ROZPRAWY INŻYNIERSKIE • ENGINEERING TRANSACTIONS • 32, 2, 275—287, 1984
Polska Akademia Nauk • Instytut Podstawowych Problemów Techniki

PROBLEM OF THE DECOHESIVE CARRYING CAPACITY OF A CYLINDRICAL SHELL UNDER A RING OF FORCES AND TENSION(*)

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Termination of the process of elastic-plastic deformations of a sandwich cylindrical shell under a simultaneous ring of radial forces and axial tension is studied in detail. The material is assumed to be perfectly elastic-plastic, incompressible and subject to the Huber–Mises–Hencky yield condition. The decohesive carrying capacity is in this case determined by an infinite increase of axial strains ε_x in the outer layer at the point $x=0$. The relevant singularity is described by generalized power series and, using those series combined with numerical integration, a concave interaction curve corresponding to the decohesive carrying capacity is determined.

1. Introductory remarks

Processes of small deformations of perfectly elastic-plastic or asymptotically perfectly plastic bodies usually terminate when reaching the limit state characterized by a mechanism of unrestricted plastic flow, and the limit load-carrying capacity.

However, there are many exceptions when the limit state cannot be reached because of earlier formation of inadmissible discontinuities due to an infinite increase of normal strains ε. Such cases were first mentioned by K. Szuwalski and M. Życzkowski [15] and the relevant loading parameter was called the "decohesive carrying capacity". Subsequently, these cases were analyzed in detail for bar systems by Szuwalski [11], for disks by the same author [12, 13], for beams by Tran-le Binh and M. Życzkowski [17], Szuwalski and Tran-le Binh [14]. Inadmissible discontinuities may also be encountered in the analysis of finite strains, though they are then due to an infinite increase of derivatives of normal stresses σ. Finite strains in disks were discussed by Życzkowski and Szuwalski [20, 16], in toroidal shells by J Skrzypek and M Życzkowski [10].

The present paper analyzes the decohesive carrying capacity of circular cylindrical shells under combined loadings: radial pressure (in general variable along the axis) $q_r^*=q_r^*(x)$, and axial tension, p^* per unit length of the perimeter, Fig. 1. Particular attention is paid to a ring of radial forces combined with axial tension. The assumptions are as follows:

1. The material is perfectly elastic-plastic with Young's modulus E and yield-point stress σ_0, incompressible and subject to the Huber–Mises–Hencky (HMH) yield condition;

(*) Grant No. 05.12—2.9 is gratefully acknowledged.

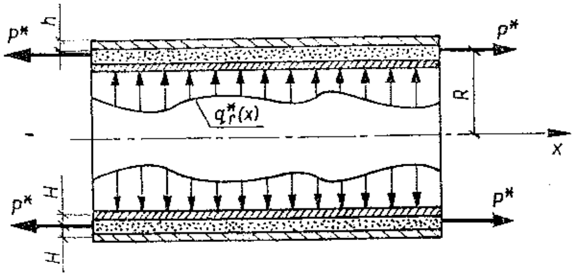

Fig. 1.

2. The analysis is restricted to small strains and small deflections;

3. The shell is of the sandwich type, with the thickness of carrying layers h, the distance between middle surfaces of the layers H, and the mean radius R;

4. The Love–Kirchhoff hypothesis of straight normals is adopted and shear deformations are neglected;

5. The Hencky–Ilyushin deformation theory is employed. The Prandtl–Reuss theory would essentially complicate the calculations, but — in view of a comparison performed in [15] for disks — only minor changes in results may be expected.

Elastic-plastic deflections of sandwich shells, based on the HMH yield condition, were first studied by Yu. N. Rabotnov [6], and A. R. Rzhanitsyn [7]. P. Klement [1] solved the problem of a cylindrical sandwich shell under a ring of radial forces. J. A. König [2] considered cylindrical shells employing the Prandtl–Reuss theory of plastic flow but approximately assumed that the whole cross-section is either fully elastic or fully plastic. More exact applications of Prandtl–Reuss equations are due to H. F. Muensterer and F. P. J. Rimrott [3] (with experimental verification) and to N. A. Schoeb and W. C. Schnobrich [9] (finite element method). A precise theory of elastic-plastic deformations of cylindrical shells with a solid cross-section is due to Y. Ohashi and T. Okouchi [4, 5]. Simultaneous axial tension and a ring of radial forces was considered by M. Sayir [8], and A. Ubaydillayev [18, 19]. However, none of the above papers studied the problem of termination of the process due to an infinite increase of strains.

2. Basic equations

The Love–Kirchhoff hypothesis determines the strains as follows:

$$\varepsilon_x^+ = \lambda_x - \varkappa_x H = \frac{du^*}{dx^*} + H\frac{d^2 w^*}{dx^{*2}},$$

$$\varepsilon_x^- = \lambda_x + \varkappa_x H = \frac{du^*}{dx^*} - H\frac{d^2 w^*}{dx^{*2}}, \tag{2.1}$$

$$\varepsilon_\theta^+ = \varepsilon_\theta^- = \lambda_\theta = -\frac{w^*}{R},$$

where the deflections w are positive inwards, and the superscripts "+" and "−" refer to the outer layer and inner layer of a sandwich shell, respectively. The dimensional variables are starred, so as to retain the usual symbols for dimensionless quantities; the expressions for the elongations λ and curvatures $\varkappa$ are substituted in linearized form. Now introduce the dimensionless variables

$$(2.2) \qquad x=\frac{x^*}{\sqrt{RH}}, \qquad u=\frac{Eu^*}{\sigma_0\sqrt{RH}}, \qquad w=\frac{Ew^*}{\sigma_0 R}$$

and rewrite Eq. (2.1) as follows:

$$(2.3) \qquad \varepsilon_x^+=(u'+w'')\frac{\sigma_0}{E}, \qquad \varepsilon_x^-=(u'-w'')\frac{\sigma_0}{E}, \qquad \varepsilon_\theta^+=\varepsilon_\theta^-=-w\frac{\sigma_0}{E},$$

where the primes denote differentiation with respect to x. The bending moments M and membrane forces N are in a sandwich shell expressed directly by the stresses as follows:

$$(2.4) \qquad M_i=(\sigma_i^- - \sigma_i^+)Hh, \qquad N_i=(\sigma_i^- + \sigma_i^+)h,$$

where $i=x, \theta$, and the positive moments correspond to decreasing curvatures.

The equilibrium equations for a cylindrical shell with introduced dimensionless variables (2.2) and neglected beam-column effect take the form

$$(2.5) \qquad N_x'=0, \qquad N_x=p_x^*=\text{const.}, \qquad M_x''+HN_Q=Rhq_r^*.$$

Expressing now M and N in terms of stresses we obtain

$$(2.6) \qquad \sigma_x^-+\sigma_x^+=2\sigma_0 p, \qquad (\sigma_x^- - \sigma_x^+)''+(\sigma_\theta^- + \sigma_\theta^+)=2\sigma_0 q_r,$$

where the dimensionless loadings

$$(2.7) \qquad p=\frac{p_x^*}{2\sigma_0 h}, \qquad q_r=\frac{Rq_r^*}{2\sigma_0 h}$$

are introduced in such a way as to give $p=1$ and $q_r=1$, corresponding to plastification under pure axial tension and pure constant internal pressure, respectively.

Further equations depend on the range of work of the cross-section under consideration. We first discuss one-side plastification, of the outer layer only. Then the stresses σ_θ^+ and σ_x^+ must satisfy the *HMH* yield condition for the case of plane stress; it is convenient to use a Nadai–Sokolovsky parametrization of that condition, namely

$$(2.8) \qquad \sigma_\theta^+=\frac{2}{\sqrt{3}}\sigma_0\sin\left(\omega_+ + \frac{\pi}{3}\right), \qquad \sigma_x^+=\frac{2}{\sqrt{3}}\sigma_0\sin\omega_+,$$

thus replacing two unknowns, σ_θ^+ and σ_x^+, by one unknown ω_+ only. The first equation of the set (2.6) determines now the stress σ_x^-,

$$(2.9) \qquad \sigma_x^-=2\sigma_0 p-\frac{2}{\sqrt{3}}\sigma_0\sin\omega_+ .$$

The Hencky–Ilyushin equation of shape change will be written in the form

$$\varepsilon_x^+ = \frac{2\sigma_x^+ - \sigma_\theta^+}{2\sigma_\theta^+ - \sigma_x^+}\,\varepsilon_\theta^+ . \tag{2.10}$$

Hence, after substitution of Eqs. (2.3) and (2.8) we obtain the following formula for the derivative u':

$$u' = \frac{w}{2} - \frac{\sqrt{3}}{2}\, w \operatorname{tg} \omega_+ - w''. \tag{2.11}$$

Finally, to the elastic inner layer we apply Hooke's law for an incompressible body:

$$\begin{aligned} \sigma_x^- &= \frac{4}{3} E\left(\varepsilon_x^- + \frac{1}{2}\varepsilon_\theta^-\right) = \frac{4}{3}\sigma_0\left(u' - w'' - \frac{w}{2}\right), \\ \sigma_\theta^- &= \frac{4}{3} E\left(\varepsilon_\theta^- + \frac{1}{2}\varepsilon_x^-\right) = \frac{4}{3}\sigma_0\left(-w + \frac{u'}{2} - \frac{w''}{2}\right). \end{aligned} \tag{2.12}$$

Comparing the first equation of the set (2.12), where Eq. (2.11) is substituted, with Eq. (2.9), we obtain

$$w'' + \frac{\sqrt{3}}{4}\, w \operatorname{tg} \omega_+ = \frac{\sqrt{3}}{4}\sin \omega_+ - \frac{3}{4} p. \tag{2.13}$$

Substitution of Eqs. (2.8), (2.9) and (2.12) into the second equilibrium equation (2.6) yields

$$\begin{aligned} w'' + \frac{\sqrt{3}}{4}\, w \operatorname{tg} \omega_+ + \frac{3}{4} w = (\omega_+'^2 \sin \omega_+ - \omega_+'' \cos \omega_+)\sqrt{3} + \\ + \frac{\sqrt{3}}{4}\sin \omega_+ + \frac{3}{4}\cos \omega_+ - \frac{3}{2} q_r . \end{aligned} \tag{2.14}$$

Equations (2.13) and (2.14) are linear in the unknown w and nonlinear in the unknown ω_+; hence w may easily be eliminated. Subtracting Eq. (2.13) from Eq. (2.14) we first obtain

$$w = \frac{4}{\sqrt{3}}(\omega_+'^2 \sin \omega_+ - \omega_+'' \cos \omega_+) + \cos \omega_+ + p - 2q_r \tag{2.15}$$

and the substitution of Eq. (2.15) into either Eq. (2.13) or Eq. (2.14) yields the governing equation for ω_+:

$$\begin{aligned} \omega_+^{\mathrm{IV}} = \frac{\sqrt{3}}{4}\omega_+'^2 \operatorname{tg}^2 \omega_+ + \left(4\omega_+' \omega_+''' + 3\omega_+''^2 - \frac{\sqrt{3}}{2}\omega_+'' - \omega_+'^4\right)\operatorname{tg}\omega_+ + \\ + 6\omega_+'^2 \omega_+'' - \frac{\sqrt{3}}{4}\omega_+'^2 + \frac{3p}{16\cos\omega_+}(\operatorname{tg}\omega_+ + \sqrt{3}) - \\ - \frac{\sqrt{3}}{8\cos\omega_+}(q_r\sqrt{3}\operatorname{tg}\omega_+ + 4q_r''). \end{aligned} \tag{2.16}$$

One-side plastification of the inner layer does not introduce major changes. Now the stresses σ_θ^- and σ_x^- are parametrized by formulae of the type (2.8) with ω_- as a parameter, and Hooke's law (2.12) is applied to the outer layer. Instead of Eq. (2.11) one obtains here

$$u'=\frac{w}{2}-\frac{\sqrt{3}}{2}w\,\mathrm{tg}\,\omega_-+w'', \tag{2.17}$$

instead of Eq. (2.15) we have

$$w=-\frac{4}{\sqrt{3}}(\omega_-'^2\sin\omega_--\omega_-''\cos\omega_-)+\cos\omega_-+p-2q_r \tag{2.18}$$

and the final governing equation for ω_- takes the form

$$\begin{aligned}\omega_-^{\mathrm{IV}}=&-\frac{\sqrt{3}}{4}\omega_-'^2\,\mathrm{tg}^2\,\omega_-+\left(4\omega_-\,\omega_-'''+3\omega_-''^2+\frac{\sqrt{3}}{2}\omega_-''-\omega_-'^4\right)\mathrm{tg}\,\omega_-+\\&+6\omega_-'^2\,\omega_-''+\frac{\sqrt{3}}{4}\omega_-'^2+\frac{3p}{16\cos\omega_-}(\mathrm{tg}\,\omega_-+\sqrt{3})-\\&-\frac{\sqrt{3}}{8\cos\omega_-}(q_r\sqrt{3}\,\mathrm{tg}\,\omega_--4q_r'').\end{aligned} \tag{2.19}$$

Two-side plastification introduces some qualitative changes. Now the stresses in both layers are parametrized by formulae of the type (2.8). The Hencky–Ilyushin equations for both layers lead to Eqs. (2.11) and (2.17); comparing these equations we arrive at

$$w''+\frac{\sqrt{3}}{4}w(\mathrm{tg}\,\omega_+-\mathrm{tg}\,\omega_-)=0. \tag{2.20}$$

The equilibrium equations (2.6) yield

$$\begin{aligned}&\sin\omega_++\sin\omega_-=p\sqrt{3},\\&(\sin\omega_--\sin\omega_+)''+\sin\left(\omega_++\frac{\pi}{3}\right)+\sin\left(\omega_-+\frac{\pi}{3}\right)=q_r\sqrt{3}.\end{aligned} \tag{2.21}$$

Equations (2.20) and (2.21) determine three unknown functions, ω_+, ω_- and w. However, Eqs. (2.21) do not contain w, and hence the system of equations is partly uncoupled. We may eliminate ω_-,

$$\sin\omega_-=p\sqrt{3}-\sin\omega_+ \tag{2.22}$$

and substituting this expression into the second equation of the set (2.21), we obtain the following second-order equation for ω_+:

$$\begin{aligned}\omega_+''=&\,\omega_+'^2\,\mathrm{tg}\,\omega_++\\&+\frac{\sqrt{3}}{4\cos\omega_+}\left(\cos\omega_+\pm\sqrt{\cos^2\omega_++2\sqrt{3}\,p\sin\omega_+-3p^2}+p-2q_r\right),\end{aligned} \tag{2.23}$$

where the sign before the root depends on the range of ω_-: one should choose "+" for $\omega_- < \pi/2$ and "−" for $\omega_- > \pi/2$. In most practical cases the sign "+" should be chosen. Further, substituting Eq. (2.22) into Eq. (2.20), we obtain a second-order equation for w:

$$w'' = \frac{\sqrt{3}}{4} w \left(\pm \frac{\sqrt{3}\,p - \sin\omega_+}{\sqrt{\cos^2\omega_+ + 2\sqrt{3}\,p\sin\omega_+ - 3p^2}} - \operatorname{tg}\omega_+ \right). \tag{2.24}$$

Partial uncoupling of Eqs. (2.23) and (2.24) facilitates the solution. Equation (2.24), in contradistinction to Eq. (2.13), is homogeneous with respect to w and hence the magnitude of deflections can be determined here only by nonhomogeneous boundary conditions or by continuity conditions.

3. Example of a shell under a ring of forces and axial loading

The problem of the decohesive carrying capacity of cylindrical shells will be shown in the example of an infinite shell under a ring of forces $2Q$ at $x=0$ and axial loading p_x^* at infinity (Fig. 2). We use the equations derived in the previous section with substituted $q_r(x) \equiv 0$.

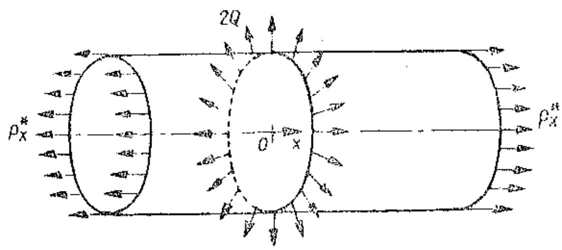

Fig. 2.

We first derive the equations of the elastic interaction curve. In the elastic range Hooke's law leads to the set (2.12) and to similar expressions for σ_x^+ and σ_θ^+; substituting these formulae into the equilibrium equations (2.5) we obtain

$$\begin{aligned} 2u' - w &= \frac{3}{2} p, \\ 2w^{IV} + 2w - u' &= 0 \end{aligned} \tag{3.1}$$

and after eliminating the axial displacement u,

$$w^{IV} + \frac{3}{4} w = \frac{3}{8} p. \tag{3.2}$$

The corresponding homogeneous equation may be regarded as a particular case ($\nu = 1/2$) of the well-known governing equation for elastic cylindrical shells under end loads which, however, is usually quoted without axial loading. The general solution of Eq. (3.2) is as follows:

$$w=e^{-\beta x}(A_1 \cos \beta x+A_2 \sin \beta x)+e^{\beta x}(A_3 \cos \beta x+A_4 \sin \beta x)+\frac{p}{2}, \tag{3.3}$$

where $\beta^4=3/16$. In view of the symmetry of the shell we consider only its right-hand side, $x \geqslant 0$. The condition at infinity, $w'(\infty)=0$, yields $A_3=A_4=0$ Two further boundary conditions are $w'(0)=0$ and

$$D \left.\frac{d^3 w^*}{dx^{*3}}\right|_{x=0}=-Q, \tag{3.4}$$

where

$$D=\frac{2EhH^2}{1-\nu^2}=\frac{8}{3}EhH^2 \tag{3.5}$$

denotes the elastic bending rigidity of the shell, and w^* and x^* are the physical (dimensional) variables.

Introducing into Eq. (3.4) the dimensionless variables (2.2) and the dimensionless ring force

$$q=\frac{1}{2\sigma_0 h}\sqrt{\frac{R}{H}}Q, \tag{3.6}$$

we rewrite that boundary condition in the form

$$w'''(0)=-\frac{3}{4}q. \tag{3.7}$$

Making use of the above boundary conditions we obtain $A_1=A_2=-3q/16\beta^3$, and hence

$$w=-\frac{3q}{16\beta^3}e^{-\beta x}(\cos \beta x+\sin \beta x)+\frac{p}{2}. \tag{3.8}$$

The corresponding elastic stress distribution is given by

$$\begin{aligned} \sigma_x &= \pm\frac{q\sigma_0}{2\beta}e^{-\beta x}(\cos \beta x-\sin \beta x)+p\sigma_0, \\ \sigma_\theta &= \frac{q\sigma_0}{4\beta}e^{-\beta x}\left[\frac{3}{4\beta^2}(\cos \beta x+\sin \beta x)\pm(\cos \beta x-\sin \beta x)\right], \end{aligned} \tag{3.9}$$

where the upper and the lower signs refer to the outer and the inner layers, respectively. The stress intensity σ_0 reaches its maximum at $x=0$, and equating this maximum to the yield-point stress σ_0, we determine the elastic interaction curve as follows:

$$\left(p\pm\frac{q}{2\beta}\right)^2+(\sqrt{3}\pm1)^2\frac{q^2}{16\beta^2}-(\sqrt{3}\pm1)\left(p\pm\frac{q}{2\beta}\right)\frac{q}{4\beta}=1. \tag{3.10}$$

These two ellipses intersect at $q=0$, $p=\pm 1$, and at $p=0$, $q=\pm\sqrt{2/\sqrt{3}}=\pm 1.0746$, and describe a curvilinear convex tetragon.

In what follows, we confine ourselves to the quadrant $q \geqslant 0$, $p \geqslant 0$. Then the maximal stress intensity in the elastic range is reached in the outer layer; exceeding the elastic carrying capacity, we arrive at a certain zone $0<x<x_1$ in which the outer layer is plastic (except for $p=0$, when the elastic range is followed immediately by plastification of both layers, and for $q=0$, when the shell becomes plastic as a whole). Under the assumption of the Hencky–Ilyushin theory of plasticity, the zone under consideration is governed by Eq. (2.23) with substituted $q_r \equiv 0$. In the elastic zone $x_1<x<\infty$ the general solution (3.3) with $A_3=A_4=0$ remains without change. This system of equations requires a total of 7 boundary conditions which determine four integration constants for Eq. (2.16), A_1, A_2, and the boundary coordinate x_1. They are as follows:

$$
\begin{aligned}
&w_p'(0)=0, \qquad m_{xp}'(0)=q,\\
&w_p(x_1)=w_e(x_1), \qquad w_p'(x_1)=w_e'(x_1),\\
&(\sigma_e^+)_e(x_1)=\sigma_0 \quad \text{or, equivalently,} \quad \varphi_p^+(x_1)=\frac{1}{2G}=\frac{3}{2E},\\
&m_{xp}(x_1)=m_{xe}(x_1), \qquad m_{xp}'(x_1)=m_{xe}'(x_1),
\end{aligned}
\tag{3.11}
$$

where the subscripts e refer to the elastic zone, and the subscripts p to the elastic-plastic zone, m_x denotes the dimensionless bending moment, and φ — the variable modulus in the Hencky–Ilyushin equations. The last two conditions make it possible to determine the constants A_1 and A_2:

$$
\begin{aligned}
A_1 &= -\frac{3}{8\beta^2}e^{\beta x_1}\left[\left(p-\frac{2}{\sqrt{3}}\sin\omega_1\right)(\sin\beta x_1+\cos\beta x_1)-\right.\\
&\qquad\left.-\frac{2}{\beta\sqrt{3}}\omega_1'\cos\omega_1\cos\beta x_1\right],\\
A_2 &= -\frac{3}{8\beta^2}e^{\beta x_1}\left[\left(p-\frac{2}{\sqrt{3}}\sin\omega_1\right)(\sin\beta x_1-\cos\beta x_1)-\right.\\
&\qquad\left.-\frac{2}{\beta\sqrt{3}}\omega_1'\cos\omega_1\sin\beta x_1\right],
\end{aligned}
\tag{3.12}
$$

where the subscripts "1" denote the values of the respective functions at $x=x_1$, and the subscript "+" of ω_+ has been dropped. In order to use the condition determining the boundary coordinate x_1, we first calculate the modulus φ:

$$
\varphi^+=\frac{\varepsilon_x^+-\varepsilon_\theta^+}{\sigma_x^+-\sigma_\theta^+}=\frac{u'+w''+w}{\sin\omega_+-\sqrt{3}\cos\omega_+}\,\frac{\sqrt{3}}{E}
\tag{3.13}
$$

and substitution of Eqs. (2.11) and (2.15) yields

$$
(2\sqrt{3}-4\omega_1'')\cos\omega_1+4\omega_1'^2\sin\omega_1+p\sqrt{3}=0.
\tag{3.14}
$$

The conditions at $x=0$, after some transformations, take the form

$$(3.15)\qquad \begin{aligned}&\left(12\omega_0'\,\omega_0''-\sqrt{3}\,\omega_0'\right)\sin\omega_0+4\left(\omega_0'^2-\omega_0'''\right)\cos\omega_0=0,\\ &q=-\frac{2}{\sqrt{3}}\,\omega_0'\cos\omega_0,\end{aligned}$$

where the subscripts "0" denote the values of the respective functions at $x=0$. Finally, the continuity conditions at $x=x_1$ may be written thus:

$$(3.16)\qquad \begin{aligned}&2\sin\omega_1+\left(\frac{4}{\sqrt[4]{3}}\,\omega_1'+2\right)\cos\omega_1-\left(\sqrt{3}-1\right)p=0,\\ &\left(12\omega_1'\,\omega_1''-\sqrt{3}\,\omega_1'+\sqrt[4]{27}\right)\sin\omega_1+\\ &\qquad+\left(4\omega_1'^3-4\omega_1'''+\sqrt{3}\,\omega_1'\right)\cos\omega_1-\frac{3\sqrt[4]{3}}{2}\,p=0.\end{aligned}$$

Numerical integration of the governing equation in the zone of one-side plastification (2.16) requires four initial conditions at the starting point $x=0$. Since Eq. (3.15) furnishes only two of them, we have to assume the remaining two and adjust them so as to satisfy after integration the system of three equations (3.14) and (3.16) with one additional unknown x_1.

4. The decohesive carrying capacity

The solution of the system of equations (3.2) and (2.16) terminates with infinitely increasing strains ε_x^+ at $x=0$: this condition determines the decohesive carrying capacity of the shell. Indeed, ε_x in the plastic zone are larger than ε_θ and they reach their upper bound at $x=0$.

Making use of Eqs. (2.3), (2.8) and (2.10) we may write

$$(4.1)\qquad \frac{\varepsilon_x^+}{\varepsilon_\theta^+}=-\frac{u'+w''}{w}=\frac{\sin(\omega-\pi/6)}{\cos\omega}$$

and hence the condition $\varepsilon_x^+\to\infty$ is equivalent to $\omega(0)=\pi/2$ at $w(0)\neq 0$ (in order to eliminate $\varepsilon_\theta^+(0)=0$). This additional condition joins p and q and describes the interaction curve of the decohesive carrying capacity of the shell. Now Eq. (2.16) becomes singular at the starting point $x=0$ and a numerical solution must be completed by an appropriate generalized power series valid in the vicinity of that point. It turns out that the following generalized power series holds (for $x\geqslant 0$):

$$(4.2)\qquad \omega(x)=\sum_{j=0}^{\infty}C_j\,x^{j/2}=C_0+C_1\,x^{1/2}+C_2\,x+\dots,$$

where $C_0=\pi/2$. The initial condition $m_x'(0)=q$ yields

$$(4.3)\qquad q=\frac{C_1^2}{\sqrt{3}},$$

whereas to satisfy the condition $w'(0)=0$ we have to put

$$C_2=0,\tag{4.4}$$
$$C_5=\frac{1}{6}C_1^2 C_3-\frac{1}{2}\frac{C_3^2}{C_1}-\frac{1}{720}C_1^5 .$$

Further, substituting Eq. (4.2) into Eq. (2.16) and equating the coefficients of $x^{j/2}$ on both sides make it possible to determine the subsequent coefficients of Eq. (4.2) in terms of C_1 and C_3:

$$C_4=\frac{\sqrt{3}}{15},\tag{4.5}$$
$$C_6=\frac{4\sqrt{3}}{105}\left(\frac{C_3}{C_1}+\frac{p\sqrt{3}}{4C_1^2}+\frac{C_1^2}{12}\right),$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

Now we can describe displacements by generalized power series. Equation (2.15) with substituted Eqs. (4.2), (4.4) and (4.5) gives

$$\frac{E}{\sigma_0}w=\left[p-\frac{4}{\sqrt{3}}\left(\frac{C_1^4}{12}-2C_1 C_3\right)\right]\left(1+\frac{1}{C_1\sqrt{3}}x^{3/2}+\ldots\right),\tag{4.6}$$

whereas Eq. (2.11) integrated with the initial condition $u(0)=0$ yields

$$\frac{E}{\sigma_0}u=\frac{\sqrt{3}}{2C_1}\left[p-\frac{4}{\sqrt{3}}\left(\frac{C_1^4}{12}-2C_1 C_3\right)\right]x^{1/2}+ \tag{4.7}$$
$$+\left[\frac{5}{4}p-\frac{\sqrt{3}}{4}-\frac{2}{\sqrt{3}}\left(\frac{C_1^4}{12}-2C_1 C_3\right)\right]x+\ldots .$$

The exponent 3/2 in Eq. (4.6) shows clearly that the curvature $\varkappa_x(0)$ increases infinitely in the case under consideration unless

$$p=\frac{4}{\sqrt{3}}\left(\frac{C_1^4}{12}-2C_1 C_3\right),\tag{4.8}$$

because then the coefficient of $x^{3/2}$ vanishes. But in this case also $w(0)=0$, and Eq. (4.1) is satisfied without $\varepsilon_x^+(0)\to\infty$. This exceptional case determines the tangency of the interaction curves of elastic carrying capacity and of decohesive carrying capacity. The exponent 3/2 coincides with that found for the decohesive carrying capacity of beams [17] (under a concentrated force).

The determination of the interaction curve of decohesive carrying capacity is carried out as follows. For an assumed value of axial loading p and for arbitrarily chosen C_1 and C_3, the subsequent coefficients C_j are found from Eqs. (4.4) and (4.5); the series (4.2) is used in the vicinity of $x=0$ and then numerical integration of Eq. (2.16) (the Runge–Kutta procedure) is performed. Finally, the continuity conditions (3.16) and the condition (3.14) are treated as a system of equations with three unknowns C_1, C_3 and x_1, and solved by means of the falsi rule. Then the second coördinate of the interaction curve, q, is found from Eq. (4.3).

The procedure described above works for $0.2695 \leqslant p \leqslant 0.9720$, the corresponding values of q being $q=1.2879$ and $q=0.3120$, respectively. For $p>0.9720$ there appears another zone of one-side plastification, namely plastification of the inner layer within a certain zone $x_2<x<x_3$; this zone is governed by Eq. (2.19). For p very close to unity, even further zones of one-side plastification are possible. On the other hand, for $p<0.2695$, there appears a zone of two-side plastification $0<x<x_1$, governed by Eqs. (2.23) and (2.24), and followed by a zone of one-side plastification $x_1<x<x_2$; in the latter zone either the outer layer is plastic (for $p>0.1$) or the inner layer is plastic (for $p<0.1$). In these cases the numerical procedures are more complicated; the details will be given in a separate paper.

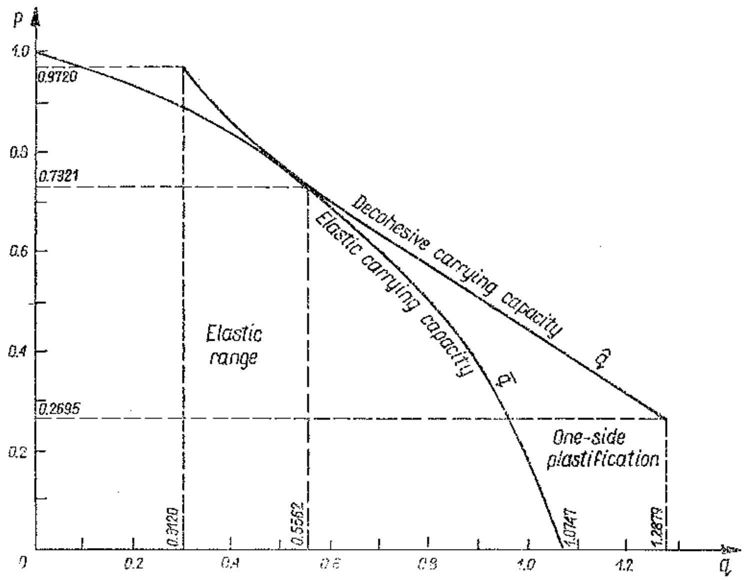

Fig. 3.

A part of the interaction curve of decohesive carrying capacity (concave) is shown in Fig. 3 together with the convex elastic interaction curve (3.10). Both curves are tangent to each other at the point

$$\bar{q}=\frac{\sqrt{3}-1}{\sqrt[4]{3}}=0.5562, \quad \bar{p}=\sqrt{2}-1=0.7321, \tag{4.9}$$

corresponding to the condition (4.8). Indeed, at this particular point of the elastic interaction curve we have $\omega=\pi/2$, but simultaneously $w=0$, $\varepsilon_\alpha^+=0$ and ε_x^+ in Eq. (4.1) may be arbitrary, not necessarily infinitely great.

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Streszczenie

ZAGADNIENIE NOŚNOŚCI ROZDZIELCZEJ POWŁOKI CYLINDRYCZNEJ OBCIĄŻONEJ PIERŚCIENIEM SIŁ I ROZCIĄGANEJ

Zbadano szczegółowo zakończenie procesu sprężysto-plastycznej deformacji sandwiczowej powłoki cylindrycznej poddanej osiowemu rozciąganiu i jednoczesnemu obciążeniu pierścieniem sił promieniowych. Założono, że materiał jest idealnie sprężysto-plastyczny, nieściśliwy i spełniający warunek plastyczności Hubera–Misesa–Hencky'ego. W tym przypadku nośność dekohezyjną określa się jako nieskończony wzrost odkształceń osiowych ε_x w warstwie zewnętrznej w punkcie $x=0$. Odpowiednią osobliwość opisuje się uogólnionym szeregiem potęgowym. Korzystając z tego szeregu w połączeniu z całkowaniem numerycznym, określono wklęsłą krzywą oddziaływania odpowiadającą nośności rozdzielczej.

Резюме

ПРОБЛЕМА ДЕКОГЕЗИОННОЙ НЕСУЩЕЙ СПОСОБНОСТИ ЦИЛИНДРИЧЕСКОЙ ОБОЛОЧКИ НАГРУЖЕННОЙ КОЛЬЦОМ СИЛ И РАСТЯГИВАЕМОЙ

Исследовано детально окончание процесса упруго-пластической деформации цилиндрической оболочки типа сандвич, подвергнутой осевому растяжению и одновременному нагружению кольцом радиальных сил. Предположено, что материал идеально упруго-пластический, несжимаемый и удовлетворяет условию пластичности Хубера–Мизеса–Генки. В этом случае декогезионную несущую способность определяется как бесконечный рост осевых деформаций ε_x во внешнем слое в точке $x=0$. Соответствующую особенность описывается обобщенным степенным рядом. Используя этот ряд в соединении с численным интегрированием, определена вогнутая кривая взаимодействия, отвечающая декогезионной несущей способности.

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Received November 21, 1983.