ROZPRAWY INŻYNIERSKIE • ENGINEERING TRANSACTIONS • 34, 3, 337—352, 1986
Polska Akademia Nauk • Instytut Podstawowych Problemów Techniki

A NOTE ON AVERAGING OF STIFFNESSES OF THIN ELASTIC PERIODIC PLATES[1]

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The homogenization problem for a nonhomogeneous plate of doubly periodic structure is discussed. It has been proved that an effective plate with constant effective homogenized stiffnesses is (in a certain meaning) energy equivalent to the considered heterogeneous plate of εY-periodic stiffnesses. Moreover, the corrector of the homogenized solution is defined and then its elementary derivation is presented, emphasis being put on the physical clearness of the procedure. In the last section a particular case of a plate with the thickness periodic in one (x^1) direction is examined. A comparison is made of two sets of formulas for effective stiffnesses proposed by Duvaut and Kączkowski, respectively. It is noted that the two formulae for D_{1212} and D_{2222} do not coincide.

1. Introduction

In the last years new mathematical methods of averaging of properties of heterogeneous solids and structures have been proposed. For the origins of these new methods the reader should refer to the book by Bensoussan, Lions and Papanicolaou [1] and the review papers [2, 3]. The so-called homogenization approach has been applied to classical as well as nonclassical problems of various fields of physics and mechanics [1—10]. The homogenization problems of plates were studied in Refs. [6, 9, 10].

On the other hand, the problem of computing effective stiffnesses for heterogeneous plates has a long history, the origin of which can be seen in early engineering papers on reinforced concrete plates. The results well known for engineers and regarded as original, were obtained by Huber (see [11] where an account of the origins of these problems can be found). The averaging methods related to Huber's approach are given in Ref. [12]. Although these techniques are more intuitive than rigorous, they have been accepted because of their simplicity and clear mechanical sense. The new mathematical homogenization methods have not yet been adapted to engi-

(1) The paper was writen during the author's stay at the Institute of Fundamental Technological Research, Polish Academy of Sciences.

neering practice. Thus it seems reasonable to bring them forward by disclosing (wherever possible) their mechanical sense.

The present paper deals with periodic plates with slowly varying elastic properties or thickness. Thus the characteristic cell of the considered structure is assumed to be a plate itself, i.e. its in-plane dimensions are assumed to be much greater than its average thickness. Such plates were considered in Refs. [6, 12] and also the present paper will be confined to this case. Thus the problem of averaging of periodic plates with the period of comparable (or much less) size to the average thickness exceeds the scope of this note; therefore the recent results due to CAILLERIE [9] and KOHN and VOGELIUS [10] will not be reviewed.

The plan of the paper is as follows. First we recall the formulae for the homogenized stiffnesses for the periodic plate. Then we show that the same formulae can be derived from a certain energy equality. In the subsequent section the theorem on the correctors is recalled and then their simplified derivation is given. In Sect. 5 two formulae for computing the effective stiffnesses tensor (due to Duvaut and Kączkowski) in the particular case of isotropic plate with the thickness varying periodically in one direction are set up and compared.

2. BENDING OF PLATES WITH PERIODIC STRUCTURE HOMOGENIZATION FORMULAE

Effective stiffnesses for plates with periodic structure as well as the fundamental homogenization theorems are formulated in Ref. [6]. In this section we state the bending problem for periodic plates and recall briefly the mentioned homogenization results.

2.1. *Basic assumptions. Formulation of the boundary value problem*

Consider an elastic thin plate symmetrical with respect to its mid-surface. Assume that the plate behaviour can be described by means of the classical Kirchhoff's theory. The mid-surface π is parametrized by the Cartesian coordinate system x^α, $\alpha = 1, 2$ and is referred to the domain $\Omega \subset R^2$. Assume for simplicity that the plate is clamped along the edge $\partial\Omega$. Let the stiffnesses $D^\varepsilon_{\alpha\beta\gamma\delta}(x)$ be εY-periodic functions, $\varepsilon Y = [0, \varepsilon Y_1] \times [0, \varepsilon Y_2]$, hence one can write $\mathbf{D}^\varepsilon(x) = \mathbf{D}(y)$, $y = x/\varepsilon$ where $\mathbf{D}(y)$ is an Y-periodic tensor. The parameter ε has been introduced in order to examine a sequence of boundary value problems of plates of different, highly oscillating stiffnesses. The plate deflection is described by the function $u: x \rightarrow u(x)$ standing for the transverse displacements of the π surface. Assume that the plate is initially stressed by the moments $M^0_{\alpha\beta}$. Thus the changes of curvature

$$\varkappa^x_{\alpha\beta}(u) = -\partial^x_\alpha \partial_\beta u, \quad \partial^x_\alpha = \partial/\partial x^\alpha, \quad \alpha, \beta = 1, 2,$$

and the moments $\mathbf{M} = (M_{\alpha\beta})$ are interrelated by

$$M_{\alpha\beta} = D^\varepsilon_{\alpha\beta\gamma\delta} \varkappa^x_{\gamma\delta} + M^0_{\alpha\beta}, \quad \alpha, \beta, \gamma, \delta = 1, 2.$$

The components of the stiffness tensor $\mathbf{D}^\varepsilon$ are assumed to satisfy the following conditions:

$$\text{(2.1)} \quad \begin{aligned} &D^\varepsilon_{\alpha\beta\gamma\delta} = D^\varepsilon_{\beta\alpha\gamma\delta} = D^\varepsilon_{\gamma\delta\alpha\beta} = D^\varepsilon_{\delta\gamma\alpha\beta}, \quad D^\varepsilon_{\alpha\beta\gamma\delta} \in L^\infty(\Omega), \\ &\exists \gamma > 0, \quad D^\varepsilon_{\alpha\beta\gamma\delta} \varkappa_{\alpha\beta} \varkappa_{\gamma\delta} \geqslant \gamma \cdot \varkappa_{\alpha\beta} \varkappa_{\alpha\beta}, \quad \forall \varkappa = (\varkappa_{\alpha\beta}). \end{aligned}$$

Assume that the plate is subjected to external vertical load p and surface couples m_α, $\alpha = 1, 2$. Under the above assumptions the virtual work principle can be formulated as a variational problem: find $w^\varepsilon \in H^2_0(\Omega)$ such that

$$\text{(2.2)} \quad a^\varepsilon(w^\varepsilon, v) = f(v), \quad \forall v \in H^2_0(\Omega),$$

where the bilinear form a (...) and linear form f (.) are defined as follows:

$$\text{(2.3)} \quad \begin{aligned} a^\varepsilon(u, v) &= \int_\Omega D^\varepsilon_{\alpha\beta\gamma\delta} \varkappa^x_{\alpha\beta}(u) \varkappa^x_{\gamma\delta}(v)\, dx, \\ f(v) &= \int_\Omega \left(-M^0_{\alpha\beta} \varkappa^x_{\alpha\beta}(v) - m_\alpha \partial^x_\alpha(v) + pv\right) dx. \end{aligned}$$

The Sobolev space $H^2_0(\Omega)$ stands here for the set of admissible deflections.

Let the initial moments and external loads be square integrable i.e.: $M^0_{\alpha\beta}$, m_α, $p \in L^2(\Omega)$. Then by virtue of these assumptions and the conditions (2.1), and according to the Lax-Milgram lemma [5], the problem (2.2) is well posed, w^ε exists and is unique; here ε is held fixed.

2.2. *Homogenization: effective stiffnesses*

Let us recall the fundamental results of the homogenization of the linear theory of plates [6]. First, define the space

$$W(Y) = \left\{ v \mid v \in H^2(Y);\ v \text{ and } \frac{\partial v}{\partial y^\sigma},\ \sigma = 1, 2, \quad \text{are equal at the opposite sides of } Y \right\}$$

and formulate the boundary value problems in the basic cell Y: find $\chi^{(\sigma\mu)} \in W(Y)$, $\sigma, \mu = 1, 2$ such that

$$\text{(2.4)} \quad a_Y(-y^{(\sigma\mu)} + \chi^{(\sigma\mu)}, v) = 0, \quad \forall v \in W(Y), \quad \sigma, \mu = 1, 2,$$

where

$$\text{(2.5)} \quad a_Y(u, v) = \int_Y D_{\alpha\beta\gamma\delta}(y) \varkappa^y_{\alpha\beta}(u) \varkappa^y_{\gamma\delta}(v)\, dy,$$

(2.6) $$y^{(\sigma\sigma)} = \frac{1}{2}(y^\sigma)^2, \quad y^{(12)} = y^{(21)} = y^1 y^2, \quad \sigma = 1, 2.$$

Obviously the functions $\chi^{(\delta\mu)}$ are determined up to an additive constant. In the case considered the linear form $f(.)$ (see Eq. $(2.3)_2$) has a more general form than the form which was considered in Ref. [6]. Nevertheless one can prove that the following theorem remains valid:

THEOREM 2.1. *Under the mentioned assumptions concerning the stiffness tensor and the loads the solution w^ε of the problem (2.2) converges in a weak sense to an element $w \in H_0^2(\Omega)$ being a solution of the following homogenized boundary value problem: find $w \in H_0^2(\Omega)$ such that*

(2.7) $$a(w, v) = f(v), \quad \forall v \in H_0^2(\Omega),$$

where

(2.8) $$a(u, v) = \bar{D}_{\alpha\beta\gamma\delta} \int_\Omega \varkappa^x_{\alpha\beta}(u)\, \varkappa^x_{\gamma\delta}(v)\, dx.$$

The effective stiffnesses read

(2.9) $$\bar{D}_{\sigma\mu\lambda\eta} = \frac{1}{|Y|} \int_Y [D_{\sigma\mu\lambda\eta} - D_{\sigma\mu\gamma\delta}\, \varkappa^y_{\gamma\delta}(\chi^{(\lambda\eta)}(y))]\, dy, \quad |Y| = \text{meas}(Y).$$

From now on the denotation w will be preserved for the homogenized solution.

Prior to explaining the mechanical sense of the formulae (2.9) it is worth considering the strong formulation of the auxiliary boundary value problem (2.4); in particular it is appropriate to reveal the natural boundary conditions resulting from the weak formulation (2.4). By virtue of Eqs. (2.0) and (2.6) the moments read

$$M_{\alpha\beta}(\chi^{(\sigma\mu)}) = D_{\alpha\beta\gamma\delta}(y)\, \varkappa^y_{\gamma\delta}(\chi^{(\sigma\mu)}) + D_{\alpha\beta\delta\mu}(y).$$

Assume the Cartesian coordinate system y^i, $i = 1, 2, 3$, to be right-handed. Denote the sides and vertices of the rectangle Y by Γ_i and O_i, $i = 1, 2, 3, 4$, respectively, i.e. $\Gamma_i = O_i O_{i+1}$ for $i = 1, 2, 3$ and $\Gamma_4 = O_4 . O_1$, see Fig. 1. The length parameters are assumed to go round ∂Y accordingly to the order O_1, O_2, O_3, O_4; s determines the unit tangent vector τ, so that $ds = \tau_\alpha\, dy^\alpha$. The unit vector $\mathbf{n}$ is normal outward to ∂Y.

In the considered case of the rectangle domain Green's identity reads

$$a_Y(\chi^{(\sigma\mu)} - y^{(\sigma\mu)}, v) \equiv \int_Y M_{\alpha\beta}(\chi^{(\sigma\mu)})\, \varkappa^y_{\alpha\beta}(v)\, dy = -\int_Y \partial^y_\alpha \partial^y_\beta M_{\alpha\beta}(\chi^{(\sigma\mu)})\, v\, dy +$$

$$+ \sum_{i=1}^4 \int_{\Gamma_i} \left[V_n(\chi^{(\sigma\mu)})\, v - M_n(\chi^{(\sigma\mu)}) \frac{\partial v}{\partial n} \right] ds + \sum_{i=1}^4 R_i(\chi^{(\sigma\mu)})\, v(O_i),$$

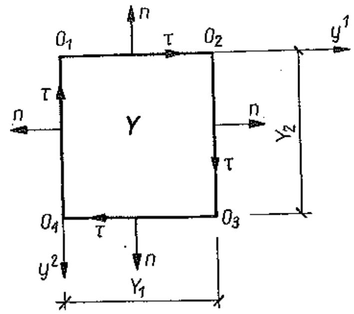

FIG. 1.

where the effective (Kirchhoff's) transverse force V_n, the normal moment M_n and the concentrated forces R_i are expressed by the formulae

$$V_n = \partial^y_\alpha M_{\alpha\beta} n_\beta + \frac{\partial M_\tau}{\partial \tau}, \qquad M_\tau = M_{\alpha\beta} n_\alpha \tau_\beta, \qquad M_n = M_{\alpha\beta} n_\alpha n_\beta,$$

$$R_i = [\![M_\tau]\!]_{O_i}.$$

The adjoining sides Γ_i, Γ_{i+1} are orthogonal, hence

$$R_i = [\![M_{\alpha\beta} n_\alpha \tau_\beta]\!]_{O_i} = -(M_{12}|_{O_i^+} + M_{12}|_{O_i^-}). \tag{2.10}$$

By taking $v \in \mathscr{D}(Y)$, one arrives at equilibrium equations:

$$\partial^y_\alpha \partial^y_\beta M_{\alpha\beta}(\chi^{(\sigma\mu)}) = 0. \tag{2.11}$$

On taking $v \in W(Y)$ such that $v = 0$ at Γ_1 and Γ_3 and $\partial v/\partial n = 0$ at ∂Y one has

$$V_n(\chi^{(\sigma\mu)})|_{\Gamma_2} = -V_n(\chi^{(\sigma\mu)})|_{\Gamma_4}. \tag{2.12}$$

Similarly one finds

$$V_n(\chi^{(\sigma\mu)})|_{\Gamma_1} = -V_n(\chi^{(\sigma\mu)})|_{\Gamma_3}. \tag{2.13}$$

By assuming $v \in W(Y)$, $v = 0$ at ∂Y and $\partial v/\partial n = 0$ at Γ_1, Γ_3 one arrives at

$$M_n(\chi^{(\sigma\mu)})|_{\Gamma_2} = M_n(\chi^{(\sigma\mu)})|_{\Gamma_4}, \tag{2.14}$$

and in a similar manner one finds

$$M_n(\chi^{(\sigma\mu)})|_{\Gamma_1} = M_n(\chi^{(\sigma\mu)})|_{\Gamma_3}. \tag{2.15}$$

Finally the variation of equation (2.4) reduces to

$$\sum_{i=1}^{4} R_i(\chi^{(\sigma\mu)})\, v(O_i) = \Big(\sum_{i=1}^{4} R_i(\chi^{(\sigma\mu)})\Big)\, v(O_1), \ \forall v \in W(Y).$$

On taking into account Eq. (2.10) we find

$$\sum_{i=1}^{4} [M_{12}(\chi^{(\sigma\mu)})|_{O_i^+} + M_{12}(\chi^{(\sigma\mu)})|_{O_i^-}] = 0. \tag{2.16}$$

If M_{12} is a continuous function of y then

$$\sum_{i=1}^{4} M_{12}(\chi^{(\sigma\mu)})|_{O_i} = 0. \tag{2.17}$$

Thus all the natural boundary conditions (2.12)—(2.15) and (2.16) or (2.17) have been established. Now we observe that by assembling the rectangles Y one to another in two directions y^1, y^2, an infinite self-equilibrated plate is constituted. Due to the natural boundary conditions, the equilibrium conditions along the joining lines $nY^\alpha = \text{const}$, $n \in Z$, and at the nodes (nY^1, mY^2), $n, m \in Z$ are fulfilled. Thus we conclude that the problem (2.4) defined at the basic cell Y can be interpreted as a statical problem for the infinite Y-periodic plate subjected to Y-periodic distortions $(-D_{\alpha\beta\sigma\mu}(y))$.

3. Mechanical interpretation of the homogenization formulae (2.9)

The formulae (2.9) have been derived in a mathematical manner based on the concept of the G-(or Γ-) convergence of functionals [1]. The latter idea is not easily interpretable from the mechanical point of view and that is why it is reasonable to interpret the final results by means of commonly known mechanical ideas. For this purpose we shall deal with an infinite plate with εY-periodic structure and the parameter ε will be fixed. We show that the effective homogenization formulae (2.9) can be derived by postulating a certain energy equality for the cell εY, without examining the behaviour of w^ε when ε tends to zero.

3.1. Equilibrium of the infinite periodic plate

Consider the case $\Omega = R^2$ and additionally assume the initial moments and external loads to be εY-periodic

$$(M^{0\varepsilon}_{\alpha\beta}, m^\varepsilon_\alpha, p^\varepsilon)(x) = (N^0_{\alpha\beta}, l_\alpha, b)(x/\varepsilon),$$

where $N^0_{\alpha\beta}$, l_α, b are Y-periodic. The problem is correctly posed provided the loads p^ε are self-equilibrated in the cells εY

$$\int_{\varepsilon Y} p^\varepsilon(x)\, dx = 0. \tag{3.1}$$

The considered infinite plate will be in an equilibrium state if the principle of virtual work for every cut out cell εY is satisfied. The right-hand side of this principle will include an influence of the moments and the effective shear forces prescribed at $\partial(\varepsilon Y)$. If one confines now the space of virtual displacements to $W(\varepsilon Y)$ (periodic functions with periodic derivatives), then the variational equilibrium condition reduces to

$$u^\varepsilon \in W(\varepsilon Y),$$

$$(3.2) \qquad a_{\varepsilon Y}(u^\varepsilon, v^\varepsilon) = \int_{\varepsilon Y} \left(-M^{0\varepsilon}_{\alpha\beta}\, \varkappa^x_{\alpha\beta}(v^\varepsilon) - m^\varepsilon_\alpha\, \partial^x_\alpha(v^\varepsilon) + p^\varepsilon v^\varepsilon\right) dx,$$

for every $v^\varepsilon \in W(\varepsilon Y)$.

Changing the coordinates we arrive at

$$u \in W(Y),$$

$$(3.3) \qquad a_Y(u, v) = \int_Y \left(-\varepsilon^2 N^0_{\alpha\beta}\, \varkappa^y_{\alpha\beta}(v) - \varepsilon^3 l_\alpha\, \partial^y_\alpha v + \varepsilon^4 bv\right) dy,$$

for every $v \in W(Y)$.

According to the Lax–Milgram lemma the solutions of (3.2) or (3.3) exist and are determined up to an additive constant.

3.2. The elementary states of deformation of the infinite periodic plate

Consider the infinite εY-periodic plate loaded at infinity in such a manner that the deformations of the plate $\varkappa = (\varkappa^x_{\alpha\beta})$ are εY-periodic. The internal forces and forces "prescribed at infinity" relevant to εY-periodic changes of curvature will be discussed later. Due to εY-periodicity of $\varkappa$ the function u which stands for the plate's transverse deflection can be composed of three elementary states

$$(3.4) \qquad u^{(\alpha\beta)}(x) = u_0^{(\alpha\beta)}(x) - \chi^{(\alpha\beta)}(x/\varepsilon), \qquad \alpha, \beta = 1, 2, \qquad (\alpha\beta) = (\beta\alpha),$$

(the rigid motions of the plate being disregarded), where

$$u_0^{(\alpha\beta)} = y^{(\alpha\beta)}(x/\varepsilon),$$

and $-\chi^{(\alpha\beta)}(x/\varepsilon)$ are εY-periodic states standing for the deflections of this infinite plate subjected to initial εY-periodic moments $M^{0(\sigma\mu)}_{\varkappa\beta} = \varepsilon^{-2} D^\varepsilon_{\alpha\beta\sigma\mu}$. If stiffnesses are differentiable, $-\chi^{(\alpha\beta)}$ can be regarded as deflections of the plate subjected to surface couples $m^{(\alpha\beta)}_\sigma = \varepsilon^{-2}\partial^x_\lambda D^\varepsilon_{\alpha\beta\sigma\lambda}$ or to vertical loads $p^{(\alpha\beta)} = \varepsilon^{-2}\partial^x_\sigma \partial^x_\mu D^\varepsilon_{\varkappa\beta\sigma\mu}$. Let us prove these facts.

As the deformations $\varkappa^x_{\alpha\beta}(u)$ are supposed to be εY-periodic, the deflection u can be decomposed into a sum of periodic and aperiodic displacements. However, by disregarding rigid motions one can notice that this latter term is at the most a polynomial of degree two. Hence the constants c_j, $j = 1, 2, 3$, exist such that $u = c_1 u_0^{(11)} + c_2 u_0^{(22)} + c_3 u_0^{(12)} + \psi^\varepsilon(x)$, where $\psi^\varepsilon(x)$ is εY-periodic: $\psi^\varepsilon(x) = \psi(y)$, $y = x/\varepsilon$; ψ is a Y-periodic function. The principle of virtual work (3.2) reduces to

$$a_{\varepsilon Y}(u, v) = 0, \quad \forall v \in W(\varepsilon Y),$$

and will be satisfied provided

$$u = c_1 u^{(11)} + c_2 u^{(22)} + c_3 u^{(12)},$$

where $u^{(\alpha\beta)}(x) = u_0^{(\alpha\beta)}(x) + \psi^{\varepsilon(\alpha\beta)}(x)$ and the functions $\psi^{\varepsilon(\alpha\beta)}$ satisfy the variational equation

$$a_{\varepsilon Y}(\psi^{\varepsilon(\sigma\mu)} v) = -\int_{\varepsilon Y} \varepsilon^{-2} D^{\varepsilon}_{\sigma\mu\gamma\delta}(x)\, \varkappa^{x}_{\gamma\delta}(v)\, dx, \quad \forall v \in W(\varepsilon Y). \tag{3.5}$$

Thus the functions $\psi^{\varepsilon(\sigma\mu)}$ are solutions to the distortion problem of the infinite εY-periodic plate subjected to initial moments $\hat{M}^{(\sigma\mu)}_{\gamma\delta} = \varepsilon^{-2} D^{\varepsilon}_{\sigma\mu\gamma\delta}$. In the case when the coefficients $D^{\varepsilon}_{\alpha\beta\gamma\delta}$ are differentiable, one can carry out an integration by parts and on taking into account the εY-periodicity of the integrands one arrives at

$$a_{\varepsilon Y}(\psi^{\varepsilon(\sigma\mu)}, v) = \int_{\varepsilon Y} m^{(\sigma\mu)}_{\alpha}(-\partial^{x}_{\alpha} v)\, dx = \int_{\varepsilon Y} p^{(\sigma\mu)} v\, dx,$$

where

$$m^{(\sigma\mu)}_{\alpha} = \varepsilon^{-2} \partial^{x}_{\gamma} D^{\varepsilon}_{\sigma\mu\gamma\alpha}, \quad p^{(\sigma\mu)} = \varepsilon^{-2} \partial^{x}_{\gamma} \partial^{x}_{\delta} D^{\varepsilon}_{\sigma\mu\gamma\delta}.$$

Note that if εY-periodic stiffnesses are twice differentiable, then their first derivatives are also εY-periodic so that the condition (3.1) is satisfied. Finally we note that Eq. (3.5) is equivalent to the variational equation

$$a_{Y}(\psi^{(\sigma\mu)} + y^{(\sigma\mu)}, v) = 0, \quad \forall v \in W(Y),$$

hence by comparing the obtained formula with Eq. (2.4) we see that $\psi^{(\sigma\mu)} = -\chi^{(\sigma\mu)}$.

By wirtue of εY-periodicity of the functions $\chi^{(\sigma\mu)}(x/\varepsilon)$ and the stiffnesses $D^{\varepsilon}_{\alpha\beta\gamma\delta}$, the moments and shear forces associated with the deflections (3.4) are εY-periodic. Thus the internal forces do not converge to a limit when $x_1 \to \pm\infty$ or $x_2 \to \pm\infty$. But we see that it is not necessary to deal with an infinite plate loaded at infinity. It is sufficient to consider an arbitrary rectangular domain: $0 \leqslant x_1 \leqslant n_1 Y_1 \varepsilon$, $0 \leqslant x_2 \leqslant n_2 Y_2 \varepsilon$. The moments $M_{\alpha\alpha}$ and effective transverse forces V_{α} are prescribed at the lines $x_{\alpha} = 0$, $x_{\alpha} = n_{\alpha} \varepsilon Y_{\alpha}$. Appropriate linear combinations of these boundary forces produce the εY-periodic states of deformation postulated at the beginning of this section.

3.3. *Physical sense of effective stiffnesses*

Let $\{a_{i,k}\}$ be a family of nodes which generate the mesh of εY rectangles. Note that the $(\alpha\beta)$-state can be treated as a superposition of $u_0^{(\alpha\beta)}$-functions of constant curvatures and nonzero values in $a_{i,k}$ points and of $\psi^{\varepsilon(\alpha\beta)}$ deflections which which take zero values in these nodes, hence $u^{(\alpha\beta)}(a_{i,k}) =$

$= u_0^{(\alpha\beta)}(a_{i,k})$. Apart from the εY-periodic plate let us consider its homogeneous counterpart of stiffnesses $\bar{D}_{\alpha\beta\gamma\delta}$ (defined by Eqs. (2.9)) in the following state of deformation $\boldsymbol{\varkappa}^{(\alpha\beta)} = (\varkappa_{\gamma\delta}^{(\alpha\beta)} = \delta_{\alpha\gamma}\,\delta_{\beta\delta})$ produced by appropriate moments prescribed at infinity. The points $a_{i,k}$ of this plate displace by the same values $u_0^{(\alpha\beta)}(a_{i,k})$ as in the case of the εY-periodic plate. Let us calculate the reciprocal works of moments in the $(\sigma\mu)$-state on changes of curvature in the $(\lambda\eta)$-state (for all combinations of $(\sigma\mu)$, $(\lambda\eta)$) referred to the arbitrary unit cells εY of both plates (the problem considered is εY-periodic with respect to deformations hence the choice of the unit cell is unimportant). The reciprocal work stored in the εY-rectangle reads

$$(3.6)\qquad a_{\varepsilon Y}^{\varepsilon}(u^{(\sigma\mu)}, u^{(\lambda\eta)}) \equiv \int_{\varepsilon Y} D_{\alpha\beta\gamma\delta}^{\varepsilon}(x)\,\varkappa_{\alpha\beta}^{x}(u^{(\sigma\mu)})\,\varkappa_{\gamma\delta}(u^{(\lambda\eta)})\,dx =$$

$$= \varepsilon^{-2}\int_{Y} D_{\alpha\beta\gamma\delta}(y)\,\varkappa_{\alpha\beta}^{y}\left(y^{(\sigma\mu)}-\chi^{(\sigma\mu)}(y)\right)\varkappa_{\gamma\delta}^{y}\left(y^{(\lambda\eta)}-\chi^{(\lambda\eta)}(y)\right)dy.$$

The above expression can be simplified to the form

$$(3.7)\qquad a_{\varepsilon Y}^{\varepsilon}(u^{(\sigma\mu)}, u^{(\lambda\eta)}) = \frac{1}{\varepsilon^2}\int_{Y}\left[D_{\sigma\mu\lambda\eta} - D_{\sigma\mu\gamma\delta}\,\varkappa_{\gamma\delta}^{y}\left(\chi^{(\lambda\eta)}(y)\right)\right]dy,$$

by virtue of the equality

$$\int_{Y} D_{\alpha\beta\gamma\delta}(y)\,\varkappa_{\alpha\beta}^{y}(\chi^{(\sigma\mu)})\,dy = \int_{Y} D_{\alpha\beta\gamma\delta}(y)\,\varkappa_{\alpha\beta}^{y}\left(\chi^{(\sigma\mu)}(y)\right)\varkappa_{\gamma\delta}^{y}\left(\chi^{(\lambda\eta)}(y)\right)dy,$$

which follows from the principle of virtual work (2.4) in the case when as virtual displacements the functions $\chi^{(\sigma\mu)}$ are taken.

On the other hand the reciprocal work stored in the εY-rectangle of the homogeneous effective plate reads

$$a_{\varepsilon Y}(u_0^{(\sigma\mu)}, u_0^{(\lambda\eta)}) \equiv D_{\alpha\beta\gamma\delta}\int_{\varepsilon Y}\varkappa_{\alpha\beta}^{x}(u_0^{(\sigma\mu)})\,\varkappa_{\gamma\delta}^{x}(u_0^{(\lambda\eta)})\,dx = \varepsilon^{-2}\,|Y|\,\bar{D}_{\sigma\mu\lambda\eta}.$$

Comparing the above result with Eq. (3.7) and bearing in mind the definitions (2.9) we conclude that

$$(3.8)\qquad a_{\varepsilon Y}^{\varepsilon}(u^{(\sigma\mu)}, u^{(\lambda\eta)}) = a_{\varepsilon Y}(u_0^{(\sigma\mu)}, u_0^{(\lambda\eta)}).$$

Thus we arrive at the simple interpretation of the formulae (2.9); we see that the effective plate with constant stiffnesses is in the above meaning energy-equivalent to the εY-periodic plate.

In order to avoid possible misunderstadings, let us emphasize here that the presented interpretation of the formulae (2.9) is not in conflict with the definition of the homogenization process given in Ref. [6]. In our consideration the period εY is fixed but both plates are infinite. The energy equivalence of both plates is required for the εY-periodic states of deformation only. In the paper [6] both plates are finite. The essence of the homogenization consists in the convergence of the energy of the periodic plate to the energy of the effective homogeneous plate:

$$J_\varepsilon = \frac{1}{2} a^\varepsilon (w^\varepsilon, w^\varepsilon) \underset{\varepsilon\to 0}{\longrightarrow} \frac{1}{2} a(w, w) = J.$$

Note that if ε tends to zero, the domain Ω occupied by the plate becomes infinite in comparison to the period δY and hence the formulae (2.9) are independent of Ω. Moreover, the external loads do not intervene to the homogenization process and that is why the resulting formulae are independent of the type of loads to which the plate is subjected. These two facts explain why the requirement (3.8) of the energy equivalence of both periodic and homogeneous infinite plates results in the same effective stiffnesses as those found in Ref. [6].

4. Correctors and their physical interpretation

The homogenized formulae (2.9) allow us to obtain the zero-order approximate solution w. However, the moments relevant to this solution do not describe variations of moments yielding from the periodical variation of stiffnesses. In order to achieve a deeper insight to the behaviour of moments, an improvement of the solution w is necessary. From the mathematical point view the improved solution $w+\theta^\varepsilon$ (where θ^ε is called the corrector) is required to converge strongly to w^ε. The formal construction of this solution is formulated in the Theorem 4.1 presented in Sect. 4.1. This theorem has not been given explicitly in the literature available to the author; however, the general results obtained in Ref. [1] make it possible to formulate the theorem and to carry over the proof, thus the proof will not be given. In the next part of this section we endeavour to clarify a mechanical sense of the corrector defined in Theorem 4.1.

4.1. *Theorem on the corrector*

The weak convergence of w^ε to w (pointed out in Theorem 2.1) can be improved by introducing the corrector as it follows from

THEOREM 4.1. *Let* $w \in H^4(\Omega) \cap H_0^2(\Omega)$ *and* $\chi^{(\sigma\mu)} \in W^{2,\infty}(\Omega)$. *Introduce cut-off functions* $m^\varepsilon \in \mathcal{D}(\Omega)$ *such that*

$$m^\varepsilon(x) = 0 \quad \textit{if} \quad \operatorname{dist}(x, \partial\Omega) \leqslant \varepsilon,$$

$$m^\varepsilon(x) = 1 \quad \textit{if} \quad \operatorname{dist}(x, \partial\Omega) \geqslant 2\varepsilon,$$

$$\textit{for every } \beta = (\beta_1, \beta_2) \quad \varepsilon^{|\beta|} |\partial^\beta m^\varepsilon(x)| \leqslant C_\beta,$$

where $|\beta| = \beta_1 + \beta_2$ *and* C_β *does not depend of* ε. *Let us define the function*

$$\theta^\varepsilon = -\varepsilon^2 m^\varepsilon(x)\, \chi^{(\alpha\beta)}(x/\varepsilon)\, \varkappa^x_{\alpha\beta}(w). \tag{4.1}$$

Then the difference $w^\varepsilon - (w + \theta^\varepsilon)$ *converges strongly to zero in* $H^2_0(\Omega)$.

4.2. Simplified derivation and mechanical interpretation of the corrector

We show a simplified derivation of the formula (4.1) and in this manner exhibit its mechanical interpretation. Let us start with some auxiliary approximations.

Let $\Phi^\varepsilon \in L^2_{\mathrm{loc}}(R^2)$ be εY-periodic function, $\Phi^\varepsilon(x) = \Phi(x/\varepsilon)$ where Φ is Y-periodic. Let $\eta \in L^2(\Omega)$, η does not depend of ε. Then [1, 5]

$$\lim_{\varepsilon\to 0} \int_\Omega \Phi^\varepsilon(x)\,\eta(x)\,dx = \mathfrak{M}_Y(\Phi) \int_\Omega \eta(x)\,dx, \qquad \mathfrak{M}_Y(\ ) = \frac{1}{|Y|}\int_Y (\ )\,dy. \tag{4.2}$$

Thus if ε is sufficiently small, the following approximation is justified:

$$\int_\Omega \Phi^\varepsilon(x)\,\eta(x)\,dx \approx \mathfrak{M}_Y(\Phi) \int_\Omega \eta(x)\,dx. \tag{4.3}$$

Assume now that the domain Ω is a sum of the rectangles εY_j each equal to εY and η is a piece-wise constant function with constant values η_j at these rectangles. Then

$$\int_\Omega \Phi^\varepsilon(x)\,\eta(x)\,dx = \sum_j \eta_j \int_{\varepsilon Y_j} \Phi^\varepsilon(x)\,dx = \sum_j \eta_j\, \varepsilon \int_Y \Phi(y)\,dy =$$
$$= \sum_j \eta_j\, |\varepsilon Y|\, \mathfrak{M}_Y(\Phi) = \mathfrak{M}_Y(\Phi) \sum_j \eta_j\, |\varepsilon Y_j| = \mathfrak{M}_Y(\Phi) \int_\Omega \eta(x)\,dx.$$

If the function η is fixed, one can choose a value ε sufficiently small for the approximation (4.3) to be justified. On the other hand, if the quantity ε is fixed, one can find a function η lying in an arbitrarily small neighbourhood of the graph of a certain piece-wise constant function η^ε; if one disregards a boundary layer, the approximation (4.3) will hold good. We shall further say that this approximation is good when the function η is ε-regular, i.e. slowly varying with respect to the εY-mesh.

Let us assume additionally that $\Phi \in \mathscr{C}^2(\bar{Y})$, $\eta \in \mathscr{C}^2(\bar{\Omega})$. There exist the constants C_i, D_i, D_{ij} such that

$$\begin{aligned} &|\Phi(y)| \leqslant C_0, && |\partial^y_\alpha \Phi(y)| \leqslant C_\alpha, && y \in Y, \\ &|\partial^x_\alpha \eta(x)| \leqslant D_\alpha, && |\partial^x_\alpha \partial^x_\beta \eta(x)| \leqslant D_{\alpha\beta}, && x \in \Omega, \quad \alpha, \beta = 1, 2. \end{aligned} \tag{4.4}$$

Let us define

$$\begin{aligned} L_\alpha(x, \varepsilon) &= \partial^x_\alpha \left(\Phi^\varepsilon(x)\,\eta(x)\right) - \partial^y_\alpha \Phi(y)\,\eta(x), \\ M_{\alpha\beta}(x, \varepsilon) &= \varepsilon^2\, \partial^x_\alpha \partial^x_\beta \left(\Phi^\varepsilon(x)\,\eta(x)\right) - \partial^y_\alpha \partial^y_\beta \Phi(y)\,\eta(x). \end{aligned} \tag{4.5}$$

Taking into account that

$$L_\alpha(x, \varepsilon) = \varepsilon \Phi(y)\, \partial^x_\alpha \eta(x),$$
$$M_{\alpha\beta}(x, \varepsilon) = \varepsilon \left(\partial^y_\alpha \Phi(y)\, \partial^x_\beta \eta(x) + \partial^y_\beta \Phi(y)\, \partial^x_\alpha \eta(x)\right) + \varepsilon^2 \Phi(y)\, \partial^x_\alpha \partial^x_\beta \eta(x),$$

and bearing in mind the estimates (4.4), we arrive at

$$|L_\alpha(x, \varepsilon)| \leqslant \varepsilon C_0 D_\alpha \xrightarrow[\varepsilon\to 0]{} 0,$$
$$|M_{\alpha\beta}(x, \varepsilon)| \leqslant \varepsilon (C_\alpha D_\beta + C_\beta D_\alpha) + \varepsilon^2 C_0 D_{\alpha\beta} \xrightarrow[\varepsilon\to 0]{} 0.$$

Thus for the small ε the following approximations can be written

$$\begin{aligned} &\partial^x_\alpha \left(\Phi^\varepsilon(x)\, \eta(x)\right) \approx \eta(x)\, \partial^x_\alpha \Phi^\varepsilon(x), \\ &\partial^x_\alpha \partial^x_\beta \left(\Phi^\varepsilon(x)\, \eta(x)\right) \approx \eta(x)\, \partial^x_\alpha \partial^x_\beta \Phi^\varepsilon(x). \end{aligned} \tag{4.6}$$

Assume now that the homogenized solution w has been found and let us try to find the corrector $\theta^\varepsilon = w^\varepsilon - w$. As it is easy to note, the function θ^ε ought to satisfy the following distorsion problem: find $\theta^\varepsilon \in H^2_0(\Omega)$ such that

$$a^\varepsilon(\theta^\varepsilon, v) = F^\varepsilon(v), \qquad \forall v \in H^2_0(\Omega), \tag{4.7}$$

where

$$\begin{aligned} &F^\varepsilon(v) = \int_\Omega \left(M^\varepsilon_{\alpha\beta}(x)\, \varkappa^x_{\alpha\beta}(v)\right) dx, \\ &M^\varepsilon_{\alpha\beta}(x) = \left(\bar{D}_{\alpha\beta\gamma\delta} - D^\varepsilon_{\alpha\beta\gamma\delta}(x)\right) \varkappa^x_{\gamma\delta}(w). \end{aligned} \tag{4.8}$$

For the fixed ε, v and w let us define

$$\begin{aligned} &\Phi^\varepsilon_{\alpha\beta\gamma\delta}(x) \equiv \Phi_{\alpha\beta\gamma\delta}(x/\varepsilon) = D^\varepsilon_{\alpha\beta\gamma\delta}(x) - D^\varepsilon_{\sigma\mu\gamma\delta}\, \varkappa^y_{\sigma\mu}(\chi^{(\alpha\beta)}), \\ &\eta_{\alpha\beta\gamma\delta}(x) = \varkappa^x_{\alpha\beta}(v)\, \varkappa^x_{\gamma\delta}(w), \end{aligned} \tag{4.9}$$

and assume that the functions $\eta_{\alpha\beta\gamma\delta}$ are sufficiently ε-regular and independent of ε (or the parameter ε is small enough) for the approximation (4.3) to hold, hence

$$\int_\Omega \Phi^\varepsilon_{\alpha\beta\gamma\delta}\, \eta_{\alpha\beta\gamma\delta}\, dx \approx \mathfrak{M}_Y(\Phi_{\alpha\beta\gamma\delta}) \int_Y \eta_{\alpha\beta\gamma\delta}\, dx,$$

and accordingly to the relations (2.9) is

$$\mathfrak{M}_Y(\Phi_{\alpha\beta\gamma\delta}) = \bar{D}_{\alpha\beta\gamma\delta}.$$

By virtue of the above one obtains

$$F^\delta(v) \approx -\int_\Omega D^\varepsilon_{\sigma\mu\gamma\delta}\, \varkappa^y_{\sigma\mu}(\chi^{(\alpha\beta)})\, \varkappa^x_{\alpha\beta}(w)\, \varkappa^x_{\gamma\delta}(v)\, dx.$$

Thus we arrive at the equation

$$\int_\Omega D^\varepsilon_{\alpha\beta\gamma\delta}\, \varkappa^x_{\gamma\delta}(v) \left[\varkappa^x_{\alpha\beta}(\theta^\varepsilon) + \varkappa^y_{\alpha\beta}(\chi^{(\sigma\mu)})\, \varkappa^x_{\sigma\mu}(w)\right] dx = 0,$$

which is satisfied provided

$$\varkappa^x_{\alpha\beta}(\theta^\varepsilon) = -\varkappa^y_{\alpha\beta}(\chi^{(\sigma\mu)})\,\varkappa^x_{\sigma\mu}(w) = -\varepsilon^2\varkappa^x_{\alpha\beta}(\chi^{\varepsilon(\sigma\mu)})\,\varkappa^x_{\sigma\mu}(w). \tag{4.10}$$

On applying the approximation $(4.6)_2$ we can write

$$\theta^\varepsilon = -\varepsilon^2\chi^{\varepsilon(\sigma\mu)}\,\varkappa^x_{\sigma\mu}(w)+p, \tag{4.11}$$

where p is a polynomial of degree one. Note that θ^ε violates the boundary conditions: i.e.: $w^\varepsilon = w+\theta^\varepsilon \notin H^2_0(\Omega)$. In order to make this function belong to $H^2_0(\Omega)$ the cut off functions have been introduced, see Eq. (4.1).

The correctness of Eq. (4.10) directly depends on ε-regularity of the functions $\eta_{\alpha\beta\gamma\delta}$. Once the homogenized solution w is given the character of these functions is determined by the virtual deflections. Therefore the approximate character of the formula (4.10) is closely relevant to approximate satisfaction of the virtual work principle (4.7) being now fulfilled only by the trial functions which are taken from a subset of ε-regular functions belonging to the space $H^2_0(\Omega)$ of admissible deflections.

Moreover, our intuitive derivation suggests that Eq. (4.10) is better justified than the formula (4.11), hence improving the moments seems to be more natural than to correct the deflections.

5. Comparison of Duvaut's effective stiffnesses with Kączkowski's formulae in the case of the plate with x^1-periodic thickness

Consider now an isotropic, εY-periodic plate whose thickness $h^\varepsilon(x) = h^\varepsilon(x^1) = h(y^1)$, $y^1 = x^1/\varepsilon$, is a periodic function of x^1 with a period ε; h is a 1-periodic function in y^1; $Y = [0, 1]\times[0, 1]$. The variation of h is such that the plate is symmetric with respect to its mid-surface.

The Young modulus E and Poisson's ratio ν are assumed to be constant. The bending stiffness reads

$$D^\varepsilon(x) = \frac{E}{12(1-\nu^2)}\,h^3(x^1/\varepsilon).$$

The effective stiffnesses $\bar{D}_{\alpha\beta\gamma\delta}$ can be found by means of Duvaut's formulae [6]

$$\bar{D}_{1111} = \frac{Eh_1^3}{12(1-\nu^2)}, \qquad \bar{D}_{1212} = \frac{Eh_2^3}{24(1+\nu)},$$

$$\bar{D}_{1122} = \frac{\nu E h_1^3}{12(1-\nu^2)}, \tag{5.1}$$

$$\bar{D}_{2222} = \frac{Eh_2^3}{12(1-\nu^2)} + \frac{E\nu^2}{12(1-\nu^2)}(h_1^3-h_2^3),$$

where

$$h_1 = \left[\int_0^1 \frac{dy^1}{h^3(y^1)} \right]^{-1/3}, \quad h_2 = \left[\int_0^1 h^3(y^1)\, dy^1 \right]^{1/3}. \tag{5.2}$$

On the other hand the same problem was considered by KĄCZKOWSKI [12]. We shall not recall here this derivation, for details the reader should refer to [12]. It is worth noting that Kączkowski did not examine the behaviour of w^ε when $\varepsilon \to 0$, but he directly showed how to replace a plate with periodic thickness by an effective plate of constant thickness. Taking into account the fact that the classical plate theory is based on Kirchoff's hypotheses which are applicable for plates of slowly varying thickness only, the author of [12] recommends his formulae only for plates whose thickness varies slowly with a small amplitude. The effective stiffnesses derived in [12] read

$$\begin{aligned} &\tilde{D}_{1111} = \bar{D}_{1111}, \quad \tilde{D}_{1122} = \bar{D}_{1122}, \\ &\tilde{D}_{1212} = \bar{D}_{1212} + \frac{E}{24(1+\nu)}(h_1^3 - h_2^3), \\ &\tilde{D}_{2222} = \bar{D}_{2222} - \frac{\nu^2}{1-\nu^2}\frac{E}{12}(h_1^3 - h_2^3). \end{aligned} \tag{5.3}$$

In the case of $h = \text{const}$, $h_1 = h_2$ and $\tilde{\mathbf{D}} = \bar{\mathbf{D}}$. The difference $\tilde{D}_{2222} - \bar{D}_{2222}$ is proportional to ν^2 so that the modification of D_{2222} ought not to be essential. The difference between the torsional stiffnesses $\bar{D}_{1212}$ and $\tilde{D}_{1212}$ seems to be greater. Thus the homogenisation method shows formulae $(5.3)_{3,4}$ to be incorrect. In particular, evaluation of the torsional stiffness can evade our intuition.

6. Concluding remarks

In the last section an interesting fact has come out. On the one hand the effective stiffnesses (5.1) are arrived at as a result of the passage to the limit ($\varepsilon \to 0$) what apparently suggests that the smaller ε, the more accurate should be the formulae (5.1). On the other hand one can apply the classical Kirchhoff's model for plates with nonconstant thickness only when this thickness varies slowly. If not, the equilibrium conditions of some parts of the plate (considered as a three-dimensional body) could be badly violated.

Nevertheless, as it has already been pointed out in Sect. 3, the formulae (2.9) can be obtained without passing to the zero limit with ε. Thus the homogenization procedure being based on the analysis of the sequence w^ε should be treated as a process which exceeds the frames of the Kirchhoff's assumptions and we can regard it as a "transcendental" procedure. In mechanics, at certain stages of the analysis, we are often compelled to handle physical quantities of such magnitudes that they cannot be physically interpreted. None the less, the mathematically confirmed correctness of the considerations as well as the firm physical foundations of the theory ensure the final results to be physically correct provided a return passage to the initial range where the assumptions hold good is carried out. In view of the above in the considered plate problem the formulae (5.1) are valid in the range of slowly varying thickness.

In the case when the latter assumption is not satisfied one should apply another model useful for the considered case, e.g. VEKUA'S theory [13] or a new model due to KOHN and VOGELIUS [10]. However, as to the present author's opinion, the problem of averaging of stiffnesses of plates with rapidly varying thickness is not closed and requires further studies including of mathematical investigations as well as numerical and physical experiments.

ACKNOWLEDGEMENTS

The author expresses his thanks to dr J. J. TELEGA for his encouragement and helpful remarks while this paper was being written. Moreover, the author is indebted to prof. Z. KĄCZKOWSKI for the valuable discussion.

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Streszczenie

NOTA NA TEMAT OBLICZANIA ZASTĘPCZYCH SZTYWNOŚCI CIENKICH PŁYT SPRĘŻYSTYCH O PERIODYCZNEJ STRUKTURZE

W pracy omówiono problem homogenizacji płyt niejednorodnych o strukturze dwuokresowej. Wykazano, że przyjęcie sztywności zastępczych wg wzorów teorii homogenizacji niejest równoważne odpowiednio rozumianej energetycznej równoważności rozważanej płyty niejednorodnej i zastępczej płyty jednorodnej. Ponadto podano wzory definiujące korektor rozwiązania zhomogenizowanego a następnie przedstawiono jego elementarne wyprowadzenie kładąc nacisk na sens fizyczny wywodu. W ostatnim punkcie pracy rozpatrzono szczególny przypadek płyty o wysokości zmiennej w jednym kierunku x^1 i porównano wzory na zastępcze sztywności znalezione przez Duvaut z wynikami Kączkowskiego. Stwierdzono niezgodność w definicjach zastępczych sztywności D_{1212} i D_{2222}.

Резюме

ЗАМЕТКА НА ТЕМУ РАСЧЕТА ЭФФЕКТИВНЫХ ЖЕСТКОСТЕЙ ТОНКИХ УПРУГИХ ПЛИТ С ПЕРИОДИЧЕСКОЙ СТРУКТУРОЙ

В работе обсуждена проблема гомогенизации неоднородных плит с двоякопериодической структурой. Показано, что принятие эффективных жесткостей по формулам теории гомогенизации эквивалентно соответственно понимаемой энергетической эквивалентности рассматриваемой неоднородной плиты и эквивалентной однородной плиты. Кроме этого приведены формулы определяющие корректор гомогенизованного решения, а затем представлен его элементарный вывод, подчеркивая физический смысл вывода. В последнем пункте работы рассмотрен частный случай плиты с переменной высотой в одном направлении x^1 и сравнены формулы для эффективных жесткостей, найденные Дюво, с результатами Кончовского Констатировано несовпадение в определениях эффективных жесткостей D_{1212} и D_{2222}.

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Received June 14, 1985.