ENGINEERING TRANSACTIONS • *Enging. Trans.* • **39**, 2, 181-190, 1991
Polish Academy of Sciences • Institute of Fundamental Technological Research

THE INFLUENCE OF TENSILE STRESS ON CREEP DISTORTION

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An experiment is presented concerning the process of creep in a plane state of stress under non-proportional loading by a torque and a tensile force. At a fixed value of the torque, the influence of a tensile force upon the shear deformation of non-steady creep process was analyzed. The PA4 aluminium alloys were tested. The plastic resistance function was used to describe the deformation. The concept of slip lines as proposed by Batdorf, Budiansky was used. The theoretical results derived comply with experimental data.

1. Introduction

The problem of creep in a complex state of stress has not yet been solved in an exhaustive manner. The problem is additionally complicated in the case of non-proportional loads. Phenomena occuring in the structure of the material influence in an essential manner the creep process taking place under constant load. This subject is treated on fairly broad grounds in the works of W. Trąmpczyński [1, 2], who points out the considerable variety of behaviour of the material in the course of a creep process preceded by plastic predeformation. The influence of predeformation may be considered to constitute a sum of two effects: a non-oriented (scalar) effect which is a function of effective strain and an oriented effect, which depends on the angle between the principal direction of the strain tensor of predeformation and that of the stress tensor in the course of the creep process.

In [2, 3] it was shown, using as an example the creep process of copper, that the scalar effect is dominant. The experimental technique used was that proposed by W. Szczepiński [4]. The classical technique of tests in a plane state of stress (based on test pieces in the form of thin-walled copper tubes subjected to tension and torsion) was used in [5]; the influence of the oriented effect was shown to be essential.

On the basis of alloy steel and aluminium-copper alloy tests, the authors of [6] and [7] suggest that plastic predeformation results in anisotropic hardening (oriented effect). According to those authors, preliminary tension of the test piece by an axial force produces no hardening effect in the case of, for instance, subsequent torsion.

The mechanism of plastic strain of metals is complex and is based chiefly on slips occurring in definite systems of planes and directions. It is assumed that the material is hardened, above all, in regions of slips. If the load increases in a proportional manner, the hardening phenomenon occurs in the same systems in which slips occur during the creep process, after the load increase has been stopped. The resulting orientation of those systems in a given slip region remains unchanged during the entire deformation process. If the loading is not proportional, the orientation of slip systems varies during the loading process. It is only after the load increase has been stopped that this orientation is stabilized in the course of creep and becomes, after a sufficiently long time, independent of the loading path. This phenomenon was observed in [8].

For mathematical description and interpretation of the mechanisms of plastic strain discussed, the use of the idea of slips in a manner as presented by Batdorf and Budiansky [9] appears to be justified. The Batdorf-Budiansky theory was at first considered to be of cognitive importance, and its development proceeded in various directions. Among other interesting papers in this domain let us mention those of P. Dłużewski [10].

One of the directions of development of the slip theory is the idea of making use of the results of investigation of physics of solid bodies, its object being to formulate a plastic resistance function in such a manner that instantaneous plastic and creep strain can be described by the same equations, both strains being treated jointly as a permanent deformation varying in time [8].

The aim of the present paper is to suggest a method of applying the Batdorf–Budiansky plastic resistance function for quantitative description of the influence of the tensile force on the distortion during nonstationary creep. This influence was recorded during experimental investigations.

2. Experimental investigation

The creep phenomenon under non-proportional loading by a torque and a tensile force was studies by experimental means. The history of the load applied is represented by a diagram in Fig.1. Table 1 contains the values of the shear stresses τ_{xz} and the normal stresses σ_z for which the research program was carried out. The test pieces used were thin-walled tubular specimens (gauge length 75 mm, external diameter 17.5 mm, wall thickness 0.75 mm) made of PA4 industrial aluminium alloy. The tests were performed at room temperature using a creep test stand ensuring a plane state of stress. The angle of twist of a test piece was measured by means of dial gauges with minimum graduation of 0.001 mm, in cooperation with using three test pieces for each state of load marked by a plus sign in Table 1. The total number of test pieces was 45. The time of creep after the tangential stress τ_{xz} was applied was 50 hours. Next, normal stress σ_z was applied and the

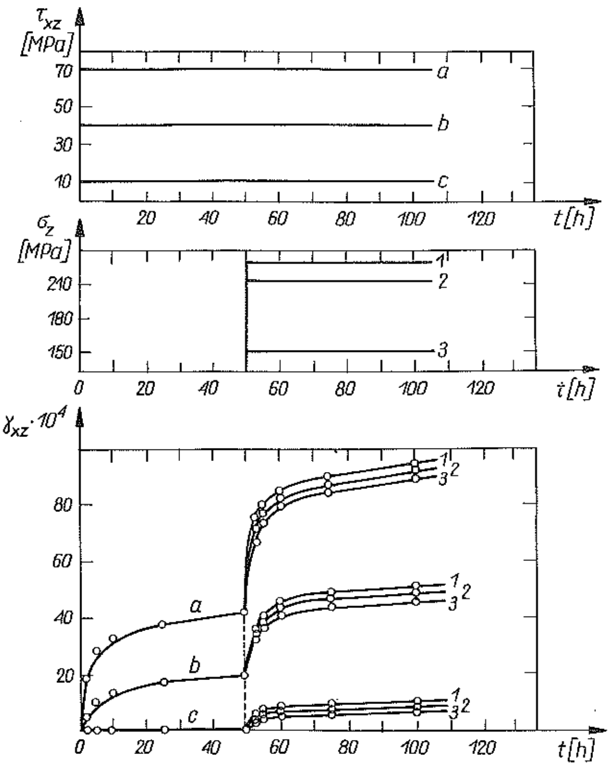

FIG. 1.

Table 1. Stresses used in the analysis.

σ_z [MPa] \ τ_{xz} [MPa]	0	10	40	70
0		+	+	+
150	+	+	+	+
212	+	+	+	+
227	+	+	+	+

variation of the distortion γ_{xz} was studied for another period of 50 hours. The average (for three test pieces) experimental values of γ_{xz} are represented, for various states of stress as functions of time, by points in Fig.1.

No influence of σ_z on γ_{xz} was observed for $\tau_{xz} = 0$.

3. Theoretical description

The Batdorf–Budiansky theory [9] is based on the assumption that microscopic deformation is a result of an infinite number of slips of every possible orientation in a slip system. The orientation of a slip system n, l can easily be determined by means of the three Eulerian angles α, β, ω, which are represented, on a hemisphere of unit radius, in Fig.2. The total deformation

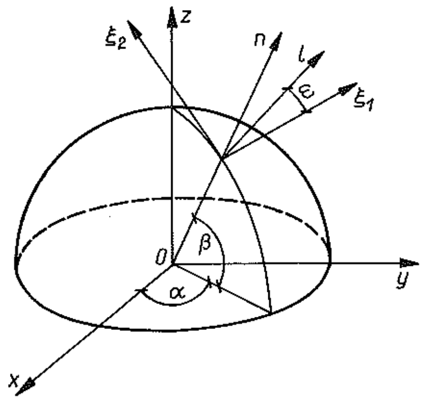

FIG. 2.

is

$$(3.1) \qquad \gamma_{ij} = \frac{1}{2} \int_{\Omega} \int_{\omega_1}^{\omega_2} \varphi(\tau)(n_i l_j + n_j l_i)\, d\Omega\, d\omega \qquad (i,j = x,y,z)\,.$$

where Ω is the region of the hemisphere in which slips occur; $d\Omega = \cos\beta\, d\alpha\, d\beta$; ω_1, ω_2 are the boundaries of the slips in planes tangent to the hemisphere within the region Ω, φ – slip density function, τ – tangential stress in the n, l system and n_i, l_j are the direction cosines of the n and l axis, respectively, with reference to the coordinates x,y,z, determined by the formulae

$$(3.2) \qquad \begin{aligned} l_x &= -\sin\alpha\cos\omega - \cos\alpha\sin\beta\sin\omega\,, \\ l_y &= \cos\alpha\cos\omega - \sin\alpha\sin\beta\sin\omega\,, \\ l_z &= \cos\beta\sin\omega\,, \qquad n_x = \cos\alpha\cos\beta\,, \\ n_y &= \sin\alpha\cos\beta\,, \qquad n_z = sin\beta; \end{aligned}$$

$$(3.3) \qquad \tau = \sigma_{ij} l_i n_j\,, \qquad (i,j = x,y,z)\,.$$

The relation (3.1) does not describe creep, nor takes into consideration the effect of slips in one system on slips in other systems.

The idea presented below will enable us to use the classical Batdorf–Budiansky theory for describing the creep phenomenon and explaining the complex character of the mechanism of plastic strain. The plastic resistance function is assumed to have the form

$$S = S(S_0, I, \varphi, \alpha, \beta, \omega) \ , \tag{3.4}$$

where S_0 is the initial plastic resistance (for $\varphi = I = 0$) and I – a parameter of concentration of internal stress expressed by the formula

$$I = B \int_0^{t_1} \frac{\partial \tau}{\partial s} Q(t-s) \, ds \ , \tag{3.5}$$

where B is a material constant, $Q(t-s)$ – a decreasing function of the time difference between t and s, and t_1 – the limit of the time of growth of the external stress.

If the stress σ_z is applied at a constant rate $\dot{\sigma}_z$, $Q(t-s) = \exp[-b(t-s)]$ (b – structural constant) is assumed, and the expressions (3.2) for the direction cosines are taken into account as well as the relation (3.3), the parameter I is expressed, by virtue of (3.5), by the relation

$$I = \frac{1}{2} I_z \sin 2\beta \sin \omega \ , \tag{3.6}$$

where

$$I_z = B\dot{\sigma}_z \int_0^{t_1} \exp[-b(t-s)] ds$$

is the value of the parameter in the z-direction. If the load is applied at a sufficiently high rate, such as for instance the processing rate, the parameter I_z , may be expressed with sufficient accuracy, in the case of tension, by the equation

$$I_z = B\sigma_z \exp(-bt) \ , \tag{3.7}$$

where σ_z is the value of the external stress and t – the time measured from the instant of application of σ_z.

It is assumed that the condition for the occurrence of slip in the system n, l is the equality

$$\tau = S \ . \tag{3.8}$$

In regions where no slips occur we have

$$\tau < S \ .$$

The relations (3.1), (3.4), (3.8) are fundamental for the idea to be presented.

Let us now determine the influence of the stress σ_z on the creep distortion γ_{xz}. Our task is to construct a plastic resistance function S such as to enable us to obtain, by virtue of the condition (3.8), a slip density function

φ which would give, on substituting into (3.1), a correct description of the deformation process. Let us assume that

$$S = S_0 \left[1 + r_1 \int_{\omega_1}^{\omega_2} \varphi \cos(\omega - \omega_0)\, d\omega_0 + I \right], \tag{3.9}$$

where $\cos(\omega - \omega_0)$ is a function describing the mutual action of slip occuring in the same plane but in different directions (that is ω and ω_0) and r_1 is a material constant.

It was observed in the Introduction that the direction of slip varies in the course of the creep process under non-proportional load. This can be proved by means of the relation (3.9). Let us divide the entire creep process into short periods p, the number of those periods being k. Then, in the k-th period, S is determined by the relation

$$S = S_0 \left[1 + r_1 \sum_{p=1}^{k-1} \int_{\omega_1^{(p)}}^{\omega_2^{(p)}} \Delta\varphi^{(p)} \cos(\omega - \omega_0)\, d\omega_0 + r_1 \int_{\omega_1^{(k)}}^{\omega_2^{(k)}} \Delta\varphi^{(k)} \cos(\omega - \omega_0)\, d\omega_0 + I \right]. \tag{3.10}$$

Making use of Eqs.(3.10),(3.8),(3.6) and (3.3), we obtain

$$r_1 \int_{\omega_1^{(k)}}^{\omega_2^{(k)}} \Delta\varphi^{(k)} \cos(\omega - \omega_0)\, d\omega_0 = A_1^{(k)} \sin\omega + A_2^{(k)} \cos\omega - 1, \tag{3.11}$$

where

$$\begin{aligned} A_1^{(k)} &= q_2 \cos\alpha\, \cos 2\beta - r_1 \sum_{p=1}^{k-1} \int_{\omega_1^{(p)}}^{\omega_2^{(p)}} \Delta\varphi^{(p)} \sin\omega_0\, d\omega_0 \\ &\quad + q_1 \sin 2\beta - \frac{1}{2} I_z \sin 2\beta, \\ A_2^{(k)} &= -q_2 \sin\alpha \sin\beta - r_1 \sum_{p=1}^{k-1} \int_{\omega_1^{(p)}}^{\omega_2^{(p)}} \Delta\varphi^{(p)} \cos\omega_0\, d\omega_0, \end{aligned} \tag{3.12}$$

and

$$q_1 = \frac{\sigma_z}{2S_0}, \qquad q_2 = \frac{\tau_{xz}}{S_0}.$$

The influence of τ_{xz} on the quantity I has not been taken into account in Eqs.(3.12), this influence being negligibly small as compared with that of σ_z.

The equation (3.11) is a Fredholm integral equation of the first kind with a degenerate kernel and has a sense only for a discrete -value of ω.

Its solution is

$$\Delta\varphi^{(k)} = \Delta\varphi_n^{(k)}\delta\left(\omega - W^{(k)}\right), \tag{3.13}$$

where δ is the Dirac function, $\Delta\varphi_n^{(k)}$ – a new, unknown function of angles α, β, and $W^{(k)}$ – the angle of direction of slip, as yet unknown, in the plane n.

The above solution makes us observe that $\Delta\varphi^{(k)}$ is different from zero only for $\omega = W^{(k)}$, which means that slips in a definite plane may occur, at any instant of time, only in one direction $\omega_1^{(k)} = \omega_2^{(k)} = \omega = W^{(k)}$.

Bearing in mind that $S \geq \tau$, the value of the angle $W^{(k)}$ can be obtained from the condition of tangency of the curves of plastic resistance $S^{(k)}$ and shear stress $\tau^{(k)}$ (the functions $S^{(k)}$ and $\tau^{(k)}$ are differentiable). Thus

$$\frac{\partial S^{(k)}}{\partial\omega} = \frac{\partial\tau^{(k)}}{\partial\omega}, \qquad \left(\omega = W^{(k)}\right). \tag{3.14}$$

In other directions $(\omega \neq W^{(k)})$ $S^{(k)} > \tau^{(k)}$ and, therefore, no slips occur. From the condition (3.14), the relations (3.10), (3.3) and (3.12) being taken into account, we obtain

$$A_1^{(k)} \cos W^{(k)} = A_2^{(k)} \sin W^{(k)}$$

and

$$\sin W^{(k)} = \frac{A_1^{(k)}}{\sqrt{\left[A_1^{(k)}\right]^2 + \left[A_2^{(k)}\right]^2}}, \quad \cos W^{(k)} = \frac{A_2^{(k)}}{\sqrt{\left[A_1^{(k)}\right]^2 + \left[A_2^{(k)}\right]^2}}. \tag{3.15}$$

From Eqs.(3.15) and (3.12) it follows that in the case of non-proportional loading the direction of the slips does not coincide with that of maximum shear stress in the plane n.

On substituting (3.13) into (3.11) and bearing in mind that $\omega = \omega_0 = W^{(k)}$ we obtain

$$r_1\Delta\varphi_n^{(k)} = A_1^{(k)} \sin W^{(k)} + A_2^{(k)} \cos W^{(k)} - 1, \tag{3.16}$$

where

$$\begin{aligned} A_1^{(k)} &= q_2 \cos\alpha \cos 2\beta + q_1 \sin 2\beta - r_1 \sum_{p=1}^{k-1} \Delta\varphi_n^{(p)} \sin W^{(p)} - \frac{1}{2} I_z \sin 2\beta, \\ A_2^{(k)} &= -q_2 \sin\alpha \cos\beta - r_1 \sum_{p=1}^{k-1} \Delta\varphi_n^{(p)} \cos W^{(p)}. \end{aligned}$$

Making use of the relation (3.1) and taking into consideration the properties of the Dirac function, the increase in creep distortion in the k-th period will be expressed by the formula

$$\Delta\gamma_{xz}^{(k)} = \frac{1}{2}\iint\limits_{\Omega^{(k)}} \left(l_x^{(k)} n_z + l_z^{(k)} n_x\right) \Delta\varphi_n^{(k)} d\Omega\,, \tag{3.17}$$

where $l_z^{(k)}$ and $l_x^{(k)}$ are values of the direction cosines according to Eq.(3.2), for $\omega = W^{(k)}$.

The boundary of the region of slips $\Omega^{(k)}$ is determined from the equation

$$\Delta\varphi_n^{(k)} = 0\,. \tag{3.18}$$

The curve (3.18) divides the hemisphere into regions, in which slips occur in the k-th period $\left(\Delta\varphi_n^{(k)} > 0\right)$ and those in which no slips takes place in that period $\left(\Delta\varphi_n^{(k)} \le 0\right)$.

Rigorous analytical determination of the creep deformation in the case of non-proportional loading is difficult. This concerns the determination of the boundaries of the slip regions (3.18) and the solution of (3.17). The problem can be easily solved by numerical methods. To this aim the hemisphere of Fig.2 is divided into a great number of sufficiently small regions $\Delta\Omega_i = \cos\beta_i \Delta\alpha_i \Delta\beta_i$. The integrals in Eq.(3.17) are approximated by sums. The creep deformation in the k-th period of time depends on the slips in the i-th region $\Delta\Omega_i$ and is expressed by the formula

$$\left(\Delta\gamma_{xz}^{(k)}\right)_i = \frac{1}{2}\Big(\cos\alpha_i\, \sin W_i^{(k)}\, \cos\beta_i\, \cos 2\beta_i - \frac{1}{2}\sin\alpha_i\, \sin 2\beta_i\, \cos W_i^{(k)} \left(\Delta\varphi_n^{(k)}\right)_i \Delta\alpha_i \Delta\beta_i\,. \tag{3.19}$$

This computation is performed for all the regions $\Delta\Omega_i$ of which the hemispherical region is composed, and the results are summed up, thus yielding the values of the strain in each particular period of the creep process

$$\Delta\gamma_{xz}^{(k)} = \sum_{i=1}^{r} \left(\Delta\gamma_{xz}^{(k)}\right)_i\,, \tag{3.20}$$

where r is the number of elementary regions $\Delta\Omega_i$ of slip $\left(\Delta\varphi_n^{(k)} > 0\right)$. The values $W^{(k)}$ and $\Delta\varphi_n^{(k)}$ for consecutive periods of time $k = 1, 2, 3, \ldots$ are found from the formulae (3.15) and (3.16), respectively.

To confront the results of the theory with those of experiment, numerical analysis was performed by dividing this hemispherical region into 1296 subregions $(\Delta\alpha_i = \Delta\beta_i = 5^0)$. The period for which the creep process was studies was divided into 10 non-equal periods $t = 0.01, 0.09, 0.2, 0.7, 1, 3, 5, 15, 25$ and 50 [h], which were then used with Eq.(3.7). For $r_1 = 9.3 \cdot 10^3$, $B = 8.8 \cdot 10^{-2}$ [MPa]$^{-1}$, $b = 0.33$ [h]$^{-1}$, $S_0 = 10$ [MPa] and for the prescribed external stresses τ_{xz} and σ_z which are given in Table 1, theoretical values of creep distortion γ_{xz} were obtained. They are represented in Fig.1 by a solid line.

4. Conclusions

The results of test of the isotropic material PA4, which is an annealed aluminium alloy, show that if $\tau_{xz} > 0$, there is a distinct influence of the normal stress σ_z on the distortion γ_{xz}. No such influence was observed for $\tau_{xz} = 0$. The test results can be described correctly by the theory discussed in the present paper. The method consisting in the plastic resistance function (3.9) being used with the Bratdorf–Budiansky theory of slip enables us to describe not only immediate plastic strain but also complex rheological processes under non-proportional load. Under simultaneous action of normal and shear stress there occur common regions of slip in the stressed material. The size of those regions may be determined from Eqs.(3.16) and (3.18) and it depends on the values and the ratio of the stresses σ_z and τ_{xz} which were applied. The slips in the common regions cause an increase in the corresponding components of plastic strain $\Delta\varepsilon_z$ and $\Delta\gamma_{xz}$. The distortion γ_{xz}, which is studies, is determined by the relation (3.20) which depends on σ_z through the medium of Eqs. (3.12), (3.16) and (3.19).

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Streszczenie

WPŁYW NAPRĘŻENIA ROZCIĄGAJĄCEGO NA ODKSZTAŁCENIE POSTACIOWE PEŁZANIA

W pracy przedstawiono eksperyment dotyczący pełzania w płaskim stanie naprężenia przy nieproporcjonalnym obciążeniu momentem skręcającym i siłą rozciągającą. Przy ustalonym momencie skręcającym zbadano wpływ siły rozciągającej na odkształcenie postaciowe pełzania nieustalonego. Badania przeprowadzono na stopie aluminium PA4. Do teoretycznego opisu odkształcenia zaproponowano wykorzystanie funkcji oporu plastycznego. Posłużono się koncepcją poślizgów w wersji przedstawionej przez Batdorfa i Budiansky'ego. Wyniki uzyskane teoretycznie wykazały zgodność z doświadczeniem.

Резюме

ВЛИЯНИЕ РАСТЯГИВАЮЩЕГО НАПРЯЖЕНИЯ НА ДЕФОРМАЦИЮ СДВИГА ПОЛЗУЧЕСТИ

В работе представлен эксперимент, касающийся ползучести в плоском напряженном состоянии, при непропорциональном нагружении скручивающим моментом и растягивающей силой. При установившимся скручивающим моменте исследовано влияние растягивающей силы на деформацию сдвига неустановившейся ползучести. Исследования проведены на сплаве алюминия PA4. Для теоретического описания деформации предложено использование функции пластического сопротивления. Послужились концепцией скольжений, в версии представленной Батдорфом и Будянским. Результаты, полученные теоретически, показали совпадение с экспериментом.

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Received May 10, 1990.