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INFLUENCE OF TANGENTIAL DISPLACEMENTS ON THE DYNAMIC BUCKLING OF VISCOPLASTIC CYLINDRICAL SHELLS

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A theory of dynamic buckling is developed for viscoplastic cylindrical shells subjected to uniform radially inward impulses. The influence of tangential displacements on the magnitude of radial displacement, buckling mode and critical impulse is investigated. Asymmetrical and axi-symmetrical mode of buckling are considered. It is shown that the asymmetrical mode occurs, what is in accord with the experimental observations reported in the literature.

1. Introduction

The experimental investigations have shown that the metal cylindrical shells, buckling plastically, exhibit a characteristic wrinkled shape when subjected to sufficiently large uniform radially inward impulses, ABRAHAMSON, GOODIER [1], LINDBERG [2], ANDERSON, LINDBERG [3], FLORENCE, VAUGHAN [4], FLORENCE [5], LYONS [6]. The papers [1–4], STUIVER [7], VAUGHAN, FLORENCE [8], JONES, OKAWA [9] dealt with the model of an elastic-plastic or rigid-plastic body with linear hardening. In the papers by FLORENCE [5, 10], WOJEWÓDZKI [11–14], PERRONE [15] the viscosity effects were considered. In most of the above mentioned papers, to explain the buckling process, the problem was described within the framework of the linearized shell theory and only the radial displacement was taken into account. The buckling mode and the threshold impulse were determined. In the paper [6] the nonlinear geometric relation for the circumferential strain component was used and in consequence the threshold impulse was found to be smaller than that obtained in [3]. FLORENCE, ABRAHAMSON [16] found that the stability of a viscoplastic cylindrical shell subjected to a large radial impulse improved during the deformation process when the increase in wall thickness was taken into account.

The aim of the present paper is to investigate the influence of the tangential displacements on the magnitude of radial displacement, buckling mode and critical impulse. The thin viscoplastic cylindrical shell is loaded by a uniform radially inward impulse.

2. Basic equations

The influence of strain rate on the material response can be described by the following equations, PERZYNA [17]:

$$(2.1)\qquad \dot{\varepsilon}_{ij}=\frac{\gamma}{2}\langle\Phi(F)\rangle\frac{s_{ij}}{J_2^{1/2}},\quad F=\frac{J_2^{1/2}}{k}-1,\quad \langle\Phi(F)\rangle=\begin{cases}\Phi(F) & \text{for}\quad F>0,\\ 0 & \text{for}\quad F\leqslant 0,\end{cases}$$

where $\dot{\varepsilon}_{ij}$ is the strain rate tensor, s_{ij} denotes the stress deviator, $J_2=\frac{1}{2}s_{ij}s_{ij}$, $k=\sigma_0/3^{1/2}$, σ_0 is the static yield stress and γ stands for the viscosity coefficient of the material. The material is incompressible. The linear function $\Phi(F)=F$ is assumed. The physical equations of the Saint Venant–Levy–Mises theory of plastic flow, $\dot{\varepsilon}_{ij}=\lambda s_{ij}$, are obtained from Eqs. (2.1) if $\gamma=\infty$ and $J_2^{1/2}=k$.

The dynamic equilibrium equations are assumed in the following form [18] Fig. 1:

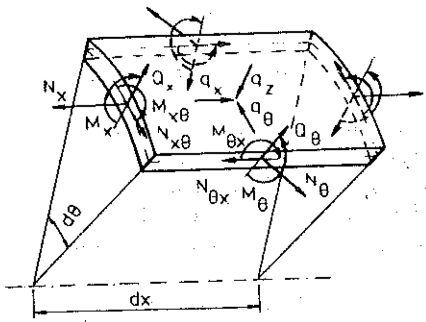

FIG. 1. Shell element—notation.

(2.2)

$$a\frac{\partial N_x}{\partial x}+\frac{\partial N_{\theta x}}{\partial\theta}-aN_{x\theta}\frac{\partial^2 v}{\partial x^2}-\left(\frac{\partial^2 v}{\partial x\,\partial\theta}-\frac{\partial w}{\partial x}\right)N_\theta-aQ_x\frac{\partial^2 w}{\partial x^2}-Q_\theta\left(\frac{\partial v}{\partial x}+\frac{\partial^2 w}{\partial x\,\partial\theta}\right)+aP_x=0,$$

$$a\frac{\partial N_{x\theta}}{\partial x}+\frac{\partial N_\theta}{\partial\theta}-Q_x\left(\frac{\partial^2 w}{\partial x\,\partial\theta}+\frac{\partial v}{\partial x}\right)-Q_\theta\left(1+\frac{1}{a}\frac{\partial v}{\partial\theta}+\frac{1}{a}\frac{\partial^2 w}{\partial\theta^2}\right)+aN_x\frac{\partial^2 v}{\partial x^2}+N_{\theta x}\left(\frac{\partial^2 v}{\partial x\,\partial\theta}-\frac{\partial w}{\partial x}\right)+aP_\theta=0,$$

$$a\frac{\partial Q_x}{\partial x}+\frac{\partial Q_\theta}{\partial\theta}+aN_x\frac{\partial^2 w}{\partial x^2}+(N_{x\theta}+N_{\theta x})\left(\frac{\partial v}{\partial x}+\frac{\partial^2 w}{\partial x\,\partial\theta}\right)+N_\theta\left(1+\frac{1}{a}\frac{\partial v}{\partial\theta}+\frac{1}{a}\frac{\partial^2 w}{\partial\theta^2}\right)+aP_z=0,$$

$$a\frac{\partial M_{x\theta}}{\partial x}-\frac{\partial M_\theta}{\partial\theta}-aM_x\frac{\partial^2 v}{\partial x^2}-M_{\theta x}\left(\frac{\partial^2 v}{\partial x\,\partial\theta}-\frac{\partial w}{\partial x}\right)+aQ_\theta=0,$$

$$a\frac{\partial M_x}{\partial x}+\frac{\partial M_{\theta x}}{\partial\theta}+aM_{x\theta}\frac{\partial^2 v}{\partial x^2}-M_\theta\left(\frac{\partial^2 v}{\partial x\,\partial\theta}-\frac{\partial w}{\partial x}\right)-aQ_x=0,$$

$$(M_x-M_\theta)\left(\frac{\partial v}{\partial x}+\frac{\partial^2 w}{\partial x\,\partial\theta}\right)+aM_{x\theta}\frac{\partial^2 w}{\partial x^2}+M_{\theta x}\left(1+\frac{1}{a}\frac{\partial v}{\partial\theta}+\frac{1}{a}\frac{\partial^2 w}{\partial\theta^2}\right)+a(N_{x\theta}-N_{\theta x})=0,$$

where

$$P_x = q_x - \rho h\ddot{u}, \quad P_\theta = q_\theta - \rho h\ddot{v}, \quad P_z = q_z - \rho h\ddot{w},$$

ρ denotes the density of the material.

The strain rate components are taken in the form (Fig. 1)

$$\dot{\varepsilon}_x = \frac{\partial \dot{u}}{\partial x} - z\frac{\partial^2 \dot{w}}{\partial x^2}, \quad \dot{\varepsilon}_\theta = \frac{1}{a}\frac{\partial \dot{v}}{\partial \theta} - \frac{\dot{w}}{a} - \frac{z}{a^2}\left(\frac{\partial^2 \dot{w}}{\partial \theta^2} + \frac{\partial \dot{v}}{\partial \theta}\right),$$

$$\dot{\varepsilon}_z = -(\dot{\varepsilon}_x + \dot{\varepsilon}_\theta), \tag{2.3}$$

$$2\dot{\varepsilon}_{x\theta} = \frac{\partial \dot{v}}{\partial x} + \frac{1}{a}\frac{\partial \dot{u}}{\partial \theta} - \frac{2z}{a}\left(\frac{\partial^2 \dot{w}}{\partial x\,\partial \theta} + \frac{\partial \dot{v}}{\partial x}\right), \quad \dot{\varepsilon}_{xz} = \dot{\varepsilon}_{\theta z} = 0.$$

3. Method of the solution

In the case of thin shells, $\sigma_z = \sigma_{33} = 0$, Eqs. (2.1) yield for $J_2^{1/2} > k$ the following nonlinear equations:

$$\sigma_{ij} = k\left(\frac{1}{\gamma_0} + \frac{1}{I_2^{1/2}}\right)(\dot{\varepsilon}_{ij} + \dot{\varepsilon}_{\alpha\alpha}\,\delta_{ij}), \quad i, j = 1, 2, 3, \quad \alpha = 1, 2, \tag{3.1}$$

where $\gamma_0 = \gamma/2$ and for the incompressible material ($\dot{\varepsilon}_{ll} = 0$, $l = 1, 2, 3$), $I_2 = \frac{1}{2}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}$ is the second invariant of the strain rate deviator, δ_{ij} denotes the Kronecker delta.

Let us consider a cylindrical shell loaded by a pressure impulse directed radially and inwards (Fig. 2). In the state of compressive plastic flow the shell buckles as a result of imperfections. A characteristic feature of dynamic buckling is the

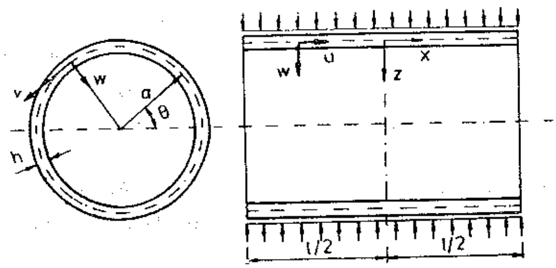

FIG. 2. Cylindrical shell; dimensions and loading.

significance of inertial effect in restraining the growth of buckling mode amplitudes at an early stage of the motion. This effect results in the yielding of the shell before the instabilities can become dominant. Analytically, the problem is formulated as a superposition of small perturbations $u_p(x, \theta, t)$, $v_p(x, \theta, t)$, $w_p(x, \theta, t)$ on the basic unperturbed motion $u_0(x, t)$, $v_0 = 0$, $w_0(t)$. The amplitudes of perturbed motion are so small that the homogeneous compressive deformation is predominant over the local bending. Also, this condition permits the constitutive equations ot be linearized by the expansion of Eqs. (3.1) into Taylor's series of five variables in the vicinity of unperturbed motion and to retain two terms only.

We get

$$\sigma_{ij}=k\left(\frac{1}{\gamma_0}+\frac{1}{\sqrt{I_2^0}}\right)(\dot{\varepsilon}_{ij}+\dot{\varepsilon}_{\alpha\alpha}\delta_{ij})-\frac{k}{2(I_2^0)^{3/2}}\dot{\varepsilon}_{lr}^0\dot{\varepsilon}_{lr}^p(\dot{\varepsilon}_{ij}^0+\dot{\varepsilon}_{\alpha\alpha}^0\delta_{ij}), \tag{3.2}$$

where $i, j, l, r=1, 2, 3$, $\alpha=1, 2$, $\dot{\varepsilon}_{ij}^0$, $\dot{\varepsilon}_{ij}^p$ denote the strain rate tensors of the unperturbed and perturbed motion and $I_2^0=\frac{1}{2}\dot{\varepsilon}_{ij}^0\dot{\varepsilon}_{ij}^0$.

According to the Kirchhoff-Love theory, $\varepsilon_{13}=\varepsilon_{23}=0$ (Eqs. (2.3)) and the indices i, j, l, r, α in Eqs. (3.2) can take on 1, 2 only. In this case Eqs. (3.2) can be rewritten in the form

$$\sigma_{ij}=\sigma_{ij}^0+\sigma_{ij}^p, \tag{3.3}$$

where

$$\sigma_{ij}^0=k\left(\frac{1}{\gamma_0}+\frac{1}{\sqrt{I_2^0}}\right)(\dot{\varepsilon}_{ij}^0+\dot{\varepsilon}_{ll}^0\delta_{ij}),$$

$$\sigma_{ij}^p=k\left(\frac{1}{\gamma_0}+\frac{1}{\sqrt{I_2^0}}\right)(\dot{\varepsilon}_{ij}^p+\dot{\varepsilon}_{ll}^p\delta_{ij})-\frac{k}{2(I_2^0)^{3/2}}(\dot{\varepsilon}_{ij}^0+\dot{\varepsilon}_{ll}^0\delta_{ij})(\dot{\varepsilon}_{st}^0\dot{\varepsilon}_{st}^p+\dot{\varepsilon}_{ss}^0\dot{\varepsilon}_{tt}^p), \tag{3.4}$$

$$I_2^0=\frac{1}{2}[\dot{\varepsilon}_{ij}^0\dot{\varepsilon}_{ls}^0\delta_{is}\delta_{jl}+(\dot{\varepsilon}_{ij}^0\delta_{ij})^2], \qquad i, j, l, s, t=1, 2;$$

$\dot{\varepsilon}_{ij}^p$ is the strain rate tensor of perturbed motion.

Equations (3.3) may now be used together with Eqs. (2.3) and (2.2) to obtain the differential equations governing the viscoplastic flow buckling of the shell. Buckling stems from the growth of small imperfections in the otherwise uniform initial displacements and loading fields. It turns out that certain harmonics grow rapidly and cause the shell to exhibit a characteristic wrinkled shape which is characterized by the critical mode numbers. This property of the amplitudes is used to determine the threshold impulse that the shell can tolerate without excessive deformation.

The constitutive equations (3.1) may be used if the loading criterion $J_2^{1/2}>k$ is satisfied. It is equivalent to $I_2^{1/2}>0$. After linearization this condition has the form

$$\dot{\varepsilon}_{ij}(\dot{\varepsilon}_{ij}^0+\dot{\varepsilon}_{\alpha\alpha}^0\delta_{ij})>0, \qquad i, j=1, 2, 3, \qquad \alpha=1, 2. \tag{3.5}$$

In the case of Eqs. (2.3) the subscripts i, j, α become 1, 2.

4. Equations of viscoplastic flow buckling

4.1. Unperturbed motion

Let us assume the unperturbed displacement functions in the form

$$u_0=u_0(x, t), \qquad v_0=0, \qquad w_0=w_0(t). \tag{4.1}$$

Making use in Eqs. (2.3) and (3.4) of the functions (4.1), we obtain the non-vanishing stress components

$$\sigma_x^0 = k\left[\frac{1}{\gamma_0} + \frac{1}{(I_2^0)^{1/2}}\right]\left(2\frac{\partial \dot{u}_0}{\partial x} - \frac{\dot{w}_0}{a}\right),$$

(4.2)

$$\sigma_\theta^0 = k\left[\frac{1}{\gamma_0} + \frac{1}{(I_2^0)^{1/2}}\right]\left(\frac{\partial \dot{u}_0}{\partial x} - 2\frac{\dot{w}_0}{a}\right),$$

where

$$I_2^0 = \left(\frac{\partial \dot{u}_0}{\partial x}\right)^2 - \frac{\dot{w}_0}{a}\frac{\partial \dot{u}_0}{\partial x} + \frac{\dot{w}_0^2}{a^2}.$$

Integrating the components (4.2) over the thickness of the shell, the membrane forces can now be computed. Inserting these forces into Eqs. (2.2), we get the set of two nonlinear differential equations. The solution of these equations is very difficult. The problem can be simplified by taking the relation

(4.3) $$\alpha = -\dot{\varepsilon}_x/\dot{\varepsilon}_\theta \quad \text{for} \quad z=0, \quad 0 \leqslant \alpha \leqslant 1/2.$$

The change of the value of α from $\alpha=1/2$ to $\alpha=0$ corresponds to variation of the length of the shell from very short to infinitely long. The relation was originally proposed by VAUGHAN and FLORENCE [8]. On the basis of experimental tests on cylindrical shells made of aluminium alloy the relation $\alpha = 0.5\exp(-l/4a)$ was determined, where l is the shell length and a is the radius. FLORENCE [10] proposed the relation $\alpha = 0.5\left[\cos h\left(\frac{\sqrt{3}\,l}{4a}\right)\right]^{-1}$. The values of α calculated from these two relations do not differ appreciably and the discrepancies involved have a negligible influence on numerical results. From Eqs. (4.3), (2.3) and (3.4), we get

$$\dot{\varepsilon}_x^0 = \alpha\frac{\dot{w}_0}{a}, \quad \dot{\varepsilon}_\theta^0 = -\frac{\dot{w}_0}{a}, \quad \dot{\varepsilon}_{x\theta}^0 = 0,$$

(4.4) $$\sigma_x^0 = (2\alpha-1)\left(\frac{k}{\gamma_0}\frac{\dot{w}_0}{a} + \frac{\sigma_0}{K_1}\right), \quad \sigma_\theta^0 = (\alpha-2)\left(\frac{k}{\gamma_0}\frac{\dot{w}_0}{a} + \frac{\sigma_0}{K_1}\right), \quad \sigma_{x\theta}^0 = 0;$$

$$N_x^0 = \int_{-h/2}^{h/2} \sigma_x^0\,dz = (2\alpha-1)\left(\frac{k}{\gamma_0}\frac{\dot{w}_0}{a} + \frac{\sigma_0}{K_1}\right)h,$$

$$N_\theta^0 = \int_{-h/2}^{h/2} \sigma_\theta^0\,dz = (\alpha-2)\left(\frac{k}{\gamma_0}\frac{\dot{w}_0}{a} + \frac{\sigma_0}{K_1}\right)h,$$

(4.5) $$N_{x\theta}^0 = N_{\theta x}^0 = 0, \quad M_x^0 = M_\theta^0 = 0, \quad M_{\theta x}^0 = -M_{x\theta}^0 = 0,$$

where $K_1 = \sqrt{3(\alpha^2-\alpha+1)}$.

The shell is subjected to the action of radial pressure impulse, thus $q_x=0$. The longitudinal deformation occurs freely during motion; no axial restraint and no axial forces exist at the ends of the shell. Longitudinal inertia will also be disregarded. In this situation Eqs. (2.2) reduce to the following equation:

$$N_\theta^0 = -a\,(q_z^0(t) - \rho h \ddot{w}_0) \tag{4.6}$$

which, combined with the expression for N_θ, Eqs. (4.5), yields

$$\ddot{w}_0 + \frac{(2-\alpha)k}{\gamma_0 \rho a^2}\dot{w}_0 + \frac{(2-\alpha)\sigma_0}{a\rho K_1} = \frac{q_z^0(t)}{\rho h}. \tag{4.7}$$

This equation of unperturbed motion will be solved for two types of the impulse. In the case of ideal impulse loading (uniform initial velocity), $q_z^0(t)=0$ and the solution of Eq. (4.7) with the initial conditions $w_0(0)=0$, $\dot{w}_0(0)=V_0$ has the form

$$w_0 = \frac{\gamma_0 \rho a^2}{(2-\alpha)k}\left(V_0 + \frac{\sqrt{3}\,\gamma_0 a}{K_1}\right)\left[1 - \exp\left(\frac{(\alpha-2)k}{\gamma_0 \rho a^2}t\right)\right] - \frac{\sqrt{3}\,\gamma_0 a}{K_1}t. \tag{4.8}$$

The unperturbed motion ceases at the instant $t=t_f$ when $\dot{w}_0(t_f)=0$. Thus

$$t_f = \frac{\gamma_0 \rho a^2}{(2-\alpha)k}\ln\left(\frac{\sqrt{3}\,\gamma_0 a}{V_0 K_1 + \sqrt{3}\,\gamma_0 a}\right)^{-1}. \tag{4.9}$$

In the case of rectangular impulse pressure

$$q_z^0(t) = \begin{cases} Q, & \text{for} \quad 0 \leqslant t \leqslant T, \\ 0, & \text{for} \quad t > T \end{cases} \tag{4.10}$$

and for the initial conditions $w_0(0)=0$, $\dot{w}_0(0)=0$ the solution of Eq. (4.7) is of the form

$$\begin{aligned} w_0 &= \frac{1}{c}\left(\frac{Q}{\rho h} - \frac{(2-\alpha)\sigma_0}{K_1 \rho a}\right)\left[t + \frac{1}{c}(e^{-ct}-1)\right], \quad \text{for} \quad 0 \leqslant t \leqslant T, \\ w_0 &= \frac{(2-\alpha)\sigma_0}{c^2 K_1 a\rho}(1 - ct - e^{-ct}) + \frac{Q}{\rho h c}\left[T + \frac{1}{c}(1-e^{cT})e^{-ct}\right], \quad \text{for } T \leqslant t \leqslant t_f, \end{aligned} \tag{4.11}$$

where $c = k(2-\alpha)/(\gamma_0 \rho a^2)$. The unperturbed motion ceases at the instant $t=t_f$,

$$t_f = \frac{1}{c}\ln\left[1 - \frac{Q a K_1}{(2-\alpha)\sigma_0 h}(1-e^{cT})\right]. \tag{4.12}$$

In Figs. 3, 4, 5 and 6 are shown the graphs of the unperturbed displacement w_0 calculated for a shell made of mild steel characterized by various viscosity coefficients. The effect of the h/a and l/a is readily observed and also a significant influence of viscosity on the values of displacements of the shell and on the duration of the deformation.

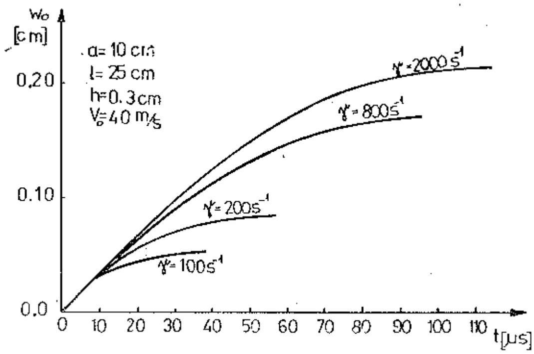

FIG. 3. Unperturbed displacement w_0 vs. time for a few values of viscosity coefficients γ

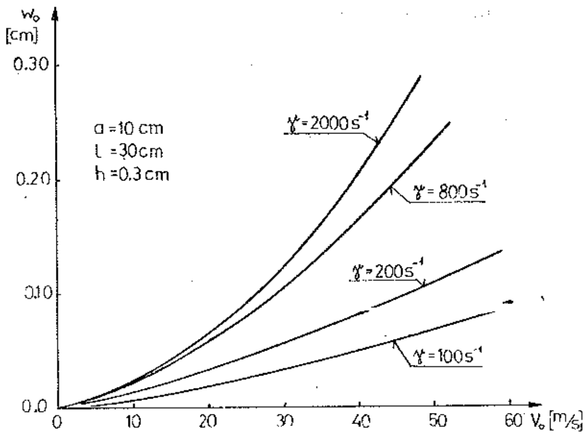

FIG. 4. Unperturbed final displacement $w_0(t_f)$ vs. ideal impulse V_0 for a few values of viscosity coefficients γ.

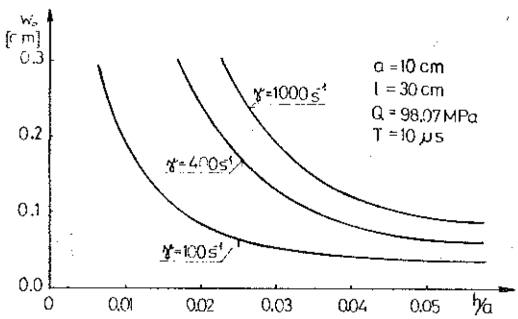

FIG. 5. Influence of h/a and γ on the magnitude of $w_0(t_f)$.

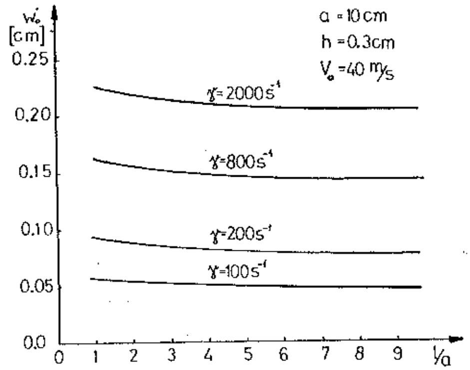

FIG. 6. Influence of l/a and γ on the magnitude of $w_0(t_f)$.

4.2. Perturbed motion

The total displacements in the perturbed motion are expressed by the following functions:

$$(4.13)\qquad u=\frac{\alpha w_0(t)}{a}x+u_p(x,\theta,t),\qquad v=v_p(x,\theta,t),\qquad w=w_0(t)+w_p(x,\theta,t).$$

Substituting these functions into the expressions (2.3) leads to the strain rates formulae

$$\dot{\varepsilon}_x=\alpha\frac{\dot{w}_0}{a}+\frac{\partial\dot{u}_p}{\partial x}-z\frac{\partial^2\dot{w}_p}{\partial x^2},\qquad \dot{\varepsilon}_z=-(\dot{\varepsilon}_x+\dot{\varepsilon}_\theta),$$

$$(4.14)\qquad \dot{\varepsilon}_\theta=\frac{\partial\dot{v}_p}{a\partial\theta}-\frac{\dot{w}_0}{a}-\frac{\dot{w}_p}{a}-\frac{z}{a^2}\left(\frac{\partial^2\dot{w}_p}{\partial\theta^2}+\frac{\partial\dot{v}_p}{\partial\theta}\right),$$

$$2\dot{\varepsilon}_{x\theta}=\frac{\partial\dot{v}_p}{\partial x}+\frac{\partial\dot{u}_p}{a\partial\theta}-\frac{2z}{a}\left(\frac{\partial^2\dot{w}_p}{\partial x\,\partial\theta}+\frac{\partial\dot{v}_p}{\partial x}\right),\qquad \dot{\varepsilon}_{xz}=\dot{\varepsilon}_{\theta z}=0$$

and hence, by Eqs. (3.4) and (3.5) the stress components are

$$\sigma_x=\sigma_x^0+\frac{k}{\gamma_0}\left[2\left(\frac{\partial\dot{u}_p}{\partial x}-z\frac{\partial^2\dot{w}_p}{\partial x^2}\right)+\frac{1}{a}\left(\frac{\partial\dot{v}_p}{\partial\theta}-\dot{w}_p\right)-\frac{z}{a^2}\left(\frac{\partial^2\dot{w}_p}{\partial\theta^2}+\frac{\partial\dot{v}_p}{\partial\theta}\right)\right]+$$
$$+\frac{9\sqrt{3}\,ak}{2\dot{w}_0K_1^3}\left[\frac{\partial\dot{u}_p}{\partial x}-z\frac{\partial^2\dot{w}_p}{\partial x^2}+\frac{\alpha}{a}\left(\frac{\partial\dot{v}_p}{\partial\theta}-\dot{w}_p\right)-\alpha\frac{z}{a^2}\left(\frac{\partial^2\dot{w}_p}{\partial\theta^2}+\frac{\partial\dot{v}_p}{\partial\theta}\right)\right],$$

$$(4.15)\qquad \sigma_\theta=\sigma_\theta^0+\frac{k}{\gamma_0}\left[\frac{2}{a}\left(\frac{\partial\dot{v}_p}{\partial\theta}-\dot{w}_p\right)-\frac{2z}{a^2}\left(\frac{\partial^2\dot{w}_p}{\partial\theta^2}+\frac{\partial\dot{v}_p}{\partial\theta}\right)+\frac{\partial\dot{u}_p}{\partial x}-z\frac{\partial^2\dot{w}_p}{\partial x^2}\right]+$$
$$+\frac{9\sqrt{3}\,ak}{2\dot{w}_0K_1^3}\left[\alpha\left(\frac{\partial\dot{u}_p}{\partial x}-z\frac{\partial^2\dot{w}_p}{\partial x^2}\right)+\frac{\alpha^2}{a}\left(\frac{\partial\dot{v}_p}{\partial\theta}-\dot{w}_p\right)-\alpha^2\frac{z}{a^2}\left(\frac{\partial^2\dot{w}_p}{\partial\theta^2}+\frac{\partial\dot{v}_p}{\partial\theta}\right)\right],$$

$$\sigma_{x\theta}=\frac{k}{2}\left(\frac{1}{\gamma_0}+\frac{\sqrt{3}\,a}{\dot{w}_0K_1}\right)\left[\frac{\partial\dot{v}_p}{\partial x}+\frac{\partial\dot{u}_p}{a\,\partial\theta}-\frac{2z}{a}\left(\frac{\partial^2\dot{w}_p}{\partial x\,\partial\theta}+\frac{\partial\dot{v}_p}{\partial x}\right)\right].$$

The stress distribution (4.15) produces the following resultant forces and moments:

(4.16)
$$N_x=\int_{-h/2}^{h/2}\sigma_x\,dz=N_x^0+kh\left[\beta_1\frac{\partial\dot{u}_p}{\partial x}+\beta_2\left(\frac{\partial\dot{v}_p}{a\,\partial\theta}-\frac{\dot{w}_p}{a}\right)\right],$$
$$N_\theta=\int_{-h/2}^{h/2}\sigma_\theta\,dz=N_\theta^0+kh\left[\beta_2\frac{\partial\dot{u}_p}{\partial x}+\beta_3\left(\frac{\partial\dot{v}_p}{a\,\partial\theta}-\frac{\dot{w}_p}{a}\right)\right],$$
$$N_{x\theta}=N_{\theta x}=\int_{-h/2}^{h/x}\sigma_{x\theta}\,dz=kh\,\beta_4\left(\frac{\partial\dot{v}_p}{\partial x}+\frac{\partial\dot{u}_p}{a\,\partial\theta}\right),$$
$$M_x=\int_{-h/2}^{h/2}\sigma_x\,z\,dz=-\frac{kh^3}{12a^2}\left[\beta_1\,a^2\frac{\partial^2\dot{w}_p}{\partial x^2}+\beta_2\left(\frac{\partial^2\dot{w}_p}{\partial\theta^2}+\frac{\partial\dot{v}_p}{\partial\theta}\right)\right],$$
$$M_\theta=\int_{-h/2}^{h/2}\sigma_\theta\,z\,dz=-\frac{kh^3}{12a^2}\left[\beta_2\,a^2\frac{\partial^2\dot{w}_p}{\partial x^2}+\beta_3\left(\frac{\partial^2\dot{w}_p}{\partial\theta^2}+\frac{\partial\dot{v}_p}{\partial\theta}\right)\right],$$
$$M_{x\theta}=-M_{\theta x}=-\int_{-h/2}^{h/2}\sigma_{x\theta}\,z\,dz=\frac{kh^3}{6a}\beta_4\left(\frac{\partial\dot{v}_p}{\partial x}+\frac{\partial^2\dot{w}_p}{\partial x\,\partial\theta}\right),$$

where

(4.17)
$$\beta_1=\frac{2}{\gamma_0}+\frac{9\sqrt{3}\,a}{2\dot{w}_0\,K_1^3},\qquad \beta_2=\frac{1}{\gamma_0}+\frac{9\sqrt{3}\,\alpha a}{2\dot{w}_0\,K_1^3},$$
$$\beta_3=\frac{2}{\gamma_0}+\frac{9\sqrt{3}\,\alpha^2 a}{2\dot{w}_0\,K_1^3},\qquad \beta_4=\frac{1}{2\gamma_0}+\frac{\sqrt{3}\,a}{2\dot{w}_0\,K_1}.$$

Eliminating shear forces from Eqs. (2.2), neglecting terms with the products of perturbation quantities and the sixth equation, we get

(4.18)
$$a\frac{\partial N_x}{\partial x}+\frac{\partial N_{\theta x}}{\partial\theta}-\left(\frac{\partial^2 v}{\partial x\,\partial\theta}-\frac{\partial w}{\partial x}\right)N_\theta^0-aph\ddot{u}=0,$$
$$a\frac{\partial N_{x\theta}}{\partial x}+\frac{\partial N_\theta}{\partial\theta}+\frac{\partial M_{x\theta}}{\partial x}-\frac{\partial M_\theta}{a\,\partial\theta}+a\frac{\partial^2 v}{\partial x^2}N_x^0-aph\ddot{v}=0,$$
$$a\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_\theta}{a\,\partial\theta^2}+\frac{\partial^2 M_{\theta x}}{\partial x\,\partial\theta}-\frac{\partial^2 M_{x\theta}}{\partial x\,\partial\theta}+a\frac{\partial^2 w}{\partial x^2}N_x^0+N_\theta+\frac{1}{a}\left(\frac{\partial v}{\partial\theta}+\frac{\partial^2 w}{\partial\theta^2}\right)\times$$
$$\times N_\theta^0+a\,(q_z-ph\ddot{w})=0.$$

On substituting the expressions (4.16) in Eqs. (4.18) and on accounting for Eq. (4.6) and the relation $q_z=q_z^0(t)+q_z^p(t,\theta,x)$, we obtain the following equations of the viscoplastic flow buckling:

(4.19)
$$\ddot{u}_p-\frac{k}{\rho a}\left[a\beta_1\frac{\partial^2\dot{u}_p}{\partial x^2}+\frac{1}{a}\beta_4\frac{\partial^2\dot{u}_p}{\partial\theta^2}+(\beta_2+\beta_4)\frac{\partial^2\dot{v}_p}{\partial x\,\partial\theta}-\beta_2\frac{\partial\dot{w}_p}{\partial x}\right]+$$
$$+\frac{1}{aph}\left(\frac{\partial^2 v_p}{\partial x\,\partial\theta}-\frac{\partial w_p}{\partial x}\right)N_\theta^0=0,$$

(4.19) [cont.]

$$\ddot{v}_p-\frac{k}{\rho a}\left[a\left(1+\frac{h^2}{6a^2}\right)\beta_4\frac{\partial^2\dot{v}_p}{\partial x^2}+\frac{1}{a}\left(1+\frac{h^2}{12a^2}\right)\beta_3\frac{\partial^2\dot{v}_p}{\partial\theta^2}+\right.$$
$$\left.+(\beta_2+\beta_4)\frac{\partial^2\dot{u}_p}{\partial x\,\partial\theta}-\frac{1}{a}\beta_3\frac{\partial\dot{w}_p}{\partial\theta}\right]-\frac{kh^2}{12\rho a^2}\left[(\beta_2+2\beta_4)\frac{\partial^3\dot{w}_p}{\partial x^2\,\partial\theta}+\right.$$
$$\left.+\frac{1}{a^2}\beta_3\frac{\partial^3\dot{w}_p}{\partial\theta^3}\right]-\frac{1}{\rho h}\frac{\partial^2 v_p}{\partial x^2}N_x^0=0,$$

$$\ddot{w}_p+\frac{kh^2}{12\rho a^2}\left[a^2\beta_1\frac{\partial^4\dot{w}_p}{\partial x^4}+2(\beta_2+2\beta_4)\frac{\partial^4\dot{w}_p}{\partial x^2\,\partial\theta^2}+\frac{1}{a^2}\beta_3\frac{\partial^4\dot{w}_p}{\partial\theta^4}\right]+$$
$$+\frac{k}{\rho a^2}\beta_3\dot{w}_p+\frac{kh^2}{12\rho a^2}\left[\frac{1}{a^2}\beta_3\frac{\partial^3\dot{v}_p}{\partial\theta^3}+(\beta_2+4\beta_4)\frac{\partial^3\dot{v}_p}{\partial x^2\,\partial\theta}-\frac{12}{h^2}\beta_3\frac{\partial\dot{v}_p}{\partial\theta}\right]-$$
$$-\frac{k}{\rho a}\beta_2\frac{\partial\dot{u}_p}{\partial x}-\frac{1}{\rho h}\left[\frac{\partial^2 w_p}{\partial x^2}N_x^0+\frac{1}{a^2}\left(\frac{\partial^2 w_p}{\partial\theta^2}+\frac{\partial v_p}{\partial\theta}\right)N_\theta^0+q_z^p\right]=0,$$

where N_x^0, N_θ^0, β_1, β_2, β_3, β_4 are given by the expressions (4.5) and (4.11). In the following equations the terms $h^2/(6a^2)$ and $h^2/(12a^2)$ occurring in the second equation of the set (4.19) will be disregarded as small ones in comparison with unity.

The displacement and loading perturbations can be assumed in the form of Fourier series:

$$\begin{aligned}
u_p(t,\theta,x)&=\sum_{n=0}^{N}\sum_{m=1}^{M}u_{nm}(t)\cos n\theta\sin\alpha_m x,\\
v_p(t,\theta,x)&=\sum_{n=1}^{N}\sum_{m=0}^{M}v_{nm}(t)\sin n\theta\cos\alpha_m x,\\
w_p(t,\theta,x)&=\sum_{n=0}^{N}\sum_{m=0}^{M}w_{nm}(t)\cos n\theta\cos\alpha_m x,\\
q_z^p(t,\theta,x)&=\sum_{n=0}^{N}\sum_{m=0}^{M}q_{nm}^p(t)\cos n\theta\cos\alpha_m x,
\end{aligned}\tag{4.20}$$

where $\alpha_m=m\pi/l$, m and $2n$ denote the number of half-waves in longitudinal and circumferential directions, respectively. The origin of the coordinates was taken in the middle-length of the shell.

Substituting the series (4.20) in Eqs. (4.19), we find the following equations for the amplitudes $u_{nm}(t)$, $v_{nm}(t)$ and $w_{nm}(t)$:

(4.21)

$$\ddot{u}_{nm}+\frac{k}{\rho}\left(\beta_1\alpha_m^2+\beta_4\frac{n^2}{a^2}\right)\dot{u}_{nm}+\frac{n\alpha_m}{\rho a}\left[k(\beta_2+\beta_4)\dot{v}_{nm}-\frac{N_\theta^0}{h}v_{nm}\right]-$$
$$-\frac{\alpha_m}{\rho a}\left(k\beta_2\dot{w}_{nm}-\frac{N_\theta^0}{h}w_{nm}\right)=0,$$

(4.21) [cont.]

$$\ddot{v}_{nm}+\frac{k}{\rho}\left(\frac{n^2}{a^2}\beta_3+\beta_4\,\alpha_m^2\right)\dot{v}_{nm}+\frac{\alpha_m^2\,N_x^0}{\rho h}v_{nm}+\frac{kn\alpha_m}{\rho a}(\beta_2+\beta_4)\,\dot{u}_{nm}-$$
$$-\frac{kn}{\rho a^2}\left[\left(1+n^2\frac{h^2}{12a^2}\right)\beta_3+\frac{h^2}{12}(\beta_2+2\beta_4)\,\alpha_m^2\right]\dot{w}_{nm}=0,$$

$$\ddot{w}_{nm}+\frac{kh^2}{12\rho a^2}\left[a^2\,\alpha_m^4\,\beta_1+2n^2\,\alpha_m^2\,(\beta_2+2\beta_4)+\left(\frac{n^4}{a^2}+\frac{12}{h^2}\right)\beta_3\right]\dot{w}_{nm}+$$
$$+\frac{1}{\rho h}\left(\alpha_m^2\,N_x^0+\frac{n^2}{a^2}N_\theta^0\right)w_{nm}+\frac{k\alpha_m}{\rho a}\beta_2\,\dot{u}_{nm}-\frac{kn}{\rho a^2}\left[\frac{h^2}{12}\alpha_m^2\,(\beta_2+4\beta_4)+\right.$$
$$\left.+\left(1+n^2\frac{h^2}{12a^2}\right)\beta_3\right]\dot{v}_{nm}-\frac{n\,N_\theta^0}{\rho a^2\,h}v_{nm}-\frac{q_{mn}^p}{\rho h}=0.$$

The coefficients of these equations are functions of time and are determined by the solution for the unperturbed motion.

In a general case the solution of Eqs. (4.21) for given initial conditions can be easily obtained by numerical integration.

At the instant $t=t_f$ when $\dot{w}_0(t_f)=0$ we have $\dot{u}_{nm}=\dot{v}_{nm}=\dot{w}_{nm}=0$.

Bearing in mind the form of the solution (4.20), the initial conditions must also be expressed in the form

(4.22)

$$u_p(0,\theta,x)=\sum_{n=0}^{N}\sum_{m=1}^{M}\bar{u}_{nm}\cos n\theta\sin\alpha_m x,$$
$$\dot{u}_p(0,\theta,x)=\sum_{n=0}^{N}\sum_{m=1}^{M}\hat{u}_{nm}\cos n\theta\sin\alpha_m x,$$
$$v_p(0,\theta,x)=\sum_{n=1}^{N}\sum_{m=0}^{M}\bar{v}_{nm}\sin n\theta\cos\alpha_m x,$$
$$\dot{v}_p(0,\theta,x)=\sum_{n=1}^{N}\sum_{m=0}^{M}\hat{v}_{nm}\sin n\theta\cos\alpha_m x,$$
$$w_p(0,\theta,x)=\sum_{n=0}^{N}\sum_{m=0}^{M}\bar{w}_{nm}\cos n\theta\cos\alpha_m x,$$
$$\dot{w}_p(0,\theta,x)=\sum_{n=0}^{N}\sum_{m=0}^{M}\hat{w}_{nm}\cos n\theta\cos\alpha_m x,$$

(4.23)

$$u_{nm}(0)=\bar{u}_{nm},\qquad \dot{u}_{nm}(0)=\hat{u}_{nm},$$
$$v_{nm}(0)=\bar{v}_{nm},\qquad \dot{v}_{nm}(0)=\hat{v}_{nm},$$
$$w_{nm}(0)=\bar{w}_{nm},\qquad \dot{w}_{nm}(0)=\hat{w}_{nm}.$$

The easiest way to solve Eqs. (4.21), complying with the given initial conditions, is by numerical integration. The analytical solution expressed in terms of power series is also obtainable but it is too complicated for practical calculations.

The numerical solution of the equations for the amplitudes $u_{nm}(t)$, $v_{nm}(t)$, $w_{nm}(t)$ were carried out for the following sets of initial values:

for loading by the ideal impulse,

$$\bar{u}_{nm}=0, \quad \bar{v}_{nm}=0, \quad \bar{w}_{nm}=a_{nm}h, \qquad \hat{u}_{nm}=0, \quad \hat{v}_{nm}=0, \quad \hat{w}_{nm}=b_{nm}V_0. \tag{4.24}$$

and for loading by the rectangular impulse,

$$\bar{u}_{nm}=0, \quad \bar{v}_{nm}=0, \quad \bar{w}_{nm}=a_{nm}h, \qquad \hat{u}_{nm}=0, \quad \hat{v}_{nm}=0, \quad \hat{w}_{nm}=0. \tag{4.25}$$

For a_{nm}, b_{nm}, $q^p_{nm}=b_{nm}Q$ the constant values were assumed.

In general, the perturbed displacement functions can be expressed as a sine or cosine sum of the θ of Fourier series. The magnitudes of amplitudes are the same for both series, thus only the functions (4.20) were taken into account.

Therefore the loading condition (3.6), making use of the relation (4.3), can be written as

$$\dot{\varepsilon}^0_\theta\,[(1-2\alpha)\,\dot{\varepsilon}_x+(2-\alpha)\,\dot{\varepsilon}_\theta]>0. \tag{4.26}$$

5. Special cases of buckling mode

5.1. *Asymmetrical mode*

The total displacements in the perturbed motion are expressed by

$$u=\frac{\alpha w_0(t)}{a}x, \quad v=v_p(\theta, t), \quad w=w_0(t)+w_p(\theta, t). \tag{5.1}$$

Substituting $u_p=0$ and $\partial(...)/\partial x=0$ into the equations of Sect. 4.2, we get the formulae for the strain rates, stress components, resultant forces and moments.

The equations of the viscoplastic flow buckling (4.19) reduce to the form

$$\begin{gathered}\ddot{v}_p-\frac{k}{\rho a^2}\beta_3\left[\left(1+\frac{h^2}{12a^2}\right)\frac{\partial^2\dot{v}_p}{\partial\theta^2}-\frac{\partial\dot{w}_p}{\partial\theta}+\frac{h^2}{12a^2}\frac{\partial^3\dot{w}_p}{\partial\theta^3}\right]=0,\\ \ddot{w}_p+\frac{k}{\rho a^2}\beta_3\left[\left(\dot{w}_p+\frac{h^2}{12a^2}\frac{\partial^4\dot{w}_p}{\partial\theta^4}\right)+\frac{h^2}{12a^2}\frac{\partial^3\dot{v}_p}{\partial\theta^3}-\frac{\partial\dot{v}_p}{\partial\theta}\right]-\\ -\frac{1}{\rho h}\left[\frac{1}{a^2}\left(\frac{\partial^2 w_p}{\partial\theta^2}+\frac{\partial v_p}{\partial\theta}\right)N^0_\theta+q^p_z\right]=0.\end{gathered} \tag{5.2}$$

Following the series (4.20) we assume the solution of the equations in the form

$$\begin{gathered}v_p(t,\theta)=\sum_{n=1}^{N}v_n(t)\sin n\theta, \quad w_p(t,\theta)=\sum_{n=0}^{N}w_n(t)\cos n\theta,\\ q^p_z(t,\theta)=\sum_{n=0}^{N}q^p_n(t)\cos n\theta.\end{gathered} \tag{5.3}$$

After substituting Eqs. (5.3) into Eqs. (5.2), the equations for the amplitudes become

$$\ddot{v}_n+\frac{kn^2}{\rho a^2}\beta_3\dot{v}_n-\frac{kn}{\rho a^2}\beta_3\left(1+n^2\frac{h^2}{12a^2}\right)\dot{w}_n=0,$$

(5.4)
$$\ddot{w}_n+\frac{k}{\rho a^2}\beta_3\left(1+n^4\frac{h^2}{12a^2}\right)\dot{w}_n+\frac{n^2}{\rho h a^2}N_\theta^0 w_n-$$
$$-\frac{kn}{\rho a^2}\beta_3\left(1+n^2\frac{h^2}{12a^2}\right)\dot{v}_n-\frac{n}{\rho h a^2}N_\theta^0 v_n-\frac{q_n^p}{\rho h}=0.$$

The obtained solution reduces to the specific simple form for $v_p=0$,

$$u=\frac{\alpha^p w_0(t)}{a}x,\qquad v=0,\qquad w=w_0(t)+w_p(\theta,t),$$

(5.5)
$$\ddot{w}_p+\frac{k}{\rho a^2}\beta_3\left(\dot{w}_p+\frac{h^2}{12a^2}\frac{\partial^4\dot{w}_p}{\partial\theta^4}\right)-\frac{1}{\rho h a^2}N_\theta^0\frac{\partial^2 w_p}{\partial\theta^2}-\frac{q_z^p}{\rho h}=0,$$
$$w_p(t,\theta)=\sum_{n=0}^{N}w_n(t)\cos n\theta,\qquad q_z^p(t,\theta)=\sum_{n=0}^{N}q_n^p(t)\cos n\theta;$$
$$\ddot{w}_n+\frac{k}{\rho a^2}\beta_3\left(1+n^4\frac{h^2}{12a^2}\right)\dot{w}_n+\frac{n^2}{\rho h a^2}N_\theta^0 w_n-\frac{q_n^p}{\rho h}=0.$$

5.2. *Axisymmetrical mode*

In this case the total displacements in the perturbed motion are described by the functions

(5.6)
$$u=\frac{\alpha w_0(t)}{a}x+u_p(x,t),\qquad v=0,\qquad w=w_0(t)+w_p(x,t).$$

Inserting $v_p=0$ and $\partial(\ldots)/\partial\theta=0$ into the equations of Sect. 4.2, we obtain the following equations of the viscoplastic flow buckling:

$$\ddot{u}_p-\frac{k}{\rho a}\left(a\beta_1\frac{\partial^2\dot{u}_p}{\partial x^2}-\beta_2\frac{\partial\dot{w}_p}{\partial x}\right)-\frac{1}{\rho h a}N_\theta^0\frac{\partial w_p}{\partial x}=0,$$

(5.7)
$$\ddot{w}_p+\frac{kh^2}{12\rho}\beta_1\frac{\partial^4\dot{w}_p}{\partial x^4}+\frac{k}{\rho a^2}\beta_3\dot{w}_p-\frac{k}{\rho a}\beta_2\frac{\partial\dot{u}_p}{\partial x}-$$
$$-\frac{1}{\rho h}\left(N_x^0\frac{\partial^2 w_p}{\partial x^2}+q_z^p\right)=0.$$

Assuming the perturbed displacements and loading in the form

(5.8)
$$u_p(t,x)=\sum_{m=1}^{M}u_m(t)\sin\alpha_m x,\qquad w_p(t,x)=\sum_{m=0}^{M}w_m(t)\cos\alpha_m x,$$
$$q_z^p(t,x)=\sum_{m=0}^{M}q_m(t)\cos\alpha_m x,\qquad \alpha_m=\frac{m\pi}{l},$$

we get from Eqs. (5.7) the following amplitude equations:

(5.9)
$$\ddot{u}_m+\frac{k}{\rho}\alpha_m^2\beta_1\dot{u}_n-\frac{\alpha_m}{\rho a}\left(k\beta_2\dot{w}_m-\frac{N_\theta^0}{h}w_m\right)=0,$$
$$\ddot{w}_n+\frac{k}{\rho}\left(\frac{h^2}{12}\alpha_m^4\beta_1+\frac{1}{a^2}\beta_3\right)\dot{w}_m+\frac{\alpha_m^2}{\rho h}N_x^0 w_m-\frac{k\alpha_m}{\rho a}\beta_2\dot{u}_m-\frac{1}{\rho h}q_m^p=0.$$

The problem may be furthermore simplified by assuming $u_p=0$. In this case we have

(5.10)
$$u=\frac{\alpha w_0(t)}{a}x,\quad v=0,\quad w=w_0(t)+w_p(x,t),$$
$$\ddot{w}_p+\frac{kh^2}{12\rho}\beta_1\frac{\partial^4\dot{w}_p}{\partial x^4}+\frac{k}{\rho a^2}\beta_3\dot{w}_p-\frac{1}{\rho h}\left(N_x^0\frac{\partial^2 w_p}{\partial x^2}+q_z^p\right)=0,$$

(5.11)
$$w_p(t,x)=\sum_{m=0}^{M}w_m(t)\cos\alpha_m x,\qquad q_z^p(t,x)=\sum_{m=0}^{M}q_m^p(t)\cos\alpha_m x,$$
$$\ddot{w}_m+\frac{k}{\rho}\left(\frac{h^2}{12}\alpha_m^4\beta_1+\frac{1}{a^2}\beta_3\right)\dot{w}_m+\frac{\alpha_m^2}{\rho h}N_x^0 w_m-\frac{1}{\rho h}q_m^p=0.$$

5.3. *General mode for $u_p=v_p=0$*

On putting $u_p=v_p=0$, the general solution given in Section (4.2) reduces to the following form:

The total displacements:

(5.11′)
$$u=\frac{\alpha w_0(t)}{a}x,\quad v=0,\quad w=w_0(t)+w_p(x,\theta,t).$$

The equation governing the problem of viscoplastic flow buckling:

(5.12)
$$\ddot{w}+\frac{kh^2}{12\rho a^2}\left[a^2\beta_1\frac{\partial^4\dot{w}_p}{\partial x^4}+2(\beta_2+2\beta_4)\frac{\partial^4\dot{w}_p}{\partial x^2\partial\theta^2}+\beta_3\left(\frac{1}{a^2}\frac{\partial^4\dot{w}_p}{\partial\theta^4}+\frac{12}{h^2}\dot{w}_p\right)\right]-\frac{1}{\rho h}\left(N_x^0\frac{\partial^2 w_p}{\partial x^2}+\frac{N_\theta^0}{a^2}\frac{\partial^2 w_p}{\partial\theta^2}+q_z^p\right)=0.$$

The displacements and loading perturbations:

(5.13)
$$w_p(t,\theta,x)=\sum_{n=0}^{N}\sum_{m=0}^{M}w_{nm}(t)\cos n\theta\cos\alpha_m x,$$
$$q_z^p(t,\theta,x)=\sum_{n=0}^{N}\sum_{m=0}^{M}q_{nm}(t)\cos n\theta\cos\alpha_m x,\qquad \alpha_m=\frac{m\pi}{l}.$$

The amplitude equation:

$$\ddot{w}_{nm}+\frac{kh^2}{12\rho a^2}\left[a^2\alpha_m^4\beta_1+2n^2\alpha_m^2(\beta_2+2\beta_4)+\left(\frac{n^4}{a^2}+\frac{12}{h^2}\right)\beta_3\right]\dot{w}_{nm}+$$
(5.14)
$$+\frac{1}{\rho h}\left(\alpha_m^2 N_x^0+\frac{n^2}{a^2}N_\theta^0\right)w_{nm}-\frac{q_{nm}^p}{\rho h}=0.$$

6. Numerical results and discussion

The equations of amplitudes have been solved numerically for a shell made of mild steel, for the data: $\sigma_0=206.9$ MPa, $\rho=7.65\times10^{-5}$ Ns^2/cm^4 and for many values of the a, h, l, V_0, Q, T, γ, a_{nm}, b_{nm}.

Some of the numerical results are presented diagrammatically. Figures 3, 4, 5 and 6 show the considerable influence of viscosity of the material, of the ratio of h/a and l/a, on the magnitude of unperturbed radial displacement w_0. In the case of loading by rectangular pressure impulses (Q-pressure, T-duration), for the same values $I=QT$, larger displacements w_0 were observed when the time of duration was shorter.

In Figs. 7 and 8 the amplitudes of perturbed displacements w_{nm}, v_{nm}, u_{nm} are shown as the functions of n, m, γ, t. The solution of Eqs. (4.21) is given in figures

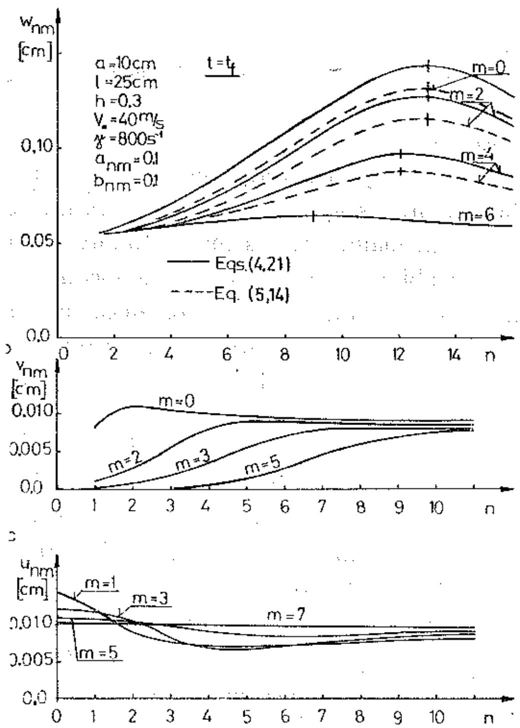

Fig. 7. Final amplitudes of perturbed displacements $w_{nm}(t_f)$, $v_{nm}(t_f)$, $u_{nm}(t_f)$ vs. the numbers of half-waves in longitudinal and circumferential direction.

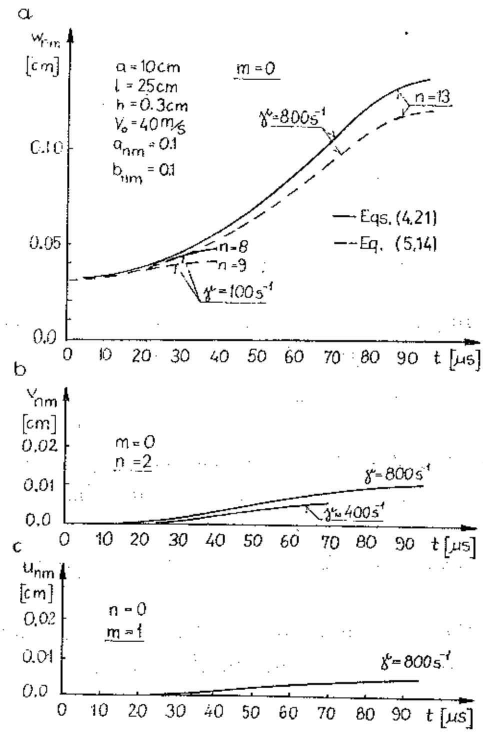

FIG. 8. Time variation of the amplitudes w_{nm}, v_{nm}, u_{nm}.

by a solid line, the solution of Eq. (5.14) accounting for the radial displacement only, by a broken line. The amplitudes w_{nm} and v_{nm} are seen to reach their largest values for $m=0$. The amplitudes w_{nm} are over ten times greater than v_{nm} and u_{nm}; it is the growth of w_{nm} that mainly causes the instability of the shell. In this type of buckling the loss of stability is not quite instantaneous, the process needs the increment of loading and some time to develop. Since the values of w_{nm}, v_{nm} are the largest and $u_p=0$ for $m=0$, Eqs. (4.20), the generators of the cylindrical shell remain straight. This conclusion is verified by the experimental observations [3, 5, 6].

The values of w_{nm} obtained from Eq. (5.14), where $v_p=u_p=0$, are smaller than those obtained from the set of equations (4.21), Figs. 8–12. For example, in the case of m=0 and for the data given in Fig. 12 and $V_0=30$ m/s the decrease of $w_{nm}(t_f)$ is equal to 12 and 10% for $\gamma=800\ s^{-1}$ and $\gamma=2000\ s^{-1}$, respectively. For loading by the rectangular impulse $Q=98.07$ MPa, $T=10\ \mu s$ and for the data: $a=10$ cm, $l=35$ cm, $h=0, 3$, $a_{nm}=b_{nm}=0.1$ the decrease of $w_{nm}(t_f)$ is for $\gamma=800\ s^{-1}$ – – 14% ($n=15$) and for $2000\ s^{-1}$ – 10% ($n=16$). This is why the threshold impulse determined from Eq. (5.14) is larger than that obtained from Eqs. (4.21), Fig. 12.

The influence of initial imperfections on the perturbed displacements was also investigated. The numerical results show the linear influence of the coefficient of initial perturbed displacement a_{nm}, Fig. 10, and the weak nonlinear influence of the coefficient of perturbed loading b_{nm} on the magnitude of $w_{mn}(t_f)$.

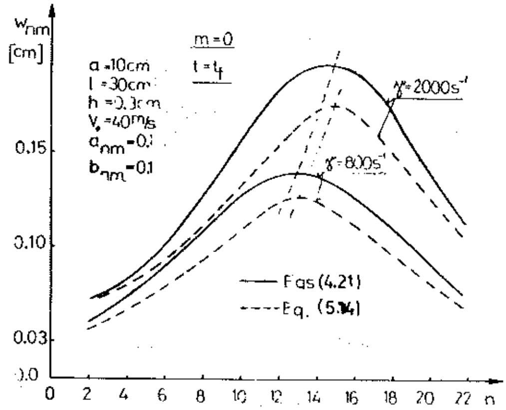

FIG. 9. Amplitudes of perturbed displacement $w_{nm}(t_f)$ vs. the number of half-waves n in circumferential direction for $m=0$ and two values of γ.

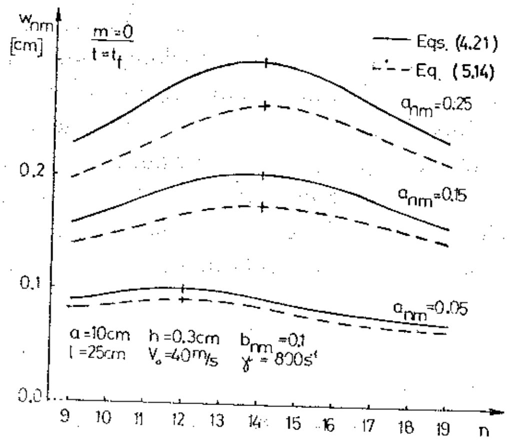

FIG. 10. Influence of the coefficient of initial perturbed displacement a_{nm} on the magnitude of $w_{nm}(t_f)$.

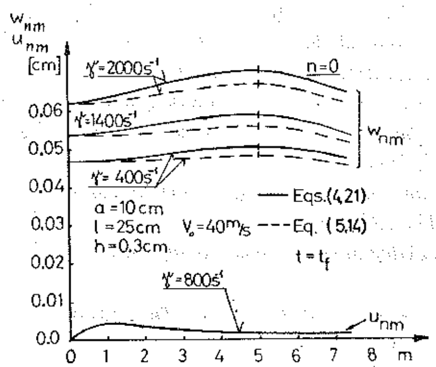

FIG. 11. Amplitudes of perturbed displacements $w_{nm}(t_f)$ and $u_{nm}(t_f)$ for $n=0$ vs. the number of half-waves m in longitudinal direction.

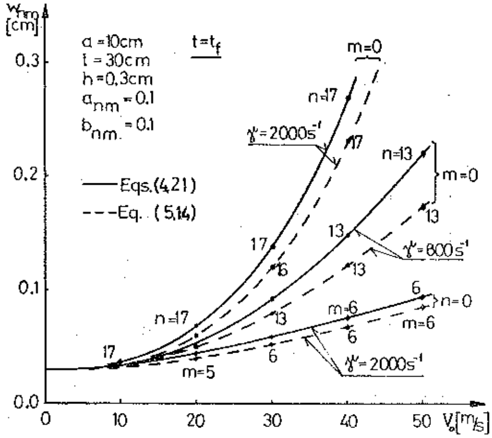

FIG. 12. Maximum amplitudes of perturbed displacement $w_{nm}(t_f)$ vs. applied impulse V_0. Asymmetrical (m=0) and axisymmetrical (n=0) buckling mode.

Figure 12 shows the variation of $w_{nm}(t_f)$ as a function of impulse applied for the symmetrical (m=0) and axisymmetrical buckling mode (n=0). It can be seen that for a given load the shell buckles exhibiting an asymmetrical wrinkled shape. The numbers at the dots distributed along the curves denote the critical modes. The function $w_{nm}(t_f)$ reach large values in a certain narrow interval of the impulse variation; hence it is natural to determine the critical value of the impulse graphically as the abcissa of such a point on the curve at which a small increment of the pulse beings to produce considerable increments of the deflection amplitude.

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STRESZCZENIE

WPŁYW PRZEMIESZCZEŃ STYCZNYCH NA DYNAMICZNE WYBOCZENIE LEPKOPLASTYCZNYCH POWŁOK CYLINDRYCZNYCH

Rozwinięte zostało zagadnienie dynamicznego wyboczenia powłok cylindrycznych poddanych działaniu promieniowego równomiernie rozłożonego impulsu ciśnienia. Zbadano wpływ przemieszczeń stycznych na wielkość przemieszczenia radialnego, postać wyboczenia i impuls krytyczny. Rozpatrzona została niesymetryczna i osiowosymetryczna postać wyboczenia. Wykazano występowanie niesymetrycznej postaci wyboczenia. Jest to zgodne z podanymi w literaturze obserwacjami doświadczalnymi.

Резюме

ВЛИЯНИЕ КАСАТЕЛЬНЫХ ПЕРЕМЕЩЕНИЙ НА ДИНАМИЧЕСКОЕ ВЫПУЧИВАНИЕ ВЯЗКОПЛАСТИЧЕСКИХ ЦИЛИНДРИЧЕСКИХ ОБОЛОЧЕК

В работе представлены проблемы динамического выпучивания цилиндрических оболочек, находящихся под действием радиального импульса давления. Исследовано влияние касательных перемещений на величину радиального перемещения, вид выпучиваний, а также величину критического импульса.

Исследованы несимметричный и осесимметричный виды выпучиваний. Доказано существование несимметричного вида деформаций. Сделанные выводы согласны с результатами экспериментов, отмеченными в литературе.

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