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CREEP-DAMAGE IN PRESSURE COMPONENTS EXPERIENCING FOLLOW-UP

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A simple method for estimating creep-damage in pressure components experiencing elastic or creep follow-up is presented. The theoretical formulation essentially relates the multiaxial relaxation process that exhibits follow-up to the traditional uniaxial stress-relaxation model. The resulting mixed-mode response is expressed in the generalized local stress-strain plot. The follow-up parameters are then determined by utilizing the slope of the relaxation-response in the generalized local stress-strain plot. Finally, a procedure for partitioning the accumulated follow-up damage into load-controlled and deformation-controlled contributions is discussed. The method is applied to a typical elevated-temperature piping system configuration that exhibits follow-up potential.

NOMENCLATURES

B, n creep-parameters for second-stage creep,
E_0 modulus of elasticity of the local system,
E_r modulus of elasticity of the remaining system,
E_1 secant modulus of the local system,
$\bar{E}_r$ normalized modulus of the remaining system,
D life-usage or damage fraction during creep-relaxation,
D_f life-usage or damage fraction during relaxation accompanied by follow-up,
D_p life-usage or damage fraction under the action of primary stresses,
D_q life-usage or damage fraction during deformation-controlled relaxation,
D_m life-usage or damage fraction during a purely load-controlled situation,
L time to rupture for a given magnitude of stress,
S^* pseudo-elastic secondary stress allowable,
S_m allowable stress for primary membrane loads,
p, q rupture parameters,
β elastic follow-up parameter,
ϕ creep follow-up parameter,
γ generalized-constraint parameter,
τ^* time-scale for relaxation with follow-up,
τ time-scale for deformation-controlled relaxation,
θ the angle included between downward vertical and the mixed-mode response trajectory, measured anticlockwise,

δ_t total system deflection,
δ_t^0 local system deflection,
δ_t^r remaining system deflection,
ε_t total system strain,
ε_t^0 local system strain,
ε_t^r remaining system strain,
ε_e^0 elastic local system strain,
ε_e^r elastic remaining system strain,
ε_p^0 plastic local system strain,
ε_0^r plastic remaining system strain,
ε_c^0 creep local system strain,
ε_c^r creep remaining system strain,
σ stress,
σ_i initial stress before relaxation,
σ_y yield stress,
$\xi_{f,l}$ load-controlled fraction of the creep-damage incurred during mixed-mode response,
$\xi_{f,s}$ deformation-controlled fraction of the creep-damage incurred during mixed-mode response.

1. Introduction

The current "design by analysis" methods outlined in several pressure vessel codes, reference [1] for instance, basically ensure that suitable margins of safety with respect to the potential failure modes are incorporated. A hierarchy of failure risks inherent in pressure components is explicitly recognized and then taken into consideration during the design process. These design methods, which are predominantly based on "linear elastic analysis", are attractive to a designer since the methods are direct and noniterative. The elastic analysis presumes that the constitutive relationship is linear and reversible. Most pressure component materials are, however, quite ductile and can therefore withstand considerable plastic deformation. Since linear elastic analysis cannot by itself account for the inelastic and nonlinear effects due to plasticity and creep, there is a need to categorize loads and stresses so that the inelastic effects are recognized [1].

The basic approach is to decompose the stresses (calculated on an elastic basis) into several parts — primary, secondary and peak stresses, and appropriate limits are then prescribed. The underlying rationale for the classification of stresses in a component is based on well-established bounding theorems of limit analysis and shakedown [2]. The rules governing the decomposition of stresses, while certainly useful, are neither very rigorous nor precise. Whereas the stress-decomposition procedure is reasonably well-documented for axisymmetric thinshell structures, it is not routine or well understood for general geometric configurations. The entire topic of stress-classification is an area of active research [3].

The distinction between primary and secondary stresses are based on the recognition of whether they are caused by load-controlled or deformation-controlled actions. However, when both actions are present, the classification process is not well-understood. This case of "mixed-mode response" is commonly referred to as "follow-up". In this paper, a theory is presented that would enable both creep-damage assessment and stress-classification in elevated temperature pressure components experiencing elastic or creep follow-up actions.

This paper essentially relates the multiaxial relaxation process (that includes follow-up) to the traditional uniaxial stress-relaxation model through the introduction of follow-up parameters. Subsequently, a method of partitioning the accumulated follow-up damage into load-controlled and deformation-controlled contributions is discussed. The method is applied to a typical piping system configuration that exhibits follow-up potential.

2. Elastic and creep follow-up

Deformation-controlled stresses are usually classified as secondary on the basis that they are "self-limiting", and that they would eventually shakedown to elastic action after a few cycles of load-application. At temperatures below the creep-range, secondary stresses are limited to twice the yield stress. In the creep range, however, deformation-controlled stresses relax with time. During the traditional relaxation process, the total strain is held constant, and the creep-strain essentially replaces the elastic strains. During relaxation accompanied by follow-up, an increase in creep strain does not result in an equal reduction in elastic strain. Therefore, the stresses would relax at a rate slower than the traditional relaxation, and induce larger creep strains. In the extreme case, the stresses might not relax at all. In any event, follow-up action induces more creep-damage during a specified period of relaxation.

Robinson [4] introduced the concept of follow-up to emphasize the importance of inelastic strain-concentration in piping systems operating at elevated temperatures. Several other authors [5, 6] have provided useful techniques to estimate follow-up in pressure vessels and piping systems.

Two types of follow-up action can occur in elevated-temperature components:

1) *Elastic follow-up*: This type of action occurs when the less stressed parts of the component act as a "spring" on the highly stressed parts, resulting in the accumulation of excessive creep deformations. In other words, the elastic strain-energy of the bulk of the structure maintains the high magnitude of stress in "localized" areas that are prone to strain-concentration.

2) *Creep follow-up*: For this type of action, the deformation-controlled stresses do not decrease to the same extent as in traditional relaxation.

Continued creep deformation of the bulk of the structure maintains a high stress in localized areas that are prone to strain-concentration.

Some typical examples where follow-up action might occur are: (a) local reduction in the size of a cross-secton, or the use of a relatively weaker material; and (b) piping system of a uniform size that has a configuration for which most of the piping lies near the imaginary line joining the two anchors, with a small portion (loops) departing from this line. The small portion of the pipe then absorbs most of the expansion stress.

Various codes stipulate that elevated-temperature components that exhibit significant follow-up action be evaluated as load-controlled. However, it is not specified how follow-up should be estimated. Consequently, in the absence of appropriate follow-up damage estimation methods, stresses are classified as primary, and this can result in an unduly conservative design relatively expensive component construction. In this subsequent sections of this paper, a theory is developed for the purpose of creep-damage assessment in the presence of follow-up, and a technique for classifying stresses is discussed.

3. Theoretical considerations

3.1. A simple method for stress-relaxation with follow-up

A one-dimensional model depicting the follow-up behaviour in pressure components is described here. Superscript 0 refers to the local part of the system where the inelastic strain-concentration occurs, and the superscript r refers to the remainder of the system. When dealing with three-dimensional systems, equivalent stresses and strains are used in conjunction with the one-dimensional model.

If end A (Fig. 1) is displaced by an amount $\delta_t(\tau)$, then

$$\delta_t(\tau) = \delta_t^0 + \delta_t^r. \tag{3.1}$$

In terms of total strains,

$$\varepsilon_t(\tau) = \varepsilon_t^0 + \varepsilon_t^r. \tag{3.2}$$

The strains are "average" quantities which depend on the size of the local system. The strains in the remainder system are defined in terms of the local system (see Eq. (3.6)).

The entire system is divided into the "local" system and the "remaining" system. The local system essentially experiences large inelastic strain-concentration. If the total strain for the local system is considered to be made up of the elastic and inelastic components, then

$$\varepsilon_t^0 = \varepsilon_e^0 + \varepsilon_p^0 + \varepsilon_c^0. \tag{3.3}$$

The total strain for the entire system can be written as

$$\varepsilon_t = \varepsilon_e^0 + \varepsilon_e^r + \varepsilon_p^0 + \varepsilon_p^r + \varepsilon_c^0 + \varepsilon_c^r. \tag{3.4}$$

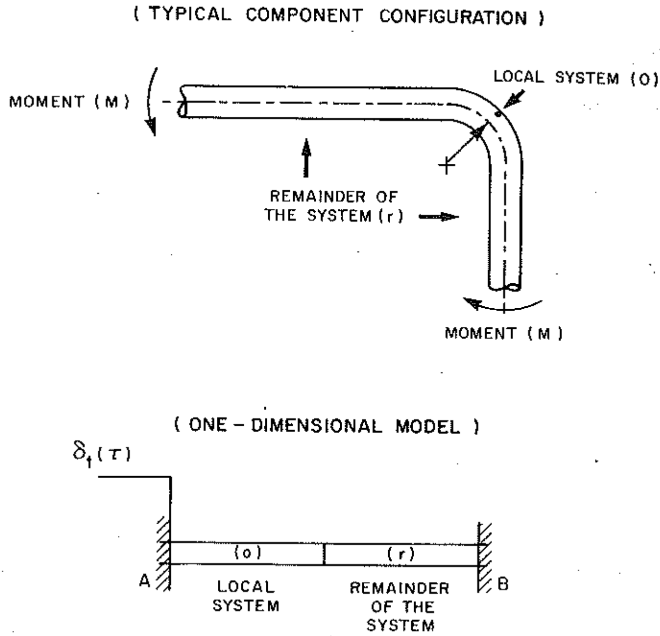

FIG. 1. One-dimensional follow-up model for pressure components.

Making the assumption that the plastic strains do not change with time during relaxation follow-up process,

(3.5) $$\dot{\varepsilon}_t = \dot{\varepsilon}_e^0 + \dot{\varepsilon}_c^0 + \dot{\varepsilon}_e^r + \dot{\varepsilon}_c^r.$$

In Eq. (3.5), $\dot{\varepsilon}$ refers to the strain-rate $\left(\frac{d\varepsilon}{d\tau}\right)$.

The following ratios can now be defined:

(3.6) $$\beta = \frac{\varepsilon_e^r}{\varepsilon_e^0}; \qquad \phi = \frac{\dot{\varepsilon}_c^r}{\dot{\varepsilon}_c^0}.$$

β and ϕ are parameters that account for elastic and creep follow-up, respectively. Theorem, Eq. (3.5) becomes

(3.7) $$\dot{\varepsilon}_t = (1+\beta)\dot{\varepsilon}_e^0 + (1+\phi)\dot{\varepsilon}_c^0.$$

Now if it is further assumed that the steady-state creep equation is given by $\dot{\varepsilon} = B\sigma^n$, where B and n are constants that depend on the material and temperature, then the decay of the initial stresses with reference to the overall system is obtained by setting $\dot{\varepsilon}_t = 0$ in Eq. (3.7).

Therefore,

(3.8) $$\frac{d\sigma}{d\tau} + \left(\frac{1+\phi}{1+\beta}\right) BE_0 \sigma^n = 0.$$

By defining $\tau^* = \left(\frac{1+\phi}{1+\beta}\right)\tau$, and substituting into Eq. (3.8), the traditional uniaxial stress-relaxation model would result, i.e.,

$$\frac{d\sigma}{d\tau^*} + BE_0\sigma^n = 0. \tag{3.9}$$

In other words, the time taken for a given initial stress to decay to a prescribed value would be $\frac{1+\beta}{1+\phi}$ times traditional uniaxial relaxation time, i.e.,

$$\tau = \frac{1+\beta}{1+\phi}\left[\frac{1}{BE_0(n-1)}\left(\frac{1}{\sigma^{n-1}} - \frac{1}{\sigma_i^{n-1}}\right)\right]. \tag{3.10}$$

Once β and ϕ are determined for an elevated temperature component, then Eq. (3.10) can be used directly to evaluate the relaxation of stresses in a component or a system that exhibits follow-up. This result has been reported by Robinson [4] in connecton with the analysis of bolted flanges without gaskets.

4. Mixed-mode response in the local system

Relaxation of stress with time in a component that is subjected to follow-up action is useful information for the assessment of creep damage and stress-classification. More specifically, the determination of β and ϕ would enable quantification of relaxation rates and damage fractions. Several authors [5, 6] have studied the mixed-mode response (corresponding to both load-controlled and deformation-controlled contributions) in the so-called "generalized structural stress-strain plots" which are essentially the normalized stress versus normalized strain plots in the local system. The mixed-mode response pertaining to follow-up action is obtained by determining the relaxation trajectory in the local stress-strain plot (Fig. 2). The mixed-mode "relaxation-modulus", E_r, can be defined as, E_r,

$$E_r = \frac{d\sigma}{d\varepsilon_t^0}. \tag{4.1}$$

Setting $\dot{\varepsilon}_t = 0$ in Eq. (3.7), the relaxation of stress (that includes follow-up) can be obtained. Therefore,

$$\dot{\varepsilon}_t^0 = -\beta\dot{\varepsilon}_e^0 - \phi\dot{\varepsilon}_c^0. \tag{4.2}$$

Since $\varepsilon_e^0 = \dfrac{\sigma}{E_0}$ and $\dot{\varepsilon}_c^0 = B\sigma^n$, Eq. (4.2) can be written as

$$\dot{\varepsilon}_t^0 = -\frac{\beta}{E_0}\frac{d\sigma}{d\tau} - \phi B\sigma^n. \tag{4.3}$$

Equation (4.3) can be expressed as

$$\sigma = f(\varepsilon_t^0; \tau), \tag{4.4}$$

where $f(\ )$ is a general function.

Therefore,

$$\frac{d\sigma}{d\tau} = \dot{\varepsilon}_t^0 \left\{\frac{d\sigma}{d\varepsilon_t^0}\right\}. \tag{4.5}$$

Using Eqs. (4.1), (4.3) and (4.5), and simplifying,

$$\frac{d\sigma}{d\tau} = -E_r\left\{\frac{\beta}{E_0}\cdot\frac{d\sigma}{d\tau} + \phi B\sigma^n\right\}. \tag{4.6}$$

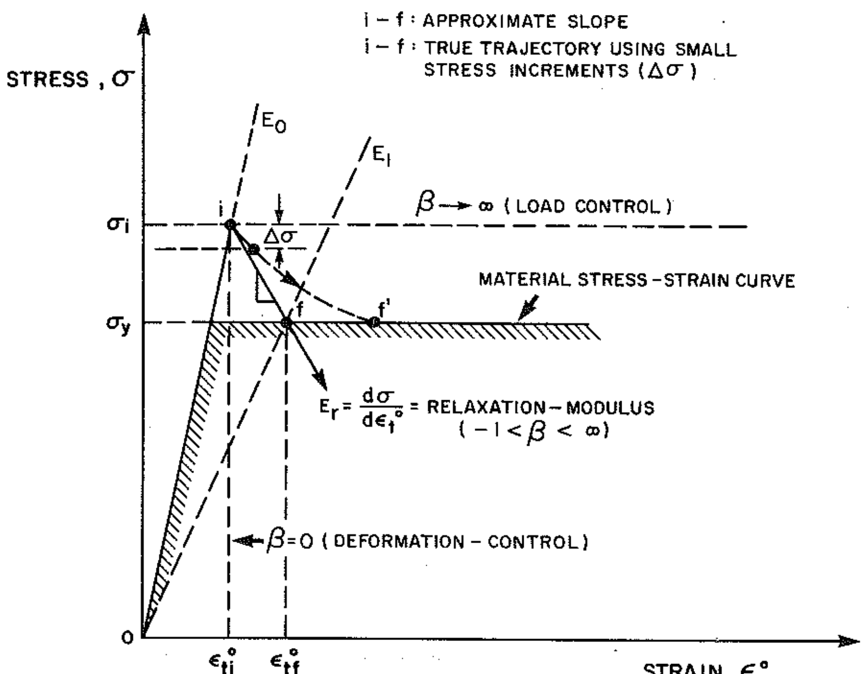

FIG. 2. Relaxation-modulus for elastic follow-up.

Substituting Eq. (3.8) and further simplifying, the following expression can be obtained:

$$\frac{1}{(1+\beta)}\left\{1+\beta\left(\frac{E_r}{E_0}\right)\right\}-\frac{\phi}{1+\phi}\cdot\left(\frac{E_r}{E_0}\right)=0. \tag{4.7}$$

Three cases of follow-up will be considered here.

Case. 1. Elastic follow-up (Fig. 2)

For this case, $\phi = 0$ in Eq. (4.7). If is non-zero, then

$$\beta = -\frac{1}{(E_r/E_0)}. \tag{4.7a}$$

Case 2. Creep follow-up (Fig. 3)

By setting $\beta = 0$ in Eq. (4.7), and assuming a nonzero, the following relationship can be obtained:

$$\phi = \frac{1}{(E_r/E_0)-l}. \tag{4.7b}$$

Case 3. Elastic as well as creep follow-up:

The entire equation (4.7) should be satisfied for this case.

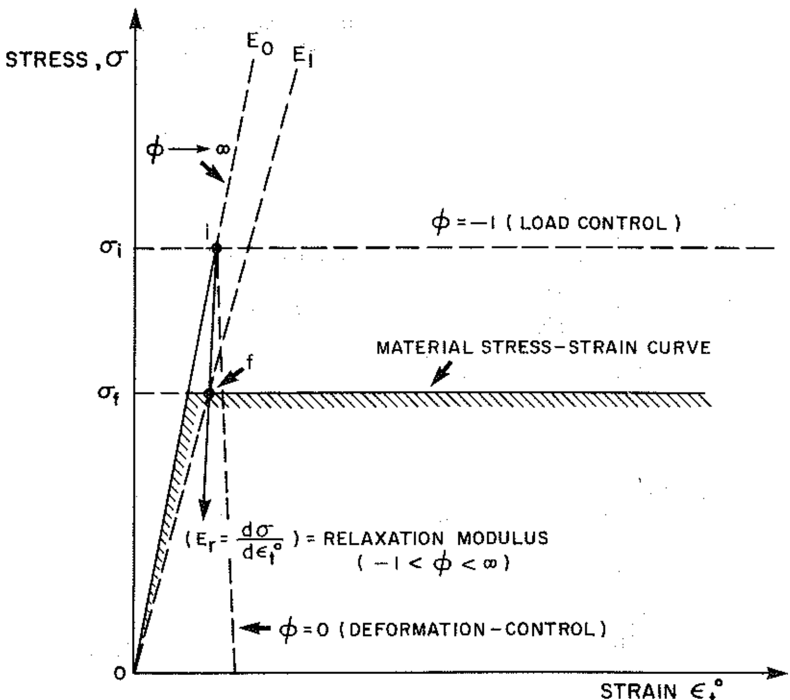

FIG. 3. Relaxation-modulus for creep follow-up.

5. Creep-damage during mixed-mode response

If $\gamma = \dfrac{1+\phi}{1+\beta}$, then Eq. (3.8) can be written as

(5.1) $$\frac{d\sigma}{d\tau} + \gamma BE_0\sigma^n = 0.$$

The variation of the relaxed stress with time can be written as

(5.2) $$\sigma(\tau) = \frac{1}{\left\{\dfrac{1}{\sigma_i n-1} + \gamma BE_0(n-l)\tau\right\}^{1/(n-1)}},$$

σ_i is the initial stress: B, E_0 and n are material constants. If $\gamma = 1$, the variation of $\sigma(\tau)$ versus τ would correspond to the traditional relaxation.

The damage-fraction or life-usage fraction during the creep-relaxation can be written as

(5.3) $$D(t) = \int_0^{t_2} \frac{d\psi}{L(\psi)},$$

where ψ is a dummy variable for τ, and $L(\psi)$ is the "time to rupture" at a given level of stress. It is assumed that the temperature remains constant.

The time to rupture at a given stress-level, σ, can be expressed, for certain range of stresses, as

(5.4) $$L(\sigma) = p\sigma^{-q},$$

where p and q are rupture parameters. Significant data on rupture parameters for pressure component materials is available in the literature.

Substituting Eq. (5.2) into Eq. (5.3), making use of the above relationship and carrying out the necessary integration, the following expression can be obtained:

(5.5) $$D_f(\tau) = \frac{1}{(q+1-n)p\gamma BE_0}\left[\sigma_i^{q+1-n} - \{\sigma_i^{1-n} + \gamma BE(n-1)\tau\}^{\frac{q+1-n}{1-n}}\right].$$

If $\gamma = 0$, $D(\tau)$ is indeterminate, using (5.3). However, if the integration is carried out using Eq. (5.3), then at $\sigma = \sigma_i$

(5.6) $$D(\tau;\ \gamma = 0) = 1$$

$\gamma = 0$ corresponds to maximum follow-up which occurs when the load is purely load-controlled. If $\gamma = 1$ in Eq. (5.5), then the damage-fraction, $D_q(\tau)$, for traditional relaxation is obtained.

5.1. Partitioning of creep damage into load-controlled and deformaton-controlled contributions

The rate of decay of stresses in a mixed-mode response would depend on the type and extent of follow-up, and is characterized by the value γ. The relaxation curve would lie between two extremes — one corresponding to the traditional relaxation ($\gamma = 1$ in Eq. 5.2), and the other corresponding to the completely load-controlled response ($\gamma = 0$). All other mixed-mode responses would correspond to values of γ between 0 and 1.
The following damage-fractions can now be defined: D_m – life-usage fraction corresponding to $\gamma = 0$, i.e., load-controlled response; D_q – life-usage fraction during traditional relaxation ($\gamma = 1$); D_f – life-usage fraction during mixed-mode response, i.e., relaxation with follow-up ($0 < \gamma < 1$).

The maximum possible "load-controlled" life-usage is given

(5.7) $$\Delta D_{\max} = D_m - D_q.$$

The load-controlled portion of life-usage during mixed-mode response is given by Eq. (5.8). Therefore the load-controlled fraction of the total damage during mixed-mode response is given by

(5.9) $$\xi_{f,l} = \frac{\Delta D_f}{\Delta D_m} = \frac{D_f - D_q}{D_m - D_q}.$$

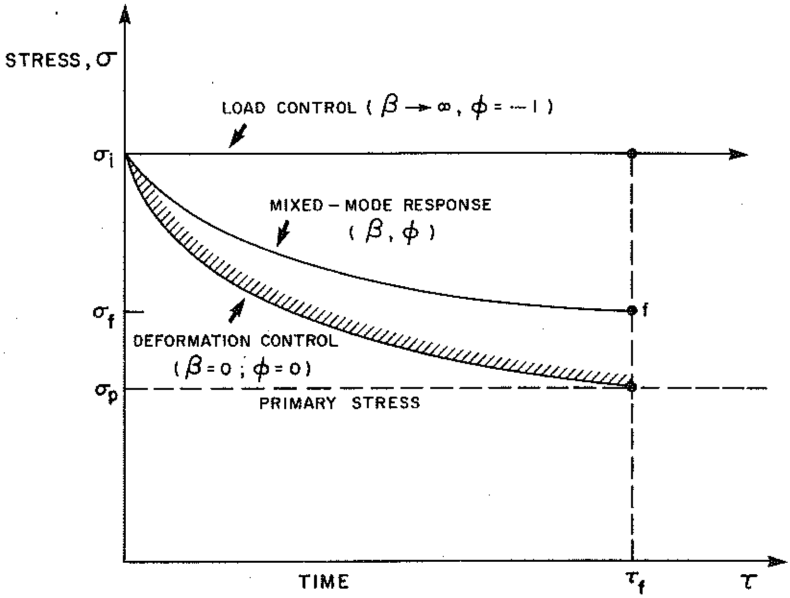

FIG. 4. Decay of initial stress for various degrees of follow-up.

From Eq. (5.9), the two limiting cases are evident:

1) when $D_f \to D_q$, $\xi_{f,l} \to 0$,

2) when $D_f \to D_m$, $\xi_{f,l} \to 1.0$.

The limit $\xi_{f,l} \to 0$ corresponds to the purely deformation-controlled response, whereas the limit $\xi_{f,l} \to 1.0$. corresponds to purely load-controlled response for which all of the creep-damage is of a primary nature. Of course, for the intermediate cases of mixed-mode response, $0 < \xi_{f,l} < 1.0$.

The deformation-controlled fraction of the total damage during mixed-mode response is given by

(5.10) $$\xi_{f,s} = l - \xi_{f,l} = \frac{D_m - D_f}{D_m - D_q}.$$

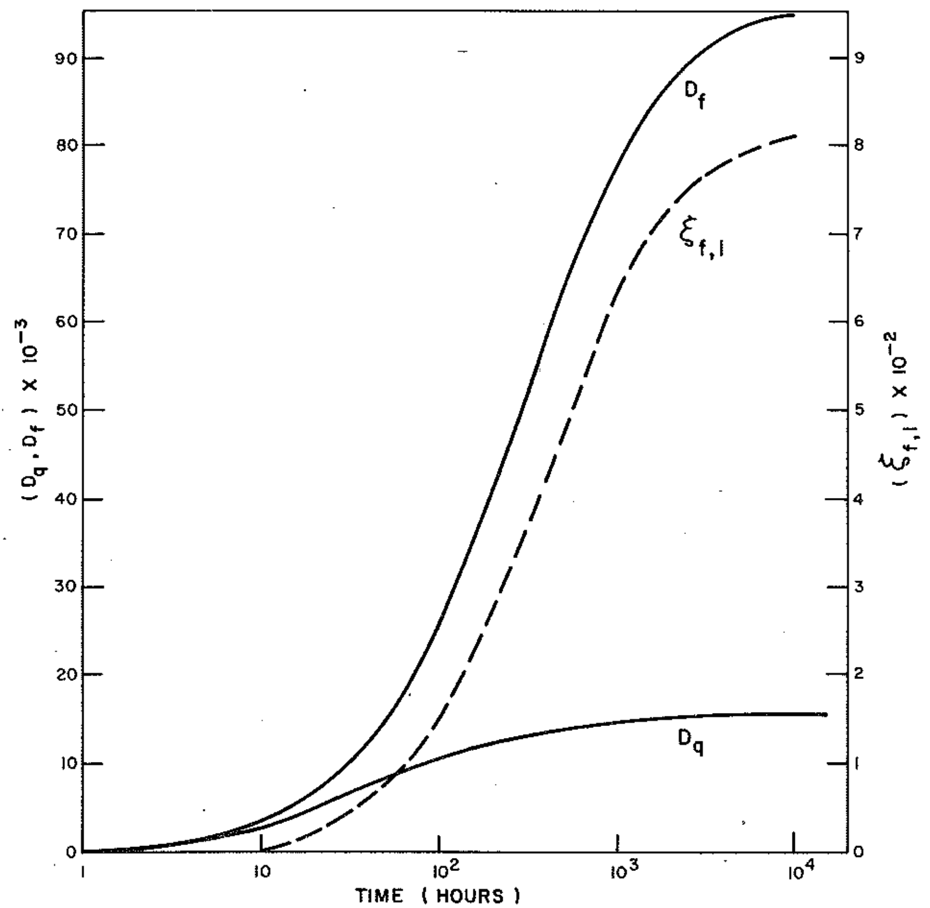

FIG. 5. D_f, D_q and $\xi_{f,l}$ versus time.

5.2. Design considerations

The total life-usage mixed-mode response is given by the expression (Fig. 4)

$$D(\tau) = D_p(\tau) + D_f(\tau), \tag{5.11}$$

$D_p(\tau)$ is the damage incurred due to the presence of the "primary stresses" in the component, and can be calculated by the expression

$$D_p(\tau) = \frac{\tau}{L(\sigma_p)}. \tag{5.12}$$

The pressure-component should be designed so that

$$D(\tau) = D_p(\tau) + D_f(\tau) < 1.0. \tag{5.13}$$

In terms of assigning a suitable stress-limit for components subjected to follow-up action, it appears reasonable to keep stress below S^* (the pseudo--elastic secondary stress allowable):

$$S^* = S_m + \xi_{f,s}(2S_m). \tag{5.14}$$

Equation (5.14) can also be expressed as

$$S^* = 3S_m - \xi_{f,l}(2S_m). \tag{5.15}$$

The term $\xi_{f,l}(2S_m)$ is the reduction in the secondary stress limit $(3S_m)$ and reflects the damage caused by follow-up action (Appendix 1).

6. The generalized structural stress-strain plot

As has been mentioned earlier, a convenient representation that has emerged in the literature, which allows evaluation of follow-up damage, is the generalized local stress-strain plot [7]. The plot is essentially a representation of the relaxation of stresses at the location of the inelastic strain-concentration, i.e., local system.

Introducing the nondimensional quantities

$$\bar{\sigma} = \frac{\sigma}{\sigma_{\text{ref}}} \quad \text{and} \quad \bar{\varepsilon}_t^0 = \frac{\varepsilon_t^0}{\varepsilon_{\text{ref}}}, \tag{6.1}$$

where σ_{ref} and ε_{ref} are reference quantities. It is convenient to set $\sigma_{\text{ref}} = \sigma_i$ and $\varepsilon_{\text{ref}} = \varepsilon_e = \dfrac{\sigma_i}{E_0}$.

The mixed-mode relaxation modulus E_r was defined in Eq. (4.1) as $E_r = d\sigma/d\varepsilon_t^0$. In order to express the response in terms of the generalized stress-strain plot, $d\bar{\sigma}/d\bar{\varepsilon}_t^0$, the following transformation can be introduced, i.e.,

$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}_t^0} = \frac{\varepsilon_{\text{ref}}}{\sigma_{\text{ref}}}\left(\frac{d\sigma}{d\varepsilon_t^0}\right). \tag{6.2}$$

Therefore, substituting appropriate reference quantities in Eq. (6.2),

$$\frac{d\sigma}{d\bar{\varepsilon}_t^0} = \frac{l}{E_0}\left(\frac{d\sigma}{d\varepsilon_t^0}\right). \tag{6.3}$$

The "relaxation-modulus" for mixed-mode response with reference to the generalized stress-strain plot can be expressed as (see Eq. (4.1))

$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}_t^0} = \frac{E_r}{E_0} = \bar{E}_r. \tag{6.4}$$

Equation (4.7) can now be written as

$$\frac{1}{(1+\beta)}[1+\beta\bar{E}_r] - \frac{\phi}{l+\phi}\cdot\bar{E}_r = 0. \tag{6.5}$$

Equation (4.7) is suitable for use in conjuntion with the generalized plot.

When only elastic follow-up occurs,

$$\beta = -\frac{1}{\bar{E}_r}. \tag{6.6}$$

When only creep follow-up occurs,

$$\phi = \frac{1}{\bar{E}_r - 1}. \tag{6.7}$$

6.1. Determination of $\bar{E}_r$ using linear elastic finite element analysis — an approximate method

With reference to Fig. 2, an elastic finite element analysis of the pressure-component configuration is carried out with the entire material specified with a modulus of elasticity of E_0. The pseudo-elastic stresses (σ_i) — the equivalent stresses at deformation-controlled locations, are determined. The corresponding point in the plot is i. A secant-modulus $(E_l < E_0)$ is chosen in an attempt to approximately simulate the lowering of pseudo-elastic stresses due to inelastic effects for all the elements that exceed the yield stress, σ_y. The secant modulus (E_l) can the determined, as a first approximation, using the expression

$$E_1 = \left(\frac{\sigma_y}{\sigma_i}\right)E_0 \quad (\sigma_i > \sigma_y). \tag{6.8}$$

Finite-element analysis using the above changes will locate point f in Fig. 2.

When only elastic follow-up occurs, Eq. (6.6) is used; however if only creep follow-up is dominant, Eq. (6.7) is used.

With reference to Fig. 2, an approximate estimate of $\bar{E}_r$ can be obtained using linear elastic finite-element analysis as follows:

1) For a given pressure component configuration, a finite-element analysis is carried out by assuming a linear elastic material with a modulus of elasticity of E_0;

2) For all elements in deformation-controlled locations in which the stresses exceed the yield stress, σ_y, point i is determined;

3) For all elements for which $\sigma_i > \sigma_y$, the modulus of elasticity (E_0) is replaced by the secant-modulus (E_1) such that $E_1 < E_0$. The approximate value of E_1 can be estimated using the expression

$$E_1 = \left(\frac{\sigma_y}{\sigma_i}\right) E_0. \tag{6.9}$$

The finite-element analysis is now carried out and points such as f are located;

4) For a given element the slope ($\bar{E}_r$) of the line i-f can be determined. Using Eqs. (6.6) and (6.7), the values of β and ϕ can be obtained;

5) Approximate damage estimates can be obtained using Eqs. (5.5) and (5.13).

Although the method is approximate, it offers the advantage that only linear elastic analysis methods are required to predict damage in an inelastic system. The method, of course, could be improved or modified since β and ϕ are, in reality, a function of stress, σ.

7. Numerical example

Consider the example of a piping system with four large bending loops in series with four smaller loops of half-size in parallel, Fig. 8, as described in the paper by Robinson [4]. For a 2 1/4 chrome-1 Molybdenum steel, pertinent data is presented in Table 1.

The steady-state creep relationship can be expressed as $\dot{\varepsilon} = \dot{\varepsilon}_0(\sigma/\sigma_0)^n$, where for the 2.25 Cr-1 Mo steel at 537.8°C (1000°F), $\dot{\varepsilon}_0 = 0.65 \times 10^{-6}$ and $n = 4.15$.

If it is assumed that for a given range of stress $L = L_0(\sigma/\sigma_0)^{-q}$, then $L_0 = 5.7116 \times 10^4$ and $q = 7.037$.

For $\phi = 2.67$ and $\beta = 22.0$, $\gamma = 0.16$. Therefore, $\tau = 6.27\tau^*$. In 3000 hours, traditional relaxation ($\gamma = 1$) would give a stress of 29.57 MPa (4288.6 psi). With follow-up present ($\gamma = 0.16$), the relaxed stress after 3000 hours is 51.61 MPa (7484.6 psi). It can be seen that the rate of decay of stresses during mixed-mode response is smaller than that for traditional relaxation.

Using Eq. (5.5), the damage-fraction during mixed-mode response at the

Table 1. Material and follow-up data

	SI (Metric) Units	U.S. Coustomary units
Material	2.25 percent Chrome — 1 percent Molybdenum	
Temperature	537.8°C	1000°F
Modulus of elasticity	15.86×10^4 MPa	23×10^6 psi
Creep parameters:		
$\dot{\varepsilon}_0$	0.65×10^{-6}	0.65×10^{-6}
n	4.15	4.15
Stress to rupture in 1000,000 hours	102.07 MPa	14800 psi
Elastic follow-up parameters:		
β	22.0	22.0
ϕ	2.67	2.67
Rupture parameters:		
p	5.7116×10^4	5.7116×10^4
q	7.037	7.037
Initial stress	104.83 MPa	152 psi

Note: 1. The data pertains to the piping loop presented in Fig. 8.

end of 3000 hours is given by $D_q = 0.0152$ and $D_f = 0.0901$. The load--controlled portion of the damage during mixed-mode response can be calculated using Eq. (5.9) as $\xi_{f,l} = 0.076$ or 7.6 percent. In other words, 92.4 percent of the creep damage during mixed-mode response can be considered secondary in nature.

The mixed-mode response modulus (E_r) in the generalized local stress-strain plot, Eq. (6.5) is used. For $\phi = 2.67$, $\beta = 22.0$, E_r can be calculated to be -0.19, what corresponds to an angle $\theta = 79.25$ degrees. If the partitioning of primary and secondary contributions is based on a method proposed by DHALLA [7], then

$$\xi_{f,l} = \frac{79.95}{90} = 0.88.$$

It can be seen that the above "linear" partitioning method is extremely conservative and would overpredict damage considerably.

Figure 7 is a plot of $\xi_{f,l}$ versus θ for $\phi = 2.67$ and $\beta = 22.0$. The damage is not significant until θ approaches 85 degrees. In other words, a high degree of follow-up (characterized by higher β and lower ϕ) is required for causing substantial damage, especially during one-time relaxation.

The allowable stress-limit for pressure components that experience follow-up can be obtained from Eq. (6.2) as $S_a = 2.85\, S_m$. Due to follow-up action, the "deformation-controlled limit" of $3 S_m$ is essentially reduced.

8. Discussion

8.1. *Mixed-mode response in the generalized local stress-strain plot*

The generalized local stress-strain plot is being increasingly used to either quantify follow-up or partition the "elastically calculated stress" as primary or secondary. Dhalla [7], for instance, calculates the "Primary Stress Fraction" using the expression $\theta/90$ (Fig. 6). A linear relationship between the primary

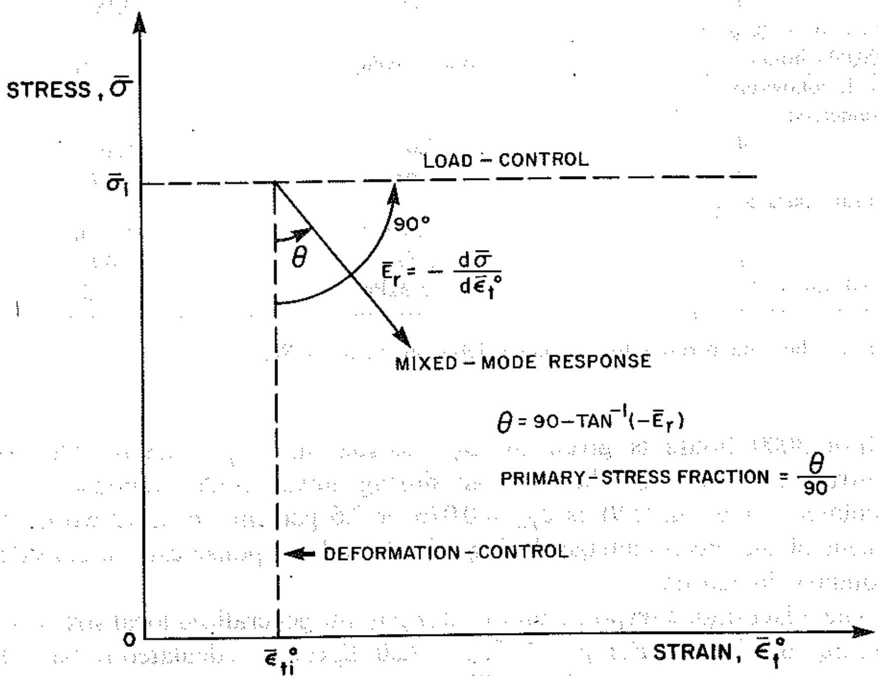

Fig. 6. Primary-stress fraction for mixed-mode response.

stress fraction and the slope of the mixed-mode response (ψ) is implied. Boyle [8] points out that it is debatable whether 30 percent follows-up for instance should imply 30 percent primary stress fraction. The entire process of creep-damage is clearly non-linear with regard to the follow-up parameters β and ϕ. This can be easily seen from Fig. 7 where damage increases dramatically as θ approaches 90 degrees, corresponding to a load-controlled situation.

In a discussion following Robinson's paper [4], Markl points out that even though creep can be localized in a component, it is difficult to visualize failure under conditions of relaxation in a piping system designed to conform to suitable stress-limits. To elaborate further, a 10 percent drop in the stress during relaxation in a 2.25 Cr – 1 Mo component at 537.6°C would reduce

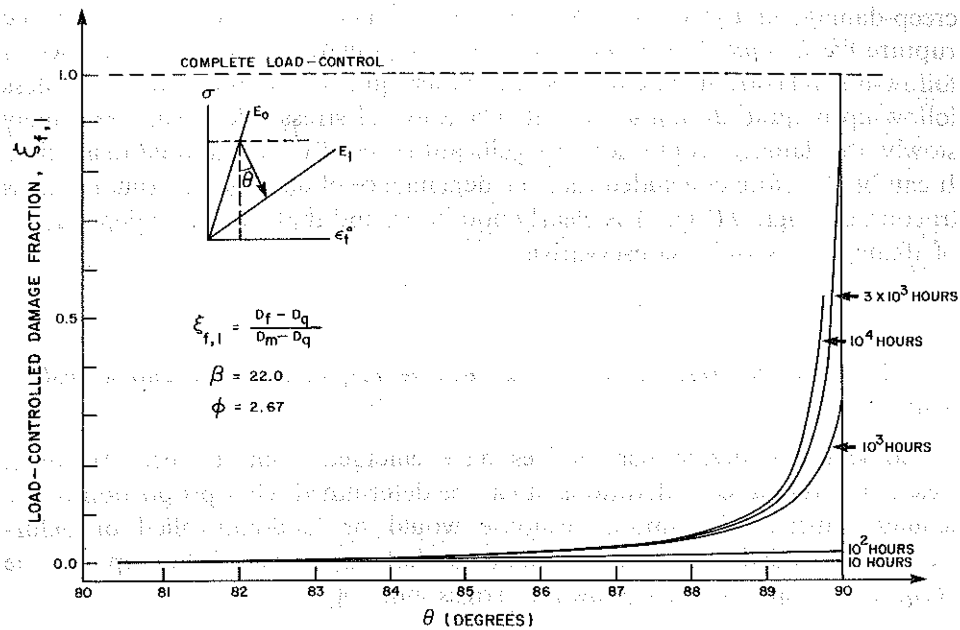

FIG. 7. Load-controlled damage fraction ($\xi_{f,l}$) versus θ.

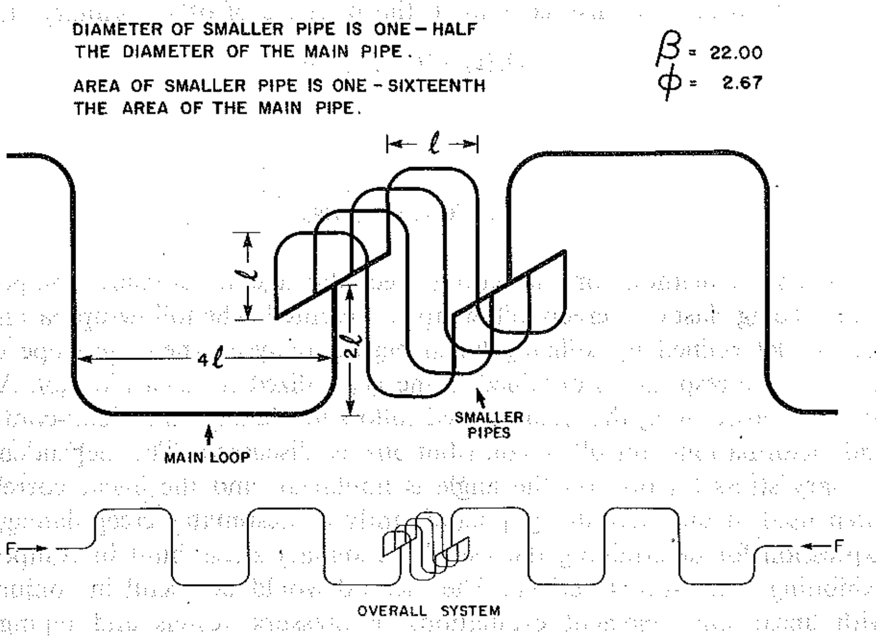

FIG. 8. A typical piping system with follow-up potential.

creep-damage in a given time by more than 50 percent. This is so because the rupture life $L = p\sigma^{-q}$. In typical pressure components, even in the presence of follow-up, relaxation would proceed rather quickly at initial times. Unless follow-up is quite dramatic in that relaxation of stress takes place extremely slowly, the damage would not be significant especially for one-time relaxation. It can be therefore concluded that the dependence of damage or primary stress fraction on angle θ (Fig. 7), is clearly nonlinear, and that the linear dependence of damage θ is quite conservative.

8.2. Allowable stress-limit for mixed-mode response — „pseudo-secondary limit"

Some useful design perspectives have emerged from the present study. Based on rational considerations, it can be determined what proportion of the damage during mixed-mode response would be load-controlled or deformation-controlled. Pressure components that experience follow-up can be designed by specifing an allowable stress-limit of

$$S_a = 3\,S_m - 2\,\xi_{f,l}(S_m),$$

where $\xi_{f,l}$ is defined in Eq. (5.10).

The term $2\,\xi_{f,l}(S_m)$ is the reduction in the traditional shakedown limit of $3\,S_m$ due to follow-up effects.

It should also be ensured that in the presence of other primary stresses,

$$D_p(\tau) + D_f(\tau) \leqslant 1.0.$$

9. Conclusions

A simple method for estimating creep-damage in pressure components experiencing elastic or creep follow-up is presented. The follow-up parameters can be determined by utilizing linear elastic analysis when the slope of the mixed-mode response is obtained in the generalized stress-strain plot. A method for partitioning the accumulated follow-up damage into load-controlled and deformation-controlled contributions is discussed. The dependence of primary stress fraction on the angle is nonlinear, and the linear correlation often used in the literature [7] significantly overestimates creep-damage. An expression for determining the pseudo secondary-stress limit in components exhibiting follow-up is derived. The method would be useful in conjunction with linear finite element evaluations of pressure vessels and piping that operate in the creep range.

APPENDIX 1

The allowable stress limit for deformation-controlled cyclic stresses is based upon the concept of shakedown. A lower bound method for predicting shakedown is based on an adaption of Melan's theorem which states "if any distribution of self-equilibrating residual stresses can be found in a structure (or a component), which when taken together with the pseudô-elastic stresses constitute a system of stresses within the yield limit, then the structure will shakedown". Therefore, if the initial loading cycle induces plasticity due to some deformation-controlled action and is the shakedown limits are satisfied, then only elastic action will result during subsequent cycles. The shakedown limit for deformation-controlled stress is $3S_m$ (or $2\sigma_y$), where S_m is the load-controlled membrane stress (no greater than $2/3\sigma_y$).

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Streszczenie

ZNISZCZENIE SPOWODOWANE PEŁZANIEM W ELEMENTACH CIŚNIENIOWYCH PODDANYCH PROCESOWI „ŚLEDZENIA"

Przedstawiono prostą metodę oceny uszkodzeń spowodowanych pełzaniem w elementach ciśnieniowych poddanych zjawisku sprężystego lub pełzającego „śledzenia". Wieloosiowy proces relaksacji odniesiony został do jednowymiarowego modelu relaksacji naprężeń, a przebieg procesu przedstawiony został na wykresie lokalnych, uogólnionych naprężeń i odkształceń. Wyznaczono następnie parametry „śledzenia" wykorzystując kąty nachylenia krzywej na tym wykresie. Przedstawiono na koniec sposób podziału zakumulowanego uszkodzenia spowodowanego pełzaniem na składniki odpowiadające kontrolowanym obciążeniom oraz odkształceniom. Metodę zastosowano do typowych układów rurowych poddanych działaniu podwyższonych tempertur.

Резюме

РАЗРУШЕНИЕ ВЫЗВАННОЕ ПОЛЗУЧЕСТЬЮ В ЭЛЕМЕНТАХ ДАВЛЕНИЯ ПОДВЕРГНУТЫХ ПРОЦЕССУ „СЛЕЖЕНИЯ"

Представлен простой метод оценки повреждений, вызванных ползучестю в элементах давления, подвергнутых явлению упругого или ползучего „слежения". Многоосевой процесс релаксации однесен к одномерной модели релаксации напражений, ход же процесса представлен на диаграмме локальных, обобщенных напражений и деформаций. Затем определены параметры „слежения", используя углы наклона кривой на этой диаграмме. Наконец представлен способ разделения накопленных повреждений, вызванных ползучестью, на компоненты, отвечающие контролированным нагружениям и деформациям. Метод применен для типичных трубчатых систем, подвергнутых действию повышенной температуры.

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