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ANALYSIS OF ELASTIC PROPERTIES OF THIN-WALLED STRUCTURES DESIGNED BY SADSF METHOD

I. Markiewicz

Kielce University of Technology
Al. Tysiąclecia Państwa Polskiego 7, 25-314 Kielce, Poland

The paper presents the results of analyses of elastic properties of thin-walled structures designed by means of the SADSF method, carried out in order to confirm its practical usefulness. The SADSF method makes it possible – without applying any iterative correction procedures – to effectively solve the problems of design of such structures. The method can be applied in cases when only boundary conditions are given. The obtained solutions are free of the structural errors which can significantly deteriorate load carrying ability of structures of this class.

Key words: design, thin-walled structures, limit analysis, FEM analyses.

1. Introduction

The results of analyses presented in this paper are a part of an extensive program aimed at investigating actual properties of thin-walled constructions, whose structure, e.g. the number of component elements, their spatial allocation and the system of mutual connections, as well as the initial shape and dimensions of the elements, are determined by using the method of statically admissible discontinuous stress fields, SADSF [1, 9, 11].

In this paper, the author concentrates on three examples of structures, which were designed by W. Bodaszewski with application of its own, original software [1, 2]. These are:

- bent box section with corners (Fig. 1a);
- constructional joint created in the area of connection between two bent sections, of box and double-tee types, whose axes coincide in one straight line (Fig. 1b);
- constructional joint created in the area of connection between a box section subjected to torsion, and a twin-tee section subjected to bending (Fig. 1c).

The FEM analyses must be carried out because the SADSF method does not concern the elastic range, which is usually the range of exploitation load of the structure. One considers only the limit state of the structure, which pertains to

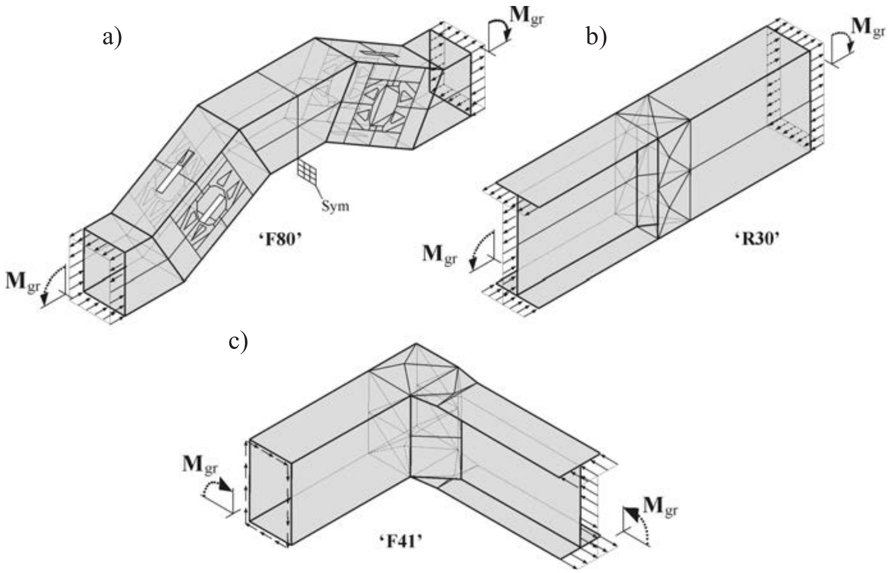

Fig. 1. Contours of statically-admissible stress fields determining shapes and dimensions of models of structures analyzed in this work ([1,2]).

the beginning of its collapse. It is also assumed that the collapse arises in the form of plastic flow, in which plane state of stress still exists in each component element. In the method, one uses a rigid-plastic model of the material, and the statically admissible stress fields which satisfy only equilibrium conditions and do not exceed the assumed yield condition at any point.

Despite these limiting assumptions, the structures designed by the SADSF method have several positive properties, which have been confirmed by numerical and experimental investigations. It has been confirmed, among other things, that membrane states of stress dominate in the elastic range, stress concentration is low, and material effort is well equalized in the whole volume of the structure – or at least along its free boundaries [1, 3, 4, 7, 8]. Such properties are difficult to obtain by traditional methods, which are based on designer's experience and intuition. On the other hand, one must be particularly careful when applying advanced methods based on consecutive iterative corrections to this class of structures. Generally speaking, de Saint Venant's principle does not apply to these cases, so that even small changes of constructional details may result in radical changes of load-carrying ability [1, 2, 8].

The fundamental advantage of systems designed by the SADSF method is that their structures are correctly selected to match the assumed loads. It means that it is possible to transmit the whole assumed load only through membrane forces. The errors made when selecting the structure can not be eliminated by changing dimensions of its elements. FEM analyses can only confirm inferior quality of the preliminarily designed structure, but they can not hint at any direction of possible improvement [1, 8].

The SADSF method can be applied already at the very beginning of the design process, when only boundary conditions are known [1, 2]. The task of the designer is reduced to selecting ready-made particular solutions from the library of the application version of the method's software [1, 2, 5, 12] and connecting them – like the Lego blocks – to form the structure. At the same time, one must keep the assumed boundary conditions and the conditions of equilibrium at the joined edges.

2. Calculational models

The analyses were carried out by means of the finite element method (FEM) using the system CosmosM. In the analyses, one assumes:

- linearly-elastic physical model of material and small strains;
- triangular shell elements of 3 nodes and 6 degrees of freedom in a node type SHELL3;
- average size of finite elements equal to 2–3 thicknesses of the element;
- loads equal to a half of the limit load value assumed in the design; distributions of loads consistent with the beam formulae used in the mechanics of materials for elastic range.

Additionally, one assumes:

- yield point of $\sigma_{\mathrm{pl}} = 300$ MPa for determining the limit load value; it means that, if one could obtain an ideal level of effort, the intensity of equivalent stress would be $\sigma_{\mathrm{eq}} = 150$ MPa at each point of the analysed structure;
- shape and dimensions of the analysed models nearly the same as those of the contours obtained from the solutions to design problems; small corrections of external contours introduced only in the vicinity of corners by rounding them with arches drawn outside of external boundaries. Within the inner contours, the inscribed circular holes are tangent to their boundaries (the problem of boundary corrections was not undertaken).

The analyses carried out in this work have an approximate character. Due to the fact that one operates on a shell model, local three-dimensional states in the vicinity of common borders between component elements are not analysed.

3. General results of analyses

In order to facilitate reviewing the obtained results, we first formulate a list of results which, because of their repeatability, seem to lead to general conclusions. Then, in all of the analysed cases one can find:

1. Domination of membrane states; the values of effort related to the bending state are small.
2. Relatively low concentrations of stress, and similar levels of maximal effort in all component elements.
3. Almost ideally-equalised fields of effort in torsion sections (Fig. 1c). In sections subjected to bending, well-equalised state of effort was found only in flanges, because of the existence of harmful states in these sections, characteristic for the bending axis.

The results obtained for all structure models are illustrated in the same way by the graphs. First, one presents the shape of analysed model with the assumed boundary conditions, then the distributions of equivalent stresses, in the Huber-Mises sense, to the component states of membrane and bending type.

4. Detailed results

4.1. Bent box-type section with corners

Because the structure is symmetrical, and so is the field of internal forces in it, we analysed only a half of the structure (Fig. 2a). On the symmetry plane β-β, we assumed appropriate boundary conditions, additionally introducing displacements that prevented the possibility of rigid motion. The load of bending moment was applied in the cross-section α-α consistently with the beam-type distribution used in mechanics of materials.

The distributions of stresses obtained for the component states, of membrane and bending type, are shown in Figs. 2b–d. By inspection of these distributions, one can see:

- In the membrane state (Figs. 2b,c):
  - formation of harmful states associated with the axis of elastic bending;
  - relatively good equalisation of effort in large areas of the flanges, and very similar levels of effort at the places of maximal effort;
  - low stress concentrations (maximal equivalent stress 215.5 Pa is not much greater than that which would exist when uniform effort was obtained in the whole volume of the structure, i.e. 150 MPa).
- In the bending state (Fig. 2d):
  - well-equalised effort field of very low value, which only locally reaches 7.75% of effort values associated with membrane state (16.7/215.5) – the maximal equivalent stress of 106.1 MPa appearing in the corner of the loaded boundary is not taken into account, because it results from the assumed boundary conditions.

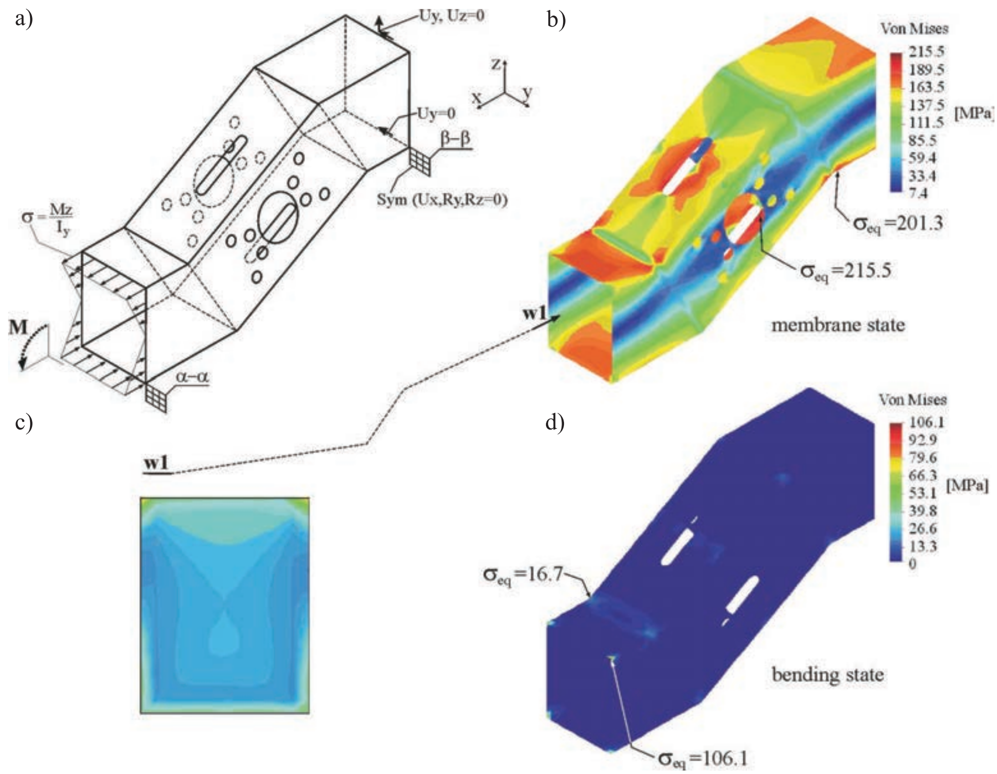

FIG. 2. Shape of a symmetric half of the analysed structure along with the assumed boundary conditions and obtained distributions of equivalent stresses.

Using the SADSF method, we obtain both the structure of the system, and shapes and dimensions of its component elements. What would happen, if one changed the structure designed by the SADSF method by removing one of its elements? Let this element be the diaphragm, for which an additional view of membrane stress distribution is shown in Fig. 2c. Inserting it into the structure (welding it in) is difficult; on the other hand, stresses in the diaphragm seem to be relatively low.

Distributions of equivalent stresses obtained for such a case are shown in Fig. 3. As it can be seen, the mentioned change in the structure caused almost a threefold increase of local equivalent stresses in membrane state (628/215.5), and over fifteenfold increase of it in bending state (260.2/16.7).

Despite the fact that such a dramatic increase of maximal stress concentrations was obtained, the changed structure still has the ability of transmitting the assumed load in membrane state (the structure remains a proper one). If such a possibility would not exist, the deterioration of load-carrying ability would have been even worse, and would affect the whole structure [1, 8].

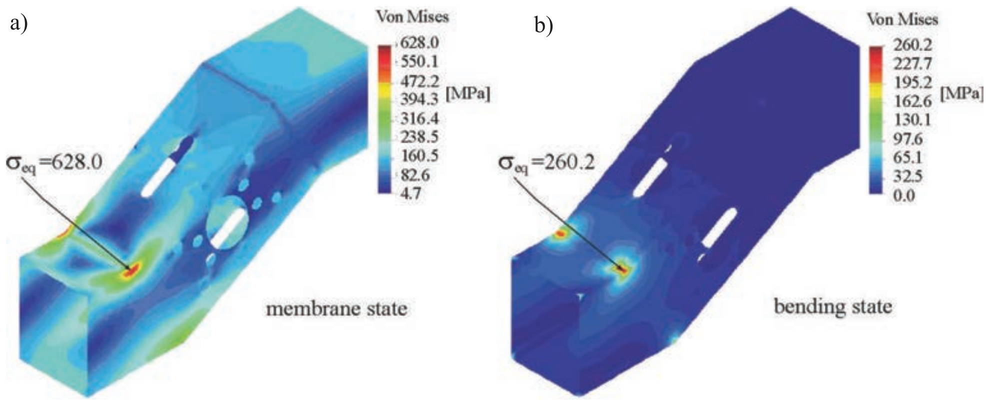

Fig. 3. Results of FEM analysis obtained for structural model with removed diaphragm.

4.2. Joint connecting bent sections of twin-tee and box types

The shapes of the analysed structure model, together with the assumed boundary conditions, are shown in Fig. 4a. Similarly as it was in the previ-

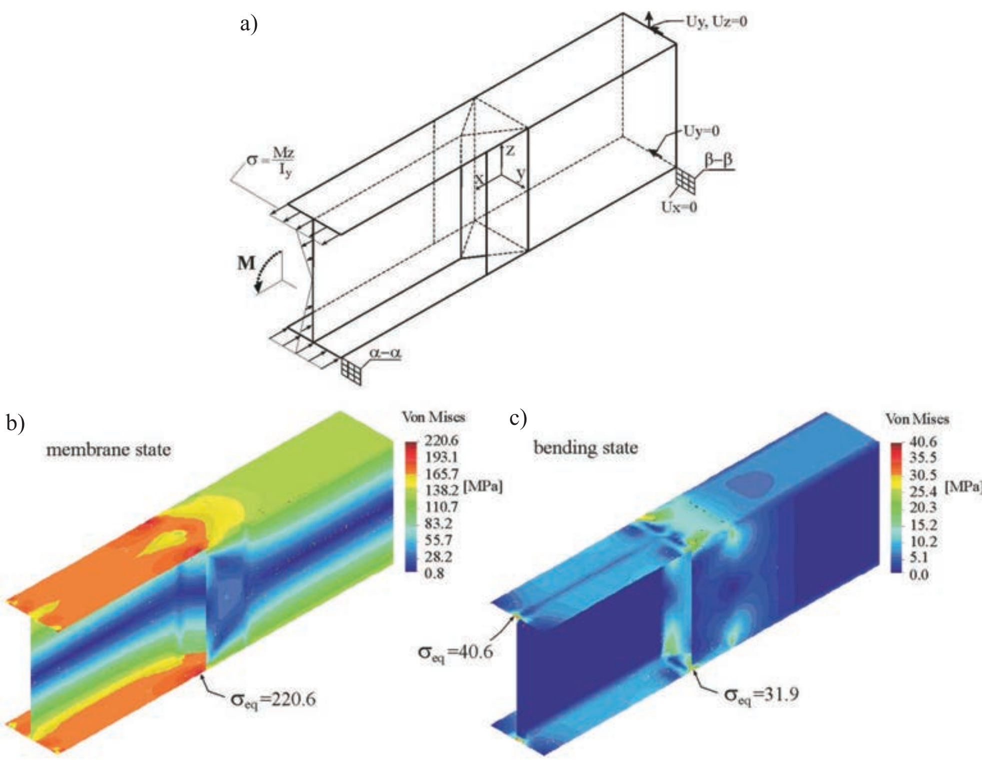

Fig. 4. Shapes and assumed boundary conditions of the analysed structure model as well as obtained distributions of equivalent stresses.

ous example, the load by a bending moment introduced in the cross-section α-α had a distribution consistent with beam-type distributions. The nodes lying in the cross-section β-β were deprived of the possibility of moving in the direction of the x axis. Additionally, one assumed displacements preventing the possibility of rigid motion.

Based on the results obtained in membrane state (Fig. 4b) one can conclude that, among other things:

- there appear harmful states associated with the axis of elastic bending;
- the level of effort is well equalised in the flanges of the structure;
- there appear low concentrations of stress locally, in the central part of flanges, where $\sigma_{\rm eq} = 220.6$ MPa.

The effort associated with bending state (Fig. 4c) is small, and maximal value of effort in this state reaches barely 14.5% of the values associated with membrane state (31.9/220.6).

4.3. Joint connecting torsional box-type section with bent section of twin-tee type

The boundary conditions and shapes of the structure model are well illustrated in Fig. 5a. The load by torsional moment was introduced in the plane α-α by means of shear forces of constant values around the whole circumference of

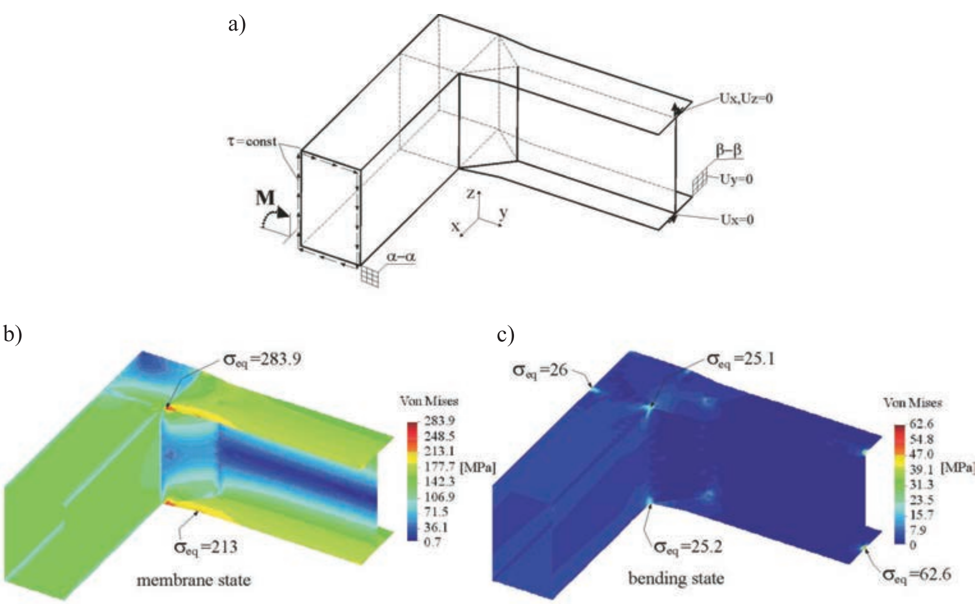

FIG. 5. Boundary conditions, shapes of the analysed structure model and obtained distributions of equivalent stresses.

the cross-section. The nodes lying in the cross-section β-β were deprived of the possibility of moving in the direction of the y axis. Additionally, one assumed displacements preventing the possibility of rigid motion.

In this case, one can conclude that in membrane state (Fig. 5b):

- the level of effort is ideally equalised in the torsional box section where pure shear load in the statically admissible stress field is assumed;
- the states characteristic for the bending axis are formed in the bent twin-tee section; equalisation of effort in the flanges of this section is good;
- local concentrations of stress in the vertices and corners of the structure are relatively low.

The effort asociated with bending state (Fig. 5c) reaches barely 10% of the values associated with membrane state (26/283.9).

5. Conclusions

In this study, the author presented a small fragment of FEM analyses carried out by him on thin-walled structures designed with the use of the SADSF method. In all cases – similarly as in the cases presented in this paper – one obtained good, and sometimes even very good load-carrying properties: domination of membrane states, low concentrations of stress and good equalization of elastic effort. Similar conclusions, based on investigations on elastic range pertaining to other cases of structure design, can be found in the whole literature of the subject [1, 3–12].

The results obtained so far allow us to confirm great practical usefulness of the SADSF method in designing thin-walled structures. The quality of stress fields realized in the systems designed in this way is absolutely incomparable to that obtained by using traditional methods.

Taking into account low level of bending forces, confirmed by the investigations, one can hardly expect large bending deformations in the exploitation range of load. However, in some fragments of certain structures, characterized by high slenderness ratio, the loss of stability at higher loads might be possible. The probability of maintaining the membrane state of stress up to the moment when limit load capacity is reached, as it is assumed in the SADSF method, seems to be low. However, as it results from investigations on other systems designed by this method, the assumed limit load capacity will most probably be obtained anyway [1, 4, 8].

The level of quality of preliminary designs of structures made by the SADSF method is good, so that these are worthy of expenses for further numerical analyses. In the cases of thin-walled structures, these systems are, first of all, free of structural errors, to which this class of structures is particularly sensitive, and the existence of which can deteriorate – even several dozens times – load carrying properties and global strength of the structure [1, 8]. The SADSF method eliminates such errors automatically. In contrast, the FEM makes it possible to notice such errors only after carrying out complete calculations, and even then it can not provide adequate hints of how to introduce the necessary corrections [1].

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