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NUMERICAL INTERPRETATIONS OF CREEP BUCKLING BEHAVIOR OF COLUMNS

J. WALCZAK and J. ZACHARZEWSKI (RZESZÓW)

Analysis of creep buckling columns is described and evaluated. Two different approaches are presented: 1) a numerical solution of a differential equation for the large deflection analysis; 2) the finite element method for material and geometric nonlinearities. An approximate energy method is also employed to evaluate the critical time. The discussion presented underlines the basic divergences in existing solution.

1. Introduction

Creep buckling analysis of columns has been studied by numerous authors. In those investigations the problem is generally defined in terms of unbounded deflections or deflection rates. Only a few of the authors included the geometrical nonlinearity in the way of large deflection analysis. However, they presented different conclusions. Życzkowski [1] found that due to the geometrical nonlinearity and a perfect viscoelastic material, the finite critical time can not exist. Huang's conclusion [2] is that the creep buckling may occur but only if the effect of plasticity is included in creep law. Both authors made use of Norton's nonlinear creep law and the collocation method. Samuelson [3] exposed, using the finite difference method, that creep buckling occurs with no additional conditions for the applied external load or the plastic strains. This conclusion is in a good agreement with experimental observations and, as it is going to be seen, with the results presented in the paper.

There are also some additional problems such as idealization of a real cross section by a theoretical I-section and creep failure analysis. The appropriate idealization is found to have a significant influence on the creep buckling time, therefore Samuelson's method is assumed as a better choice in the numerical procedure presented in the paper. The study of creep failure requires a creep failure criterion. One of them may be a dissipated barrier [4—6]. This criterion has also been used in the creep buckling analysis of columns to evaluate the critical time, on the basis of the small deflection theory [7]. The dissipated barrier is handy in case of constant stresses. In creep buckling problems where stress redistributions take place it is not efficient enough. Therefore we will use it as an additional approach since in practical analysis the achievement of an approximate critical time t^*_{cr} with no computational efforts is very useful. It should be recognized and underlined that in reality the criterion is a creep failure criterion and the time t^*_{cr} is a creep failure time. Having obtained this value or at least the upper and lower bounds of the critical time, we thus indirectly have an answer to the problem whether or not creep failure occurs prior to creep bucklung.

2. A NUMERICAL SOLUTION OF CREEP BUCKLING COLUMNS

The creep buckling behaviour of a column may be described in the natural coordinates (s, θ) (Fig. 1) which are related to the Cartesian coordinates (x, y) in the well-known forms:

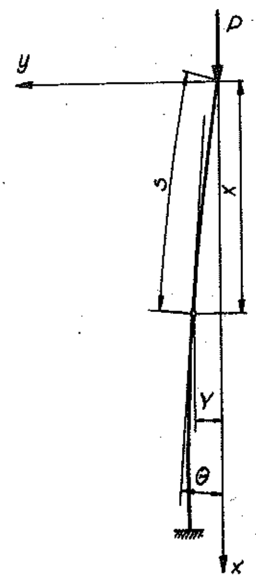


FIG. 1.

$$x = \int_0^s \cos\theta \, ds,$$

(2.1)

$$y = \int_0^s \sin\theta \, ds,$$

where $\theta = \theta(s, t)$ is our investigated solution.

To avoid numerical integration along the thickness of the column we will make an idealisation of the cross section, similar to SAMUELSON'S assumptions [4]. The cross section is divided into a number of flanges and cores (Fig. 2), where the cores do not take normal forces. All dimensions shown in Fig. 2 can be obtained from a comparison of inertia moduli

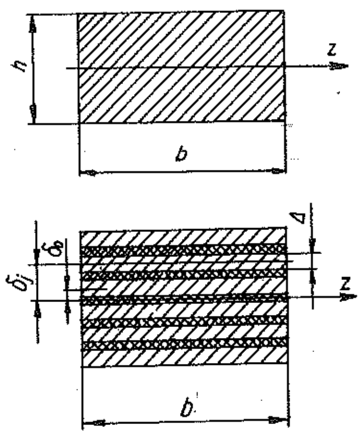


FIG. 2.

$$\delta_0 = \frac{h\sqrt{m}}{\left[12\sum_{i=1}^{m}\left(\frac{m-2i+1}{2}\right)^2\right]^{1/2}},$$

(2.2)
$$\delta_i = \delta_0 \cdot \frac{m-2i+1}{2},$$

$$\Delta = h/m,$$

where m is the number of flanges.

The constitutive equation is accepted in the form of the Norton-Hooke viscoelastic material law:

(2.3)
$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + B\,\sigma^n,$$

where the creep term has the following meaning:

(2.4)
$$B\cdot\sigma^n = B\,|\sigma|^{(n-1)}\,\sigma.$$

To find a bending line of the column we have to consider a deformed element (Fig. 3). One can find from the figure

(2.5)
$$\varkappa = \frac{1}{R} = \frac{\varepsilon_z - \varepsilon_w}{H},$$

where ε_z and ε_w denote strains of extreme fibres, H is the distance between the fibres and $\varkappa$ is the curvature of the deformed column, and

(2.6)
$$\varkappa = \frac{d\theta}{ds}.$$

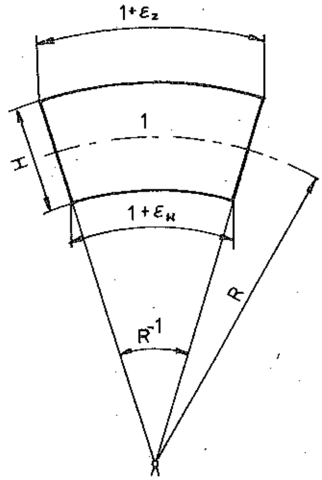


FIG. 3.

Taking the first derivative of Eq. (2.5) with respect to time, we have

$$\dot{\varepsilon}_z - \dot{\varepsilon}_w = \dot{\varkappa}\, H. \tag{2.7}$$

Equation (2.6) holds for any pair of flanges, thus it may be written in a system of equations:

$$\dot{\varepsilon}_z^i - \dot{\varepsilon}_w^i = \dot{\varkappa}\, H_i, \quad i = 1, 2, \ldots, m \quad \text{and} \quad H_i = 2\delta_i. \tag{2.8}$$

Making a linear combination of Eqs. (2.7) we have

$$\dot{\varepsilon}_z^i - \dot{\varepsilon}_w^i - \sum_{i=2}^{m} \dot{\varepsilon}_z^i + \sum_{i=2}^{m} \dot{\varepsilon}_w^i = \dot{\varkappa}\,(H_1 - \sum_{i=2}^{m} H_i). \tag{2.9}$$

For any flange, one can evaluate stresses as follows:

$$\sigma_{z,w}^i = -\sigma_0 \cos\theta \pm \frac{PH_i}{2I}\, y, \tag{2.10}$$

where $\sigma_0 = P/A$, $I = bh^3/12$ (rectangular cross section).

Substitution of Eqs. (2.3) and (2.10) into Eq. (2.9) leads to a differential equation which may be written in the form

$$\frac{PH_1}{IE}\frac{\partial}{\partial t}\left[\int_0^s \sin\theta\, ds\right] + B\left[\frac{PH_1}{2I}\int_0^s \sin\theta\, ds - \sigma_0\cos\theta\right]^n + \tag{2.11}$$

$$+\sum_{i=2}^{m}\left[(-1)^n \cdot B\left(\frac{PH_i}{2I}\int_0^s \sin\theta\, ds + \sigma_0\cos\theta\right)^n - B\left(\frac{PH_i}{2I}\int_0^s \sin\theta\, ds - \right.\right.$$

$$\left.\left. -\sigma_0\cos\theta\right)^n - \frac{PH_i}{IE}\frac{\partial}{\partial t}\left(\int_0^s \sin\theta\, ds\right)\right] - (-1)^n \cdot B\left[\frac{PH_1}{2I}\int_0^s \sin\theta\, ds + \right.$$

$$\left. + \sigma_0\cos\theta\right]^n = \dot{\varkappa}\,(H_1 - \sum_{i=2}^{m} H_i).$$

Equation (2.11) can be solved by the Newton–Raphson iteration method. To achieve the goal it would be convenient to rewrite it as follows:

$$(2.12)\qquad {}^{t+\Delta t}\dot{\mathbf{K}} = {}^{2t+\Delta t}\dot{\mathbf{F}}_1 + B\,({}^{t+\Delta t}\mathbf{F}_1 - {}^{t+\Delta t}\mathbf{S})^n + \sum_{i=2}^{m}\Big[(-1)^n B\,({}^{t+\Delta t}\mathbf{F}_i + {}^{t+\Delta t}\mathbf{S})^n - {} + B\,({}^{t+\Delta t}\mathbf{F}_i - {}^{t+\Delta t}\mathbf{S})^n + \frac{2}{E}\,{}^{t+\Delta t}\dot{\mathbf{F}}_i\Big] + (-1)^n B\,({}^{t+\Delta t}\mathbf{F}_1 + {}^{t+\Delta t}\mathbf{S})^n;$$

where

$$(2.13)\qquad \begin{aligned} {}^{t+\Delta t}\dot{\mathbf{K}} &= {}^{t+\Delta t}\dot{\varkappa}\,(H_1 - \sum_{i=2}^{m} H_i),\\ {}^{t+\Delta t}\mathbf{S} &= \sigma_0 \cos({}^{t+\Delta t}\theta),\\ {}^{t+\Delta t}\mathbf{F}_i &= \frac{PH_i}{2I}\int_0^s \sin({}^{t+\Delta t}\theta)\,ds. \end{aligned}$$

All the integrals above have been carried out numerically by the trapezoid rule. The columns have been divided into twenty elements and more subdivisions did not change results significantly. To solve Eq. (2.12) we have to replace all differentials by finite differences:

$$(2.14)\qquad \begin{aligned} {}^{t+\Delta t}\dot{\mathbf{K}}^{(k)} &= ({}^{t+\Delta t}\mathbf{K}^{(k)} - {}^{t}\mathbf{K})/\Delta t = [{}^{t+\Delta t}\mathbf{K}^{(k-1)} + \Delta\mathbf{K} - {}^{t}\mathbf{K}]/\Delta t,\\ {}^{t+\Delta t}\dot{\mathbf{F}}^{(k-1)} &= [{}^{t+\Delta t}\mathbf{F}_i^{(k-1)} - {}^{t}\mathbf{F}_i]/\Delta t,\\ {}^{t+\Delta t}\dot{\theta}^{(k-1)} &= [{}^{t+\Delta t}\theta^{(k-1)} - {}^{t}\theta]/\Delta t, \end{aligned}$$

where the superscript k denotes iterations.

Substituting the above formulas into Eq. (2.12), we have

$$(2.15)\qquad \Delta\mathbf{K} = \frac{2}{E}\,({}^{t+\Delta t}\mathbf{F}_1^{(k-1)} - {}^{t}\mathbf{F}_1) + B\,({}^{t+\Delta t}\mathbf{F}_1^{(k-1)} - {}^{t+\Delta t}\mathbf{S}^{(k-1)})^n\,\Delta t + {} + \Big\{\sum_{i=2}^{m}\big[(-1)^n B\,({}^{t+\Delta t}\mathbf{F}_i^{(k-1)} + {}^{t+\Delta t}\mathbf{S}^{(k-1)})^n - B\,({}^{t+\Delta t}\mathbf{F}_i^{(k-1)} - {} - {}^{t+\Delta t}\mathbf{S}^{(k-1)})^n - \frac{2}{E}\,({}^{t+\Delta t}\mathbf{F}_i^{(k-1)} - {}^{t}\mathbf{F}_i)/\Delta t\big]\Big\}\,\Delta t - {} - (-1)^n B\,({}^{t+\Delta t}\mathbf{F}_1^{(k-1)} + {}^{t+\Delta t}\mathbf{S}^{(k-1)})^n + {}^{t}\mathbf{K} - {}^{t+\Delta t}\mathbf{K}^{(k-1)},$$

which may be written as follows

$$(2.16)\qquad \Delta\mathbf{K} = {}^{t+\Delta t}\mathbf{R}^{(k-1)}\,\Delta t + {}^{t}\mathbf{K} - {}^{t+\Delta t}\mathbf{K}^{(k-1)},$$

where ${}^{t+\Delta t}\mathbf{R}^{(k-1)}$ denotes the right hand side of Eq. (2.12).

Since ware seeking the curvature of the column corresponding to time $(t+\Delta t)$, it is natural to require that the curvature at the end of each iteration be within a tolerance of the true solution. Hence the convergence criterion is [8]

$$\frac{\|\mathbf{K}\|_2}{\|^{t+\Delta t}\mathbf{K}\|_2} < \varepsilon_K, \tag{2.17}$$

where $\|\mathbf{K}\|_2$ denotes the Euclidean norm of $\mathbf{K}$ [9] and ε_K is a curvature tolerance. Unfortunately, the vector $^{t+\Delta t}\mathbf{K}$ is unknown, so we use the modified criterion

$$\frac{\|\mathbf{K}\|_2}{\|^{t+\Delta t}\mathbf{K}^{(k-1)}\|_2} < \varepsilon_K, \tag{2.18}$$

which was effective in the analysis.

To show some of the problems mentioned above we will study two kinds of columns for two different material sets. One of them (column number 1) is relatively short and thick with a slenderness ratio $\lambda = 36.9$. The other one (column number 2) is long and slender with $\lambda = 554.2$.

All numerical results are presented in graphic forms. Figure 4 shows the predicted displacement response for column number 1. As expected, it can be seen from the figure that by increasing the number of flanges we get a better solution, and the approximation by six flanges is a computational optimum. Figure 5 presents solutions for column number 2. In both cases the idealisations by two flanges give worse results. It may also be said that the idealisation by the theoretical I-section is difficult to implement. If we set a restriction to compare inertia moduli only, then the compressive stresses will be too high and the obtained critical time will be several

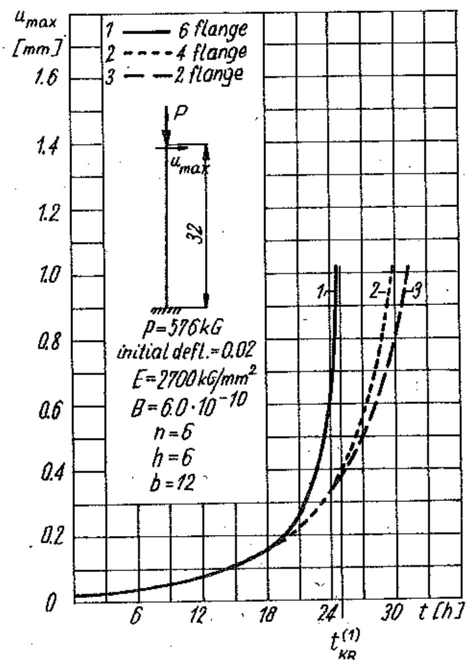


FIG. 4.

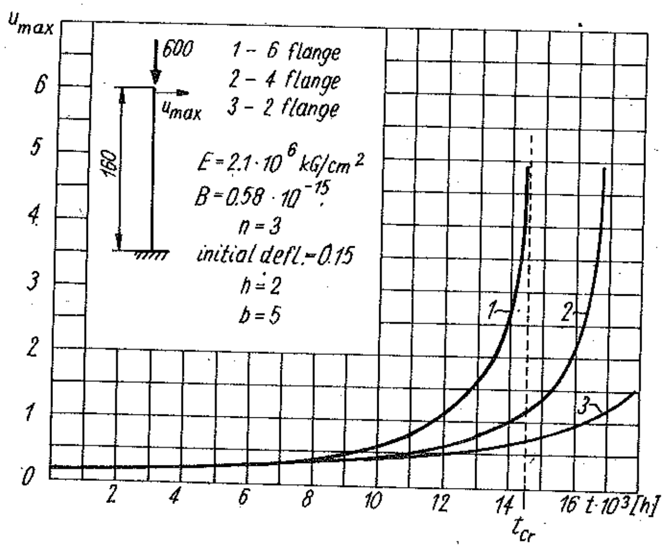


FIG. 5.

times too low. If we set up the same cross section areas for the real and I-section, then the time to buckle will be several times too high. It is admittedly possible to combine the compressive and bending stresses getting quite good results, but the combination is artificial. Therefore the I-section does not represent any real section with the except of I-section. This conclusion is not in agreement with other papers. As pointed out by N. J. HOFF [10], the idealization of the cross section by two identical flanges separated by a web would not introduce any significant error to the creep buckling analysis on the basis of the small deflection theory. But as stressed by HUANG [2] the small deflection theory is questionable. Other authors did not even mention the real cross section assuming the theoretical I-section by the default. A good answer to the problem will be presented on the basis of the finite element analysis.

As can be seen from Figs. 4 and 5 for the materials obeying Norton's creep law and the large deflection theory, the finite critical time can exist. To be positively sure we will present another solution based on a different methodology and assumptions.

3. FINITE ELEMENT ANALYSIS OF CREEP BUCKLING COLUMNS

As pointed out by K. J. BATHE [11], the nonlinear finite element analysis still encounters difficulties, particularly in creep buckling problems. Some of the questions have been studied by J. WALCZAK [12]. In this paper we are going to present the finite element solution based on the ADINA program [8] with no intention to analyse the finite element difficulties but to focus on the issue of existence of the critical time. To make our discussion more valuable, let us summarize the basic equations used in the ADINA program [13—14].

Since the column can undergo the large deflection analysis, we have used the total Lagrangian formulation (T. L.) [14] and the thermo-elastic-plastic and creep material model for $2/D$ solid elements.

The equation of motion in the T. L. formulation may be written as follows [14—15]:

$$\int_{{}^0V} {}^{t+\Delta t}_{0}S_{ij}\, \delta\, {}^{t+\Delta t}_{0}\varepsilon_{ij}\, {}^0dV = {}^{t+\Delta t}R, \tag{3.1}$$

where ${}^{t+\Delta t}_{0}S_{ij}$ is the component of the 2nd Piola–Kirchhoff stress tensor in the configuration at time $t+\Delta t$, referred to configuration at time 0, ${}^{t+\Delta t}_{0}\varepsilon_{ij}$—component of the Green–Lagrange strain tensor in the configuration at time $t+\Delta t$, referred to the configuration at time 0, ${}^0V$—volume of the body in the configuration at time 0, δ denotes "variation in", ${}^{t+\Delta t}R$—external virtual work at time $t+\Delta t$.

Taking incremental decompositions of stresses and strains we have

$$\begin{aligned} {}^{t+\Delta t}_{0}S_{ij} &= {}^{t}_{0}S_{ij} + {}_{0}S_{ij}, \\ {}^{t+\Delta t}_{0}\varepsilon_{ij} &= {}^{t}_{0}\varepsilon_{ij} + {}_{0}\varepsilon_{ij}, \end{aligned} \tag{3.2}$$

where ${}_0S_{ij}$ and ${}_0\varepsilon_{ij}$ denote the components of the stress and strain increments, respectively. The component of the strain increment may be decomposed into linear and nonlinear parts:

$$ {}_0\varepsilon_{ij} = {}_0e_{ij} + {}_0\eta_{ij}, \tag{3.3}$$

where

$$\begin{aligned} {}_0e_{ij} &= \frac{1}{2}\left({}_0u_{i,j} + {}_0u_{j,i} + {}^{t}_{0}u_{k,i}\, {}_0u_{k,j} + {}_0u_{k,i}\, {}^{t}_{0}u_{k,j}\right), \\ {}_0\eta_{ij} &= \frac{1}{2}\, {}_0u_{k,i}\, {}_0u_{k,j}. \end{aligned}$$

To find the stress increment let us write the constitutive equation in the incremental form for the Cauchy stress tensor and engineering strains:

$$\sigma = \mathbf{C}^E\left(\mathbf{e} - \mathbf{e}^{th} - \mathbf{e}^{p} - \mathbf{e}^{c}\right), \tag{3.4}$$

where σ denotes the increment in the Cauchy stress tensor in a vector notation, $\mathbf{e}$—increment in the total infinitesimal strain tensor, $\mathbf{e}^{th}$—increment in thermal strains, $\mathbf{e}^{p}$—increment in plastic strains and $\mathbf{e}^{c}$—increment in creep strains, $\mathbf{C}^E$—elastic property matrix. The increment in creep infinitesimal strains may be written as follows:

$$\mathbf{e}^c = {}^tk\, {}^t\mathbf{s}, \tag{3.5}$$

where $\mathbf{s}$ denotes the deviatoric stress components in a vector notation. To evaluate ${}^tk$ let us take the Norton–Odqvist creep law

(3.6) $$ {}^{t}\dot{\mathbf{e}} = B\,{}^{t}\sigma_e^{n-1}\,{}^{t}\mathbf{s}, $$

thus

(3.7) $$ {}^{t}k = B\sigma_e^{n-1}, $$

or

(3.8) $$ {}^{t+\Delta t}k = B\,{}^{t+\Delta t}\sigma_e^{n-1}, $$

hence Eq. (3.4) can be rewritten in the form

(3.9) $$ \boldsymbol{\sigma} = \mathbf{C}^{EP}\,\mathbf{e} - \boldsymbol{\sigma}^{CTH}, $$

where $\mathbf{C}^{EP}$ is the tangent elastic plastic material property matrix and $\boldsymbol{\sigma}^{CTH}$ is the incremental solution containing the creep and thermal terms.

For the large deflection analysis we can rewrite Eq. (3.9) into the form

(3.10) $$ \mathbf{S} = \mathbf{C}^{EP}\,\mathbf{e} - \mathbf{S}^{CTH}. $$

The reason for being able to use in the T.L. formulation the above equation is that the 2nd Piola–Kirchhoff stresses and the Green–Lagrange strains are approximately equal in magnitude to the Cauchy stresses and engineering strains measured in a convected coordinate system. Hence Eq. (3.1) takes the form

(3.11) $$ \int_{^0V} \delta\,{}_0\mathbf{e}^T\,\mathbf{C}^{EP}\,{}_0\mathbf{e}\,{}^0dV + \int_{^0V} \delta\,{}_0\eta^T\,{}_0^t\mathbf{S}\,{}^0dV = {}^{t+\Delta t}\mathbf{R} - \int_{^0V} \delta\,{}_0\mathbf{e} \times \times ({}_0^t\mathbf{S} - {}_0\mathbf{S}^{CTH})\,{}^0dV. $$

Assuming finite element isoparametric discretization for displacement increments, we have

(3.12) $$ \begin{aligned} \mathbf{u}(x, y) &= \mathbf{H}(x, y)\,\mathbf{U}, \\ \mathbf{e}(x, y) &= \mathbf{B}(x, y)\,\mathbf{U}, \end{aligned} $$

where $\mathbf{U}$ denotes the increment in nodal displacements, $\mathbf{H}$ is an isoparametric interpolation matrix, $\mathbf{B}$ is the strain matrix. The finite element discretization leads to a form

(3.13) $$ \int_{^0V} {}_0^t\mathbf{B}_L^T\,\mathbf{C}^{EP}\,{}_0^t\mathbf{B}_L\,{}^0dV\,\mathbf{U} + \int_{^0V} {}_0^t\mathbf{B}_{NL}^T\,{}_0^t\mathbf{S}\,{}_0^t\mathbf{B}_{NL}\,{}^0dV \cdot \mathbf{U} = = {}^{t+\Delta t}\mathbf{R} - \int_{^0V} {}_0^t\mathbf{B}_L^T ({}_0^t\hat{\mathbf{S}} - {}_0\mathbf{S}^{CTH})\,{}^0dV, $$

where the subscripts L and NL denote the linear and nonlinear parts of the strain matrix, respectively, ${}_0^t\mathbf{S}$ is the 2nd Piola–Kirchhoff stress matrix, ${}_0^t\hat{\mathbf{S}}$ is the 2nd Piola–Kirchhoff stress vector, ${}^{t+\Delta t}\mathbf{R}$ is the vector of external loads.

Equation (3.13) can be solved with or without iterations. To solve it without iterations (reforming the nonlinear stiffness matrix every time step) we have to consider Eq. (3.7) in our solution scheme. Hence, Eq. (3.13) can be written as follows:

$$({}^{t}_{0}\mathbf{K}^{EP}_{L} + {}^{t}_{0}\mathbf{K}^{EP}_{NL})\,\mathbf{U} = {}^{t+\Delta t}\mathbf{R} - {}^{t}_{0}\mathbf{F}, \tag{3.14}$$

where ${}^{t}_{0}\mathbf{K}^{EP}_{L}$ is the first and ${}^{t}_{0}\mathbf{K}^{EP}_{NL}$ is the second integral of Eq. (3.14). By using the Newton–Raphson iteration method for the implicit time integration scheme (Eq. (3.8)) we have

$$({}^{t}_{0}\mathbf{K}^{EP}_{L} + {}^{t}_{0}\mathbf{K}^{EP}_{NL})\,\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{R} - {}^{t+\Delta t}\mathbf{F}^{(i-1)}, \tag{3.15}$$

where in the iteration process the incremental equations are in the form

$${}^{t+\Delta t}\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{U}^{(i-1)} + \mathbf{U}^{(i)}. \tag{3.16}$$

As an alternative to forms of Newton iteration, the BFGS method has been developed by MATHIES and STRANG [15] and implemented by BATHE [8, 16] and this method appears to be the most effective.

As can be seen from the above equations, ADINA is an excellent program for the nonlinear creep buckling analysis of columns with arbitrary cross section areas. Let us take the same columns as before with rectangular cross sections. The columns were modelled with six and eight, eight-node elements respectively, for a plain stress analysis. In all cases we used 4×4 Gauss integration points.

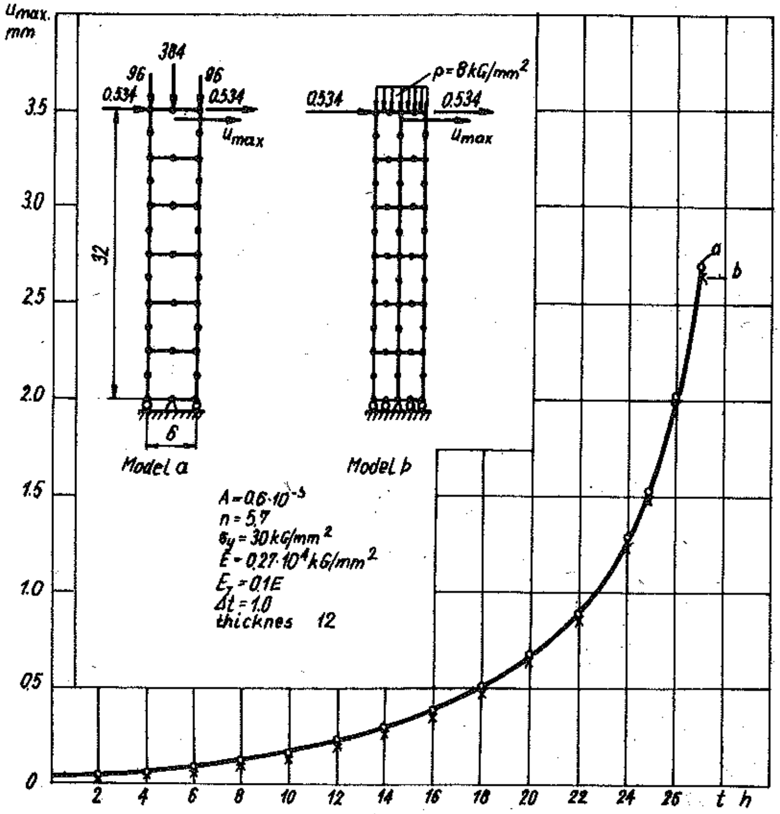


FIG. 6.

Numerical results shown in Fig. 6 and 7 refer to the finite element mesh. It has been apprehended that the nonlinear stress distribution along the thickness of the column may cause some perturbations in the analysis. As it turned out, it is enough to model the transversal dimension of the column by one eight-node element with 4×4 Gauss points (¹). Figure 8 represents solutions for column number 1 and Fig. 9 for column number 2.

Figures 10 and 11 present comparisons of these results and those obtained from the numerical solution. Additionally, there one of SAMUELSON'S solutions [4] is presented. The minor differences are due to varied assumptions, but in general the agreement is good and the clue is the fact that in the above solutions the finite critical time exists. In using Eq. (3.13) to solve the problem, at least one of the pivot elements in the nonlinear stiffness matrix becomes nonpositive at time $t = t_{cr}$, what means that the stiffness matrix is nonpositive definite and this in turn means that the column becomes unstable at the final time equal to t_{cr}. This is the same when the Newton–Raphson or the BFGS iteration methods are used; one can never go beyond the critical time.

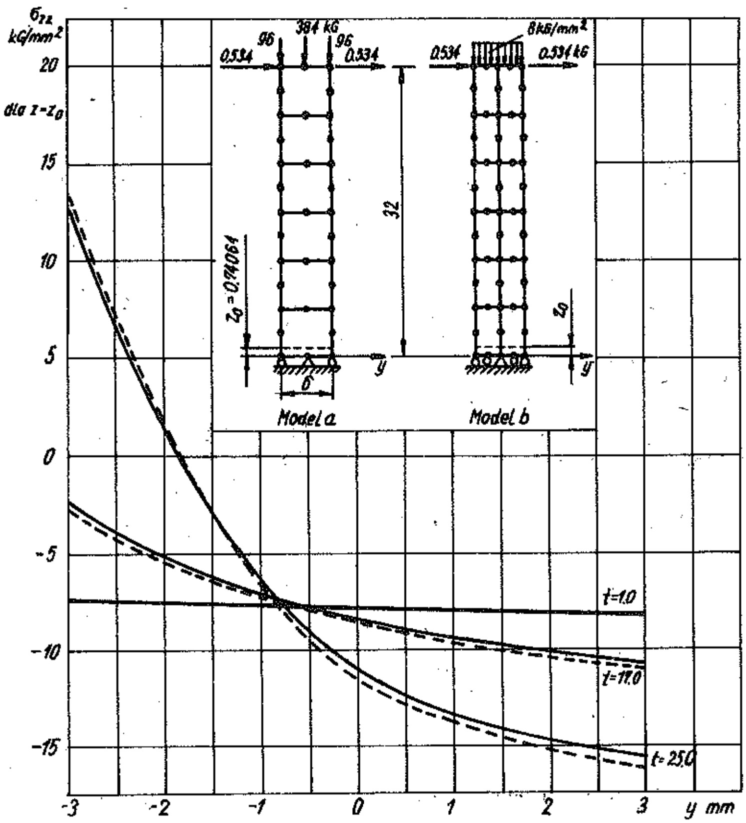
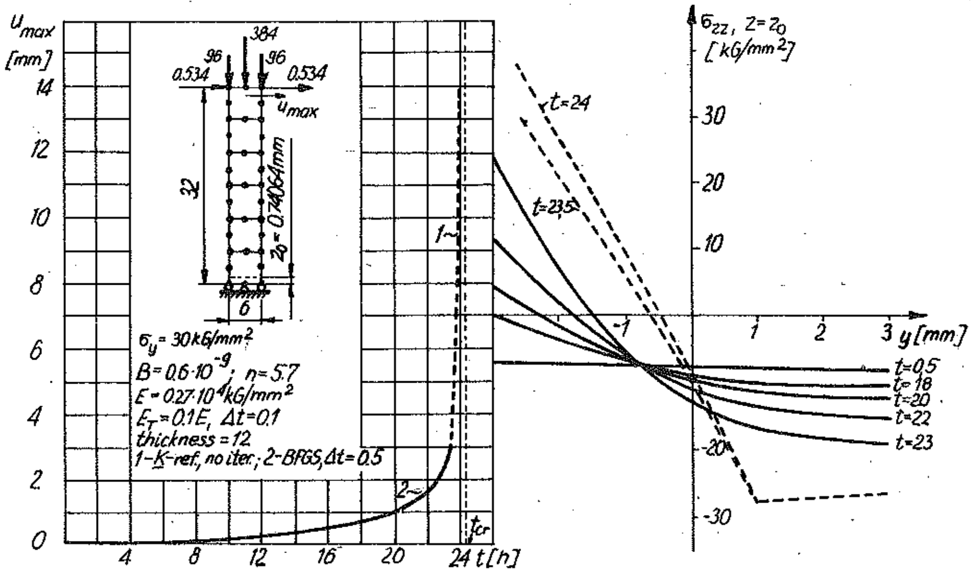


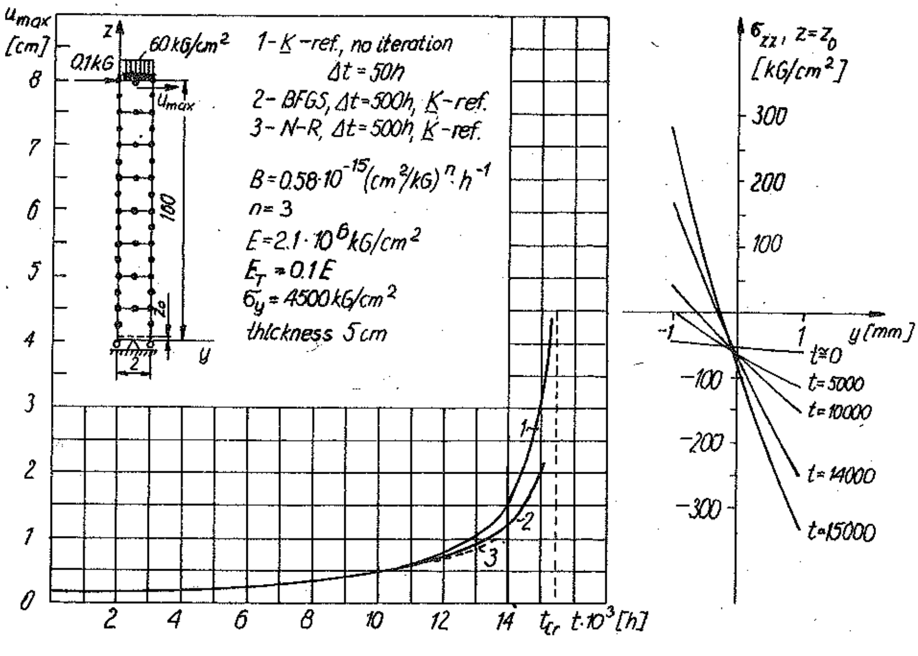
FIG. 7.

(¹) All calculations were performed on a IBM 370 machine in Stalowa Wola.



u_{max} [mm]
14
12
10
8
6
4
2
0
4 8 12 16 20 24 t [h]
384
96 96
0.534 0.534
u_{max}
32
z_0 = 0.74064mm
6
σ_y = 30kG/mm²
B = 0.6·10⁻⁹; n = 5.7
E = 0.27·10⁴ kG/mm²
E_T = 0.1E, Δt = 0.1
thickness = 12
1-K-ref, no iter.; 2-BFGS, Δt = 0.5
1~
2~
t_{cr}
σ_{zz}, z = z_0
[kG/mm²]
30
20
10
-20
-30
-1 1 2 3 y [mm]
t=24
t=23.5
t=0.5
t=18
t=20
t=22
t=23

FIG. 8.



u_{max} [cm]
8
7
6
5
4
3
2
1
0
2 4 6 8 10 12 14 t_{cr} t·10³ [h]
z
60 kG/cm²
0.1kG
u_{max}
180
z_0
2
y
1-K-ref., no iteration
Δt = 50h
2-BFGS, Δt = 500h, K-ref.
3-N-R, Δt = 500h, K-ref.
B = 0.58·10⁻¹⁵ (cm²/kG)ⁿ·h⁻¹
n = 3
E = 2.1·10⁶ kG/cm²
E_T = 0.1E
σ_y = 4500 kG/cm²
thickness 5cm
1
2
3
σ_{zz}, z = z_0
[kG/cm²]
300
200
100
-100
-200
-300
-1 1 y [mm]
t≈0
t=5000
t=10000
t=14000
t=15000

FIG. 9

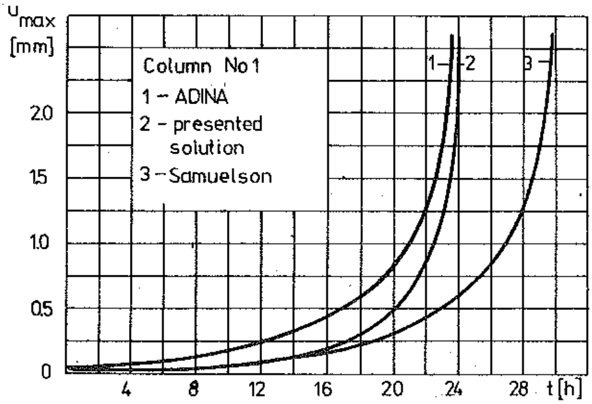


FIG. 10.

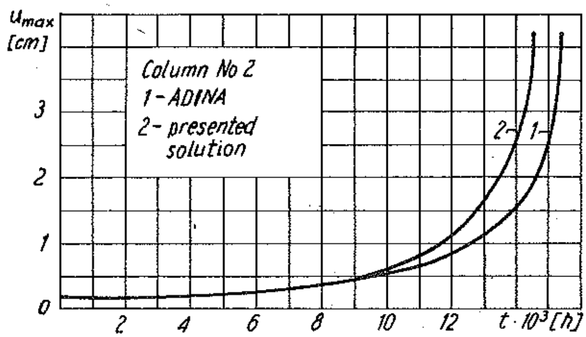


FIG. 11.

4. APPROXIMATE EVALUATION OF CRITICAL STATES

The dissipated energy criterion has been used effectively in the creep failure analysis of rotating discs [5—6]. The form of a dissipative barrier may be written as follows [6]:

(4.1)
$$f(\sigma_e, E_d) = \frac{E_d^\beta}{\sigma_e^{1/\beta}} = C = \text{const.}$$

Hence we can evaluate the critical time integrating the equation

(4.2)
$$B^\beta \int_0^{t_{cr}^*} \sigma_e^{[(n+1)\beta - \frac{1}{\beta}]} \, dt = C,$$

where C and β are material constants, which for a chromium-molybdenum-silicon steel at 540 degrees centigrade, are found to be [6]

(4.3)
$$\beta = 0.665, \quad C = 1.00127 \cdot 10^{-4}.$$

Equation (4.2) is convenient to use in the case of constant stresses. In creep buckling analysis, however, this is not the case, and the response of the column is unknown at the beginning of the solution process; therefore we are not able to use this criterion efficiently. We can use it in a reduced form to estimate the range of the critical time for a certain value of the effective stress which we might call a representative stress.

The representative stress σ_r may be chosen in two ways:

a) as a mean stress:

$$\sigma_{r1} = (\sigma_0 + \sigma_{\max})/2 = \left(\sigma_0 + \frac{Pf}{W}\right)/2; \tag{4.4}$$

b) as a reasonable maximum stress

$$\sigma_{r2} = \sigma_0 + \frac{Pf}{W}, \tag{4.5}$$

where σ_0 denotes the initial compressive stress, P—applied external load, W—section modulus and f—assumed maximum deflection of the column. One way to find f may be as follows:

$$f = \frac{\lambda}{\lambda_0} \frac{h}{2}, \tag{4.6}$$

where λ_0 is a limiting slenderness ratio, h—transversal dimension of the column. In our example $\lambda = 554.2$, $\lambda_0 = 88.0$, $h = 2.0$, hence $f = 6.3$.

The representative stresses are found to be

$$\sigma_{r1} = 627.0, \tag{4.7}$$

$$\sigma_{r2} = 1134.0. \tag{4.8}$$

Taking into account Eq. (4.2) one can find the range of the critical time. For our investigated column number 2 the critical time should be

$$8152.3 \leqslant t^*_{cr} \leqslant 22748.0. \tag{4.9}$$

As can be seen from Figs. 5 and 9 this estimation is excellent. With no computational effort we evaluated the upper and lower bound of the life time of the column. The method is simple; it actually confirms the fact that the dissipation energy may play a major role in the creep buckling process.

5. Discussion

The problem of existence of creep buckling time is naturally very important and often leads to misunderstandings of creep buckling behaviour, especially of columns. As the presented analyses reveal, in any case of the applied external load larger than zero and smaller than the instantaneous buckling load, the finite critical time as defined at the beginning of the paper is likely to appear due to the large deflection analysis. The above idea is in good agreement with experimental observations but diverges with Życzkowski's and Huang's studies on that problem. This fact leads to the doubt whether or not different solution methods or definitions of critical time lead to discrepancies. It is characteristic to note that Życzkowski as well as Huang used in their papers the collocation method, while Samuelson's solution and the one presented in the paper are based on three different approaches with one of them, the finite element method, based on a different philosophy. The existing solution convergence of these methods allows us to draw some conclusions.

The critical time can exist due to geometric nonlinearity, independently of plastic strains in the constitutive equations. In the presented solutions the plastic strains have been included only in the case of columns number 1 in the ADINA solution.

The idealisation of cross sections by the Samuelson model is good and simple and may be applied to any cross section with one symmetry axis. Using the theoretical I-section the basic problem is to find the correlation between the real and theoretical sections.

The presented approximate method to find the upper and lower bounds of the critical time is very useful. The dissipated barrier is simple and convenient to use but has one basic disadvantage—two more material constants. This is not a problem, however, when we deal with a known material, i.e. for which the creep curves are known. Since it is a creep failure criterion, the creep buckling behaviour is likely to occur prior to creep failure of columns, in general, for materials obeying the Norton creep law. This arises from the fact that the representative stress σ_{r1} which is actually responsible for he creep failure of the column gives a critical time larger than the time to buckle. As is shown by Walczak [6], the dissipated barrier lowers the results more than other rupture theories. From the physical point of view, the obtained critical time based on this criterion is a transition time into the third stage of creep.

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Streszczenie

NUMERYCZNA INTERPRETACJA ZAGADNIENIA WYBOCZENIA REOLOGICZNEGO PRĘTÓW

Przedstawiono analizę zagadnienia stateczności w sensie reologicznym prętów ściskanych. Przedstawiono numeryczne rozwiązanie linii ugięcia pręta w zakresie dużych deformacji oraz rozwiązanie na podstawie metody elementów skończonych, wykorzystując system programowy ADINA. Szczególnie zwrócono uwagę na problem istnienia skończonej wartości czasu krytycznego oraz sposobu modelowania przekroju poprzecznego pręta. Przedstawiono również przybliżoną metodę oszacowania krytycznego czasu pracy opierając się na kryterium bariery dysypacji. W pracy przedstawiono istotne wnioski dotyczące tego zagadnienia.

Резюме

ЧИСЛЕННАЯ ИНТЕРПРЕТАЦИЯ ПРОБЛЕМЫ РЕОЛОГИЧЕСКОЙ УСТОЙЧИВОСТИ СЖИМАЕМЫХ СТЕРЖНЕЙ

В работе представлен анализ реологической устойчивости сжимаемых стержней двумя методами: методом численного решения дифференциального уравнения больших деформаций и закона Нортона-Гука, методом конечных элементов для физически и геометрически нелинейных проблем. Представлен тоже приближенный метод определения критического времени работы конструкций (стержня), при использовании энергетического критерия. Широкий анализ приведенный в работе экспонирует принципиальные разницы в существующих решениях.

TECHNICAL UNIVERSITY OF RZESZÓW

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