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INVESTIGATION OF TECHNICAL STOCHASTIC STABILITY OF LATERAL VIBRATIONS OF MATHEMATICAL MODEL OF RAIL VEHICLE

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The authors present a method for investigating the technical stochastic stability of the mathematical model of a mechanical system with real input. The paper includes an algorithm which can be used in numerical applications. Sample results are given for a model of a four-axial luggage-car with coach 25 TN (Y25 Cs).

1. Introduction

Studies on lateral vibrations of rail vehicles often deal with an analysis of stability. The question of stability does not arise when vertical vibrations with a structurally stable mathematical model are investigated. The problem of stability is essential for a proper guiding of the wheelset in the track, i.e. for such a guiding in which a contact between the track and the wheelset is ensured at two points. Most of the authors [1, 2, 3], while analysing the stability, restrict their investigation to the wheelset or to a part of the rail vehicle and the stability definition given by Lapunov [4]. This paper presents a method for analysing the technical stochastic stability of a linear mathematical model describing the whole vehicle. Such an approach seems to be more useful for analyzing the dynamics of a real technical system with natural limitations.

2. Basic definitions and theorems

Let us consider a differential equation (2.1):

$$\frac{d\bar{x}}{dt} = f\left(\bar{x}, t, \bar{\xi}(t, \gamma)\right), \qquad \bar{x}(0) = \bar{x}_0, \tag{2.1}$$

where

$$\bar{x} = [x_1, x_2, ..., x_n]^T,$$
$$f = [f_1, f_2, ..., f_n]^T,$$
$$\bar{\xi} = [\xi_1, \xi_2, ..., \xi_n]^T,$$

t — time, γ — element of a set of elementary events. Assuming that the stochastic process $f(0, t, \bar{\xi}(t, \gamma))$ is absolutely integrable, i.e.

(2.2) $$P\{\int_0^t |f(0, t, \bar{\xi}(t, \gamma))|\, dt < \infty\} = 1 \quad \text{for every } \tau,$$

where P — probability, f, $\bar{\xi}$, $\bar{x}$ — vectors, and that there exists a stochastic process $\eta(t, \gamma)$ absolutely integrable in the considered interval $\langle 0, T\rangle$ such that there is the inequality

(2.3) $$|f(\bar{x}, t, \bar{\xi}(t, \gamma)) - f(\bar{x}, t, \bar{\xi}(t, \gamma))| \leqslant \eta(t, \gamma)\, |\bar{x}' - \bar{x}| \quad \text{for} \quad t \in \langle 0, T\rangle;$$

(i.e. the Lipshitz condition is fulfilled with respect to $\bar{x}$ with a stochastic process $\bar{\eta}(t, \gamma)$). It is possible to formulate the theorem [4, 5] that there exists one solution of the set (2.1) and that this solution is an absolutely continuous stochastic process with a probability one, for $t \geqslant t_0$.

Let us consider in turn two sets ω and Ω contained in Euclidean space E, where ω is a limited, open and coherent set containing the origin of the system, while Ω is a limited and closed set and, moreover, $\omega \subset \Omega$. Let us designate as ε the number fulfilling the inequality $0 < \varepsilon < 1$. The technical stochastic stability is defined as follows:

If every solution of the system (2.1) $\bar{x}(t, t_0, \bar{x}_0)$ with initial conditions belonging to the region ω belongs with a probability of $1-\varepsilon$ to the area Ω, then the system is technically stochastically stable in relation to the regions ω, Ω and the process $\bar{\xi}(t, \gamma)$ with a probability $1-\varepsilon$, i.e.

(2.4) $$P\{\bar{x}(t, t_0, \bar{x}_0) \in \Omega\} > 1-\varepsilon \quad \text{for} \quad \bar{x}_0 \in \omega.$$

Let us assume that Eq. (2.1) may be presented in the following form:

(2.5) $$\dot{\bar{x}} = F(\bar{x}, t) + R(\bar{x}, t, \gamma),$$

where

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \\ x_n \end{bmatrix},$$

R — vector of random disturbances (the product of the determined function and of the stochastic process), and that $F(\bar{x}, t) + R(\bar{x}, t, \gamma)$ fulfills the same conditions as for the system (2.1). Let us designate as $V(\bar{x}, t)$ Lapunov's function fulfilling the following conditions:

$$(2.6)\quad \begin{aligned} &1)\ V(\bar{x},t) \text{ belongs to class } C^2 \text{ in relation to } \bar{x}, \text{ and to class } C^1 \text{ in relation to } t,\\ &2)\ V(\bar{x},t) > 0 \text{ for } \bar{x} \neq 0,\\ &3)\ V(\bar{x},t) = 0 \Leftrightarrow \bar{x} = 0,\\ &4)\ |V(\bar{x},t) - V(\bar{x},t)| < B^s|\bar{x}' - \bar{x}|,\ |\bar{x}'| < R_0,\ |\bar{x}| < R_0, \end{aligned}$$

where R_0 — radius of the area Ω.

Let us designate as $d^0 V/dt$ the derivative of the function $V(\bar{x}, t)$ along the solutions of the systems

$$(2.7)\qquad dx/dt \doteq F(\bar{x}, t),$$

and let us assume that

$$\bar{\eta}(t) = \sup_{\bar{x}\in E^n} |R(\bar{x}, t, \gamma)|.$$

With these assumptions, the sufficient conditions for the technical stochastic stability of the system (2.5) may be formulated as follows [4, 5]:

If the following inequalities are true:

$$(2.8)\quad \begin{aligned} &\frac{d^0 V}{dt} \leqslant -C^* V(\bar{x}, t) \quad \text{for} \quad r < |\bar{x}| < R_0, \quad t_0 \geqslant 0, \quad C^* > 0,\\ &\sup_{t\geqslant 0} |\bar{\eta}(t)| \leqslant \delta,\\ &B_r^s + \frac{\delta}{C^*} \leqslant M \times \varepsilon, \quad 0 < \varepsilon < 1, \quad M = \inf V(\bar{x}, t),\\ &\qquad |\bar{x}| = R_0, \quad t \geqslant 0. \end{aligned}$$

then the system (2.5) is technically stochastically stable, i.e.

$$P\{|\bar{x}(t, t_0, \bar{x}_0)| < R_0\} = 1 - \varepsilon, \quad t \geqslant 0, \quad |\bar{x}_0| < r,$$

where r — radius of the ω region.

3. Method of investigation of technical stability of a model of side vibration of a rail vehicle with stochastic disturbances

3.1. Mathematical model — denotations and designations. Formulation of the problem

The subject of the study is a mathematical model of a coal car with 25 TN trucks. The equations of motion are contained in the works [3, 6].

They were derived on the basis of Lagrange equations of the second type, assuming a unit transformation matrix between an inertial and noninertial system, and assuming that the present bodies (wheelset), truck frames and car body) are ideally rigid. Summing up, the system describing later vibration has 11 degrees of freedom and may be presented in the form of a matrix equation:

$$A(\bar{p})\,\ddot{\bar{q}}+B(\bar{p})\,\dot{\bar{q}}+C(\bar{p})\,\bar{q}=R(\bar{p},t), \tag{3.1}$$

where

$$\bar{q}=\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_{11} \end{bmatrix} \quad \text{vector of generalized coordinates,}$$

$$\bar{p}=\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_m \end{bmatrix} \quad \text{vector of parameters,}$$

A, B, C — matrices of inertia, attenuation and rigidity, $R(\bar{p}, t)$ — vector of random disturbances. The adopted rated model of a car and the coordinate systems with origins at points of the centre of the mass of the bodies are presented in Fig. 1.

It should be stressed that the matrices B and C are asymmetrical. Although the results presented in the next part of the work refer to a specific structure (coal car with 25 TN trucks), the considerations have a general character and may refer to any dynamic system described by Eq. (3.1). The parameters and the matrices A, B, C are present in Appendix 1. Due to some limitations in the system track — rail vehicle, the coordinates q_i, $i = 1, \ldots, 11$, are subject to the following constraints:

$$|q_i| \leqslant a_1, \tag{3.2}$$

where q_i, $i = 1, 4$ — lateral shifts of wheelset (along the OY axis)

$$|q_i| \leqslant a_2,$$

where q_i, $i = 5, 6$ — lateral shifts of truck frames (along the OY axis)

$$|q_i| \leqslant a_3,$$

where q_i, $i = 7, 8$ — rotation of the truck frames (along the OZ axis)

$$|q_i| \leqslant a_4,$$

where q_i, $i = 9, 10$ — rotations of the truck frames around the OX axis

$$|q_{11}| \leqslant a_5,$$

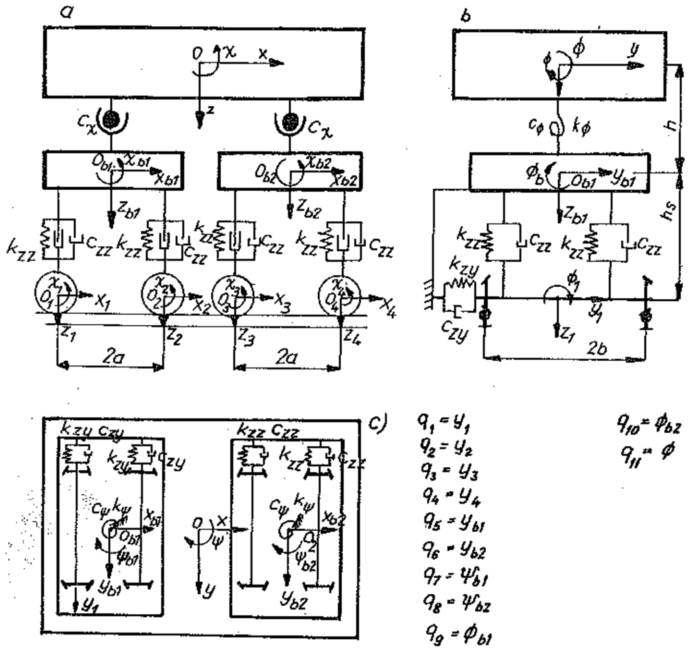

FIG. 1. The nominal model of a rail-vehicle.

where q_{11} — rotation of the car body around the OX axis. The problem of analyzing technical stochastic stability may be formulated as follows:

Let us define the region Ω by the inequalities (3.2). For a given vector of disturbances $R(\bar{p}, t)$ obtained, for example, by an experimental method and the assumed number ε, it should be checked whether the system (3.1) is technically stochastically stable.

It is often interesting not only to state whether the system is stable or not, but also to impose some constraints on the parameters — $\bar{p}$ for which the system is stable.

Therefore the problem may be formulated in a similar way: our purpose here is to determine the boundary values of some parameters with fixed values of other parameters. These boundary values separate the stable region from the unstable region in the space of parameters.

3.2. *Problem solution*

Let us introduce, for a standard, a diagonal matrix A^1 defined as follows:

$$[a_{ii}^1] = a_1 \quad \text{for} \quad i = 1, 4,$$

$$[a_{ii}^1] = a_2 \quad \text{for} \quad i = 5, 6,$$

$$[a_{ii}^1] = a_3 \quad \text{for} \quad i = 7, 8,$$
$$[a_{ii}^1] = a_4 \quad \text{for} \quad i = 9, 10,$$
$$[a_{ii}^1] = a_5 \quad \text{for} \quad i = 11.$$

This means that the matrix A^1 is a diagonal matrix with elements on the main diagonal equal in value to limitations imposed on the relevant coordinates.

Next, we shall consider the following system:

$$AA^1 \ddot{\bar{z}} + BA^1 \dot{\bar{z}} + CA^1 \bar{z} = R(t, \bar{p}), \tag{3.3}$$

where

$$q_i = a_1 z_i, \quad i = 1, 4,$$
$$q_i = a_2 z_i, \quad i = 5, 6,$$
$$q_i = a_3 z_i, \quad i = 7, 8,$$
$$q_i = a_4 z_i, \quad i = 9, 10,$$
$$q_i = a_5 z_i, \quad i = 11$$

for which R_0 of the region Ω is equal to 1, and therefore it is transformed into an n-dimensional sphere.

Multiplying the system (3.3) on the left side by $(AA^1)^{-1}$ and bringing it to the state coordinates, we obtain the following set of equations:

$$\begin{aligned} \dot{\bar{z}} &= \bar{u}, \\ \dot{\bar{u}} &= -C_1 \bar{z} - B_1 \bar{u} + P(t) \end{aligned} \tag{3.4}$$

(where $P(t) = (AA)^{-1} R(t, \bar{p})$), which, finally, may be presented in the following form:

$$\dot{\bar{W}} = D\dot{\bar{w}} + F(t, \bar{p})$$

where

$$D = \begin{bmatrix} 0_{11} & \vdots & I_{11} \\ \cdots\cdots & \vdots & \cdots\cdots \\ -C_1 & \vdots & -B_1 \end{bmatrix},$$

(3.5) block matrix 22×22,
0_{11} — zero matrix 11×11,
I_{11} — unit matrix 11×11,

$$F(t, \bar{x}) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ P_1(t, \bar{x}) \\ \vdots \\ P_{11}(t, \bar{x}) \end{bmatrix},$$

$$B_1 = (AA^1)^{-1} BA^1, \quad \bar{w} = \begin{bmatrix} z \\ u \end{bmatrix}, \quad C_1 = (AA^1)^{-1} CA^1.$$

It is easy to show that for Eq. (3.5) the conditions of the quoted theorem of the existence of a solution are fulfilled. This means that

$$\int_0^T |D \cdot 0 + F(t, \bar{x})| \, dt = \int_0^T |F(t, \bar{p})| \, dt \leqslant \Lambda\left((AA^1)^{-1}\right) |R(t, \bar{p})|,$$

where $\Lambda(X)$ — the largest characteristic root of the matrix X, $R(t, \bar{p})$ is assumed to be restricted and therefore

$$|D\bar{w}_2 + F(t, \bar{p}) - D\bar{w}_1 - F(t, \bar{p})| = |D(\bar{w}_2 - \bar{w})| \leqslant \|D\| \, |\bar{w}_2 - \bar{w}_1|,$$

$\|D\|$ — norm of the matrix D for determined values of the parameters.

Lapunov's function for the system (3.5) is defined as follows:

$$V(\bar{w}) = V(\bar{z}, \bar{u}) = \frac{1}{2} \bar{u}^T \bar{u} + \frac{1}{4} \bar{z}^T (C_1 + C_1^T) \bar{z}, \tag{3.6}$$

where the index T means transpositions.

It is easy to show that the function $V(\bar{u}, \bar{z})$ defined by the relation (3.6) fulfills the conditions for Lapunov's function, while the constant B^s of the condition 4) is defined by the following relation:

$$B^s = \max\left\{R_0, \frac{1}{2} R_0 \Lambda(C_1^T + C_1)\right\}. \tag{3.7}$$

This results from the following estimate:

$$|V(\bar{z}_2, \bar{u}_2) - V(\bar{z}_1, \bar{u}_1)| = \frac{1}{2} \Big| \bar{u}_2 T \bar{u}_2 - \bar{u}_1 T \bar{u}_1 + \frac{1}{2} \bar{z}_2 T \cdot$$

$$\cdot (C_1 + C_1^T) \bar{z}_2 - \frac{1}{2} \bar{z}_1^T (C_1^T + C_1) \bar{z}_1 \Big| \leqslant \Big| \frac{1}{2} |\bar{u}_2|^2 -$$

$$- \frac{1}{2} |\bar{u}_1|^2 \Big| + \left\{ \frac{1}{4} |(\bar{z}_2 + \bar{z}_1)^T (C_1^T + C_1)(\bar{z}_2 - \bar{z}_1)| \right\} \leqslant$$

$$\leqslant \frac{1}{2} (|\bar{u}_2| + |\bar{u}_1|) |\bar{u}_2 - \bar{u}_1| + \frac{1}{4} |(\bar{z}_2 + \bar{z}_1)^T (C_1^T + C_1)(\bar{z}_2 - \bar{z}_1)| \leqslant$$

$$\leqslant R_0 |\bar{u}_2 - \bar{u}_1| + \frac{1}{2} R_0 \Lambda(C_1^T + C_1) |\bar{z}_2 - \bar{z}_1| = B^s |\bar{w}_2 - \bar{w}_1|$$

with the assumption that the matrix $C_1 + C_1^T$ is positively determined.

The derivative of Lapunov's function along the solutions of the system is determined by the following relation:

$$\frac{d^0 V}{dt} = -\bar{u}^T (B_1 + B_1^T)\,\bar{u}. \tag{3.8}$$

Moreover, the following relations are true:

$$V(\bar{z}, \bar{u}) \leqslant \frac{1}{2}\,|\bar{u}|^2 + \frac{1}{4}\,\Lambda\,(C_1 + C_1^T)\,|\bar{z}|^2, \tag{3.9}$$

$$V(\bar{z}, \bar{u}) \geqslant \frac{1}{2}\,|\bar{u}|^2 + \frac{1}{4}\,\lambda^*\,(C_1 + C_1^T)\,|\bar{z}|^2, \tag{3.10}$$

where $\lambda^*(x)$ is the smallest characteristic root of the matrix X.

This is the result of bringing the positively determined square form defined by the equality (3.6) to a canonic form. Moreover these relations also follow from the fact that for orthogonal transformations the module of the vector remains unchanged.

If the matrix $B_1 + B_1^T$ is positively determined, then the following inequality may be derived in a similar way:

$$\frac{d^0 V}{dt} \leqslant -\lambda^*\,(B_1 + B_1^T)\,|\bar{u}|^2. \tag{3.11}$$

Now it is necessary to find such a number C^* that

$$\frac{d^0 V}{dt} \leqslant -C^*\,V(\bar{z}, \bar{u}). \tag{3.12}$$

Considering the set

$$r < |\bar{z}| < R_0,$$
$$r < |\bar{u}| < R_0,$$

it is possible to write

$$\frac{d^0 V}{dt} < -r^2\,\lambda^*\,(B_1 + B_1^T). \tag{3.13}$$

To satisfy the inequality (3.12), and taking into account the inequalities (3.9), (3.10) and (3.11), we may write

$$C^* \leqslant \left(\frac{r}{R_0}\right)^2 \frac{\lambda^*\,(B_1 + B_1^T)}{\frac{1}{2} + \frac{1}{4}\,\Lambda\,(C_1 + C_1^T)}. \tag{3.14}$$

The input $F(t, \bar{p})$ is a function of the parameters v and λ (v — velocity of the vehicle, λ — equivalent conicity). With fixed v and λ we may write

$$\sup |F(t, p)| \leqslant \delta, \quad t \geqslant 0. \tag{3.15}$$

Let us also note that

$$M = \inf V(W) = R_0^2 \left[\left(\frac{1}{2} + \frac{1}{4} \lambda^* (C_1 + C_1^T) \right] , \quad |w| = R_0, \quad t \geqslant 0. \tag{3.16}$$

Let us now analyse the third condition of the theorem of the technical stochastic stability, namely the inequality

$$Br + \frac{\delta}{C^*} \leqslant M\varepsilon. \tag{3.17}$$

Checking the inequality (3.17) with respect to the relations (3.14) and (3.16) consists in checking the following inequalities:

$$\varepsilon \geqslant \frac{R_0 r + \frac{\delta}{C^*}}{R_0^2 \left(\frac{1}{2} + \frac{1}{4} \lambda^* (C_1 + C_1^T) \right)},$$

when

$$\Lambda (C_1^T + C_1) \geqslant 2.$$

and

$$\varepsilon \geqslant \frac{\frac{1}{2} R_0 r \Lambda (C_1 + C_1^T) + \frac{\delta}{C^*}}{R_0^2 \left(\frac{1}{2} + \frac{1}{4} \lambda^* (C_1 + C_1^T) \right)};$$

when

$$\Lambda (C_1^T + C_1) \geqslant 2.$$

Introducing the following estimates:

$$\Lambda (C_1^T + C_1) \leqslant \frac{\Lambda (C + C^T)}{\lambda^* (AA^1)}, \quad \lambda^* (C_1^T + C_1) \geqslant \frac{\lambda^* (C + C^T)}{\Lambda (AA^1)},$$

$$\lambda^* (B_1 + B_1^T) \geqslant \frac{\lambda^* (B + B^T)}{\Lambda (AA^1)}$$

the last condition may also consist in checking the following inequalities:

$$(3.18)\qquad \varepsilon \geqslant \frac{rA\,(AA^1)}{R_0\left(0.5A\,(AA^1)+0.25\lambda^*\,(C+C^T)\right)}+ \\ +\frac{\delta A^2\,(AA^1)\left(0.5\lambda^*\,(AA^1)+0.25A\,(C+C^T)\right)}{r^2\,(AA^1)\,\lambda^*\,(B+B^T)\,[0.5A\,(AA^1)+0.25\lambda^*\,(C+C^T)]}$$

when

$$A\,(C_1^T+C_1)<2$$

or

$$(3.19)\qquad \varepsilon \geqslant \frac{A\,(AA_1^1)}{\lambda^*\,(AA^1)}\left\{\frac{0.5rA\,(C+C^T)}{R_0\left(0.5A\,(AA_1^1)+0.25\lambda^*\,(C+C^T)\right)}+ \right. \\ \left. +\frac{\delta A\,(AA^1)\left(0.5\lambda^*\,(AA^1)+0.25A\,(C+C^T)\right)}{r^2\,\lambda^*\,(AA^1)\,\lambda^*\,(B+B^T)\cdot\left(0.5A\,(AA^1)+0.25\lambda^*\,(C+C^T)\right)}\right.$$

when

$$A\,(C_1^T+C_1)\geqslant 2$$

the above results may be utilized for a numerical programme of stability analysis.

4. Algorithm of calculations. Exemplary results

In stability analysis, three parameters are found to have the greatest impact on stability areas. These are: k_ψ — rigidity between the truck and the body of the car around the OZ axis, λ — equivalent conicity, v — velocity. The selection of the above mentioned parameters follows from the analysis of sensitivity and from the practical possibilities of modifying elements of the real construction and conditions of operation. Assuming λ to be constant for stable profiles [6, 3] and equal to 0.038, we shall analyse the influence that the parameters k_ψ and v exert on stability areas in a rectangle defined by the following inequalities:

$$(4.1)\qquad \begin{aligned} k_{\psi\,\min} &< k_\psi < k_{\psi\,\max}, \\ v_{\min} &< v < v_{\max}. \end{aligned}$$

The algorithm of the procedure may be presented by a diagram shown in Fig. 2. It should be stressed that for the stability of a linear system defined this way it is easy to prove that a necessary condition for a system to be stable in the technical stochastic meaning is its stability in Lapunov's meaning. Sample results of numerical calculation are presented in Table 1.

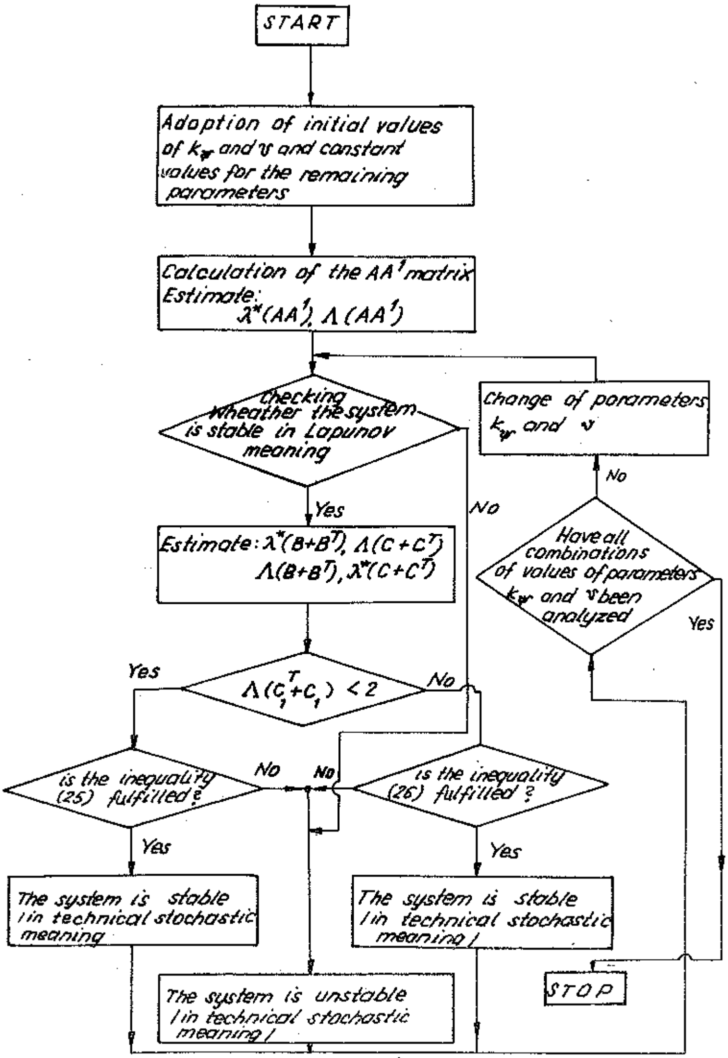

FIG. 2. The algorithm of calculation.

Table 1.

$v = 60$ [km/h]	$\lambda = 0.038$	$k_{\psi\,max} = 10^6$ [Nm/rad] $k_{\psi min} = 10^6$ [Nm/rad]
Boundary values of k_ψ for stability in Lapunov's meaning		Boundary values of k_ψ for stability in the technical and stochastic meaning
$k_\psi = 5.7 \cdot 10^6$ [Nm/rad]		$k_\psi = 6.5 \cdot 10^7$ [Nm/rad]
$k_\psi = 2 \cdot 10^7$ [Nm/rad]	$\lambda = 0.038$	$v_{min} = 20$ [km/h] $v_{max} = 150$ [km/h]
Boundary values of v for stability in Lapunov's meaning		Boundary values of v for stability in the technical and stochastic meaning
$v = 71$ [km/h]		$v = 47$ [km/h]

5. Conclusions

The regions of parameters which ensure technical stochastic stability of the system are narrower than those ensuring stability in Lapunov's meaning. From the point of view of analysis of the stability of real physical systems, which are as a rule subject to limitations and random inputs, the determination of stability in the technical stochastic meaning seems evidently more useful.

The presented method used for investigations in the stability of the mathematical model of a specific structure may also be applied to any physical system, what can be described by the relation (3.1).

Apendix 1. Specification of parameters

No	Designation	Name	Value
1	2	3	4
	m	mass of wheelset	1400 kg
	m_w	mass of truck frame	1600 kg
	m_N	mass of car body	70000 kg
	J_{zx}		1747 kg m^2
	J_{zy}	moments of inertia of the wheelset	131 kg m^2
	J_{zz}		1747 kg m^2
	J_{wz}		790 kg m^2
	J_{wy}	moments of inertia of the truck frame	1000 kg m^2
	J_{wz}		1090 kg m^2
	J_{NX}		426800 kg m^2
	J_{NY}	moments of inertia of the car body	976500 kg m^2
	J_{NZ}		921900 kg m^2
	C_{zy}		$1{,}38\times10^5$ kg/s
	C_{zz}		2.33×10^4 kg/s
	C_φ	coefficients of damping	600 kg m^2/s
	C_χ		200 kg m^2/s
	C_ψ		500 kg m^2/s
	K_{zz}		22.8×10^5 N/m
	K_{zy}		55.6×10^5 N/m
	K_φ	coefficients of rigidity	1.15×10^3 Nm/rad
	K_χ		0
	K_ψ		(table 1)
	a		0.9 m
	b		0.75 m
	a_N	geometrical parameters	3.5 m
	l		0.75 m
	h_b		0.2 m
	h		0.86 m
	r		0.46 m

dalszy ciąg tabeli

1	2	3	4
	λ		0.038 rad
	ε		0.038 rad
	δ_0		0.038 rad
	K_{px}	Kalker's coefficient	1.69×10^7 N
	K_{py}		1.93×10^7 N
	K_{ps}		7.83×10^4 Nm

q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}
1										
	1									
		1								
			1							
				2	3					
				3	2					
						$2J_{zz}+J_{wz}$				
							$2J_{zz}+J_{wz}$			
								J_{wx}		
									J_{wx}	
										J_{Nx}
y_1	y_2	y_3	y_4	y_{b1}	y_{b2}	ψ_{b1}	ψ_{b2}	Φ_{b1}	Φ_{b2}	Φ

1. $m+\mu_w^2 J_{zx}$

2. $m_w+\frac{m_N}{4}+\frac{J_{Nz}}{4a_N^2}$

3. $\frac{m_N}{4}-\frac{J_{Nz}}{4a_N^2}$

Matrix A (matrix of inertia)

q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}
4				$-c_{zy}$		1		2		
	4			$-c_{zy}$		1		2		
		4			$-c_{zy}$		1		2	
			4		$-c_{zy}$		1		2	
$-c_{zy}$	$-c_{zy}$			3	$-\frac{c_\psi}{2a_N^2}$	$\frac{c_\psi}{2a_N}$	$\frac{c_\psi}{2a_N}$	$-2h_b c_{zy}$		
		$-c_{zy}$	$-c_{zy}$	$-\frac{c_\psi}{2a_N^2}$	3	$-\frac{c_\psi}{2a_N}$	$-\frac{c_\psi}{2a_N}$		$-2h_b c_{zy}$	
$-J_{zy}\frac{V}{r}\mu_W$	$-J_{zy}\frac{V}{r}\mu_W$			$\frac{c_\psi}{2a_N}$	$-\frac{c_\psi}{2a_N}$	5				
		$-J_{zy}\frac{V}{r}\mu_W$	$-J_{zy}\frac{V}{r}\mu_W$	$\frac{c_\psi}{2a_N}$	$-\frac{c_\psi}{2a_N}$		5			
2	2			$-2h_b c_{zy}$				6		$-c_\phi$
		2	2		$-2h_b c_{zy}$				6	$-c_\phi$
								$-c_\phi$	$-c_\phi$	$2c_\phi$
y_1	y_2	y_3	y_4	y_{b1}	y_{b2}	ψ_{b1}	ψ_{b2}	ϕ_{b1}	ϕ_{b2}	ϕ

1. $\frac{2rk_{ps}}{V} + J_{zy}\frac{V}{r}\mu_W$

2. $h_b c_{zy} + 2l^2 \mu_W c_{zz}$

3. $2c_{zy} + \frac{c_\psi}{2a_N^2}$

4. $c_{zy} + 2l^2 \mu_W^2 c_{zz} + \frac{2k_{py}}{V_\phi}$

5 $c_\psi + \frac{2b^2 k_{px}}{V}$

6. $c_\phi + 2c_{zz} l^2 + 2c_{zy} h_b^2$

Matrix B (matrix of damping)

q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}
1				$-k_{zy}$		$-2k_{py}$		4		
	1			$-k_{zy}$		$-2k_{py}$		4		
		1			$-k_{zy}$		$-2k_{py}$		4	
			1		$-k_{zy}$		$-2k_{py}$		4	
$-k_{zy}$	$-k_{zy}$			6	$-\frac{k_\psi}{2a_N^2}$	$\frac{k_\psi}{2a_N}$	$\frac{k_\psi}{2a_N}$	$-2h_b k_{zy}$		
		$-k_{zy}$	$-k_{zy}$	$-\frac{k_\psi}{2a_N^2}$	6	$-\frac{k_\psi}{2a_N}$	$-\frac{k_\psi}{2a_N}$		$-2h_b k_{zy}$	
5	5			$\frac{k_\psi}{2a_N}$	$-\frac{k_\psi}{2a_N}$	2				
		5	5	$\frac{k_\psi}{2a_N}$	$-\frac{k_\psi}{2a_N}$		2			
4	4			$-2h_b k_{zy}$				3		$-k_\phi$
		4	4		$-2h_b k_{zy}$					
								$-k_\phi$	$-k_\phi$	$2k_\phi$
y_1	y_2	y_3	y_4	y_{b1}	y_{b2}	ψ_{b1}	ψ_{b2}	ϕ_{b1}	ϕ_{b2}	ϕ

1. $k_{zy} + 2l^2 \mu_w^2 k_{zz} + \frac{2N\varepsilon}{b} - \mu_w k_{ps}$
2. $k_\psi - 8Nb\delta_0$
3. $2k_{zy} h_b^2 + 2k_{zz} l^2 + c_\phi$
4. $h_b k_{zy} + 2b^2 \mu_w^2 k_{zz}$
5. $2b k_{px} \frac{\lambda}{r}$
6. $k_{zy} + \frac{k_\phi}{2a_N^2}$

Matrix C (matrix of rigidity)

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Streszczenie

ANALIZA TECHNICZNEJ STATECZNOŚCI STOCHASTYCZNEJ DRGAŃ POPRZECZNYCH MODELU MATEMATYCZNEGO POJAZDU SZYNOWEGO

Rozważania przedstawione w artykule dotyczą analizy stateczności technicznej stochastycznej liniowych modeli matematycznych układów mechanicznych. Zaproponowano sposób oraz algorytm postępowania przy badaniu wyżej wymienionych stateczności oparty na II metodzie Lapunowa. Zaprezentowane w pracy wyniki obliczeń numerycznych odnoszą się do analizy liniowego modelu matematycznego wagonu towarowego — platformowęglarki z wózkami 25TN.

Резюме

ИССЛЕДОВАНИЕ ТЕХНИЧЕСКОЙ СТОХАСТИЧЕСКОЙ УСТОЙЧИВОСТИ ЛИНЕЙНЫХ КОЛЕБАНИЙ В МАТЕМАТИЧЕСКОЙ МОДЕЛИ ВАГОНА

Рассуждения, представленные в статье, касаются анализа технической стохастической устойчивости линейных математических моделей механических систем. Предложен способ и алгоритм поступания при исследовании вышеупомянутой устойчивости, опирающейся на II метод Ляпунова. Представленные в работе результаты численных расчетов относятся к анализу линейной математической модели товарного вагона — угольной вагон-платформы с тележками 25TN.

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