material idealization will probably be apparent unless other important effects such as material elasticity, strain hardening etc. are taken into account.

An important result is that the formulae for the average strain rate in all three types of deformations discussed in the preceding section, are the same. This means that the relative contribution of bending, rolling and corner deformation to the mean crushing force is the same in static and dynamic crushing. We have thus obtained confirmation of the hypothesis introduced by T. Akerström and one of the present authors (T.W.), where only the bending type of deformation was considered to predict the dynamic crushing force [5].

The strain rate varies linearly with the distance from the middle surface of the shell according to $\dot{\varepsilon}=\dot{K}z$. A good mean value of $\dot{\varepsilon}$ is obtained by taking $z=t/4$, where t is the gauge thickness

$$\dot{\varepsilon}_{av}=\frac{Vt}{4hr}, \tag{3.1}$$

in which r is the radius of rolling or bending. The strain rate is proportional to the crushing velocity V and depends on the geometrical parameters involved.

Experiments show that the practical range in which t/r falls is $1/5\div 1$. Hence

$$\dot{\varepsilon}_{av}=\frac{V}{20h}\div\frac{V}{4h}. \tag{3.2}$$

The control parameter in the present problem is the half-length of the local buckling wave h. The scale effect is clearly observed since at the same impact velocity V structures with smaller h deform with a higher strain rate. For example, in compressed thin-walled square columns with $a\times a=50\times 50$ mm, $t/r=1/5$ and $h\simeq a/4=$ $=0.125$ m,

$$\dot{\varepsilon}_{av}=0.4V\,[\mathrm{m/sec}]=1.44V\,[\mathrm{km/h}]. \tag{3.3}$$

The strain rate (measured in s^{-1}) is approximately equal to the impact velocity taken in km/h. For example, at $V=30$ km/h the average strain rate in the compressed column is of the order $10^1\div 10^2\ s^{-1}$. Such strain rates produce an appreciable increase of the strength of dynamically loaded mild steel members compared to static crushing.

From Eq. (3.3) we learn that in order to produce the strain rate $\dot{\varepsilon}=10^{-4}$ s, the crushing velocity should be $0.5\cdot 10^{-4}$ m/s$=3\cdot 10^{-3}$ m/min$=3$ mm/min. This strain rate should be maintained in the so called static crushing tests on thin-walled columns.

4. Dynamic correction factor

In the existing theoretical solutions concerning the static compression of thin-walled columns of various cross-section, the mean crushing force P_m^s was found to be proportional to the fully plastic bending moment $M_0=(\sigma t^2)/4$, [4, 5, 7]. This result has been confirmed in numerous experimental works [8, 9].

A simple method of computing the dynamic crushing force P_m^d would be to adjust the value of the stress by taking an average strain rate σ ($\dot{\varepsilon}_{av}$). This technique is similar to that developed by Perrone for solving a class of initial-boundary value problems for strain rate sensitivity structures [10]. It can be used in the presently considered problems provided the defoımation fields in static and dynamic crushing processes are similar.

In view of a linear dependence of P_m on σ, the ratio of the dynamic to static crushing forces is simply

$$R=\frac{P^d_m}{P^s_m}=\frac{\sigma(\dot{\varepsilon})}{\sigma_0}. \tag{4.1}$$

This ratio is called in the literature the dynamic correction factor (dynamic gain).

In order to evaluate R, a particular choice of the constitutive equation describing the $\sigma-\dot{\varepsilon}$ dependence should be made. A simple but realistic power type formula was suggested by Cowper and Symonds [11]:

$$\frac{\sigma}{\sigma_0}=1+\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\frac{1}{n}}, \tag{4.2}$$

where $\dot{\varepsilon}$ and n are constants determined to get the best fit of experimental data over a certain range of the strain rate. These constants depend in general on the level of strains. It has been observed that the lower yield stress is more sensitively dependent on the strain rate than is the flow stress at higher strains and, in particular, the ultimate stress, [1, 12].

It is easy to estimate the maximum strain at outer fibers of a locally crushed element

$$\varepsilon_{\max}=\frac{t}{2}K=\frac{t}{2r}. \tag{4.3}$$

Since t/r can be as large as unity, the maximum strain can reach 50%. This is an indication that all material constants appearing in Eqs. (4.2) or (4.5) should be determined by fitting the variation of the ultimate rather than lower yield stress with the strain rate. According to Eq. (4.2), the stress equals to the static value strictly at $\dot{\varepsilon}=0$. In practice, as a reference or „static" stress we understand a stress attained at $\dot{\varepsilon}=10^{-4}-10^{-5}\ \mathrm{s}^{-1}$. Using Eqs. (3.1) and (4.2), the ratio can be expressed in terms of the crushing speed

$$R=1+\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\frac{1}{n}}=1+\left(\frac{Vt}{4hr\dot{\varepsilon}_0}\right)^{\frac{1}{n}}. \tag{4.4}$$

Another possibility of representing the dependence of σ on the strain rate is the nonlinear homogeneous viscous type of constitutive equation

$$\frac{\sigma}{\sigma_0}=\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\frac{1}{n}}, \tag{4.5}$$

where σ, $\bar{\varepsilon}$ and $\bar{n}$ is a new set of material constants. One disadvantage of Eq. (4.5) is that with $\dot{\varepsilon}\to 0$ it does not reduce to the form describing a perfectly plastic material and becomes increasingly inaccurate for smaller strain rates. Secondly, a good fit of experimental data can be obtained only over a small range of the strain rate, probably one or, at the most, two orders of magnitude of $\dot{\varepsilon}$. Despite these shortcomings, Eq. (4.5) has widely been used in analyzing dynamic response of strain rate sensitive structures [13, 14]. An obvious advantage of Eq. (4.5) is its homogeneity. Suppose experiments are run at two crushing speeds V_1 and V_2 with associated strain rate $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$. Using Eqs. (3.1) and (4.5), the corresponding ratio of mean crushing forces is equal to

$$R=\frac{P_m^{(1)}}{P_m^{(2)}}=\left(\frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_2}\right)^{\frac{1}{\bar{n}}}=\left(\frac{V_1}{V_2}\right)^{\frac{1}{\bar{n}}}. \tag{4.6}$$

We have thus arrived at an extremely interesting and useful result that the dynamic correction factor does not involve any geometrical and material parameters except one constant $\bar{n}$.

5. Comparison with experiments

The available experimental results concerning the correlation between static and dynamic mean crushing forces are in many respects incomplete. First, the „static" crushing force has never been precisely defined. According to the present simplified analysis, the static strength of thin-walled columns should be determined by compressing specimens with cross-head speeds equal or less than 3 mm/min. In none of the reported tests was the rate of compression specified.

Another difficulty in interpreting experimental data is that columns are usually dynamically crushed not in a universal testing machine but are subjected to a mass impact using a drop tower or pendulum. During these tests the crushing speed is not constant and changes from its initial value V_0 to zero causing the mean crushing force and hence R to vary in the course of the deformation process.

For practical application it is highly desirable to relate P_m^d or R to a single parameter which is the initial impact velocity V_0. A good measure of the mean force or mean correction factor R_m is

$$P_m=\frac{1}{V_0}\int_{-V_0}^{0} P(V)\,dV, \quad R_m=\frac{1}{V}\int_{-V_0}^{0} R(V)\,dV. \tag{5.1}$$

This is not the only possibility of defining the mean correction factor R_m since at least two alternative definitions exist as to how to compute P_m. An extensive discussion of this problem can be found in a series of recent papers [5, 15, 16]. It has been found that differences resulting from the application of various definitions may reach $20\%\div 50\%$.

Planning and performing a series of tests with properly measured static and dynamic crushing forces should not be a great problem. A comparison with the prediction of the formula (4.4) suitably averaged over the deformation process would then be straightforward. Such experiments will hopefully be published in the near future as evidenced by the results released at the Euromech Colloquium No. 121, [17, 18].

It is interesting to note that the application of the nonlinear viscous constitutive equation (4.5) brings major simplifications to the averaging procedure. One can easily show that the ratio R becomes the same as before

$$R_{av}=R=\left(\frac{V_1}{V_2}\right)^{\frac{1}{n}}=\left(\frac{V_0^{(1)}}{V_0^{(2)}}\right)^{\frac{1}{n}}. \tag{5.2}$$

Moreover, all three definitions of the mean dynamic crushing force yield the same final result, Eq. (5.2). Consequently, some comparison of the prediction of Eq. (5.2) with test results can be made even though the precise meaning of P_m^s or P_m^d is not known.

Several authors presented test results concerning dynamically crushed columns in the range of impact velocities 3÷7 m/sec. These results were gathered in Refs. [5, 15], (Fig. 5). However, the experimental scatter is so large that any comparison with the theory is meaningless unless more comprehensive and reproduceable results are obtained.

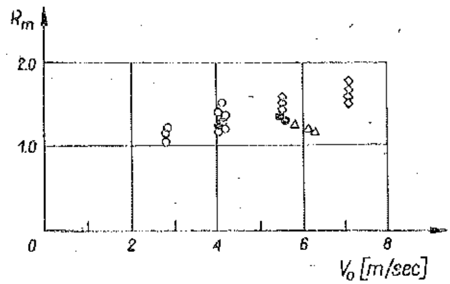

Fig. 5. Dynamic crushing strength of compressed thin-walled columns of rectangular cross-section.

6. Conclusions

A simplified analysis is presented for evaluating the average strain rate in the compression process of shell structures. The crushing strength of thin-walled members is computed and shown to depend on the crushing speed, three geometrical parameters describing the local buckling process and mechanical constants of the material. Assuming that the whole difference between the force level in static and dynamic compression is due to the viscoplastic properties of the material two alternative formulae are derived for the dynamic correction factor using different forms of constitutive equations. The paper gives guidelines as to how static and dynamic crushing tests should be run and interpreted. While in many instances reference was made to the axial deformation of columns, the results obtained are believed to be applicable to some other crushing problems in which the local plastic buckling problem can be identified.

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Streszczenie

ZGNIATANIE CIENKOŚCIENNYCH KONSTRUKCJI WRAŻLIWYCH NA PRĘDKOŚĆ ODKSZTAŁCENIA

Przedstawiono przybliżoną metodę wyznaczania dynamicznej wytrzymałości cienkościennych konstrukcji poddanych obciążeniom ściskającym. Wyodrębniono trzy podstawowe mechanizmy deformacji i w każdym przypadku wyprowadzono przybliżone wyrażenie na średnią prędkość krzywizny w procesie deformacji. W zależności od przyjętej formy równań konstytutywnych opisujących wpływ prędkości odkształcenia podano dwa alternatywne wzory na tzw. «dynamiczny współczynnik korekcji» dla ściskanych elementów cienkościennych. Podano wskazówki dotyczące właściwego przeprowadzenia eksperymentów w celu sprawdzenia rozwiązania teoretycznego.

Резюме

СДАВЛЕНИЕ ТОНКОСТЕННЫХ КОНСТРУКЦИЙ ЧУВСТВИТЕЛЬНЫХ НА СКОРОСТЬ ДЕФОРМАЦИИ

Представлен приближенный метод определения динамической прочности тонкостенных конструкций подвергнутых сжимающим нагрузкам. Выделены три основных механизма деформации и в каждом случае выведено приближенное выражение для средней скорости кривизны в процессе деформации. В зависимости от принятой формы определяющих уравнений, описывающих влияние скорости деформации, приведены две альтернативные формулы для т. наз. „динамического коэффициента коррекции" для сжимаемых тонкостенных элементов. Даются указания, касающиеся правильного проведения экспериментов с целью проверки теоретического решения.

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