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ON RELIABILITY ANALYSIS AND OPTIMIZATION OF PLASTIC PLANE FRAMES

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The paper deals with a static approach to the probabilistic analysis and optimization of structures. A rigid-perfectly-plastic material is assumed and the two types of problems are discussed. The first one corresponds to the so-called chance constrained approach in the theory of stochastic programming. It allows either to evaluate the ultimate load factor or to find the optimum distribution of plastic moduli under the prescribed probability that the stress state remains statically admissible. The second approach gives the lower bound on the reliability index of a structure taking into account possible correlation between stochastic parameters (resistances or loads). In a discrete model of a structure all the cases considered turn out to be reduceable to appropriate deterministic linear or nonlinear mathematical programming problems. Several examples of the analysis and optimization of framed structures illustrate the proposed procedure.

1. Introduction

The deterministic description of a structure is often inadequate since in practice many factors are random. Such are wind and snow loads, actions caused by sea waves, properties of a material and even geometrical dimensions of a structure. It is obvious that in any comprehensive model of structural reliability, the structure itself and its environment must be described in a probabilistic way.

Practice shows that the intensity of load and the properties of a material have a much broader dispersion of values than that of geometrical dimensions. Hence we assume in the sequel that dimensions of a structure are deterministic.

We confine ourselves to the study of rigid-perfectly-plastic structures under the assumptions of small displacements and quasi-static proportional loading. Our aim is to take into account the random character of the load components and/or of the plastic moduli. We assume that all random parameters have normal distribution. It simplifies considerably the mathematical description and seems to be not too restrictive since a non-Gaussian distribution can be reduced in many cases to an equivalent normal one.

A probabilistic approach to the analysis of structure was considered by many authors. It is well known that the failure probability of a system can be bounded from below and above by assuming perfect dependence or statistical independence of failure modes [1, 2]. Some authors suggest that an upper bound obtained in such a way is not only safe but sufficient for practical purposes [3]. This approach has been used also for the optimization of structures [4, 5, 6, 7]. The more advanced methods take into account correlation of failure modes [8, 9]. Other authors use the theorem of total probability [10] or the Monte-Carlo simulation [11]. A review of these methods including comparisons of their accuracy and efficiency can be found in [11].

In our paper two types of problems are considered: the evaluation of the ultimate load and the optimum plastic design. Before dealing with the probabilistic formulations of such problems we recall in the following sections the usual deterministic models. Then each problem is formulated in a stochastic way and reduced later to a proper deterministic counterpart. This is achieved by means of the chance constrained method. The method was developed by Charnes and Cooper [12] and discussed later by Koblin [13], Kall [14], Rao [15], West [16] and others. The resulting deterministic models turn out to fall into the category either of linear or of nonlinear mathematical programming problems.

The chance constrained approach is simple and corresponds to the semi-probabilistic evaluation of structural safety recommended by most codes in civil engineering, e.g. [17]. An alternative way is to estimate the reliability of a structure under prescribed statistical parameters of random loads and/or random resistances. Correlations between random variables are also taken into account. Our aim is to find a lower bound on the reliability index by solving an appropriate linear programming problem.

In order to illustrate the proposed procedure several examples of the analysis and of the optimization of frames are given.

2. Ultimate load factor

Let a structure be descretized in such a way that its static state is determined by a one-column matrix of generalized stresses $\mathbf{s} \in R^m$ and a one-column matrix of loads $\mathbf{p} \in R^n$. Let load increase in a proportional way:

$$\mathbf{p} = \mu \mathbf{p}^0, \tag{2.1}$$

where μ is the load factor and $\mathbf{p}^0 \in R^n$ contains the nodal forces corresponding to the reference level of load. The equilibrium equation reads

$$\mathbf{p} = \mathbf{C}^T \mathbf{s}, \tag{2.2}$$

where $\mathbf{C} \in R^{m \times n}$ is the compatibility matrix. A general solution of Eq. (2.2) is

$$\mathbf{s} = \mathbf{C}^{-} \mathbf{p} + \mathbf{C}^{0} \mathbf{z}. \tag{2.3}$$

Here $\mathbf{C}^{-} \in R^{n \times m}$ is the generalized inverse and $\mathbf{C}^{0} \in R^{n \times r}$ is the kernel matrix of $\mathbf{C}$. Since we consider hyperstatic systems only, there holds $r = m - n > 0$ and $\mathbf{z} \in R^{r}$ contains free parameters (redundant stresses).

The generalized stresses cannot violate the yield locus in a particular cross section. Assuming piecewise-linear yield loci, the admissibility condition for the whole structure can be written as

$$\mathbf{N}^{T} \mathbf{s} \leqslant \mathbf{k}. \tag{2.4}$$

Here $\mathbf{k} \in R^{l}$ contains the plastic moduli, e.g. yield axial forces or yield bending moments, and $\mathbf{N} \in R^{m \times l}$ is the gradient matrix. It collects outward unit normals to the facts of the yield locus. Combining Eqs. (2.1), (2.3), (2.4), one obtains

$$\mu \mathbf{A} \mathbf{p}^{0} + \mathbf{B} \mathbf{z} \leqslant \mathbf{k}, \tag{2.5}$$

where

$$\mathbf{A} = \mathbf{N}^{T} \mathbf{C}^{-}, \quad \mathbf{B} = \mathbf{N}^{T} \mathbf{C}^{0}. \tag{2.6}$$

According to the static theorem of the ultimate load analysis, the ultimate value μ^* of the load factor is the largest one for which the static admissible state of stresses exists. In matrix description μ^* can be found solving the linear programming problem [18]:

$$\max \{\mu | \mu \mathbf{A} \mathbf{p}^{0} + \mathbf{B} \mathbf{z} \leqslant \mathbf{k}\}. \tag{2.7}$$

If random properties of the material and of the loading are taken into account, then the stochastic counterpart of the relation (2.7) is obtained:

$$\max \{\mu | \mu \mathbf{A} \tilde{\mathbf{p}}^{0} + \mathbf{B} \mathbf{z} \leqslant \tilde{\mathbf{k}}\}. \tag{2.8}$$

Let us denote

$$\tilde{\mathbf{h}} = \mu \mathbf{A} \tilde{\mathbf{p}}^{0} + \mathbf{B} \mathbf{z} - \tilde{\mathbf{k}}. \tag{2.9}$$

Since $\tilde{h}_i$ is a linear combination of the normally distributed random variables $\tilde{p}_j^0$ and $\tilde{k}_i$, it will also have normal distribution. Then mean value $\bar{h}_i$ and the variance $\mathscr{H}_i$ of $\tilde{h}_i$ are defined in a usual way:

$$\bar{h}_i = \mu \sum_{\alpha=1}^{n} A_{i\alpha} \bar{p}_{\alpha}^{0} + \sum_{\beta=1}^{r} B_{i\beta} z_{\beta} - \bar{k}_1, \tag{2.10}$$

$$\mathscr{H}_i = \mu^2 \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} A_{i\alpha} \mathscr{P}_{\alpha\beta} A_{\beta i} + \mathscr{K}_i, \tag{2.11}$$

where: $i=1,2,\ldots,l$ and the matrix $\mathscr{P}$ is as follows:

$$\mathscr{P}=\begin{bmatrix} \mathscr{P}_1, & \operatorname{cov}(\tilde{p}_2^0,\tilde{p}_1^0), & \ldots, & \operatorname{cov}(\tilde{p}_n^0,\tilde{p}_1^0) \\ \operatorname{cov}(\tilde{p}_1^0,\tilde{p}_2^0), & \mathscr{P}_2, & \ldots, & \operatorname{cov}(\tilde{p}_n^0,\tilde{p}_2^0) \\ \vdots & & & \\ \operatorname{cov}(\tilde{p}_1^0,\tilde{p}_n^0), & \operatorname{cov}(\tilde{p}_2^0,\tilde{p}_n^0), & \ldots, & \mathscr{P}_n \end{bmatrix};$$

$\bar{k}_i$ is the mean value of $\tilde{k}_i$ and $\mathscr{K}_i$ is the variance of $\tilde{k}_i$.

In order to solve the problems (2.8), we use the chance constrained method. As its name indicates, the method introduces the notion of chance constraints. The problem (2.8) is reformulated as follows [15]:

$$\max\{\mu|P[\tilde{h}_i\leqslant 0]\geqslant\gamma_i,\quad i=1,2,\ldots,l\}. \tag{2.12}$$

This means that the i-th admissibility condition must be fulfilled with a probability not less than a given γ_i.

The constraints of the problem (2.12) can be written as

$$P\left[\frac{\tilde{h}_i-\bar{h}_i}{\sqrt{\mathscr{H}_i}}\leqslant-\frac{\bar{h}_i}{\sqrt{\mathscr{H}_i}}\right]\geqslant\gamma_i, \tag{2.13}$$

where $(\tilde{h}_i-\bar{h}_i)/\sqrt{\mathscr{H}_i}$ is the standard normal form of h_i. Let β_i denote the value of the standard normal variable at which

$$\Phi(\beta_i)=\gamma_i, \tag{2.14}$$

where Φ is the Laplace's function. Then the inequality (2.13) can be written as

$$\Phi\left(-\frac{\bar{h}_i}{\sqrt{\mathscr{H}_i}}\right)\geqslant\Phi(\beta_i). \tag{2.15}$$

These inequalities will be satisfied only if there holds

$$\bar{h}_i+\beta_i\sqrt{\mathscr{H}_i}\leqslant 0. \tag{2.16}$$

The deterministic inequality (2.16) replaces the initial stochastic one (2.13). Hence the problem (2.12) can be reduced to

$$\max\{\mu|\mu\mathbf{A}\bar{\mathbf{p}}^0+\mathbf{Bz}+\sqrt{\mu^2\mathbf{P}+\mathbf{K}}\,\boldsymbol{\beta}\leqslant\bar{\mathbf{k}}\}, \tag{2.17}$$

where

$$\mathbf{P}=\operatorname{diag}(\mathbf{a}_i\mathscr{P}\mathbf{a}_i^T), \tag{2.18}$$

$$\mathbf{K}=\operatorname{diag}(\mathscr{K}_i). \tag{2.19}$$

Here $\bar{\mathbf{p}}^0\in R^n$ is the column matrix containing the mean values $\tilde{p}_i^0$, $\boldsymbol{\beta}\in R^l$ collects the given reliability indices β_i and $\mathbf{a}_i$ is the i-th row of A. Moreover it is understood that $\mathbf{a}\leqslant\mathbf{b}$ means $a_i\leqslant b_i$ and $\mathbf{A}=\sqrt{\mathbf{B}}$ means $A_{ij}=\sqrt{B_{ij}}$.

Let us compare the problem (2.7) which neglects the stochastic nature of the problem and (2.17). Such a comparison shows that in addition to the rather obvious replacement of $\mathbf{p}^0$, $\mathbf{k}$ by the means $\bar{\mathbf{p}}^0$, $\bar{\mathbf{k}}$ one has to reduce the admissible domain for stresses. The scale of such a reduction depends upon the given $\boldsymbol{\beta}$, $\mathbf{P}$ and $\mathbf{K}$, i.e. upon the reliability indices assumed as well as upon variances of random variables.

3. Optimum plastic design

Contrary to the problem discussed in the previous section, we assume now that both the reference load $\mathbf{p}^0$ and the safety factor against the plastic collapse μ^* are given. The aim is to find an optimum distribution of plastic moduli such that a given cost function attains its minimum and the structure is at the ultimate equilibrium state under the load $\mathbf{p} = \mu^* \mathbf{p}^0$.

Let modifications be governed by t design variables which form the column matrix $\mathbf{c} \in R^t$. These variables determine uniquely the distribution of plastic moduli

$$\mathbf{k} = \mathbf{G}\mathbf{c} \tag{3.1}$$

through the matrix of configuration $\mathbf{G} \in R^{l \times t}$. One has to find an optimum $\mathbf{c}^*$ such that a given cost function $f = f(\mathbf{c})$ attains its constrained minimum over the set of statically admissible stresses.

The simplest is the case of the linear cost function

$$f = \mathbf{g}^T \mathbf{c}, \tag{3.2}$$

where $\mathbf{g} \in R^t$ is the column matrix of the given cost factors. Then the optimum plastic design problem is of the linear programming type [19]:

$$\min \{\mathbf{g}^T \mathbf{c} | \mathbf{G}\mathbf{c} - \mathbf{B}\mathbf{z} \geqslant \mathbf{A}\mathbf{p}, \mathbf{c} \geqslant \mathbf{0}\}. \tag{3.3}$$

We slip a detailed explanation of the model (3.3) because this problem seems to be commonly known. If the vectors $\mathbf{p}$, $\mathbf{k}$ are random, then the model (3.3) transforms into the following stochastic programming problem:

$$\min \{\mathbf{g}^T \tilde{\mathbf{c}} | \mathbf{G}\tilde{\mathbf{c}} - \mathbf{B}\mathbf{z} \geqslant \mathbf{A}\tilde{\mathbf{p}},\ \tilde{\mathbf{c}} \geqslant \mathbf{0}\}. \tag{3.4}$$

Using the chance constrained method in a similar way as it was done in the previous section, we replace the relation (3.4) by the following equivalent deterministic problem:

$$\min \{\mathbf{g}^T \bar{\mathbf{c}} | \mathbf{G}\bar{\mathbf{c}} - \mathbf{B}\mathbf{z} \geqslant \mathbf{A}\bar{\mathbf{p}} + \sqrt{\mathbf{P} + \mathbf{D}}\ \boldsymbol{\beta}, \bar{\mathbf{c}} \geqslant \mathbf{0}\}, \tag{3.5}$$

where

$$\mathbf{D} = \operatorname{diag}(\mathscr{C}_i) \tag{3.6}$$

with $\mathscr{C}_i$ — variances of $\tilde{c}_i$ and $\mathbf{P}$ defined in Eq. (2.18).

Note that the optimum distribution of the mean values of plastic moduli is looked for whereas the variances of these moduli are considered to be given. A similar formulation was proposed by Frangopol and Rondal [7]. It appears to be quite useful in practical applications.

Comparison of the resulting problem (3.5) with the problem (3.3) formulated regardless of the random nature of $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{k}}$ leads to similar conclusions as those presented in Sect. 2. Randomization reduces the admissible domain for stresses and the scale of such a reduction depends upon $\boldsymbol{\beta}$, $\mathbf{P}$, $\mathbf{D}$.

4. Reliability problem

Let us consider now the problem of the lower bound on the reliability of a structure. Note that in Sects. 2 and 3 the reliability of each cross section of a structure was assumed to be given. This time we want to estimate the reliability of the whole structure. The mean values and the variances of loads and plastic moduli are assumed to be known.

Following Ditlevsen's idea we introduce a notion of the safety margin m_i of the i-th cross section. In matrix description this margin is determined by the column matrix $\mathbf{m} \in R^l$:

$$\mathbf{m} = \mathbf{k} - \mathbf{N}^T \mathbf{s}. \tag{4.1}$$

Then $\mathbf{m} \geqslant \mathbf{0}$ is the stress admissibility condition. Obviously stresses and loads have to satisfy the equilibrium equation. Let us determine the probability of failure of the i-th cross section as $P(m_i < 0)$. Then the upper bound on the probability of the plastic failure of the whole structure is

$$p_f \leqslant \sum_{i=1}^{r+1} P(m_i < 0). \tag{4.2}$$

The strategy of choosing the best estimation proposed by Ditlevsen consists in making an iterative search of self-stresses that minimize the largest of the probabilities $P(m_i < 0)$ on the right-hand side of the relation (4.2). By means of the mathematical programming the same goal can be achieved directly. Namely, the solution of the LP-problem

$$\min \{ \mathbf{1}^T \boldsymbol{\gamma} | \boldsymbol{\gamma} - P[\tilde{\mathbf{k}} - \mathbf{A}\tilde{\mathbf{p}} - \mathbf{B}\mathbf{z} < \mathbf{0}] \geqslant \mathbf{0} \}, \tag{4.3}$$

gives the optimum γ^* which corresponds to the minimum upper bound (4.2).

Using Laplace's function Φ the problem (4.3) can be transformed into the following one:

$$\max \{ \mathbf{1}^T \boldsymbol{\beta} | \mathbf{B}\mathbf{z} + \sqrt{\mathbf{P} + \mathbf{K}}\, \boldsymbol{\beta} \leqslant \bar{\mathbf{k}} - \mathbf{A}\bar{\mathbf{p}} \}, \tag{4.4}$$

where the matrix $\mathbf{K} = \operatorname{diag} \mathscr{K}_i$ collects the variances of plastic moduli.

Let $\boldsymbol{\beta}^*$, $\mathbf{z}^*$ be the optimum of the problem (4.4). Then a sum of $r+1$ largest β_i^* gives the upper bound on probability of plastic failure according to the upper bound (4.2). Contrary to the method in [20], the self-stress state $\mathbf{z}^*$ is found automatically by solving the problem (4.4).

The model (4.4) estimates the reliability of a given structure; its main application, however, lies in the possibility to compare the reliability of several concurrent structures.

5. Numerical examples

5.1. *Ultimate load factor of a single-storey frame*

This example shows the dependence of the ultimate load factor on the enforced level of reliability. The two-bay portal frame (Fig. 1) made of I 200 PE rolled steel is considered. The plastic modulus is assumed to be random with the given mean value $\bar{M}^0 = 5390$ kNm and the given standard deviation $\sigma_M = 428.0$. The results shown in Fig. 2. were obtained by means

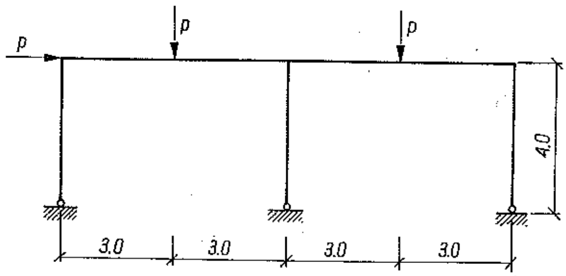

Fig. 1.

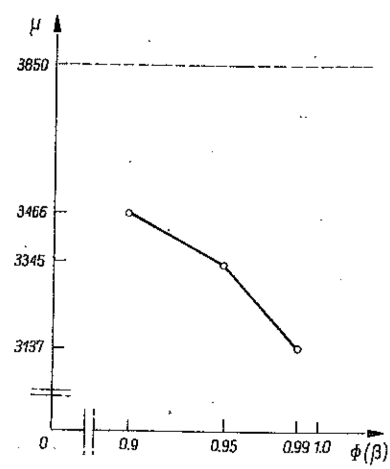

Fig. 2.

of the model (2.17). It is seen that higher reliability can be achieved at the expense of a lower ultimate load factor. The dashed line corresponds to the deterministic ultimate load factor, i.e. calculated without taking into account

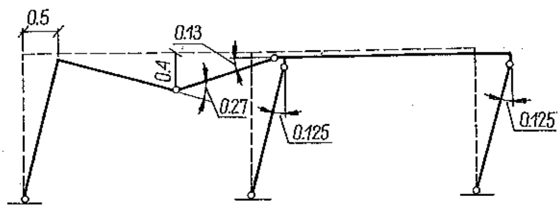

FIG. 3.

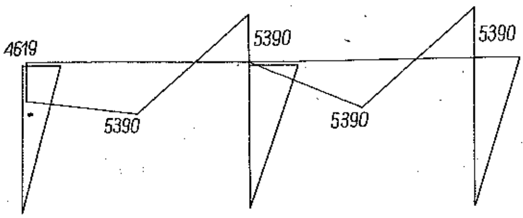

FIG. 4.

the random nature of the yield moments. It is clearly seen that the purely deterministic approach is not conservative. Figures 3 and 4 show the collapse mechanism and the distribution of bending moments at collapse respectively.

5.2. *Ultimate load factor of a two-storey frame*

The frame shown in Fig. 5. is considered. The cross section parameters are the same as in the previous example. Neglecting the random character

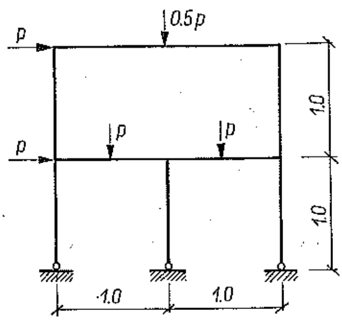

FIG. 5.

of plastic resistance, one obtains $\mu^* = 8085.0$. On the other hand $\mu^* = 7278.0$ if the reliability for each cross section is taken as 0.90. Hence, the more accurate approach decreases the load carrying capacity by about 10.3%. The collapse mechanism and the distribution of bending moments at the collapse are given in Figs. 6 and 7.

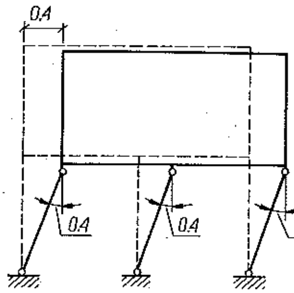

FIG. 6.

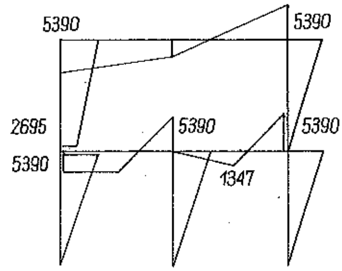

FIG. 7.

5.3. *Optimum design of a portal frame*

Consider the optimum design of the portal frame shown in Fig. 8. It is subject to the loads H, V and the design variables are the yield moment of the beam $\bar{M}_B$ and the yield moment of the column M_C (both

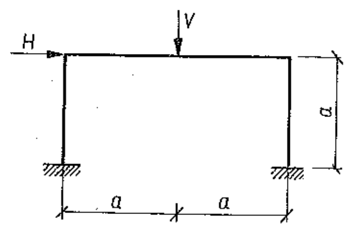

FIG. 8.

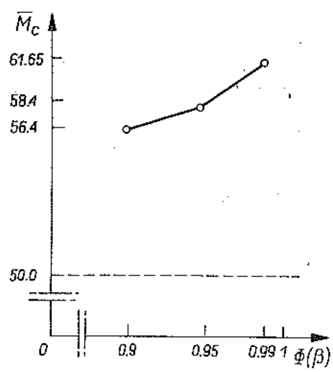

FIG. 9.

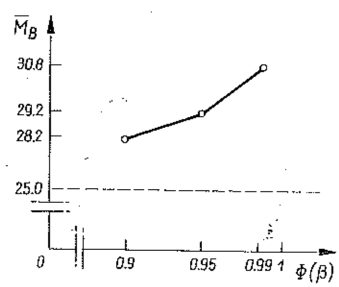

FIG. 10.

columns are assumed to be identical). The load is deterministic: $H = V = 100.0$ kN. The linear function of the mean values of yield moments

$$f = \bar{M}_C + \bar{M}_B$$

is to be minimized, i.e. the cost is assumed to be proportional to the quantity $2(\bar{M}_C+\bar{M}_B)a$. The coefficient of variation of the yield moment at each cross section is taken to be 0.10.

The results $\bar{M}_C^*$, $\bar{M}_B^*$ obtained by means of the model (3.5) for various reliability levels are plotted in Figs. 9 and 10. It is clearly seen that higher reliability leads inevitably to higher cost of the structure. The dashed lines show the values of yield moments calculated in a purely deterministic way (model 3.3).

5.4. *Reliability of portal frame*

The overall reliability of the same portal frame as in the previous example is considered. The load is the same, the beam and the column have identical mean value of the yield moment $\bar{M} = 70.0$ kNm whereas the coefficient of variations of the yield moment is 0.20. The model (4.4) gives $\beta^* \geqslant 2.73$ as the lower bound on the reliability index. It corresponds to the probability of failure $p_f \leqslant 0.0027$.

6. Conclusive remarks

We considered two possible approache to the probabilistic ultimate load analysis and plastic optimization of structures. The first one is simpler: a certain safety level for each admissibility constraint is assumed and an equivalent deterministic problem is formulated and solved. The admissibility domain for stresses depends on the assumed safety indices.

The second approach is more advanced: one tries to estimate the reliability of the whole structure. It is possible to find a lower bound on such a global reliability index using the static approach. Application of linear programming for the automatic search of self-stresses allows to find the best possible bound.

It is obvious that further investigations are necessary in order to evaluate an upper bound on the reliability index from the kinematic approach.

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STRESZCZENIE

NIEZAWODNOŚCIOWA ANALIZA I OPTYMALIZACJA PLASTYCZNYCH RAM PŁASKICH

Praca dotyczy statycznego podejścia do probabilistycznej analizy i optymalizacji konstrukcji sztywno-plastycznych. Rozważono dwa rodzaje problemów. Pierwszy odpowiada tzw. metodzie ograniczeń losowych w teorii programowania stochastycznego. Pozwala on oszacować mnożnik obciążenia granicznego lub znaleźć optymalny rozkład modułów plastycznych dla określonego prawdopodobieństwa statycznej dopuszczalności stanu naprężeń. W drugim zagadnieniu podano dolną granicę współczynnika niezawodności uwzględniając możliwe korelacje między wielkościami losowymi (wytrzymałość lub obciążenie). Pokazano, że dla dyskretnego modelu konstrukcji rozważane przypadki mogą zostać sprowadzone do odpowiednich deterministycznych liniowych lub nieliniowych problemów programowania matematycznego. Podano przykłady numeryczne analizy i optymalizacji ram ilustrujące proponowane procedury.

Резюме

АНАЛИЗ НАДЕЖНОСТИ И ОПТИМИЗАЦИЯ ПЛАСТИЧЕСКИХ СТЕРЖНЕВЫХ КОНСТРУКЦИЙ

Работа касается статического подхода к пробабилистическому анализу и оптимизации жестко-пластических конструкций. Рассмотрены два типа проблем. Первая отвечает т.наз. методу случайных ограничений в теории стохастического программирования. Позволяет он оценить множитель предельной нагрузки или найти оптимальное распределение пластических модулей для определенной вероятности статической допустимости напряженного состояния. Во второй проблеме приведен мижний предел коэффициента надежности, учитывая возможные корреляции между случайными (прочность или нагружение). Показано, что для дискретной модели конструкции рассматриваемые случаи могут быть сведены к соответствующим детерминистическим линейным или нелинейным проблемам математического програмирования. Приведены численные примеры анализа и оптимизации рам, иллюстрирующие предлагаемые процедуры.

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