ENGINEERING TRANSACTIONS • *Enging. Trans.* • **39**, 3-4, 389-418, 1991
Polish Academy of Sciences • Institute of Fundamental Technological Research

STRAIN ENERGY GOVERNED DAMAGE LAW FOR A VISCO-PLASTIC MATERIAL

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The inelastic strain energy which was shown by other authors to correlate fatigue data is used as an independent variable in a differential law of damage growth. A special form of this law is discussed, and material constants are evaluated to fit experimental data for low-cycle fatigue and creep of 304 stainless steel at 650°C. Since the main goal of this study is to develop the method of life prediction for a wide range of practical applications, both uniaxial and multiaxial states of stress are evaluated. In the former case the effects of rate of loading, strain/stress control, wave shape, mean strain and hardening are considered. The multiaxial state of stress has been illustrated by a special case of combined axial-torsional deformation. In all these situations the inelastic strain energy was calculated according to the viscoplasticity theory based on overstress (VBO), which was shown to describe the cyclic deformation of 304 stainless steel in room as well as in elevated temperatures.

1. Introduction

During the last few decades low cycle fatigue (LCF) has been the subject of extensive investigations. It is reflected by the very large amount of experimental data gathered, and scientific papers published. Proceedings of special conferences and symposia (e.g., [1, 2, 3]) contain up-to-date sources of information. However, the recent rewiev papers on this subject [4, 5] demonstrate that not all problems in this field have been solved and that the accumulated knowledge is still insufficient to describe the complexity of the phenomenon.

Two main open problems in LCF analysis are:
description of deformation of a material subjected to LCF,
prediction of lifetime of a structural member.

These problems are, of course, interdependent since life prediction is dependent on a choice of proper constitutive equations. On the other hand, damage processes which lead to final failure may influence the deformation response of a material.

In both cases the complexity induced by path dependence found in metallic alloys is increased when time-dependent effects come to play an essential role. For structural steels and alloys this normally happens at elevated temperatures, but may also occur at room temperature as well [6]. In these circumstances the effects of:

rate of loading,

hold times (creep-fatigue interaction),

wave-shape effect (slow/fast loading/unloading),

have to be taken into account. Among other problems which still deserve investigation are the effects of cyclic hardening, proportional and non-proportional cyclic loading. Each of the listed problems in fact require a separate study and experimental verification, as the observed behaviour may vary for different materials and temperatures.

The aim of the present study is to consider a new damage law for life prediction in these complex situations. Due to the discussed complexity of the phenomenon, such a study has to be considered a qualitative verification of the formulated hypothesis. Quantitative results in this paper are meant rather as an illustration of the proposed law than as the precise description of the behaviour of the material chosen (stainless steel of the 304 type; material data for room temperature and for temperature of 650^0 C were used). The main goal of this paper is to propose a law which can be used in complex situations to predict lifetime.

In this study failure will be understood as the onset of crack initiation, i.e., only a part of the lifetime of the specimen will be considered, which is spent for incubation and growth of microstructural defects. Since a phenomenological approach will be used, no microstructural considerations are involved. Such an approach would deserve a separate study (see, e.g., the review paper [7]), and is beyond the scope of the present report.

There are three approaches used in the description of material deterioration according to [5]:

the classical, which makes use of parametric equations relating the number of cycles to crack initiation to different parameters describing the local cyclic loading,

the future, which employs Continuum Damage Mechanics (CDM), and makes use of complete inelastic analysis, including coupling of damage with constitutive equations,

the intermediate, which uses CDM on the basis of the independent analysis of states of stress and strain.

The last one will be used in the present paper. Continuum Damage Mechanics (term coined by HULT, c.f. [8]) introduces a new state variable responsible for material deterioration, with a differential law of growth for this variable. This makes it possible to take into account the whole loading history which can be an important factor in life predictions.

It can be mentioned here that the CDM has already formed a new branch of fracture mechanics. Though many questions remain open in this approach, such as the damage definition itself [9] and its measurement [10], it has found wide application in many important situations including fatigue and creep-fatigue interaction [11, 12, 13, 14, 15]. The recent review papers [16, 17] can be used as a source of information.

One of the most important problems in formulating a damage growth law is the choice of governing variables. In early approaches [11, 12] stress was used as a governing variable. OSTERGREN and KREMPL [13] introduced the forcing factor and its time derivative, MAYIA and MAJUMDAR [14] – inelastic strain and strain rate; SATOH and KREMPL [15] used stress and inelastic strain rate. The bench-mark project, carried out in Japan [18], to compare these and other damage theories, revealed quite a large discrepancy between predictions and experimental results, and demonstrated that further investigations aiming at developing useful damage accumulation laws are still necessary.

In the present paper the inelastic strain energy will be used as a candidate for a governing variable. As the energetic concepts have been widely applied to fatigue data correlation, this subject will be discussed in a separate chapter. It can be mentioned here that calculation of the inelastic energy requires the use constitutive equations which can reproduce the observed deformation behaviour. The theory of Viscoplasticity Based on Overstress (VBO) developed by KREMPL in a series of papers (cf.,e.g., [19, 20]) was chosen for this purpose. This theory falls into the framework of unified theories which do not account for creep and plasticity separately. The outline of this theory necessary for the purpose of this paper will be given in Sect. 2.3.

Finally, let us remark that the method used in this paper is computer-oriented in the sense that a set of stiff differential equations which represent constitutive equations of VBO and proposed damage law was solved by using a computer even in the case of uniaxial loading. Thus the verification of the proposed method is called numerical simulation and is presented in Sect. 3. The flexibility in choosing a different form of damage equation for different loading conditions may be considered as an advantage of this method. In this aspect the present work is an extension to the time-dependent deformation of the approach by OTT, NOWACK and PEEKEN [21].

2. ENERGETIC APPROACHES TO FATIGUE FAILURE

The use of the energetic interpretation of the fatigue failure phenomenon has a long history. For example, the background for the well-known MINER'S life fraction rule [22] was an assumption that the work absorbed at failure is proportional to the number of cycles to failure. This concept of referring failure data to energy used for the production of changes in a material structure will be briefly discussed below.

2.1. Strain energy as a parameter correlating fatigue data

One of the recent approaches to material failure using the notion of entropy (WHALEY [23]) expresses the main idea behind all energetic methods. The energy supplied by the external agencies dW_{sup} is split into two parts: energy stored in the material and the heat loss dQ by convection and conduction. In turn, the energy stored in a material can be separated into two components: recoverable (elastic) dW_{rec} and irreversible dW_{irr} ones. This irreversible energy is spent for structural changes in a material and may contribute to both deformation and deterioration of a material. In an energy balance

$$dW_{\text{sup}} = dQ + (dW_{\text{rec}} + dW_{\text{irr}}) \tag{2.1}$$

all components but dW_{irr} are measurable though the technological problems to overcome measurement difficulties are still essential (cf. e.g. [24]). As far as an irrecoverable part of stored energy is considered, the partitioning of it into inelastic deformation and damage contributions

$$dW_{\text{irr}} = dW_{\text{in}} + dW_{\text{dam}} \tag{2.2}$$

depends on the microstructural mechanisms which control these processes. On the phenomenological level, only inelastic deformation can be measured and the part of dW_{irr} corresponding to the inelastic deformation dW_{in} can be assessed. Thus the remaining portion of dW_{irr}, which is responsible for damage growth dW_{dam}, can be evaluated from the balance (2.1) if an accurate measurement of heat loss and inelastic deformation W_{in} is performed. In the absence of the results of such a measurement, the simplest assumption is that the part of stored energy responsible for damaging is proportional to the stored inelastic energy:

$$W_{\text{dam}} \sim W_{\text{in}} \, . \tag{2.3}$$

The above assumption is hidden behind all developed approaches to material failure. Consequently, the inelastic strain energy W_{in} is used as a parameter determining the number of cycles to failure. Such an approach was first proposed by FELTNER and MORROW [25], and by MARTIN [26] for low cycle fatigue. In these papers the number of cycles to failure was assumed to be a function of inelastic energy calculated for a hysteresis loop. However, the assumption made in [25] that "the total damaging energy required to cause fatigue failure is equal to the area under the static stress-true-strain curve" was not verified experimentally. HALFORD [27] has shown that the plastic strain hysteresis energy to fatigue failure at 500.000 cycles is about 100 times that required for failure by monotonic tension for some metals. The relationship between the stress range $\Delta\sigma$, plastic strain range $\Delta\epsilon_p$, and hysteresis energy developed in [27] for cyclic hardening material in the uniaxial case is

$$\Delta W = ((1-m)/(1+m)) \; \Delta\sigma \; \Delta\epsilon_p \, , \tag{2.4}$$

where m is the cyclic strain hardening exponent. The total strain energy required for failure is

$$W_f = \Delta W \; N_f \, . \tag{2.5}$$

provided ΔW is constant for each cycle. Normally, stabilized values of the stress range $\Delta\sigma$ and plastic strain range $\Delta\epsilon_p$ are used or those at half the fatigue life. Such a procedure is typical for the parametric approach which does not take into account the history of loading.

The parametric approach for high temperatures was developed by FONG [28], OSTERGREN [29] and LEIS [30] to include creep-fatigue interaction. Because of the complexity of the phenomenon, different assumptions were made to partition the hysteresis loop. These assumptions, not well motivated even in the uniaxial case, have to be revalidated for the multiaxial state of stress/strain.

An energetic multiaxial approach to fatigue failure in the context of the parametric method proposed by ELLYIN [31], was recently developed in the series of papers [32 - 35], including non-Masing behaviour and the mean strain effect. The important observation made is that hysteresis energy has to be related to the fatigue life in such a way that the total energy to failure must be an increasing function of N_f .

If the assumption

$$\Delta W = K\, N_f^{\alpha} \tag{2.6}$$

made by ELLYIN *et al.* [36] holds, where K and α are material constants, then using Eq. (2.5) we have

$$W_f = K\, N_f^{\alpha+1} \,. \tag{2.7}$$

Since ΔW must increase with decreasing W_f there is $\alpha < 0$. On the other hand, W_f should increase when N_f increases. Therefore

$$-1 < \alpha < 0 \,. \tag{2.8}$$

The problems of multiaxial fatigue, dealt with by LEIS and LAFLEN [37], were recently reviewed by GARUD [38]. Still, most of the research effort is directed to describe the deformation and effect of hardening during nonproportional loading (cf., e.g., [39]) rather than to life prediction. The energetic method was offered by GARUD [40], who proposed to correlate the plastic work per cycle W_h and fatigue life to crack initiation N_f by the equation

$$N_f = F(W_h) \,, \tag{2.9}$$

where F is a monotonically decreasing function of W_h. Here

$$W_h = \int\limits_{\text{cycle}} \Delta W_p \,, \tag{2.10}$$

and

$$\Delta W_p = \boldsymbol{\sigma} : \Delta \boldsymbol{\epsilon}^p \,, \tag{2.11}$$

σ is the stress tensor, $\Delta\epsilon^p$ the plastic strain increment tensor calculated in [40] using the MRÓZ model [41, 42]. This theory gave rise to a computer-oriented model of fatigue failure developed by OTT *et all.* [21]. For variable loading amplitude the damage is calculated on the basis of the hysteresis loop corresponding to the current yielding surface. Since the time-independent plasticity model was used, this method is unable to predict the rate effect and associated phenomena such as the slow/fast loading or frequency effect. The method proposed in this study constitutes the next step by introducing a differential damage law governed by inelastic energy and by taking into account the time-dependent deformations.

2.2. *Strain energy as a variable governing the proposed incremental damage law*

If the life fraction N/N_f (N is the current number of cycles) is considered a measure of damage, then in the parametric approach discussed in the previous section the damage is related directly to the inelastic energy. From Eqs. (2.6) and (2.7) (with a more general definition of the hysteresis energy W_h instead of ΔW) follows:

$$W_f = W_h \, N_f \, , \tag{2.12}$$

that is, W_h is constant in each cycle. Thus, inelastic energy at the N-th cycle is

$$W_{\text{in}} = W_h \, N \, , \tag{2.13}$$

and from Eqs. (2.12) and (2.13) we have

$$W_{\text{in}}/W_f = N/N_f \, , \tag{2.14}$$

i.e.

$$W_{\text{in}} \sim D \, , \tag{2.15}$$

which was the base for Miner's rule. Such a relationship which relates only current values of damage and inelastic energy can no longer hold when the time-dependent behaviour of a material has to be taken into account. The prior loading history can be included if Eq. (2.15) is generalized to have the form

$$D \sim \int W_{\text{in}} \, dt \, , \tag{2.16}$$

or, in differential form,

$$dD/dt \sim W_{\text{in}} \, , \tag{2.17}$$

where both dD/dt and W_{in} are the functions of time. Inelastic energy is to be calculated according to Eq. (2.11) which, expressed in terms of the stress tensor and inelastic strain rate tensor, takes the form

$$W_{\text{in}} = \int \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^{\text{in}} \, dt \, . \tag{2.18}$$

Since generally W_{in} is nondecreasing, the rate of damage given by Eq. (2.17) will also be nondecreasing with time. The healing process is thus excluded from the proposed description. Further, the so-called "dormant damage mechanism" (HULT [43]), when $dD/dt = 0$, occurs only in the absence of inelastic energy. Since in the following use will be made of the viscoplastic theory which predicts the existence of inelastic deformation at any stress level (which may be very small at a low stress level), there will always be nonzero growth of damage.

With a functional form of Eq. (2.17), the postulated law becomes

$$dD/dt = F(W_{\text{in}}) \,. \tag{2.19}$$

This equation will be modified to meet particular phenomena in subsequent sections. Here we introduce only the damage influence on its rate by an equation:

$$dD/dt = F\,(W_{\text{in}}\,, D)\ . \tag{2.20}$$

In this study the power function will be chosen for F and variables will be separated to give

$$dD/dt = C\,(W_{\text{in}}/(1-D))^n\,, \tag{2.21}$$

where C and n are material constants to be determined from suitable tests.

The proposed law can be viewed as a generalization of some existing proposals, especially that by MAJUMDAR [44], who made use of both the inelastic strain rate and inelastic strain combined to govern the damage rate. With some simplifying assumptions, this theory can be reduced to the relationship given by Eq. (2.17). The damage law proposed by SATOH and KREMPL [15] can be reduced (again by some simplifications not intended by the authors) to

$$D \sim W_{\text{in}}\,,$$

and thus to parametric interpretation, cf. Eq. (2.15). Both of the above theories give very good predictions including creep-fatigue interaction. However, the aim of this study is to check whether an explicit use of a physical (measurable) quantity in a damage law may be accepted for practical applications.

In the present formulation there were no assumptions made according to damage development under compression. Such assumptions which are often made (cf. e.g. CHRZANOWSKI [12], SATOH and KREMPL [15]) are aimed to fit experimental observations. In parametric approaches which use the notion of energy, the assumption most frequently made is that only the tensile part of the hysteresis loop contributes to damage (OSTERGREN [29], LIU *et al.* [45]). Such an assumption needs reformulation when applied to multiaxial cycling and, according to the author's knowledge, has not been pursued. The damage law used here was not modified for this purpose. Thus the effects of tensile versus compressive cycling can be reflected only if the inelastic energy in tension and compression is different.

2.3. Inelastic strain energy calculated by means of the VBO theory

The far-reaching goal of this investigation, which goes beyond the present study, is to relate the damage variable to a physical process which occurs in a material. As a portion of dissipated energy is assumed to be responsible for material damaging, it should be evaluated from direct or indirect measurement. In the absence of such experimental data, a substitute assumption was made that this energy can be related to the energy used for the production of inelastic deformation. This energy can again be measured experimentally or calculated by using a constitutive equation which describes the deformation process with a required degree of accuracy. The theory of Viscoplasticity Based on Overstress (VBO) [19, 20] was shown to describe the cyclic behaviour of AISI Type 304 Stainless Steel and was used here to calculate W_{in}. One should remember, however, that this is only an approximate procedure, and that both steps in this approach (i.e., from inelastic strain energy calculated to the measured one, and from inelastic strain energy measured to dissipated damaging energy) should be verified experimentally.

The formulation of VBO used in this investigation corresponds to the isotropic model of a body which may exhibit cyclic hardening. The fundamentals of this theory can be found elsewhere ([20, 46]) and will not be reformulated here. Let us mention only that this theory does not distinguish between plastic and viscous components of deformation and does not use the yield surface notion (it is a "unified" theory). The important assumption is that the inelastic strain rate is a function of the overstress (the difference between the stress corresponding to the given loading rate and equilibrium stress, i.e., the stress reached asymptotically when the loading rate tends to zero). The theory is formulated for small strains and assumes that the stress-strain curve ultimately approaches a constant slope given by a tangent modulus E_t .

The constitutive equations for the multiaxial state of stress are given by the coupled system of nonlinear differential equations:

$$
\begin{aligned}
\dot e_{ij} &= \dot e^{\rm el}_{ij} + \dot e^{\rm in}_{ij} = \frac{1+\nu}{E}\,\dot s_{ij} + \frac{3}{2}\,\frac{1}{Ek(\Gamma)}\left(s_{ij} - g^{d}_{ij}\right) ,\\
\dot g^{d}_{ij} &= \frac{2}{3}\,\psi(\Gamma)\,\dot e_{ij} - \left(g_{ij} - f^{d}_{ij}\right)\,\frac{\dot\phi}{b(\Gamma)} ,\\
\dot\epsilon_{kk} &= \frac{1-2\nu}{E}\,\dot\sigma_{kk} ,
\end{aligned} \tag{2.22}
$$

where e_{ij}, s_{ij}, g_{ij} are deviatoric parts of the total strain ϵ_{ij}, stress σ_{ij} and equilibrium stress g_{ij}, respectively, E is Young's modulus, ν Poisson's ratio, and

$$
\begin{aligned}
f^{d}_{ij} &= \frac{2}{3}\,E_t e_{ij} ,\\
\dot\phi &= \left\{\frac{2}{3}\,\dot e^{\rm in}_{ij}\,\dot e^{\rm in}_{ij}\right\}^{\frac{1}{2}},\\
b(\Gamma) &= \frac{A}{\psi(\Gamma) - E_t} ,
\end{aligned} \tag{2.23}
$$

where

$$
\Gamma = \left\{\frac{3}{2}\,\left(s_{ij} - g^{d}_{ij}\right)\,\left(s_{ij} - g^{d}_{ij}\right)\right\}^{\frac{1}{2}} , \tag{2.24}
$$

$\psi(\Gamma)$ is a function controlling the shape of the stress-strain curve, and $k(\Gamma)$ is a viscoelasticity function.

The material constant A which controls the stress level, in the case of a cyclic hardening material is considered to be a function of the accumulated inelastic strain ϕ. In the most general case which includes also hardening during nonproportional loading, a rate of A is proposed in [46] in the form

$$
\begin{aligned}
\dot A^{*} &= a_1 \dot P_i^{a_2} + a_5^{*}\,\dot P_0^{a_6} - a_3\dot\phi\left|\frac{A - A_0}{A_0}\right|^{a_4^{*}} ,\\
\dot A &= a_7|\dot A^{*}| + a_8\,\dot A^{*} ,
\end{aligned} \tag{2.25}
$$

with

$$
\begin{aligned}
a_4^{*} &= a_4 + a_9\,\left(1 - \exp\left(-a_{10}\,\dot P_0\right)\right) ,\\
a_5^{*} &= a_5\,\left(1 - \exp\left(-a_{11}\,\phi\right)\right) ,
\end{aligned} \tag{2.26}
$$

where $a_1 \div a_{11}$ and A_0 are positive constants and

$$
\dot P_i = \left\{\mathrm{tr}\;\dot e^{\rm in}\;e^{\rm in}\;\dot e^{\rm in}\;e^{\rm in}\right\}^{1/2}, \qquad \dot P_0 = \left\{\mathrm{tr}\;\Omega\Omega^{T}\right\}^{1/2} , \tag{2.27}
$$

where

$$
\Omega = e^{\rm in}\,\dot e^{\rm in} - \dot e^{\rm in}\,e^{\rm in}.
$$

Table 1. Material constants and functions for AISI type 304 stainless steel at room temperature (cyclic hardening version of VBO).

$E = 195000$ [MPa] $E_t = 2000$ [MPa]	$A_0 = 115$ [MPa]
Viscosity function:	$k(x) = k_1 \left(1 + \frac{x}{k_2}\right)^{-k_3}$
$k_1 = 314200$ [s]	
$k_2 = 60$	
$k_3 = 21.98$ [MPa]	
Shape modulus functions :	$\psi(x) = C_1 + (C_2 - C_1)\exp(-C_3 x)$
$C_1(A) = H_1 + H_2 A$	
$H_1 = 74740$ [MPa]	$C_2 = 182500$ [MPa]
$H_2 = 37.04$	$C_3 = 0.0783$ [MPa^{-1}]
Hardening parameters:	
$a_1 = 6 \times 10^5$ [MPa]	$a_7 = 0.495$
$a_2 = 1.0$	$a_8 = 0.505$
$a_3 = 3.6215 \times 10^3$ [MPa]	$a_9 = 1.1722$
$a_4 = 0.76$	$a_{10} = 1 \times 10$ [s]
$a_5 = 3.14 \times 10$ [MPa]	$a_{11} = 4.85$
$a_6 = 1.0$	

The path-like quantitaties $\dot{P}_0$ and $\dot{P}_i$ are responsible for hardening in out-of-phase and in-phase cycling, respectively. It is to be observed that whenever e_{ij}^{in} and $\dot{e}_{ij}^{\text{in}}$ are coaxial, P_0 is zero; P_i will accumulate always.

The specific form of the viscosity function k and shape modulus function ψ, as well as the material constants for AISI type 304 stainless steel for room temperature, are given in Table 1 quoted from [46].

For elevated temperature (650^0 C) the hardening data were unavailable, and the cyclic neutral version of the VBO theory was used, assuming A=const (i.e., $dA/dt = 0$) and C_1 in the shape of the modulus function being constant (i.e., also independent of A). The set of material constants for the cyclic neutral behaviour of AISI type 304 stainless steel at 650^0 C is given in Table 2 after [46].

Two stress states will be considered in the sequel: the uniaxial state of stress and the axial/torsion case.

The set of constitutive equations together with the equation for the rate of inelastic energy

$$\dot{W}_{\text{in}} = \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^{\text{in}} \tag{2.28}$$

and damage equation (2.20) modified for certain cases of loading (see the following chapter) were integrated numerically. The integrations made use of computer codes developed by the author of [46], slightly modified to fit the present purpose. The IMSL routine DEGAR was used to perform integration on the IBM 3081D computer of RPI. The calculations were time consuming since the life prediction was aimed, and the number of cycles to failure varied between 200 and about 2500 depending on the applied strain range. When a large number of cycles was involved, the computer time increased. For 2500 cycles a CPU time of 650 s on IBM 3081D was required.

Table 2. Material constants and function for AISI type 304 stainless steel at 650^0 C (cyclic neutral version of VBO).

$E = 150000$ [MPa]	$E_t = 2400$ [MPa]	$A = 170$ [MPa]
Viscosity function: $k_1 = 314200$ [s] $k_2 = 60$ [MPa] $k_3 = 17.59$	$k(x) = k_1\left(1 + \frac{x}{k_2}\right)^{-k_3}$	
Shape modulus function: $C_1 = 79500$ [MPa] $C_2 = 147000$ [MPa] $C_3 = 0.216$ [MPa^{-1}]	$\psi(x) = C_1 + (C_2 - C_1)\exp(-C_3 x)$	

3. Numerical simulation

3.1. Uniaxial cycling

3.1.1. Evaluation of material constants. The material constants which appear in Eq. (2.21) have to be evaluated from experiments corresponding to the phenomenon which is going to be described. In general, the loading situations loading to failure can be grouped into four categories: 1) monotonic loading, 2) creep (constant stress condition), 3) relaxation (constant strain condition), 4) cyclic loading.

In all these cases one can expect that different microstructural mechanisms will be involved. Consequently, the material constants in the proposed damage law can be different for different loading cases.

Since the immediate field of interest of this study is related to cyclic loading, let us consider this case first.

For cyclic loading, the results of experiments are usually reffered to inelastic energy per cycle (hysteresis energy) and to number of cycles to failure

N_f. For this purpose we reformulate Eq. (2.21) relating damage increment per cycle to inelastic energy per cycle W_h:

$$(l - D)^n \frac{dD}{dN} = C\, W_h^n \,. \tag{3.1}$$

In such a formulation a part of the loading history (that within one cycle) is lost. The conversion to time and total inelastic energy is possible with the substitutions

$$\begin{aligned} W_{\text{in}} &= N\, W_h \,, \\ t &= NT \,, \end{aligned} \tag{3.2}$$

where W_{in} is a function of time to be calculated for a given loading history,

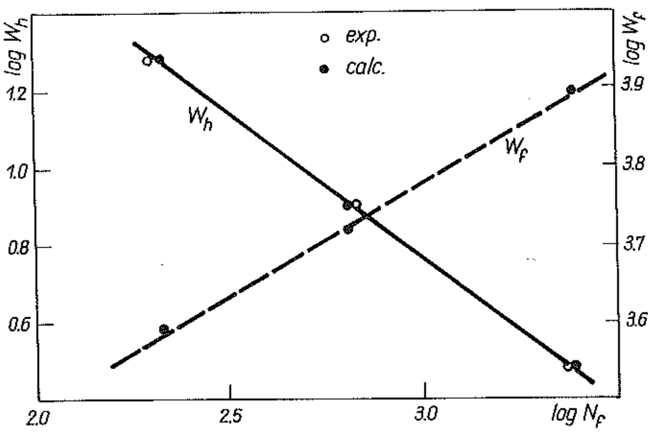

FIG. 1. Experimental [47] and calculated data for LCF, AISI Type 304 Stainless Steel at 650° C.

and T is a period of cycling. Experimental results reported by CHENG *et al.* [47] were used as a reference for low cycle fatigue. First, for given strain amplitudes $\epsilon_a = 2\%, 1\%$ and 0.5%, the hysteresis energy W_h was calculated according to VBO, and then a plot of W_h versus the experimental values of the number of cycles to failure (in double logarithmic scale) was made, Fig.1, to determine the constants C and n. With these equal to $C = 3.46 \times 10^{-5}$, $n = 1.393$, the computations were performed to calculate the number of cycles to failure $N_{f\text{calc}}$ and total inelastic energy to failure $W_{f\text{calc}}$. The results of calculations are given in Table 3. Then the calculated energy $W_{f\text{calc}}$ was plotted versus $N_{f\text{calc}}$. The almost linear dependence is in agreement with the experimental observations by other authors [32], as discussed in Sect. 2.1. A value of constant α reported by the authors of this paper (for A-516 Cr 70 carbon low alloy at room temperature) is $\alpha = -0.605$, which corresponds to $n = -1/\alpha = 1.653$ and which differs very slightly from that evaluated for the purpose of this study ($n = 1.393$).

Table 3. Neutral cycling simulation under strain control ($\epsilon = 4 \times 10^{-3}$ [s^{-1}] at 650^0 ($C = 3.46 \times 10^{-5}$), $n = 1.393$).

Strain ampl.	ϵ_a =2%	ϵ_a =1%	ϵ_a =0.5%
W_h (VBO) [MPa]	18.2	8.0	3.1
$N_{f\,exp}$	212	651	2472
$N_{f\,calc}$	200	680	2400
$W_{f\,calc}$ [MPa]	3870	5303	7725

The damage accumulation process is of a nonlinear character. Table 4 gives the values of the damage parameter and inelastic energy for a chosen number of cycles within a life span of a specimen subjected to cycling with the strain amplitude $\epsilon_a = 2\%$ and strain rate $d\epsilon/dt = 4 \times 10^{-3}$ [1/s] under strain control.

Table 4. Damage and inelastic energy growth during cyclic straining with $\epsilon_a = 2\%$ and $\dot{\epsilon} = 4 \times 10^{-3}$ [1/s].

t [s]	D	W_{in} [MPa]	N
500	0.0507	454	25
1000	0.1057	909	50
1500	0.1660	1363	75
2000	0.2329	1818	100
2500	0.3093	2272	125
3000	0.3997	2727	150
3500	0.5147	3181	175
4000	0.6922	3636	200
4250	0.4996	3863	212

For other types of loading (i.e., non-cyclic), the original version of the damage law Eq. (2.21) will be used with material constants denoted C' and n', and evaluated for different types of loading separately.

For monotonic loading (which may be viewed as a part of cyclic loading), the evaluation of material constants was not possible because of the limitation of the used theory to the small strains. Since elongation at static

Table 5. Creep rupture life for AISI 304 type stainless steel at 650^0 C, $C' = 1.55 \times 10^{-5}$, $n' = 1.85$.

σ [MPa]	$t_{f\,exp}$ [s]	$t_{f\,calc}$ [s]
97	0.360×10^7	0.396×10^7
69	0.360×10^8	0.326×10^8
48	0.360×10^9	0.393×10^9

failure for Type 304 Stainless Stell at 650^0,C [48] is about 35%, the use of the small strain theory is precluded.

The creep rupture data of [48] were used to evaluate the material constants for creep damage. Table 5 gives calculated and experimental values of time to failure for three stress levels, and for material constants $C' = 1.55 \times 10^{-5}$ and $n' = 1.85$. This good agreement does not necessarily have to be extrapolated for higher stress levels for which the experimental data were unavailable. In cycling with strain amplitude of order of 2%, the stress can be as high as 400 – 500 MPa. At such stresses again the different mechanisms can play a decisive role, and material constants have to be reevaluated. The data for failure in the single relaxation test are very scarce, and the phenomenon is not well investigated. However, it is known that the period of relaxation during strain-controlled cycling may reduce the lifetime of a specimen. In this situation an interaction of different mechanisms may occur and deterioration should be reffered to this particular situation. For this reason the values of the material constants in the damage law will be given when the effect of hold times is discussed in Sect. 3.1.4.

3.1.2. Rate/frequency effect. The rate of loading has a considerable influence upon material response when viscous effects are taken into account. This effect is included into the VBO theory. The increasing rate of loading under strain control causes an effect of hardening. In Fig. 2 three hysteresis loops are shown as calculated according to VBO for strain amplitude $\epsilon_a = 2\%$ and strain rates $4 \times 10^{-3}, 4 \times 10^{-4}$, and 4×10^{-5} [1/s]. As a consequence, the higher the rate of loading, the higher the hysteresis energy and, according to the proposed damage law, the higher the rate of damage. However, in elevated temperature an opposite effect is observed: more damaging is cycling with a lower rate (cf. Fig. 4 in [13]). To describe this behaviour, Eq. (3.1) has to be modified by including the frequency effect. Such a modification is similar to that proposed by OSTERGREN [29] for a parametric representation of the energetic approach. This can be accomplished by

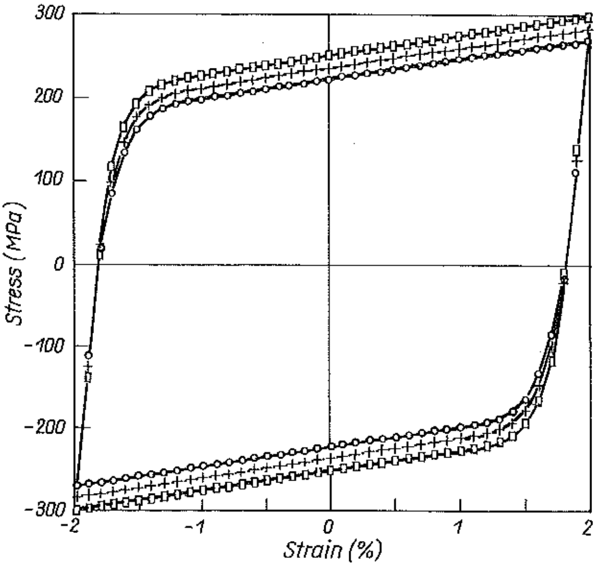

FIG. 2. Strain rate effect on hysteresis loops according to VBO (□ – strain rate 4×10^{-3}, + – strain rate 4×10^{-4}, ○ – strain rate 4×10^{-5}).

assuming:

$$(1-D)^n \frac{dD}{dN} = T^k \, C W_h^n, \tag{3.3}$$

where $k > 0$ is a constant whose value determines the range of life reduction. For rates of loading $4\times10^{-3}, 4\times10^{-4}$ and 4×10^{-5} [1/s] and strain amplitude $\epsilon_a = 2\%$ with corresponding T equal to 20, 200 and 2000 sec., respectively, the calculated reduction of a life for two values of the constant k is shown in Table 6. By adjusting to a proper value of k, the experimentally observed life reduction can be met.

Table 6. Life reduction due to rate effect (at $\epsilon_a = 2\%$).

$\dot{\epsilon}$ [s^{-1}]	T [s]	σ_a [MPa]	W_h [MPa]	k=0.1		k=0.2	
				N_f	W_f	N_f	W_f
4×10^{-3}	20	299.0	18.2	212	3863	212	3863
4×10^{-4}	200	283.6	17.1	181	3097	143	2449
4×10^{-5}	2000	270.0	16.1	152	2524	93	1511

3.1.3. Wave-shape effect. A triangular wave-shape was assumed throughout the numerical simulation, but different rates of loading and unloading may influence the lifetime of a specimen. This effect is called a fast/slow - slow/fast (FS/SF) effect, and it is experimentally observed that in elevated temperature the SF case has a more damaging effect. The shape of the hysteresis loop calculated by the VBO theory is different for FS and SF cases (Fig. 3) but the area of hysteresis loop remains unchanged. Since this quantity governs a damage growth, the FS/SF effect can be described only after some modifications in Eq. (3.1) are made. Let us denote the loading

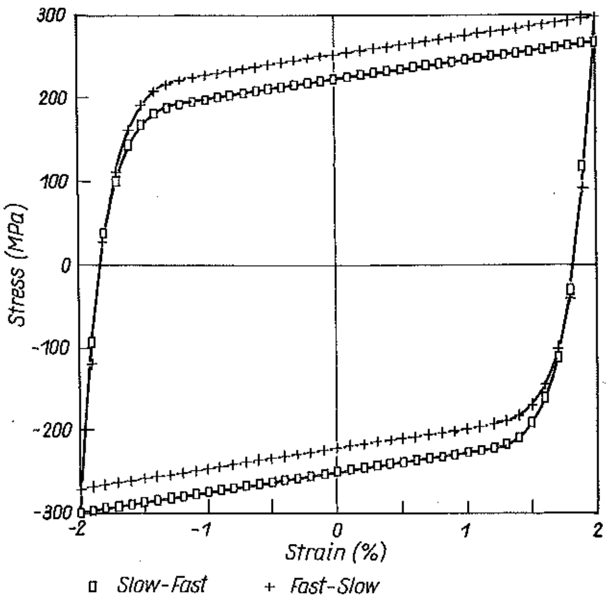

FIG. 3. Slow/fast and fast/slow loading/unloading effect.

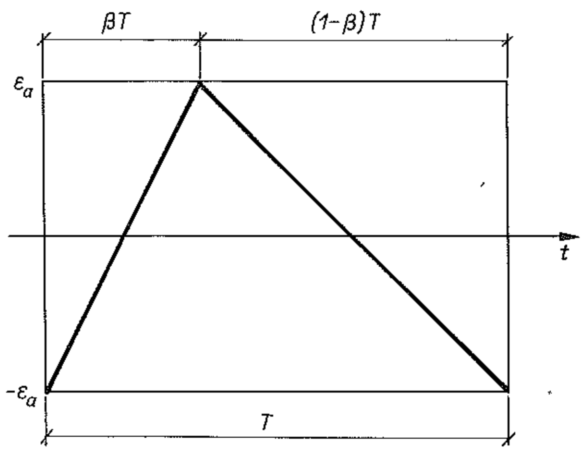

FIG. 4. Wave-shape factor β definition.

part of a cycle by $\beta T\,(0 < \beta < 1)$, whereas the unloading part becomes

$(1-\beta)T$ (Fig. 4). Equation (3.1) is postulated to have the form

$$(1-D)^n \frac{dD}{dN} = T^k \; [2/3\,(l+\beta)]^b \; CW_h^n \,. \tag{3.4}$$

For $\beta = 1/2$ the effect of wave-shape is reduced to the previously discussed rate effect, when the more damaging slow loading case was demonstrated. The influence of β on a lifetime is given in Table 7 for $\epsilon_a = 2\%, d\epsilon/dt = 4 \times 10^{-3}$ [1/s] ($T = 20$ sec) and $b = 1$.

Table 7. Fast/slow – slow/fast effect ($\epsilon_a = 2\%$, $\dot{\epsilon} = 4 \times 10^{-3}$ [s^{-1}]).

Type of loading	Fast/Slow		F/F&S/S	Slow/Fast	
β	0.1	0.3	0.5	0.7	0.9
N_f	240	235	212	170	136
% of life reduction	113	111	100	80	64

A considerable reduction of a life is predicted for SF loading, and the percentage of life reduction is higher in this case than the beneficial effect of FS loading. Again, the proper choice of b can allow the fitting experimental data.

3.1.4. Hold-time effect. The life reduction due to hold times under strain control depends on both the length of the time period when strain is kept constant and the position along a hysteresis loop. Since no distinction was made according to damage accumulation in the tensile or compressive part of hysteresis loop, the latter effect is not included.

The length of a hold-time period has an obvious influence on a lifetime as damage growth according to Eq. (2.21) depends on accumulated inelastic

Table 8. Hold-time effect strain control $\epsilon_a = 2\%$, $\dot{\epsilon} = 4 \times 10^{-3}$, $C' = 1.55 \times 10^{-12}$).

t_h [s]	t_f [s]	N_f
0	4250	212
300	57700	180
600	88090	142

energy. The material constants in Eq. (2.21) suitable for creep failure overestimate the influence of hold times in cycling with the strain amplitude

$\epsilon_a = 2\%$ and $d\epsilon/dt = 4\times10^{-3}$ [1/s]. With the corresponding stress amplitude $\sigma_a = 299$ [MPa], the numerical calculations exhibit almost instantaneous failure. A new value of material constant C' was chosen to predict realistic life reduction shown in Table 8 for two hold times. This value of C' differs from that for creep failure prediction in the order of 7. A similar situation can be encountered for different loading conditions, like monotonic loading preceding cycling under strain control (mean strain effect).

3.1.5. Mean strain effect. The area of a hysteresis loop does not depend on mean strain but on the strain range and strain rate applied. This behaviour is confirmed by experimental observation [34] as well as by calculation according to the VBO theory. One possible way to incorporate the effect of mean strain into an energetic approach is to assume that the elastic energy also contributes to damage creation. This method was used in [34], and in [45] for fatigue crack growth description. As the elastic, reversible energy contribution to a damaging process is not physically justified, this method will not be used for the purpose of this study. Instead, Eq. (2.21) will be used again for a loading up to the maximum strain with a new value for the material constant $C' = 1.55\times10^{-4}$. Table 9 gives values of the number of cycles to failure for several values of the mean strain ϵ_m and strain amplitude $\epsilon_a = 2\%$, as previously.

Table 9. Mean strain effect ($\epsilon_a = 2\%$, $\dot{\epsilon} = 4\times10^{-3}$ [s^{-1}])

$C' = 1.55\times10^{-4}$, $n' = 1.85$.

ϵ_m [%]	t_f [s]	W_f [MPa]	N_f
0	4250	3868	212
1	4088	3717	204
2	3820	3468	191
3	3343	3039	167

3.1.6. Hardening effect. Material data necessary to perform calculation according to VBO for hardening were available for 304 stainless steel for room temperature only (Table 1). Since the material constants for a damage law were evaluated for elevated temperature, the analysis in this section has only a qualitative character.

The calculation for strain cycling with the strain amplitude of 1% and

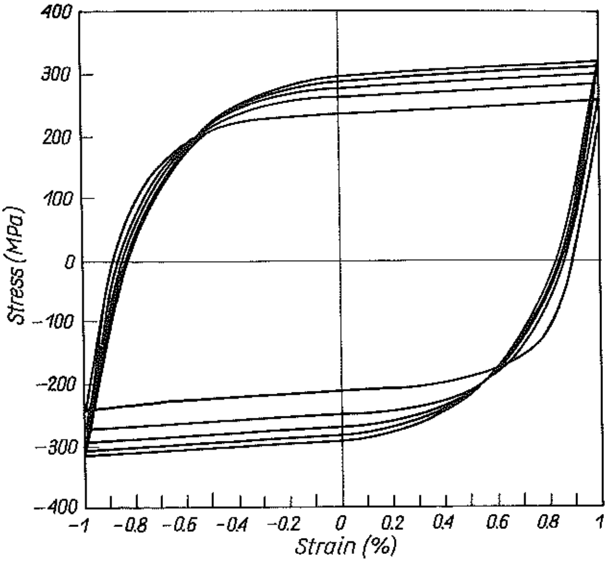

FIG. 5. Cyclic hardening within first five cycles.

strain rate 4×10^{-3} [1/s] exhibited pronounced hardening. Figure 5 shows hysteresis loops for the first 5 cycles during which stress amplitude rises from 260.3 to 320 [MPa] with a saturated value of 362 [MPa] after about 60 cycles. Such a behaviour is expected to have an essential influence on life-time, but it can be viewed as both harmful or advantageous with respect to classical (parametric) life evaluation. The standard procedure employs half-life characteristrics, i.e., saturated values of stress amplitude. In this case an underestimate of a lifetime can be expected. However, in the opposite case the use of unsaturated characteristics can yield considerable overestimation. Table 10 gives the number of cycles to failure calculated under

Table 10. Cyclic hardening effect ($\epsilon_a = 1\%$, $\dot{\epsilon} = 4 \times 10^{-3}$).

Hysteresis loop	σ_a [MPa]	t_f [s]	N_f	W_f
Initial	213.3	8388	839	5693
Actual	213.3 – 362.6	5078	507	4994
Saturated	362.6	4990	499	4919

these assumptions together with the result of the calculation for a whole loading history. The underestimation caused by the use of saturated characteristics proved to be negligible, which is a consequence of the fact that during the first cycles the damage increment is small, even with a number of cycles to saturation of about 10% of a lifetime. In contrast, the overestimation of a lifetime may be essential when prediction is based on the initial characteristics of a hysteresis loop. In the case illustrated by Table 10 this overestimation is about 165% compared with the calculations based on the actual history of loading.

3.1.7. Stress/strain control effect. All of the above calculations illustrating the influence of different effects upon the lifetime of a material were performed under the assumption that a strain was chosen as the forcing parameter.

Under stress control one can expect that the shape of the hysteresis loop will change, affecting damage accumulation. However, the comparison must be made under clearly precised conditions. Figures 6(a-c) show hysteresis loops for cyclic neutral behaviour calculated by the VBO theory under the following assumptions:

in strain control: $\epsilon_a = 2\%, d\epsilon/dt = 4 \times 10^{-3}$ [1/s] (T =20s), with correspondng stress amplitude of $\sigma_a = 299$ [MPa];

in stress control mode 1: $\sigma_a = 299$ [MPa], $d\sigma dt = 59.8$ [MPa/s] (T =20 s), with corresponding strain amplitude of about 1.7%;

in stress control mode 2: $\sigma_a = 306$ MPa, $d\sigma/dt = 61.2$ [MPa/s] (T =20 s), with corresponding strain amplitude 2%.

The differences in the shape and area of different hysteresis loops are easily seen. The results of lifetime calculation for these three cases are given in Table 11. The difference in number of cycles is small when in stress control the same strain is enforced as in strain control. However, cycling under strain control with the same stress as in stress control can reduce the life by a factor of more than 1.5. It is also to be observed that a small difference in stress amplitude under stress control may cause a great difference in the lifetime.

Table 11. Strain/stress control effect ($T = 20$ s).

Control	σ_a [MPa]	max ϵ_a [%]	t_f [s]	N_f	W_f [MPa]
Strain	299	2	4250	212	3868
Stress	299	1.7	7005	350	5732
	306	2	3920	196	3792

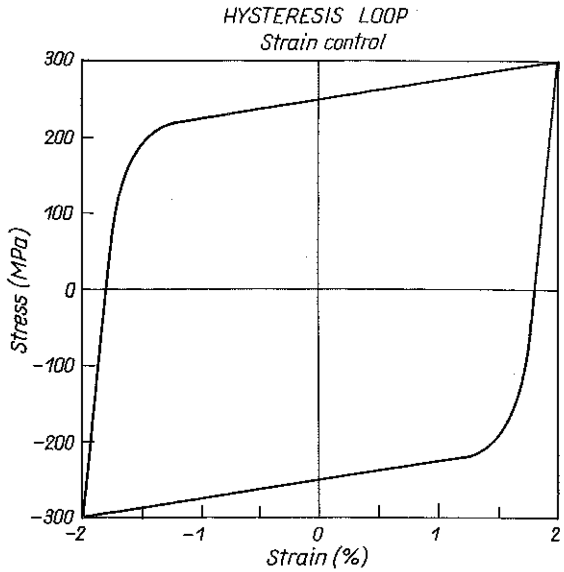

FIG. 6. a. Hysteresis loop under strain control.

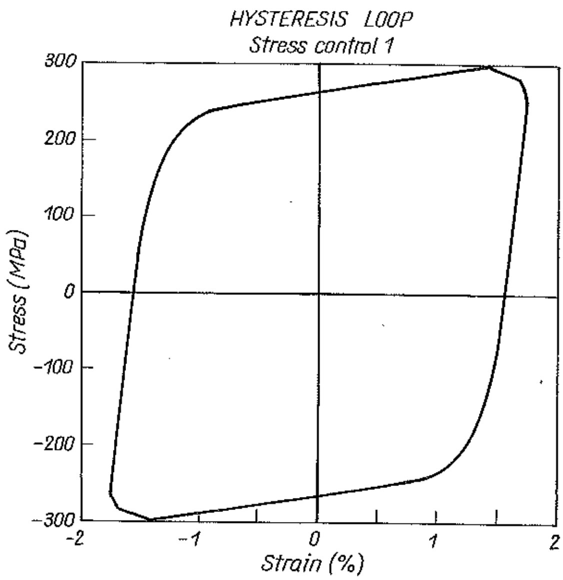

FIG. 6. b. Hysteresis loop under stress control mode 1.

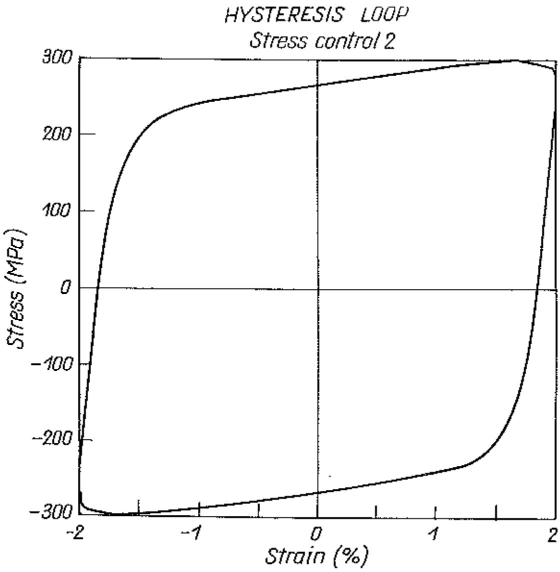

FIG. 6. c. Hysteresis loop under stress control mode 2.

3.2. *Multiaxial cycling*

3.2.1. Proportional loading. The case of simultaneous axial and torsional cycling was chosen as an example of multiaxial loading. The comparison of different states of stress was made for cycling under strain control with the same effective strain amplitude and effective strain rate defined as follows:

$$(3.5) \qquad \dot{\epsilon}_{a,eff} = \left(\epsilon^2_{a,11} + \frac{4}{3}\,\epsilon^2_{a,12}\right)^{1/2},$$

$$(3.6) \qquad \dot{\epsilon}_{eff} = \left(\dot{\epsilon}^2_{11} + \frac{4}{3}\,\dot{\epsilon}^2_{12}\right)^{1/2}.$$

Under these circumstances the VBO theory predicts accumulation of inelastic energy independent of the path of loading, i.e., of the ratio $\epsilon_{11}/\epsilon_{12}$. In consequence, there is no difference in a lifetime for all proportional paths (including pure torsional and pure axial cases) according to the proposed damage law. Indirect comparison with experimental data made by GARUD [38] did not confirm this result. The calculated plastic energy for torsional cycling with effective strain amplitude $\gamma = 1.5\epsilon_{11}$, i.e. $\epsilon_{12} = 0.75\epsilon_{11}$ was shown to be smaller than that for axial cycling in the experimental results by KANAZAWA *et al.* [49]. In this case the life prediction based on axial data would be conservative. A weighted factor of 1/2 was suggested in [38] to be applied to the plastic work done by the applied shear strain to correlate failure data independently of the loading path. However, the calculations by Garud were made for time-independent plasticity, and for effective strain amplitude, different from that used here ($\epsilon_{12} = 0.866\epsilon_{11}$). The present analysis which makes use of the definitions (3.5) and (3.6) allows to estimate the lifetime on the basis of the uniaxial test only. Thus the experiments relating damage directly to the dissipated energy during loading along different paths are necessary to verify the results of calculation based on the proposed damage accumulation law.

The above observations hold for both cyclic neutral and cyclic hardening materials, and the conclusions of Sect. 3.1.6 apply to multiaxial proportional loading as well.

3.2.2. Non-proportional loading. For the purpose of this study a special case of 90^0 out-of-phase cycling was chosen as an example of non-proportional loading. The comparison with proportional loading was made using the definitions of strain amplitude and strain rate given by Eqs. (3.5) and (3.6).

Table 12. In-phase/out-of-phase loading for $\epsilon_{a,eff} = 1\%$, $\dot{\epsilon}_{eff} = 4 \times 10^{-3}$ [s^{-1}].

Material behaviour	Loading cond.		$N=1$	$N=2$	$N=3$	$N=4$	$N=5$	N_f
Neutral	In-phase	W_{in}	10.0	(constant $W_h = 10$)			51.8	
	(satur.)	D	.0008	.0016	.0024	.0032	.0040	499
	Out-of-	W_{in}	11.1	23.6	34.7	45.8	56.9	
	-phase	D	.0070	.0017	.0026	.0035	.0045	425
Hardening		W_{in}	9.3	17.6	26.4	35.3	44.5	
	In-phase	D	.0005	.001	.0017	.0023	.0030	507
	Out-of-	W_{in}	23.7	53.4	85.2	118.1	151.7	
	-phase	D	.0014	.0039	.0070	.0105	.0142	91

The formulation of the VBO theory (Sect. 2.3) allows to take into account the effect of additional hardening which results from the non-coaxiality of strain and strain rate tensors. For cyclic neutral material, the out-of-phase loading results in some hardening (reflected by higher accumulation of the inelastic work) due to a loading path different from that during in-phase cycling. The quantitative results depend on the applied effective strain amplitude and effective rate. The plastic work calculated in [38] for time-independent plasticity and for 90^0 out-of-phase loading is about 4 times as much as for axial cycling with the strain amplitude 0.2%. The character of the diagram showing the dependence of the plastic work on applied strain amplitude (Fig. 8 of [38]) suggests that this factor can be even higher for higher strain amplitudes. To illustrate this effect, the strain amplitude of 1% with the effective rate of loading 4×10^{-3} [1/s] was chosen. The results of calculation are summarized in Table 12. The values of accumulated inelastic energy and damage are given for the first five cycles to demonstrate the different behaviour for different materials (neutral/hardening) and loading conditions (in-and out-of-phase). The last column of Table 12 gives the numbers of cycles to failure. A drastic life reduction is observed when both effects are combined. The stress amplitudes in this case rise from $\sigma_{11} = 213.5$ [MPa], $\sigma_{12} = 62$ [MPa] to $\sigma_{11} = 560.2$ [MPa], $\sigma_{12} = 324.4$ [MPa], whereas, in the case of proportional loading with hardening, the stress amplitude reaches its saturated value of 362 [MPa] at about 60 cycles (about 10% of lifetime).

4. Conclusions and final remarks

The aim of this study was to verify the application of the proposed damage law for a wide range of loading conditions. As these also included high temperature application, the rate effects were included by means of the use of appropriate constitutive equations (the VBO theory). With such a formulation of the main goal of the present report, a detailed study of individual cases of loading was beyond the scope of the investigation. However, material constant evaluation and comparison of predictions were based on experimental data for AIS type 304 stainless steel at 650° C.

The proposed damage law, with the inelastic energy as a governing variable chosen for its universality in different loading situations, has proved to be able to predict some typical behaviour and lifetime of this type of material. Though some of these predictions were only of a qualitative character, the others show a good agreement with experimental data.

The formulation of the damage law used in this study was as simple as possible and it resulted in several drawbacks of the proposed description:

1. Two different types of equations for a damage law were used for cyclic loading and for other situations. Both were assumed to be governed by inelastic energy; however, inelastic energy per cycle W_h was used for cyclic loading, whereas the total inelastic energy governed the damage law for other types of loading.

2. To fit experimental data for different phenomena (monotonic loading, creep, relaxation), it was necessary to apply different values of the material constants even for the same type of damage law. Since numerical simulation

was used, the fact that different constants had to be used did not complicate calculations. However, the consistent set of experimental data should be generated to determine material constants in all damage equations.

3. For some phenomena like wave shape or frequency effects, the damage law had to be modified by introducing some prescribed characteristics of a process (period of cycling T, wave shape factor β).

4. The damage accumulation was, in general, assumed to depend on the total accumulated energy and, therefore, the linear summation rule did not apply. For each given loading history, the numerical integration had to be perfomed to predict the lifetime.

5. The assumption was made that damage accumulation for cyclic loading was affected by the hysteresis loop area only, but not by its shape nor position. In consequence, some other assumptions were necessary to correlate experimental observations.

As far as the future development is concerned, the following directions may be worthy to consider:

1. The experimental support of the assumption that a part of the energy supplied is used for damage growth is of prime importance. A relation of this energy to the inelastic strain energy is another step in such an investigation.

2. Other variables connected to the energy should be considered as governing variables. These may include both the inelastic strain and inelastic strain rate.

3. The coupling between the deformation process and damage process is the next step in improving the proposed method, if some deformation behaviour may not be explained by the deformation theory itself. Softening, which often occurs in the very early stages of the deformation process (cf. [4]), is one such phenomenon.

4. The deformations which are connected with a failure in certain circumstances (instantaneous loading, creep failure) are often out of the range of small strain deformation theories. The finite strain theory developed along with an appropriate modification of the proposed law is an important factor of the applicability of the obtained results.

Acknowledgements

This work was performed under the financial support of the U.S. Department of Energy.

The author appreciates the encouragement and fruitful discussion with Professor E.KREMPL and his kind invitation to a joint work in the Mechanics of Materials Laboratory, RPI, during the academic year 1987/88. Mr S.H.CHOI is acknowledged for the preparation of computer codes and helpful suggestions on numerical calculations.

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Streszczenie

PRAWO USZKODZEŃ DLA ZMĘCZENIA NISKOCYKLICZNEGO W WYSOKICH TEMPERATURACH, KONTROLOWANE ENERGIĄ ODKSZTAŁCENIA

Energia niesprężysta odkształcenia, która była wielokrotnie używana jako wielkość korelująca wyniki badań zmęczeniowych, została w pracy wykorzystana jako niezależna zmienna w prawie kumulacji uszkodzeń. Przedyskutowane zostały specjalne postacie tego prawa, a także oszacowane stałe materiałowe w nim występujące dla stali nierdzewnej typu 304 w warunkach niskocyklicznego zmęczenia i pełzania w temperaturze 650^0 C. Dla jednoosiowego stanu naprężenia rozważono możliwości opisu różnych efektów, jak prędkość obciążania i odciążania, kształt cyklu, efekt sterowania odkształceniami bądź naprężeniami, wpływ naprężenia średniego, czy wreszcie wpływ umocnienia na żywotność elementu konstrukcyjnego. Stan wieloosiowy został zilustrowany łącznym obciążeniem osiowym i skrętnym, z uwzględnieniem efektu obciążenia nieproporcjonalnego. Energia niesprężysta odkształcenia była obliczana według teorii VBO (Viscoplasticity Based on Overstress), która, jak to pokazano w szeregu prac E.Krempla, dobrze opisuje deformację w warunkach obciążeń cyklicznych.

Резюме

ЗАКОН ПОВРЕЖДЕНИЙ, ДЛЯ НИЗКОЦИКЛИЧЕСКОЙ УСТАЛОСТИ В ВЫСОКИХ ТЕМПЕРАТУРАХ, КОНТРОЛИРОВАННЫЙ ЭНЕРГИЕЙ ДЕФОРМАЦИИ

Неупругая энергия деформации, которая многократно использовалась как величина коррелирующая результаты усталостных исследований, использована в работе как независимая переменная в законе кумуляции повреждений. Обсуждены специальные виды этого закона, а также оценены выступающие в ним материальные постоянные для нержавеющей стали типа 304 в условиях низкоциклической усталости и ползучести в температуре 650^0 С. Для одноосного напряженного состояния рассмотрены возможности описания разных эффектов, таких как скорость нагружения и разгружения, форма цикла, эффект управления деформациями или напряжениями, алияние среднего напряжения, или наконец влияние упрочнения на живучесть конструкционного элемента. Многоосевое состояние иллюстрируется общим осевым и скручивающим нагружением, с учетом эффекта непропорционального нагружения. Неупругая энергия деформации рассчитана согласно теории VBO (Viscoplasticity Based on Overstress,) которая, как это показано в ряде работ Э.Кремпля, хорошо описывает деформацию в условиях циклических нагружений.

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Received November 21, 1990.