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NUMERICAL ANALYSIS OF THE EFFECT OF TORISPHERICAL HEAD ON THE BUCKLING OF PRESSURE VESSEL

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The shape of end heads of a pressure vessel is usually torispherical. Buckling of this head is one of the most important points for designing of pressure vessels. This subject has been studied extensively since last years. In this field, the experimental methods are expensive and need a lot of time. In addition, because of lack of accuracy in the producing procedure, sometimes two models with identical geometry show different buckling behavior. Hence the use of finite element method in analyzing of buckling behavior of heads has a lot of benefits. In this dissertation, the finite element method has been used. Firstly with nonlinear buckling analysis, the effects of geometrical parameters such as thickness, knuckle radius and diameter of cylindrical part, on the buckling of heads have been studied, then the buckling behavior of different kinds of heads with identical geometry have been analyzed. For the nonlinear analysis we used the Arc Length method which can control the load level, the length of the displacement increment and the maximum displacement. The most important characteristic of this method is its ability to converge, even when the behavior is highly nonlinear. From the verification performed with the European Convention for Constructional Steelwork (ECCS) code, it has been confirmed that the nonlinear buckling analysis could assure accurate results for the buckling strength. In the case of internal pressure, it has been shown that initial imperfection had no effect on the pre-buckling behavior and buckling pressure of head; it just affects the post-buckling behavior.

1. Introduction

The theories of thin-walled structures applied on the pressure vessel were reviewed by Teng *et al.* [1]. Their results concerning linear and non-linear theories of thin-walled shells of revolution for numerical evaluation of buckling have been presented.

Regarding to existence of non-continuous stress distribution in cylinder-head intersection, the choice of head considering the geometrical limitation and production facilities is the most important point in designing of a pressure vessel.

Torispherical heads are used commonly in pressure vessels because of their simple manufacturing and good strength in high pressure conditions (Fig. 1). The buckling strength is one of the most important points in the design of pressure vessel [2]. Internal pressurization is often an important loading condition for pressure vessels. Finite element method is often used in the buckling analysis of pressure vessels due to its capability.

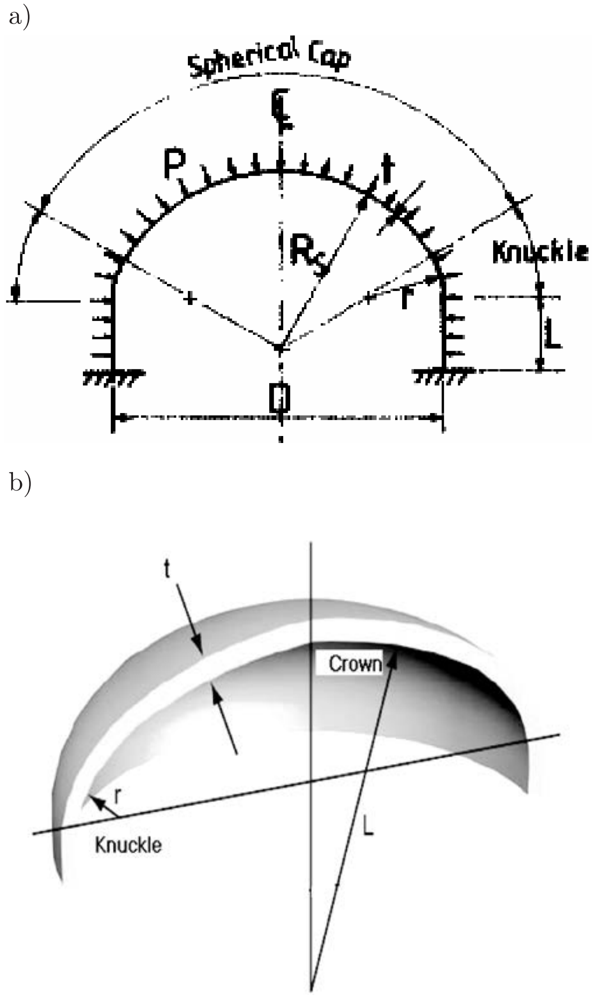

Fig. 1. Geometry of torispherical head. a) 2D-view with parameters and boundary conditions, b) view a the half-crown.

The hydrostatic buckling of shells under different boundary conditions (B.C.) has been investigated using the energy method [3]. Results have shown that in shells with medium height under different B.C., buckling load is obtained by applying a scalar coefficient to the buckling load of the pin-ended case, but this method is not applicable to the long shells, for which the circumferential waves occurred from buckling are higher than 3.

Teng *et al.* [4] have introduced a numerical model, aided by the method of eigenmode-affine, in the non-linear analysis of elastic shells. As the shells are sensitive to the initial geometric imperfections, predicting of their buckling resistance would be precise if those imperfections were taken into account.

In the torispherical heads, by increasing the ratio of knuckle radius per vessel diameter (r/D), dimension of the spherical part decreases. Thus, the spherical part as a part of the head becomes weaker and in a defined r/D, a notable fall in buckling resistance occurs [5].

European recommendation ECCS [6] introduced several experimental relations for design of spherical shells. In a recent analysis, Wanderlich [7] analyzed the buckling behavior of spherical shells under external pressure.

Here we will discuss the buckling load and influence of different parameters on it. We will try to suggest some propositions for limitation of buckling.

2. Numerical simulation methods

Buckling analysis can be carried out by numerical methods such as eigenvalue buckling analysis or non-linear buckling analysis, using the finite element approach. Eigenvalue buckling analysis predicts the theoretical buckling strength (the bifurcation point) of an ideal linear elastic structure. Bifurcation buckling refers to the unbounded growth of a new deformation pattern. Imperfection and material non-linearities can not be included in this analysis. Thus, the buckling strength obtained by eigenvalue buckling analysis may differ from that of a real structure and often yields unconservative results. Therefore, care is needed when using this method in actual evaluation of buckling strength.

Non-linear buckling analysis, including geometric and material non-linearities, is usually the more accurate approach and is therefore recommended for design or evaluation of actual structures. There are two methods for obtaining buckling strength by means of non-linear buckling analysis. One basic approach is to constantly increase the applied loads until the solution begins to diverge, which can be obtained by means of the load-controlled buckling analysis. Using this approach, a simple static analysis will be done, with large deflection extended to a point where the structure reaches its limit load. Another approach is to constantly increase the displacement to obtain the snap-through buckling curve shown in Fig. 2. Increasing of the displacement can be obtained from displacement-controlled buckling analysis. In non-linear buckling analysis, a sufficiently small load or displacement increment should be used to obtain the expected buckling strength.

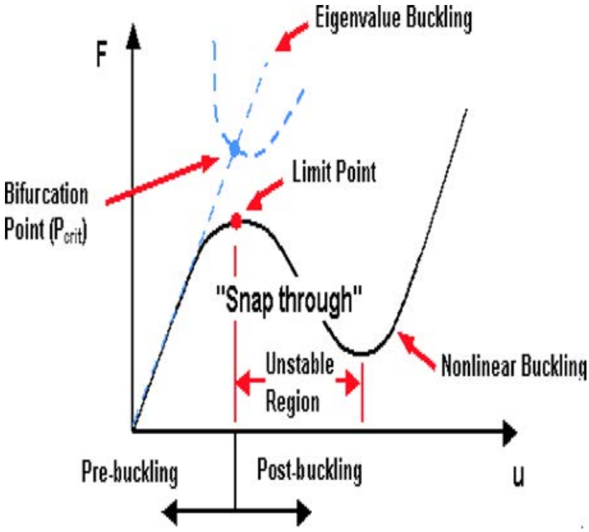

FIG. 2. Curve of non-linear buckling behaviour [8].

3. Modeling

Non-linear finite element method with large deflection analysis was performed using the commercial Ansys software. A three-dimensional finite element model was generated using Ansys 9.0 as shown in Fig. 3. To studying the buckling of pressure vessel with torispherical head, we modeled the intersection of cylinder-head. The influence of welding and forming on a material property were neglected while the effect of welding can be accounted for by modifying the yield stresses. The length of the cylinder was kept at 4λ (λ is the linear elastic meridional bending half-wave length given by $2.44\sqrt{Rt}$), to ensure that the boundary effects at the far end of the cylinder do not interfere with the behavior of the intersection [9]. The model was meshed by means of SHELL93 element. SHELL93 is particularly well-suited to model curved shells. The element has six degrees of freedom at each node: translations in the nodal x, y, and z-directions and rotations about the nodal x, y and z-axes. The element has the properties of plasticity, stress stiffening, large deflection and large strain capabilities. The material of the intersections was assumed to have typical properties of steel: an elastic modulus of 1.9×10^5 MPa; Poisson's ratio of 0.26, and yield stress of 206 MPa, and exhibits an elastic-perfectly plastic behavior.

For modeling of the geometrical imperfections in ANSYS package, we applied them in the form of initial deformations on the model [8, 10, 11]. For this reason, first we analyzed the model by using the linear method of buckling and then, by using the "Update Geom" order, we assumed the values of the magnification

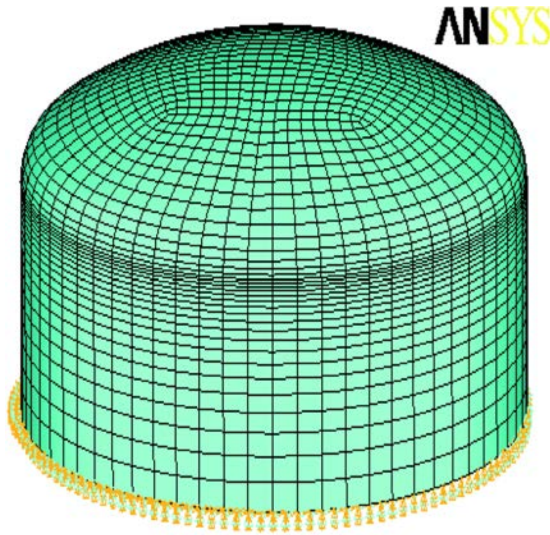

Fig. 3. Finite element model before buckling.

factor. In fact, by the resulting displacement of different buckling resolutions, a new model with geometrical imperfection was obtained. This factor of geometrical imperfection, which is in fact a deviation from the perfect model or the initial deformation, was presented by W_0. The buckled model is illustrated in Fig. 4.

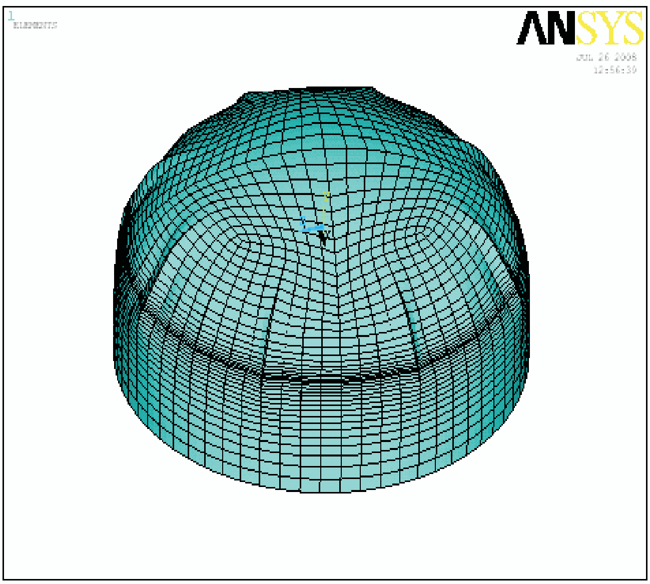

Fig. 4. Finite element model after buckling.

4. Choice of the resolving method

For the solution of a nonlinear problem, the choice of solution method and load step (referred to as the time step in the ANSYS software), is very important. It should take into account the anticipated structural behavior and the characteristics of the specific solution method.

Prior to carrying out a nonlinear buckling analysis, it is often beneficial to undertake a linear (eigenvalue) buckling analysis, in order to obtain some appreciation of the buckling behavior. It may also help to identify those regions in the model that will first exhibit nonlinear response, and at what load levels these nonlinearities will develop.

There are several methods available in ANSYS for the solution of the nonlinear buckling equations. They include the Newton–Raphson and Arc Length methods. For geometrically nonlinear analysis, the Newton–Raphson method has been shown to be one of the best methods available. The most important characteristic of this method is its ability to converge even when the behavior is highly nonlinear. The method we start with is also extremely accurate and generally converges quite rapidly, provided a realistic initial estimate of the displacement vector. With this method it is also possible to control the solution error and estimate the rate of convergence, since for any particular load step, the iterations continue until the specified solution error is achieved. Preliminary ANSYS FE analyses of the columns, in which compression loads were applied, showed that the Newton–Raphson method converged quite rapidly. For the nonlinear buckling analysis, coarse time steps may be used in the pre-buckling regime, but fine steps are required close to the buckling load and in the post-buckling regime. Different time steps may be used in the pre- and post-buckling regimes through the multiple 'load steps' option within ANSYS. However, it is not easy to choose the appropriate maximum load level in load-controlled analysis using the Newton–Raphson method.

Moreover, the Newton–Raphson method fails when a snap-through occurs. The Arc Length method does not have this drawback and allows one to control the load level, the length of the displacement increment and the maximum displacement. Therefore, in the nonlinear buckling analyses, the Arc Length method was used [12].

5. Determination of the buckling pressure

The Southwell plot technique is usually an effective approach in determining of the buckling load of the corresponding perfect structure [11]. This method was also adopted in this study, but unfortunately, it was not found to be applicable to our problem. The reason is probably that in our research, the load-displacement curves did not have generally the rectangular hyperbolic nature, which is basically necessary for application of the Southwell method. In our study we used a similar method as Theng's study on the cone-cylinder intersection [9]. In this approach, the curves of load-displacement for nodes in one circumferential path near the cylinder-head intersection were plotted. In the initial stage of loading, the curves for all nodes were similar, indicating a dominantly axi-symmetric behavior. As the pressure reached a certain value, the curves of nodes at different locations started to diverge from each other. The divergence of these curves is an indication of the growth of non-symmetric buckling deformations. The load corresponding to the divergence point is the critical buckling load (Fig. 5).

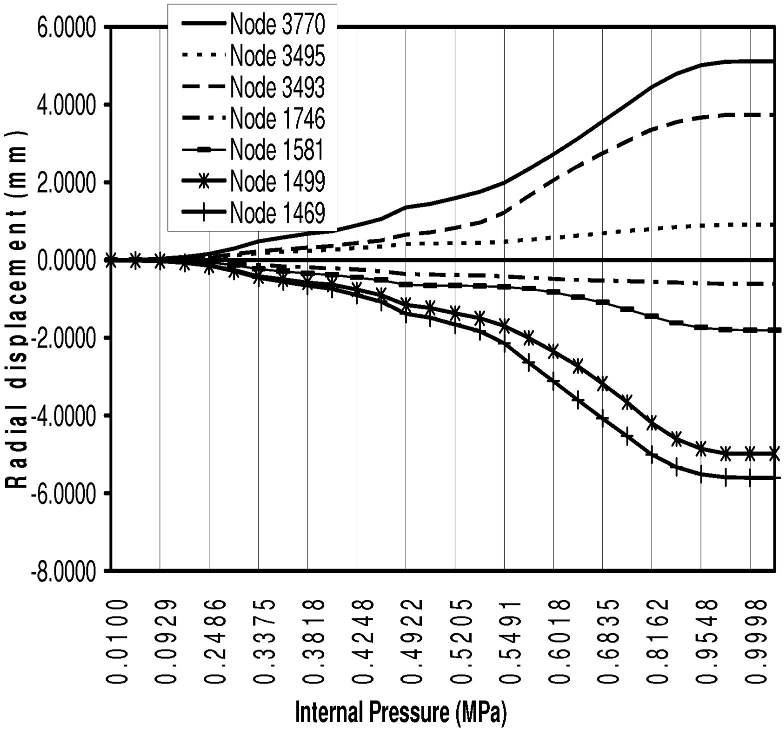

Fig. 5. Determination of buckling pressure (curve of load-radial displacement for torispherical head; $t/L = 0.002$, $r/L = 0.06$).

6. Parametric study

The geometry of torispherical head was introduced with the t/L, L/D and r/L parameters, in which t is the thickness of vessel which has identical values in heads and cylindrical part, L is the radius of spherical section, r is the knuckle radius and D is the diameter of cylindrical section. The common heads used for pressure vessels which have radii of sphere equal to diameter of the cylinder ($L/D = 1$). Our study was limited to heads with

$$t/L \leq 500 \qquad \text{and} \qquad r/L \geq 0.06.$$

The result was compared with the design rules given in European Convention for Constructional Steelwork (ECCS) [6]. The ECCS rules are based on buckling of the knuckle and the limit pressure is as given in Eq. (6.1):

$$\frac{P_b}{F_y} = \frac{120c\left(\frac{r}{D}\right)^{0.825}}{(D/t)^{1.5}\left(\frac{L}{D}\right)^{1.15}}, \tag{6.1}$$

where $c = 1.0$ for crown and segment steel heads and $c = 1.6$ for cold-spun steel heads.

By verification performed with the ECCS code, as Table 1, Fig. 6 and Fig. 7 illustrate, it was confirmed that the nonlinear buckling analysis could assure accurate results for buckling strength. The discrepancy between the numerical analysis (FE) and ECCS [6] correspond to the geometrical imperfection and residual stress, which were taken into account by the ECCS code and not by the FE.

Table 1. Buckling pressure of vessel with torispherical head ($L/D = 1$).

t/L	r/L	P_{Ansys} (MPa)	P_{ECCS} (MPa)	Error %
0.002	0.06	0.20872	0.179	16
	0.08	0.22807	0.276	9
	0.1	0.34149	0.332	2.8
	0.14	0.43883	0.438	0.09
	0.17	0.48337	0.514	5.9
	0.2	0.542663	0.588	7
0.003	0.06	0.397296	0.400	0.67
	0.08	0.46551	0.507	8
	0.1	0.53235	0.610	12.7
	0.14	0.8196	0.805	1.7
	0.17	0.95637	0.944	1.2
	0.2	1.058	1.08	1.9

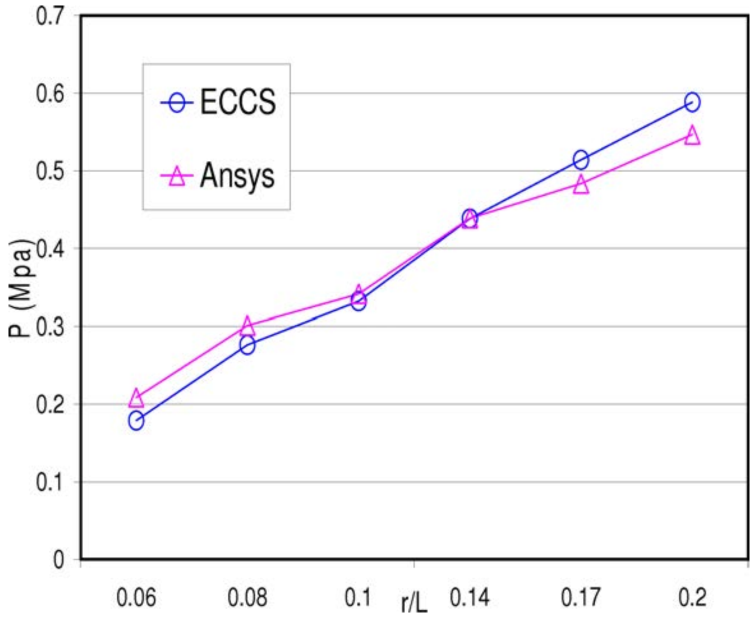

FIG. 6. Curve of critical pressure versus r/L ($t/L = 0.002$).

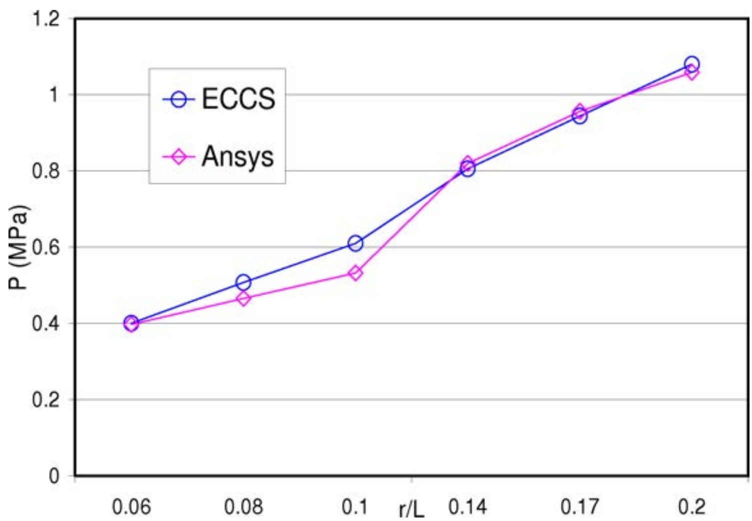

FIG. 7. Curve of critical pressure versus r/L ($t/L = 0.003$).

6.1. Influence of knuckle radius on buckling pressure

In the internal pressure vessels, due to the existence of circumferential tensile stresses in both the cylindrical and spherical parts, the intersection is deformed

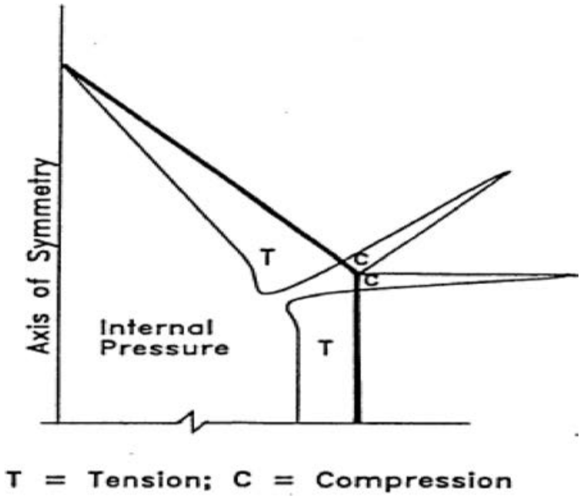

FIG. 8. Circumferential membrane stress in intersection [13].

to the internal side. Thus, both of the spherical and cylindrical parts near the intersection, as illustrated in Fig. 8, was subjected to circumferential compressive stresses, and so the buckling deflection occurred in both of them.

Figure 9 shows the buckling modes predicted by finite element analysis for the sphere-cylinder intersection with $t/L = 0.002$ and $r/L = 0.06$. These deformations are periodic around the circumference.

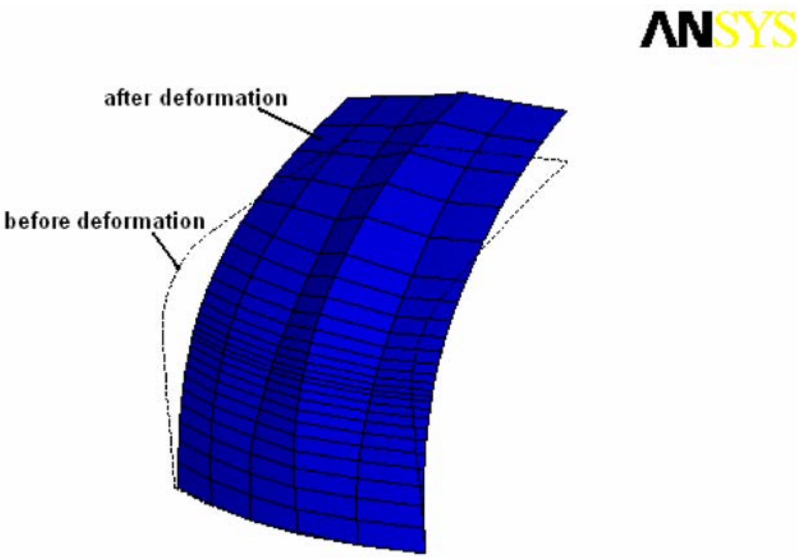

FIG. 9. Buckling modes.

The growths of buckles can now be clearly seen in Fig. 10. The number of periodic waves on the ring can be counted from this plot to be 39. It should be noted that this counted number is only a rough indication, as the buckling waves are not so uniform.

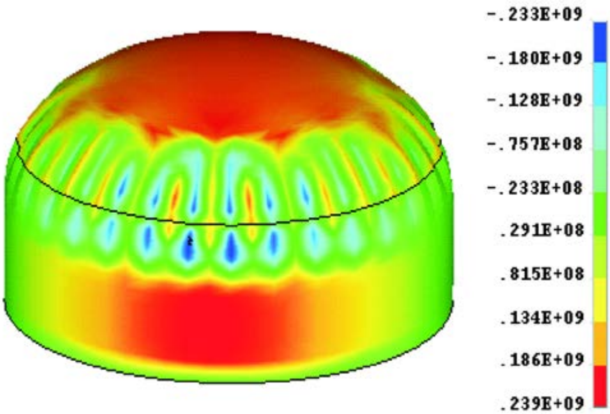

FIG. 10. Influence of knuckle radius ($t/L = 0.002$ and $r/L = 0.06$).

Numerical results for all the points of intersection between the head and cylinder have shown clearly that the post-buckling behavior of internally pressurized sphere–cylinder intersections is stable (Fig. 10).

The curves of Fig. 11 show the influence of knuckle radius on the pressure buckling with different thicknesses. In the analysis, the value of the radius of

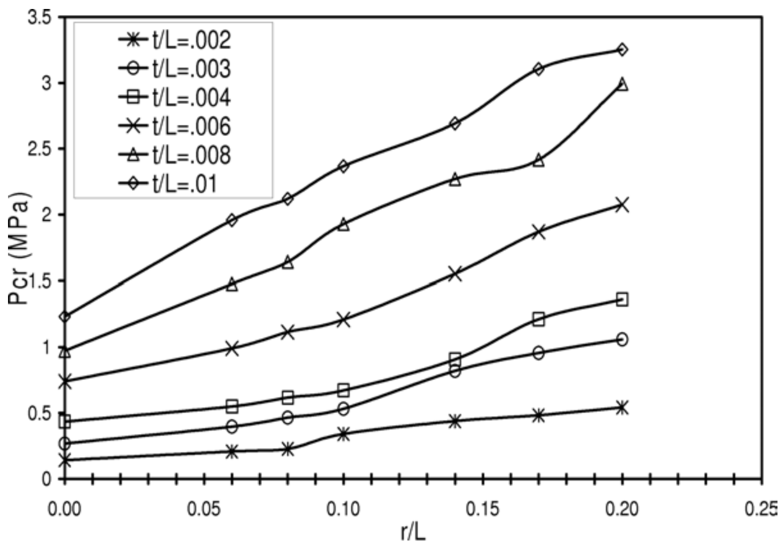

FIG. 11. Influence of the knuckle radius on the buckling behavior of the torispherical head.

spherical part (L) was kept constant ($L = 0.5$ m). When varying the ratio r/L, only the knuckle radius (r) was varied. We observe that for all the thicknesses, increasing of a radius leads to increasing of the buckling pressure. So the knuckle radius is an influential parameter for increasing of buckling resistance.

6.2. Influence of the thickness on the buckling pressure

Figure 12 illustrates the influence of t/L on the buckling pressure for different ratios of r/L. Increasing of t/L leads to increasing of the buckling pressure. The rate of increase, compared to increasing resulting from the ratio r/L, is higher. The slope of the curves of Fig. 12 as compared to those of the Fig. 11 shows it. Thus the buckling pressure is more sensitive to the thickness.

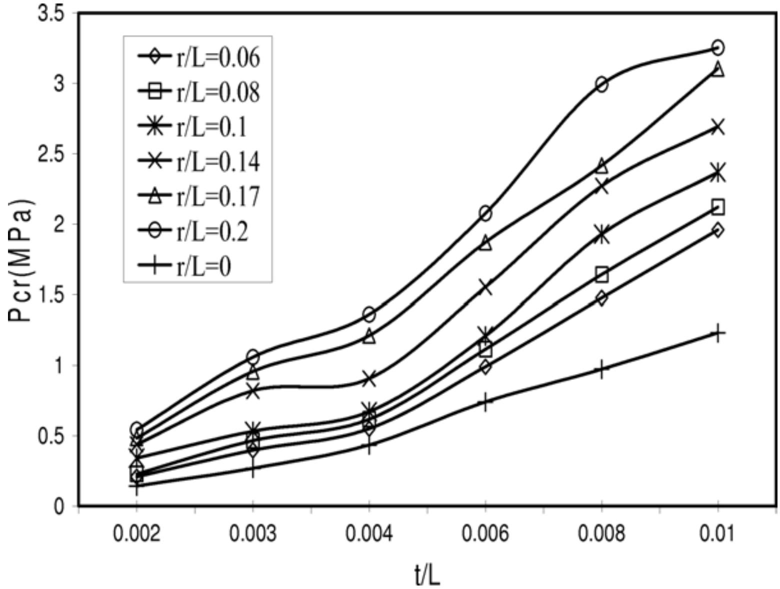

FIG. 12. Influence of t/L on the buckling pressure of the torispherical head.

6.3. Influence of the radius of the spherical part on the pressure buckling

In torispherical heads, usually the radius of sphere is the same as the cylinder ($L/D = 1$). Buckling of this kind of heads is discussed empirically. For the heads with different radii mentioned, result of FE for different ratios of L/D is shown in Fig. 13. The curve shows that by increasing of the radius of the spherical part, the buckling pressure decreases. So by decreasing of the curvature of the crown, the buckling pressure of the vessel decreases. In the same way, buckling pressure of the vessels with a plate head is much lower than of the vessels with a clear spherical head.

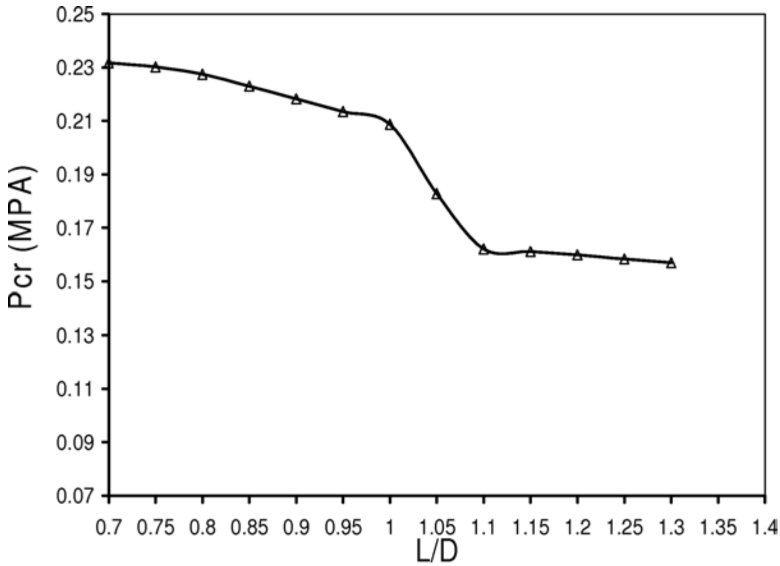

FIG. 13. Curve of the buckling pressure versus L/D for torispherical head.

While these observations may suggest that the direct use of bifurcation loads in design may be somewhat unconservative, a closer examination of the results has revealed that for many of these intersections, the plastic limit load of Eq. (8.1) is lower than the one-third stiffness load, or both the one-third and half-stiffness loads. This indicates that the strengths of these intersections are controlled by the plastic limit load, and the fact that the bifurcation load exceeds the stiffness-based buckling loads is not of a real concern. The direct use of the bifurcation loads in design is therefore generally safe.

7. Choice of the equivalent geometric imperfections

For a numerical buckling analysis it is essential, that well comparable and most general applicable equivalent imperfections should be used. To achieve this, the shape and the amplitude of the applied equivalent geometric imperfections have to reflect the effect of the really existing initial imperfections. But currently, there are no sufficient guidelines for the imperfection parameters of a numerical analysis. In [5] it is only stated, that the pattern of the equivalent geometric imperfection shall be chosen in such a form which leads to the most unfavorable effect in the buckling behavior of the shell. The 'worst' imperfection is not specified in detail and presently, no practically verified theory exists, which could indicate it. If the instability problem is caused by geometrical nonlinearity and the prebuckling behavior is almost linear, the eigenmode-affine imperfection patterns, which are derived from the classical linear eigenvalues analysis, are applicable. Besides the fact that these imperfections are unrealistic, we use eigen-affine imperfection in our study.

8. Results and discussion

8.1. Analysis of development of the circumferential wrinkles

Due to the buckling, circumferential wrinkles are developed. The number and amplitude of theses wrinkles increase by increasing of the internal pressure. The number and amplitude of the developed wrinkles are a criterion to evaluate the buckling resistance [14]. Creation and development of wrinkles could be followed by a curve of radial displacement versus the distance in a circumferential path, in the vicinity of intersection of the cylinder and head of the vessel (Fig. 14). Although by accounting of the number of wrinkles in this curve, the number of circumferential wrinkles in the head could be obtained.

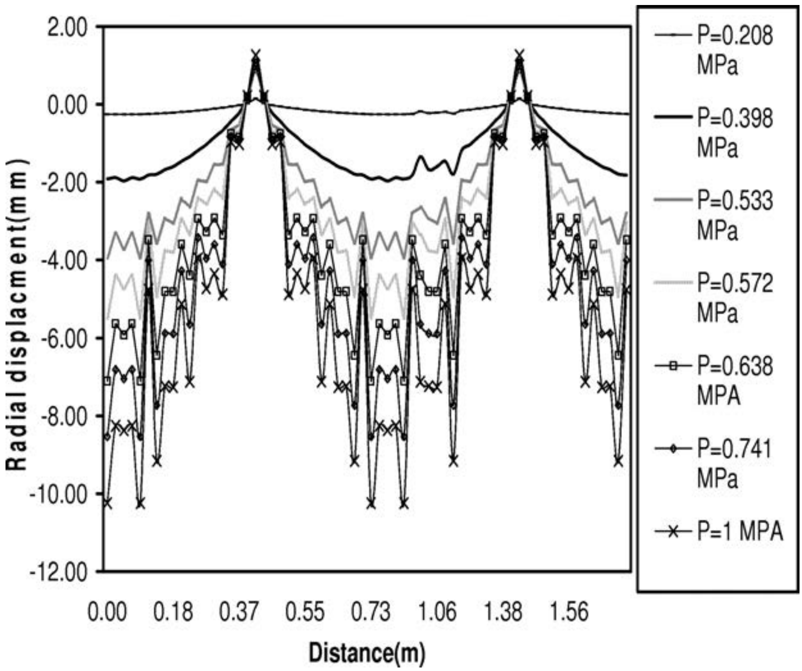

FIG. 14. Development of buckling wrinkle $r/L = 0.06$ and $t/L = 0.002$.

8.2. Comparison of buckling pressure with limit pressure

The limit pressure of the vessel (P_L) with torispherical head is computed using Eq. (8.1) [15] derived by SHIELD and DRUCKER [16]:

$$\frac{P_{SD}}{F_y} = \left(0.33 + 5.5\frac{r}{D}\right)\left(\frac{t}{L}\right) + 28\left(1 - 2.2\frac{r}{D}\right)\left(\frac{t}{L}\right)^2 - 0.0006, \tag{8.1}$$

in which P_{SD} is the limit pressure (psi), $P_{SD} = P_L$, F_y is the yield stress (ksi).

Figure 15 shows the critical buckling pressure (P_{cr}) versus the pressure obtained from Eq. (8.1). The curve shows that the buckling pressure is most often higher than the limit pressure. It means that the limit pressure is more critical than the buckling pressure. Thus for the vessels with torispherical head, the buckling pressure as a design criterion will not be sufficient.

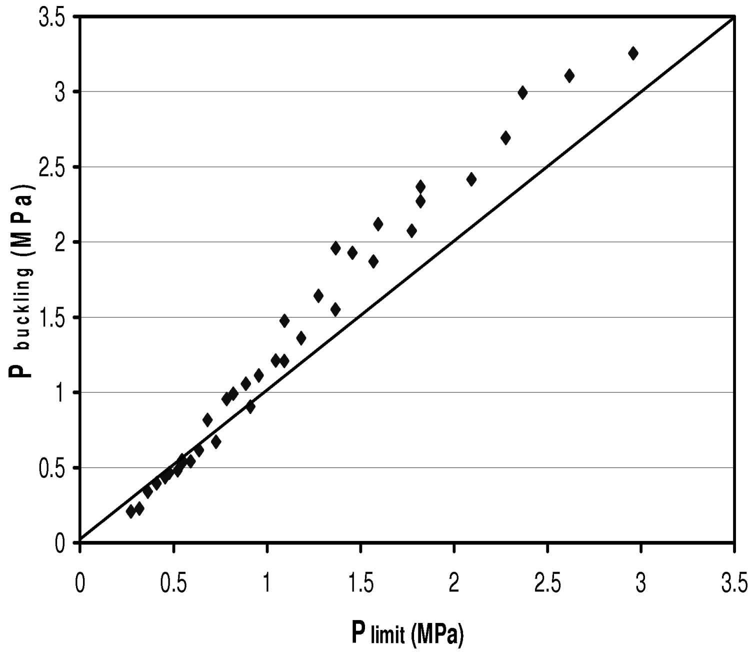

Fig. 15. Curve of buckling pressure versus limit pressure.

8.3. Influence of initial geometrical imperfection on buckling behavior

The analysis of imperfection sensitivity was performed with the initial imperfection values of $W_0/t = 1$, $W_0/t = 0.2$ and $W_0/t = 0.5$. The perfect model had $t/L = 0.002$ and $r/L = 0.06$. The buckling loads of all imperfect models, as illustrated in Fig. 16, were equal to the buckling load of the perfect model and had $P_{\mathrm{cr}} = 0.2087$ MPa. But the post-buckling behavior of imperfect models was different from post-buckling of the perfect model. The nearly perfect model ($W_0 = 0.2$) had the same value as the buckling load and the post-buckling behavior was such a perfect model. For the most imperfect model ($W_0 = 1$), the post-buckling behavior had the most important deviation from the perfect model.

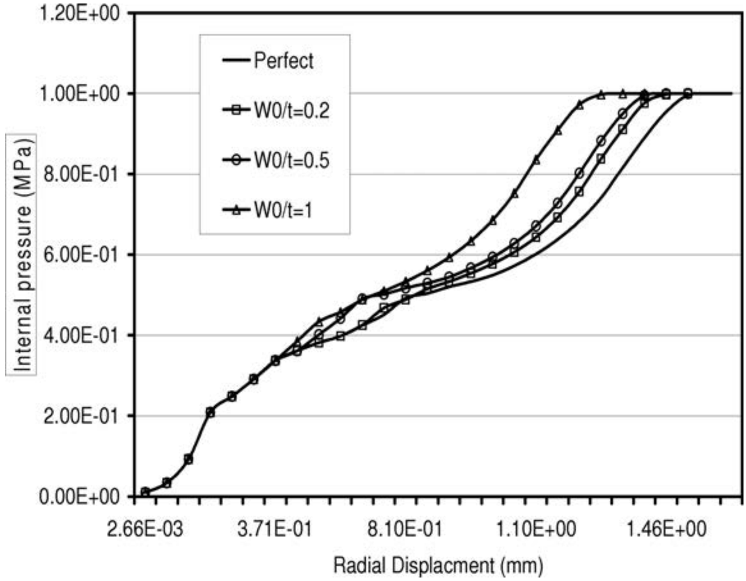

FIG. 16. Influence of imperfections on the buckling behavior.

The buckling pressure of all imperfect models being 0.208 MPa, we can conclude that imperfections do not influence the critical buckling pressure and pre-buckling behavior of the vessel. Only the post-buckling behavior is influenced by imperfections. Thus the buckling behavior of pressure vessels with torispherical head, because of their stable behavior, is not sensitive to initial geometrical imperfections.

9. Conclusion

- The non-linear FE analysis brings the numerical results in vicinity of the experimental ones. Scatters are generally due to the geometrical imperfections and residual stresses in a vessel.
- Buckling pressure is influenced by the thickness and height of the vessel. Larger thickness and height lead to a better buckling resistance.
- The influence of knuckle radius in decreasing of the compression stresses and also increasing of the buckling pressure is evident.
- In the case of uniform internal pressure, the initial geometrical imperfections have a small influence on the pre-buckling behavior and buckling load of torispherical heads. Their influence is considerable on its post-buckling behavior.
- Using of the buckling pressure criterion in design of torispherical heads would be conservative.

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