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BRIEF NOTES

KINEMATICALLY ADMISSIBLE SOLUTIONS FOR THE INCIPIENT STAGE OF EARTH MOVING PROCESSES IN THE CASES OF VARIOUS PUSHING WALL FORMS

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Kinematically admissible solutions for earth-moving processes due to various pushing wall forms (similar to the various tool shapes of loading machines as loaders, excavators, bulldozers) are presented in the case of plane strain conditions. On the basis of the mathematical theory of plasticity, theoretical solutions are obtained assuming the associated flow rule and the Coulomb–Mohr limit state, regarding incipient plastic motion. The analysis of several boundary problems shows that using only three types of simple kinematically admissible mechanisms it is possible to obtain results close to complete solutions or statically admissible solutions (obtained using the method of characteristics) for both the pushing force estimation and the motion area range.

1. Introduction

The problem of the active pressure exerted by soil on rigid walls of different shape in plane strain conditions is widely surveyed in literature [1, 2, 3, 4, 5]. It can be treated as a model for such important processes as soil shoving by the tools of such machines as bulldozers, excavators and loaders. Several theoretical solutions (for statics and for kinematics as well) were obtained within the theory of plasticity under the assumption of rigid-perfectly plastic soil behaviour [6, 7, 8] using the method of characteristics. Although such an assumption is a rather rough approximation of the real behaviour, it makes it possible to obtain effective

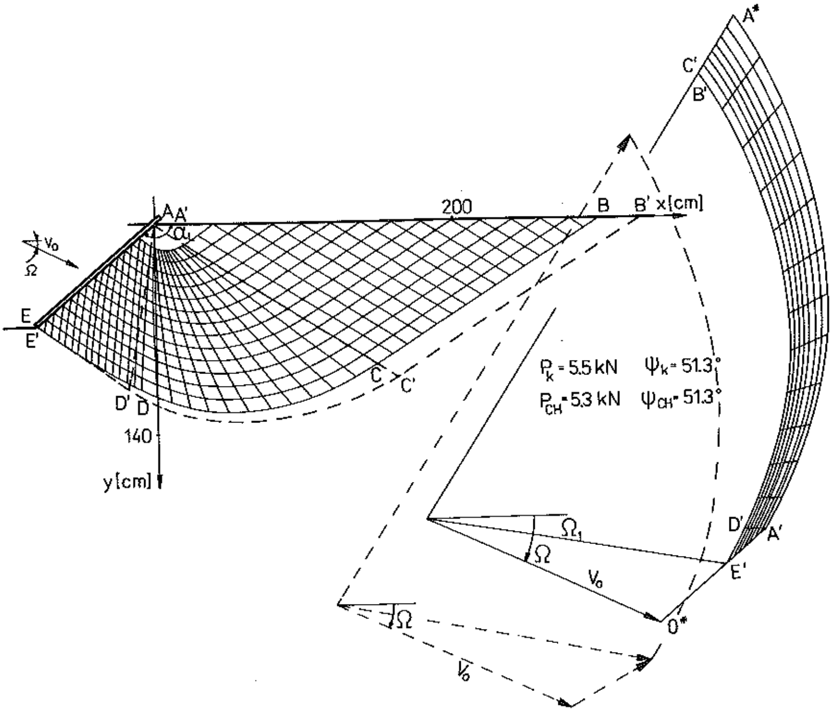

Fig. 1. Characteristics net and slip lines for kinematically admissible solutions for a particular problem.

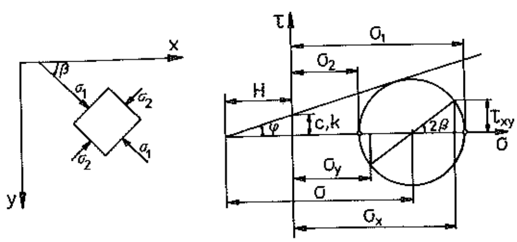

Fig. 2. Coulomb–Mohr yield criterion for plane strain conditions.

the incipient motion of a plane wall, within soil, is shown in Fig.1 (the solid lines) assuming the Coulomb–Mohr yield criterion (Fig.2) and the associated flow rule proposed by Drucker and Prager [13] (the coefficient μ describes the friction between the wall and the medium).

If the solutions obtained this way satisfy the condition of non-negativeness of the energy dissipation

$$\dot{\varepsilon}_{ij}\sigma_{ij} \geq 0 \tag{1.1}$$

as well as the velocity boundary conditions, the solution appears to be kinematically admissible and the calculated stresses determine the upper bound (limit load theorems). In the case where additionally the extension of the stress state into a rigid region [9] can be determined and the stress boundary conditions are fulfilled, the solution is complete.

A broad discussion of the kinematically admissible solutions for different wall shapes, free boundary shapes and different medium is presented in [7, 10]. It appears that, apart from the fact that a lot of problems can be solved this way, there are certain limits within which the proper solutions can be obtained. These limits have to do with the direction of the rigid wall motion, the rigid wall shapes and the free boundary shapes. The last one restricts also the possible solutions only to the initial rigid wall motion.

For example, in the cases shown in Figs.3, 4, 5 (solid lines) the energy dissipation appears to be negative in some points (calculations were made for the soil described by the following parameters: $\gamma = 22\text{kN/m}^3$, $\varphi = 25°$, $\mu = 0.2$, $c = 49\text{kPa}$) and the obtained solutions can be treated as the only statically admissible ones.

In several papers [14, 15] it was shown that the associated flow rule, for the Coulomb–Mohr material, is not a good approximation of the real material behaviour. It also concerns both the dilatation effect, which this theory overestimates, and the calculated strain zone range. So the non-associated flow rules should be used for proper material description. On the other hand, the limit load theorems which are used for the estimation of solutions presented in this paper can be properly formulated only for the associated flow rule.

The aim of this paper is to compare the solutions obtained using the method of characteristics (assuming the associated flow rule) with kinematically admissible solutions (the upper bound according to the limit load theorems) for the same problem.

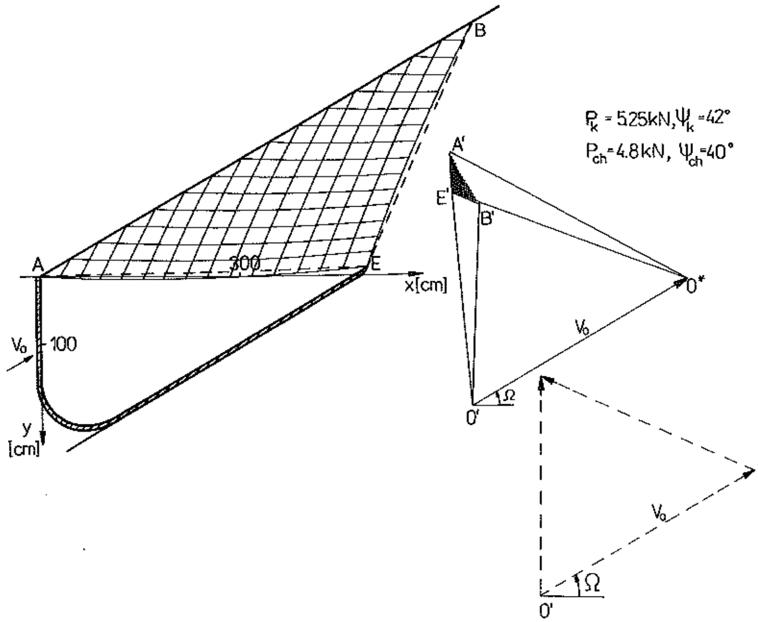

Fig. 3. Characteristics net and slip lines for kinematically admissible solutions for a particular problem.

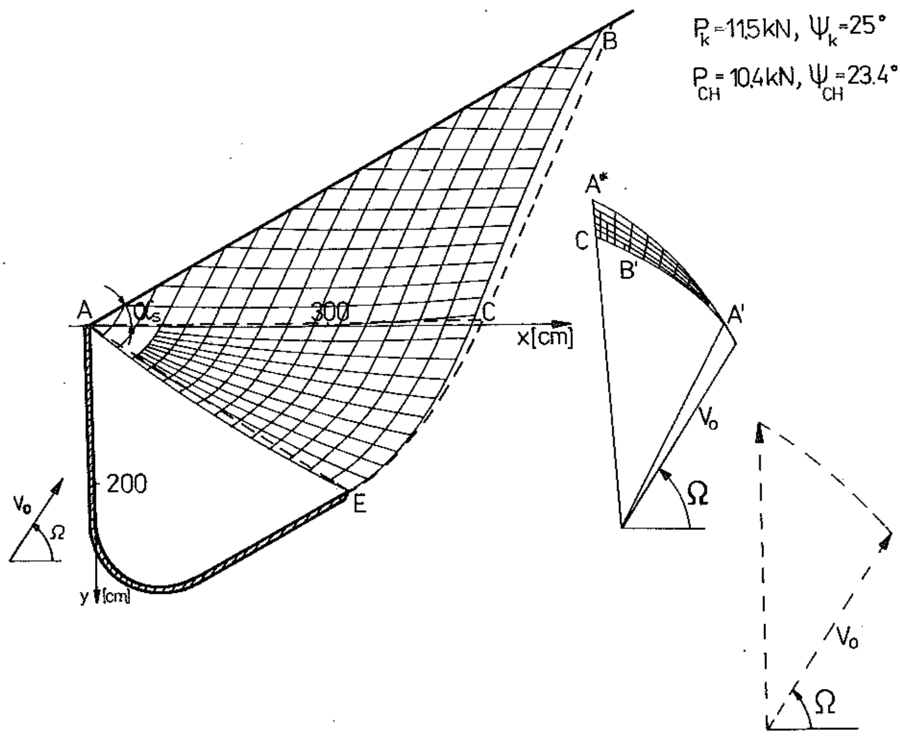

Fig. 4. Characteristics net and slip lines for kinematically admissible solutions for a particular problem.

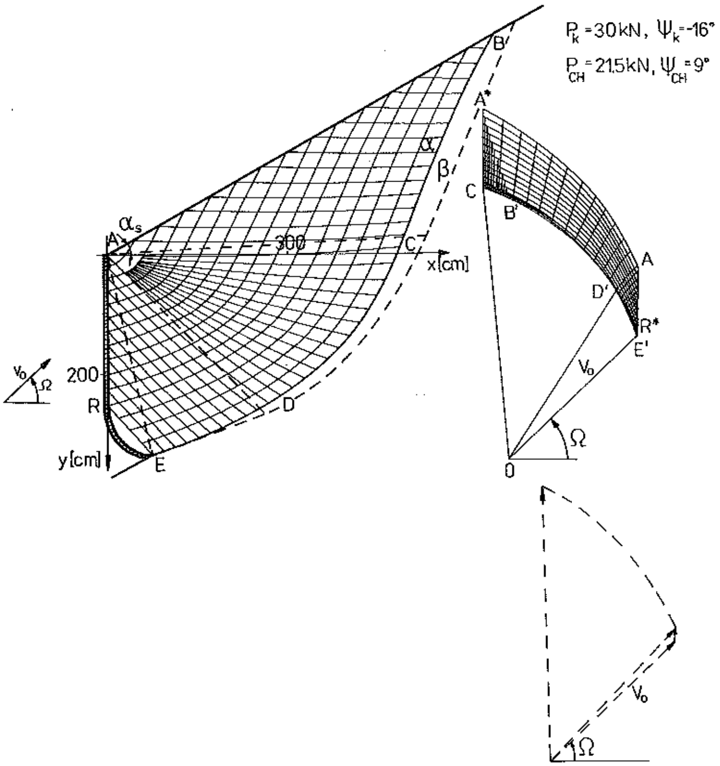

Fig. 5. Characteristcs net and slip lines for kinematically admissible solutions for a particular problem.

Taking into account the remarks mentioned above, such a comparison should be treated as a quantitative one. In some cases the difference between kinematically admissible solutions and complete ones is shown; in others only the difference between the upper and lower bound.

2. Kinematically admissible mechanisms for rigid wall shoving assuming the associated flow rule and the Coulomb–Mohr yield criterion

Let us discuss the problem of rigid wall shoving [5] shown schematically in Fig.6. The upper bound of the force P acting on the wall can be obtained assuming an arbitrary, kinematically admissible mechanism (limit load theorems). Comparing the energy due to the force P with

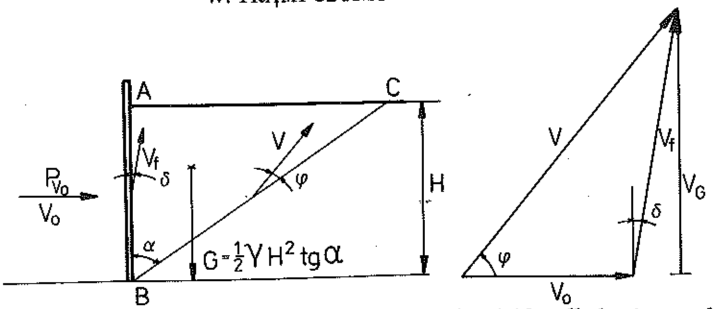

Fig. 6. Kinematically admissible mechanisms for the rigid wall shoving problem.

that necessary to overcome the gravity force G and that dissipated on the slip lines, the P value can be calculated [5].

In Fig.6 the kinematically admissible mechanism consisting of the stiff wedge ABC sliding along the discontinuity line BC and BA is shown. Assuming the associated flow rule, it is possible to show that for the Coulomb–Mohr yield criterion (Fig.2) the energy dissipation (1.1) for plane strain conditions is described by the following relation [5]:

$$D = c \cdot \cos\varphi(\dot{\varepsilon}_1 - \dot{\varepsilon}_2) \tag{2.1}$$

and along the discontinuity line BC (Fig.7) the unit energy dissipation is

$$D_L = c \cdot \cos\varphi \cdot \Delta V_L. \tag{2.2}$$

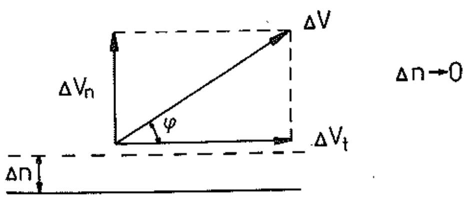

Fig. 7. Slip line velocity conditions.

Taking the friction rule between the wall and the medium in the form [12]

$$|\tau_t| = c_f + \sigma_f \mathrm{tg}\delta, \tag{2.3}$$

where τ_t is the shear stress along a contact line, σ_t is the normal stress along the contact line, $\mu = \mathrm{tg}\delta$ denotes the friction coefficient, c_f is the unit adhesive force, the unit energy dissipation along AB is described by the following relation:

$$D_f = c_f \cdot \Delta V_f \cdot \cos\delta. \tag{2.4}$$

So the upper force can be calculated from the following energy equation:

$$P_v \cdot V = G \cdot V_G + c_f V_f \cdot \cos\delta + c \cdot V_L \cdot \cos\varphi, \tag{2.5}$$

where P is the upper force, G is the gravity force, V_G is the convection velocity.

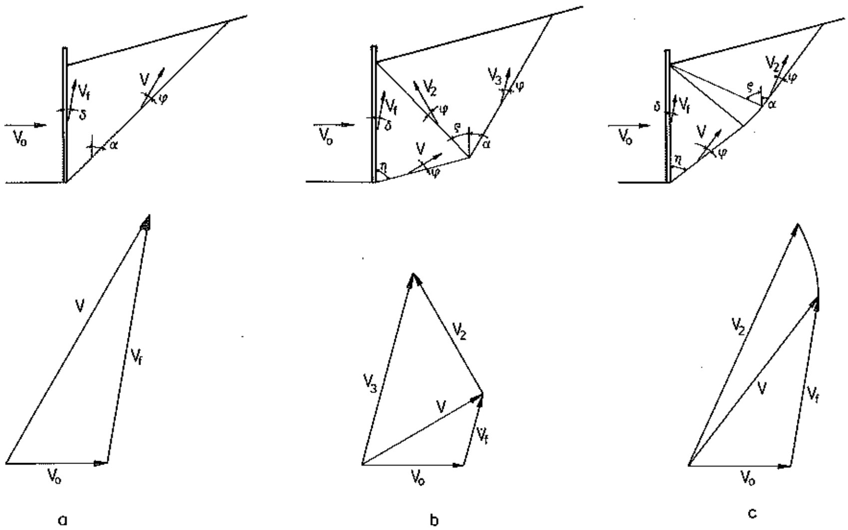

Fig. 8. Three basic kinematically admissible mechanisms.

In this paper three following types of mechanisms (Fig.8) will the be considered:

1) with one discontinuity line (Fig.8a),
2) with three discontinuity lines (Fig.8b),
3) with a logarithmic discontinuity line (Fig.8c)

(kinematically admissible solutions for such mechanisms are discussed in detail in [5]).

In every case the values of α, ρ and η (which describe the solutions) were found to give minimal energy dissipation.

3. Calculated results

The theoretical results in the case of incipient plane wall motion obtained using the method of characteristics (the complete solutions [9]) are compared with kinematic ones (drawn by the dashes lines —

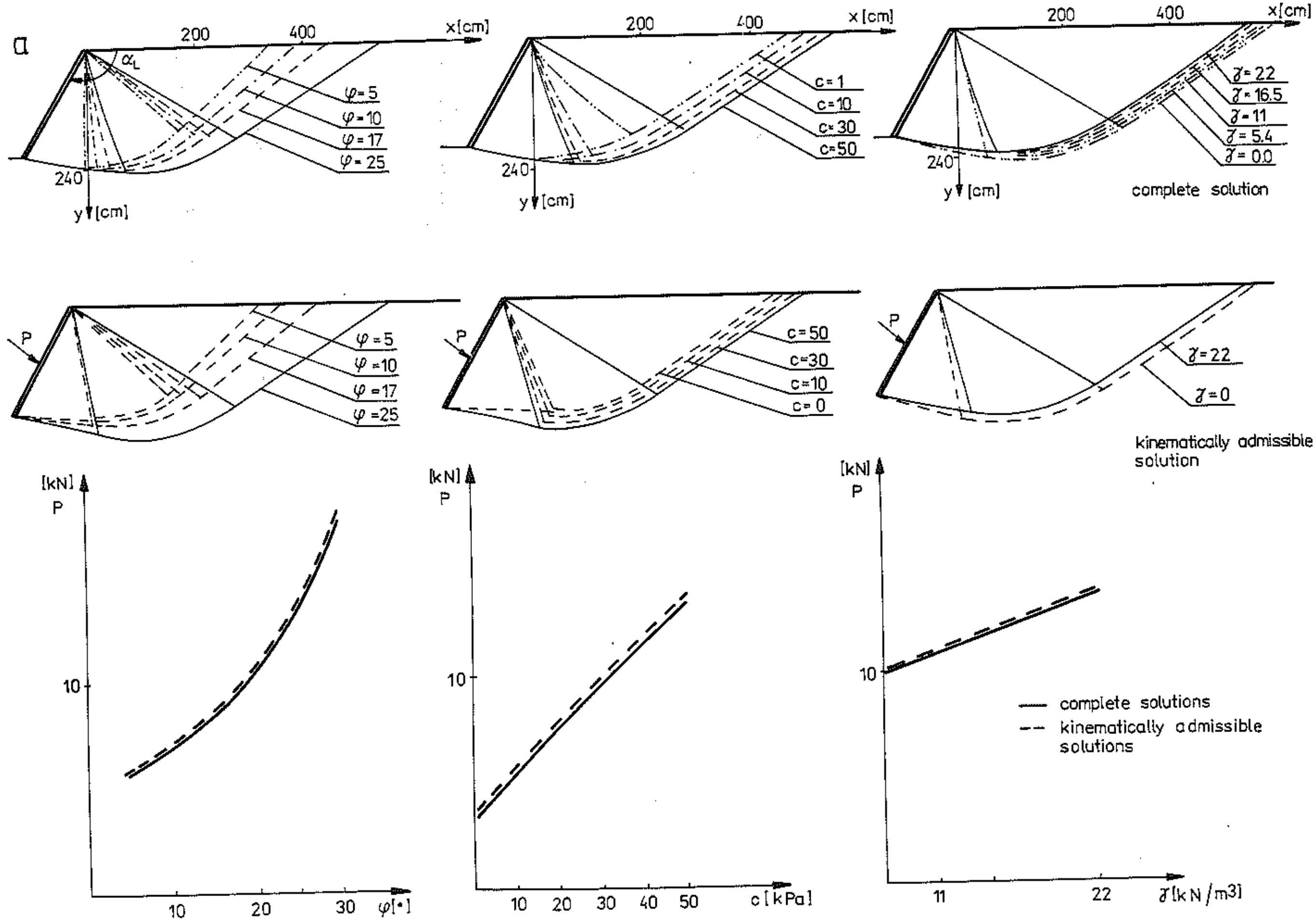

Fig. 9a. Comparison between complete solutions obtained using the method of characteristics and kinematically admissible solutions.

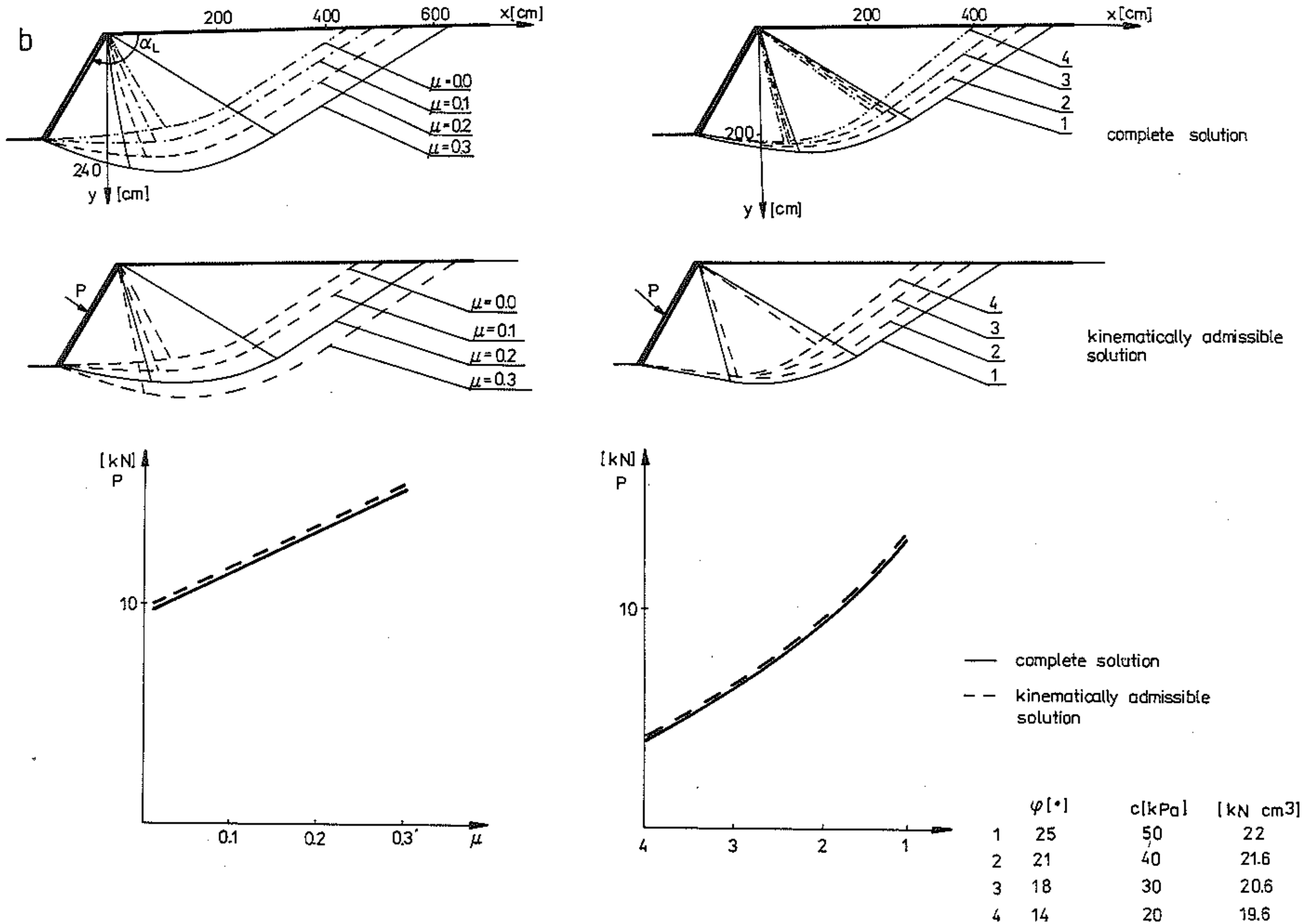

Fig. 9b. Comparison between complete solutions obtained using the method of characteristics and kinematically admissible solutions.

kinematic mechanisms and hodographs) for various force P directions and values and for the moved soil range and shown in Fig.9a and b, and in Fig.1 (P_{CH} — P force value calculated by the characteristics method, P_K — P force value calculated from the kinematically admissible mechanisms, the angle ψ_{CH} — P force direction calculated from the kinematically admissible mechanisms).

In Fig.9 this comparison is made for different material parameters. It is shown that in all cases the difference between the complete solutions and the kinematically admissible ones is less than 5%.

In Figs.3 and 4 statically admissible solutions for different rigid wall shapes obtained using the method of characteristics (solid lines), (in this case the method gives negative energy dissipation values and the presented solutions are only statically admissible – the upper bound can be calculated), are compared with kinematically admissible ones (drawn by the dashed lines – kinematic mechanisms and hodographs). It concerns both the plastic zone range (statically admissible solution) and the moved soil zone (kinematically admissible solution) comparison and the acting force direction (the angle ψ) and values. A difference less than 10% between the upper and lower bounds was observed.

In the case shown in Fig.5 a difference of more than 30% was observed. This is due to the fact that in the case of statically and kinematically admissible solutions two different problems were solved. In the former one – the problem of a limit state imposed by the wall ARE (Fig.5: $AR - \delta = 0.2$, $c = 0$; $RE - \delta = 25°$, $c = 49$kPa) was under consideration. In the second one – the kinematically admissible mechanism for the AE wall motion ($AE - \delta = 25°$, $c = 49$kPa) was found.

4. Conclusions

The simple kinematically admissible solutions were used to solve the problem of rigid wall shoving. It is shown that, using three types of mechanisms (Fig.8), results close to complete solutions (Figs.1 and 9) and to statically admissible ones (Fig.3 and 4) can be obtained this way. So one can suggest that they can be treated as a relatively simple and good approximation for earth-moving processes (one has to be sure that the same problem is solved in the case of statically and kinematically admissible solutions — Fig.5). Hence it is possible to obtain upper bounds in cases where the method of characteristics does not provide complete solutions or its use is very complicated. In particular, it is expected that not only incipient motion but the whole rigid wall shoving process can be described in such a way.

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