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MAGNETOHYDRODYNAMIC UNSTEADY FREE CONVECTION FLOW OF A VISCOELASTIC FLUID ALONG A VERTICAL PLATE

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The unsteady free convection flow of an incompressible electrically conducting viscoelastic fluid past an oscillating vertical plate in the presence of a transverse magnetic field has been studied. The flow phenomena have been characterized by the nondimensional numbers P (Prandtl number), G (Grashof number), m (magnetic number), ω (frequency number) and k (viscoelastic parameter). It is found that the temperature profile may be compared with a damped harmonic wave propagating in a direction perpendicular to the plate. The dimensionless temperature as well as the thermal boundary layer δ_T increase with the decrease of P. The effects of P, G and m on the velocity field u_r and velocity boundary layer thickness δ for the viscoelastic fluid are similar to that for a Newtonian fluid. As k increases, u_r and δ increase simultaneously. Opposite effects are noticed at a certain distance away from the plate. The skin friction τ_r at the plate is estimated for different values of P, G, m and k.

1. Introduction

The laminar flow behaviour of a non-Newtonian fluid set into motion by temperature-induced buoyancy forces is of importance in a number of geophysical and other engineering applications, such as petroleum drillings. Previous studies for free convenction flow along a vertical flat plate were restricted, in general, to Newtonian fluid only. In a series of papers discussed below, the unsteady laminar boundary layers of a Newtonian fluid on a semi-infinite plate are considered. Williams [1] assumed wall temperature that vary with time and position and found possible semisimilar solutions for a variety of classes of wall temperature distribution. Wang [2] studied the uncoupled boundary-layer problem where a fluid on a finite heated plate is suddenly set into motion. NANBU [3] estimated the limit of pure conduction for unsteady free convection on a vertical flat plate. Unsteady free convection from an infinite vertical plate has been considered by ILLINGWORTH [4], RAO [5], NANDA and SHARM [6], MENOLD and YANG [7], SCHETZ and EICHHORN [8], GOLDSTEIN and BRIGGS [9]. The unsteady free convection flow of a NEWTONIAN fluid in the presence of a magnetic field has been studied by GUPTA [10], CHAWLA [11], SOUNDALGEKAR [12] and MISHRA [13]. The unsteady free convection of a non-Newtonian power law fluid along a vertical wall is studied numerically by HAG, KLEINSTREUER and MULLIGAN [14]. Other approximate solutions for the laminar free convection of a power-law fluid along an isothermal vertical wall have been reviewed by SHENOY and MASHELKAR [15].

Our aim in this work is to study the free convection flow of a WALTER'S B' viscoelastic fluid [16] past a vertical plate whose velocity and temperature fluctuate with time harmonically. The method of solution is suggested by LIGHTHILL [17], STUART [18] and MESSIHA [19]. The effects of the Prandtl number P, Grashof number G, magnetic number m, frequency ω and the viscoelastic parameter k on the velocity, skin friction and temperature have been studied.

2. FORMULATION OF THE PROBLEM

Here the origin of the coordinate system is taken to be at any point of a flat vertical infinite plate. The x' axis is chosen along the plate vertically upwards and the y' axis perpendicular to the plate. It is assumed that the temperature difference between the plate and the fluid is small, so that the fluid properties may be taken as constant. In the special case when the flow is independent of x' and the velocity normal to the plate v' vanishes everywhere, the unsteady free convection flow of an incompressible viscoelastic fluid in the presence of a magnetic field is governed by the following equations of momentum and energy [13, 16]:

$$\rho'\frac{\partial u'}{\partial t'} = \rho' f_x\beta(T'-T'_\infty) - \sigma B_o^2 u' + \mu\frac{\partial^2 u'}{\partial y'} - K'\frac{\partial^3 u'}{\partial t'\partial y'^2}, \tag{2.1}$$

$$\rho' c'\frac{\partial T'}{\partial t'} = \gamma'\frac{\partial^2 T'}{\partial y'^2}. \tag{2.2}$$

In these equations ρ' is the density, u' the velocity in the x'-direction, t' the time variable, f_x the acceleration due to gravity, β the coefficient of volume expansion, T' the temperature in the boundary layer, T'_∞ the temperature far away from the plate, σ the conductivity of medium, B_o the applied magnetic field strength, μ the coefficient of viscosity, k' the viscoelastic parameter, c' the specific heat at constant pressure and γ' the thermal conductivity. In the energy equation (2.2) the terms representing viscous and Joule dissipation are neglected as they are really very small in free convection flows.

The boundary conditions of the problem are

at

$$y' = 0 : u' = u'_m e^{i\omega' t'}, \qquad T' = T'_\infty + (T'_m - T'_\infty)e^{i\omega' t'}, \tag{2.3}$$

at

$$y' \to \infty : u' = 0, \qquad T' = T'_\infty. \tag{2.4}$$

where u'_m and T'_m are the maximum velocity and the maximum temperature of the plate, respectively, and ω' is the frequency of fluctuation.

Let us introduce the following dimensionless variables

$$\begin{aligned} &\eta = \frac{y' u'_m}{\nu}, \quad t = \frac{u'^2_m t'}{4\nu}, \quad \omega = \frac{4\nu\omega'}{u'^2_m}, \quad u = \frac{u'}{u'_m}, \quad \theta = \frac{T' - T'_\infty}{T'_m - T'_\infty}, \\ &P = \frac{\mu c'}{\gamma'}, \quad G = \frac{\nu f_x \beta (T'_m - T'_\infty)}{u'^3_m}, \quad m = \frac{\sigma B_o^2 \nu}{\rho' u'^2_m}, \quad K = \frac{k' u'^2_m}{4\nu^2 \rho'}, \end{aligned} \tag{2.5}$$

and $\nu = \frac{\mu}{\rho'}$.

Equations (2.1), (2.2), (2.3) and (2.4) under the transformations (2.5) reduce to

$$\frac{\partial^2 u}{\partial \eta^2} - \frac{1}{4}\frac{\partial u}{\partial t} - mu - k\frac{\partial^3 u}{\partial t \partial \eta^2} = -G\theta, \tag{2.6}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} - \frac{P}{4}\frac{\partial \theta}{\partial t} = 0, \tag{2.7}$$

at

$$\eta = 0 : u = e^{i\omega t}, \qquad \theta = e^{i\omega t}, \tag{2.8}$$

at

$$\eta \to \infty : u = 0, \qquad \theta = 0. \tag{2.9}$$

3. Solution of equations

To solve Eqs.(2.6) and (2.7) subject to the boundary conditions (2.8) and (2.9) in the neighbourhood of the plate, we take

$$u(\eta, t) = u_1(\eta)e^{i\omega t}, \tag{3.1}$$
$$\theta(\eta, t) = \theta_1(\eta)e^{i\omega t}. \tag{3.2}$$

Substituting Eqs.(3.1) and (3.2) in Eqs.(2.6), (2.7), (2.8) and (2.9), we get

$$(1 - i\omega k)\frac{d^2u_1}{d\eta^2} - \left(\frac{i\omega}{4} + m\right)u_1 = -G\theta_1, \tag{3.3}$$

$$\frac{d^2\theta_1}{d\eta^2} - \frac{i\omega P}{4}\theta_1 = 0. \tag{3.4}$$

The boundary conditions on u_1 and θ_1 are

at

$$\eta = 0 : u_1 = 1, \qquad \theta_1 = 1, \tag{3.5}$$

at

$$\eta \to \infty : u_1 = 0, \qquad \theta_1 = 0. \tag{3.6}$$

The solution of Eq.(3.4) subject to the boundary conditions $(3.5)_2$ and $(3.6)_2$ is

$$\theta_1 = e^{-\alpha\eta}, \tag{3.7}$$

where

$$\alpha = \frac{1}{2}\sqrt{iP\omega}, \tag{3.8}$$

the real and imaginary parts of $\alpha = \alpha_r + i\alpha_i$ are given by

$$\alpha_r = \alpha_i = \sqrt{\frac{P\omega}{8}}. \tag{3.9}$$

From Eqs.(3.2) and (3.7)

$$\theta = e^{(i\omega t - \alpha\eta)}, \tag{3.10}$$

the real and imaginary parts of $\theta = \theta_r + i\theta_i$ are given by

$$\theta_r = e^{-\alpha_r\eta}\cos(\omega t - \alpha_i\eta), \tag{3.11}$$
$$\theta_i = e^{-\alpha_r\eta}\sin(\omega t - \alpha_i\eta). \tag{3.12}$$

Solving Eq.(3.3) with the conditions $(3.5)_1$ and $(3.6)_1$ and the expression for θ_1 from Eq.(3.7), we get

$$u_1 = e^{-\lambda\eta} + \frac{G}{\alpha^2(1-i\omega k) - m - \frac{i\omega}{4}}(e^{-\lambda\eta} - e^{-\alpha\eta}), \tag{3.13}$$

where

$$\lambda = \lambda_r + i\lambda_i,$$

$$\lambda_r = \left[\frac{1}{2}(A + \sqrt{A^2+4B})\right]^{\frac{1}{2}}, \qquad \lambda_i = \frac{\omega(1+4mk)}{8\lambda_r(1+\omega^2k^2)}, \tag{3.14}$$

$$A = \frac{m - \frac{\omega^2 k}{4}}{1+\omega^2k^2}, \qquad B = \frac{\omega^2(1+4mk)^2}{64(1+\omega^2k^2)^2}.$$

The real and imaginary parts of $u_1 = u_{1r} + iu_{1i}$ are

$$u_{1r} = G\frac{DE - FH}{E^2+H^2} + e^{-\lambda_r\eta}\cos\lambda_i\eta, \tag{3.15}$$

$$u_{1i} = -G\frac{DH + FE}{E^2+H^2} - e^{-\lambda_r\eta}\sin\lambda_i\eta, \tag{3.16}$$

where

$$D = e^{-\lambda_r\eta}\cos\lambda_i\eta - e^{-\alpha_r\eta}\cos\alpha_i\eta, \qquad E = 2\omega k\alpha_r^2 - m,$$

$$F = e^{-\lambda_r\eta}\sin\lambda_i\eta - e^{-\alpha_r\eta}\sin\alpha_i\eta, \qquad H = 2\alpha_r^2 - \frac{\omega}{4}. \tag{3.17}$$

From Eq.(3.1) the real and imaginary parts of the velocity $u = u_r + iu_i$ are

$$u_r = u_{1r}\cos\omega t - u_{1i}\sin\omega t, \tag{3.18}$$

$$u_i = u_{1i}\cos\omega t + u_{1r}\sin\omega t. \tag{3.19}$$

The shearing stress at the plate is

$$\tau' = \left[\mu\frac{\partial u'}{\partial y'} - k\frac{\partial^2 u'}{\partial t'\partial y'}\right]_{y'=0}. \tag{3.20}$$

Using Eqs.(2.5), (3.1) and (3.20), the skin friction τ can be written as

$$\tau = \frac{\tau'}{\rho u_m'^2} = \left(\frac{\partial u}{\partial\eta} - k\frac{\partial^2 u}{\partial t\partial\eta}\right)_{\eta=0} = e^{i\omega t}(1-i\omega k)\left(\frac{du_1}{d\eta}\right)_{\eta=0}. \tag{3.21}$$

From Eqs.(3.15) and (3.16) the real and imaginary parts of $\tau = \tau_r + i\tau_i$ are

$$(3.22) \qquad \tau_r = \left[\frac{G\{E(\alpha_r - \lambda_r) - H(\lambda_i - \alpha_i)\}}{E^2 + H^2} - \lambda_r\right](\cos\omega t + \omega k \sin\omega t) + \left[\frac{G\{H(\alpha_r - \lambda_r) + E(\lambda_i - \alpha_i)\}}{E^2 + H^2} - \lambda_i\right](\sin\omega t - \omega k \cos\omega t),$$

$$(3.23) \qquad \tau_i = -\left[\frac{G\{H(\alpha_r - \lambda_r) - E(\lambda_i - \alpha_i)\}}{E^2 + H^2} - \lambda_i\right](\cos\omega t + \omega k \sin\omega t) + \left[\frac{G\{E(\alpha_r - \lambda_r) + H(\lambda_i - \alpha_i)\}}{E^2 + H^2} - \lambda_r\right](\sin\omega t - \omega k \cos\omega t),$$

4. Conclusions

In this work we have studied the different effects of the magnetic field and heat transfer on the flow of a Walter's B' viscoelastic fluid past a vertical plate. Both the velocity and temperature of the plate fluctuate harmonically with time.

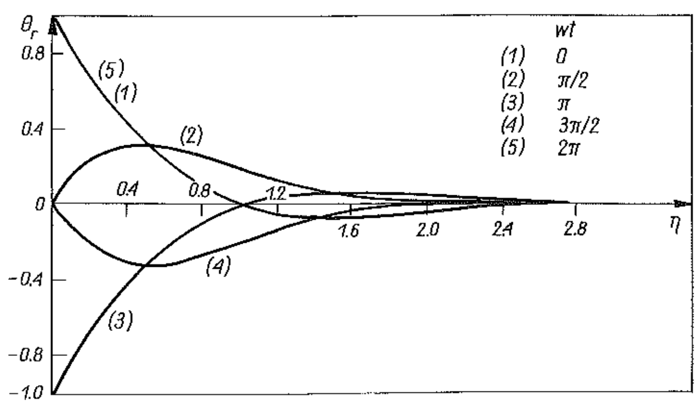

Fig. 1. The variation of θ_r with η at different time instants ωt with $P = 2$ and $\omega = 10$.

It is seen from Eq.(3.11) and Fig.1 that the temperature profile may be compared with a damped harmonic wave of wavelength $2\pi/\alpha_i$ propagating in the η – direction with a phase velocity $\sqrt{8\omega/P}$. The damping is such that the amplitude of oscillation decreases by $e^{-\alpha_r \eta}$.

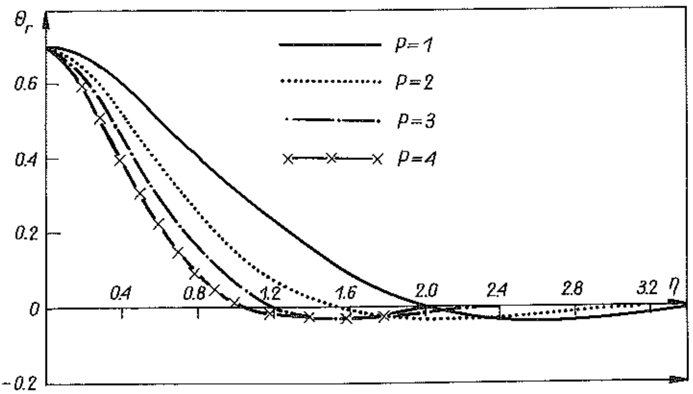

Fig. 2. The variation of θ_r with η for different values of P with $\omega t = \pi/4$ and $\omega = 10$.

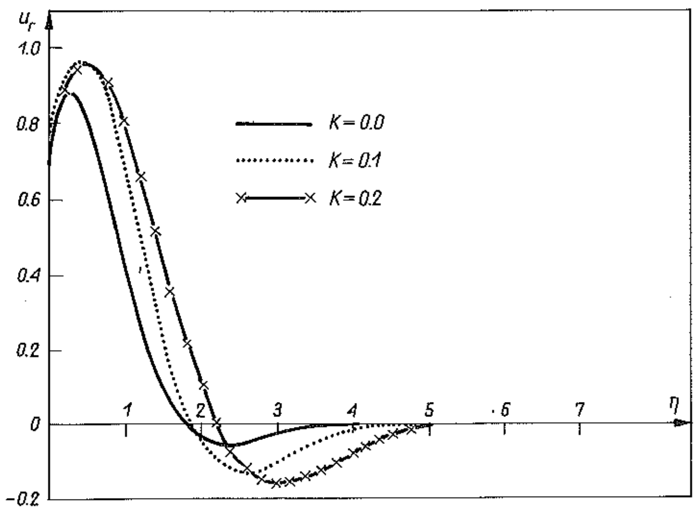

Fig. 3. Velocity field u_r against η for different values of K with $P = 2$, $\omega = 10$, $\omega t = \pi/4$, $G = 10$ and $m = 3$.

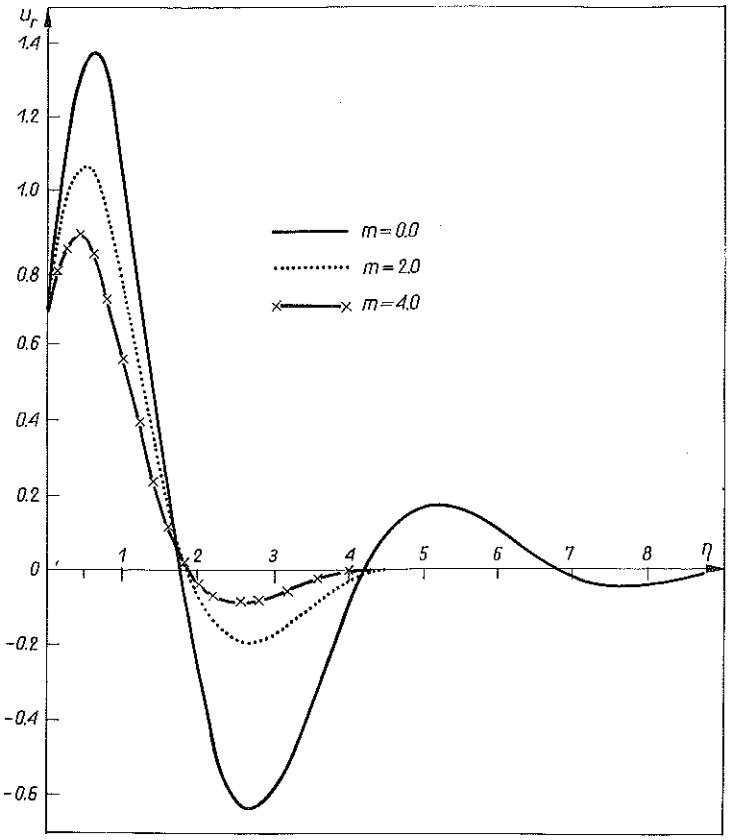

Fig. 4. Velocity field u_r against η for different values of m with $P = 2$, $\omega = 10$, $\omega t = \pi/4$, $G = 10$ and $K = 0.1$.

From Fig.2, it is clear that the temperature distribution in the boundary layer for the viscoelastic fluid increases with the decrease of the Prandtl number P, as for the case of the Newtonian fluid [13]. But an opposite effect is noticed at a certain distance away from the plate. Also the thermal boundary layer thickness δ_T increases with the decrease of P.

From Figs.3-6 it is clear that the effects of the magnetic field m, the Grashof number G and the Prandtl number P on the velocity field for the viscoelastic fluid are similar to that of a Newtonian fluid [13]. The positive value of the velocity at any point increases with the increase of either G or K individually, keeping the other parameter constant. However, the velocity gives a reverse relation by increasing P or m. Almost at the edge of the boundary layer an opposite effect takes place.

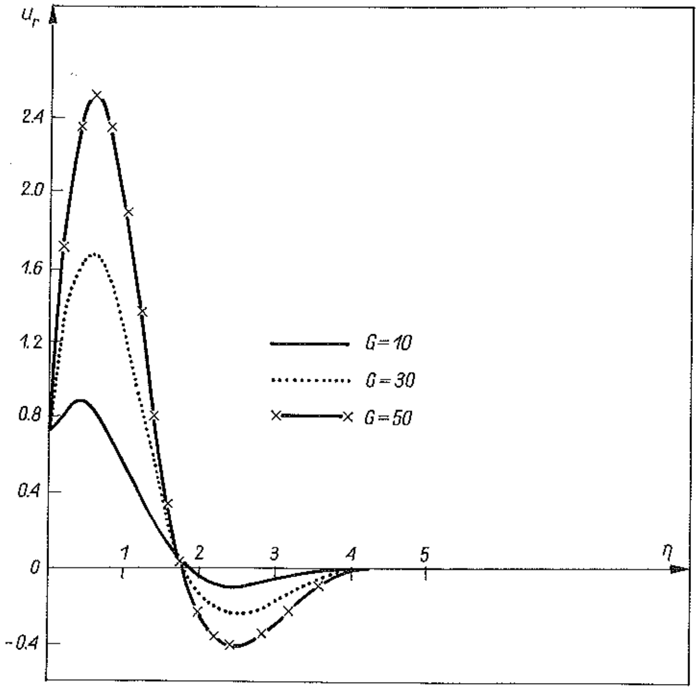

Fig. 5. Velocity field u_r against η for different values of G with $P = 2$, $\omega = 10$, $\omega t = \pi/4$, $m = 4$ and $K = 0.1$.

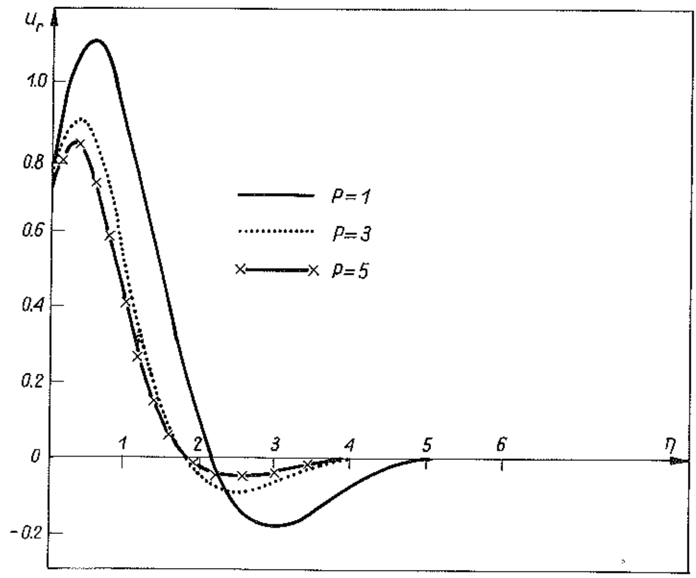

Fig. 6. Velocity field u_r against η for different values of P with $\omega = 10$, $\omega t = \pi/4$, $G = 10$, $m = 3$ and $K = 0.1$.

Table 1.

$P=2$, $G=10$, $m=3$		$P=2$, $G=10$, $K=0.10$		$P=2$, $m=4$, $K=0.10$		$G=10$, $m=3$, $K=0.10$	
K	τ_r	m	τ_r	G	τ_r	P	τ_r
0.00	1.2427	0.0	2.1059	10	0.4411	1.0	1.3483
0.05	1.1409	1.0	1.6965	20	2.9034	2.0	0.8304
0.10	0.8304	2.0	1.2545	30	5.3658	3.0	0.5374
0.15	0.5713	3.0	0.8304	40	7.8281	4.0	0.3379
0.20	0.3396	4.0	0.4411	50	10.2904	5.0	0.1889

The velocity boundary layer thickness δ increases with the increase of K and the decrease of (P or m) individually, keeping the other parameter constant.

Table 1 gives the variation of the skin friction τ_r with the parameters of the flow P, G, m and k for $\omega = 10$ and $\omega t = \pi/4$. Thus the skin friction τ_r decreases as one of the parameters k, m and P increases keeping the other two parameters constant. τ_r increases with the increase of G, keeping the other parameters constant.

The transient free convection flow of a Newtonian fluid with and without a magnetic field can be derived from the above analysis by taking $k = 0$ and $k = m = 0$, respectively.

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STRESZCZENIE

NIEUSTALONY MAGNETOHYDRODYNAMICZNY PRZEPŁYW SWOBODNY PŁYNU ŁEPKOSPRĘŻYSTEGO WZDŁUŻ ŚCIANKI PIONOWEJ

Przeanalizowano swobodny nieustalony przepływ konwekcyjny nieściśliwego płynu przewodzącego elektryczność wzdłuż drgającej ścianki pionowej w obecności poprzecznego pola magnetycznego. Przepływ jest scharakteryzowany przez bezwymiarowe liczby P (Prandtla), G (Grashofa), m (parametr magnetyczny), ω (częstość) oraz k (parametr lepkosprężysty). Stwierdzono, że profil temperatury porównać można do tłumionej fali harmonicznej poruszającej się w kierunku prostopadłym do ścianki. Zarówno bezwymiarowa temperatura jak i grubość warstwy przyściennej δ_T wzrastają przy spadku wartości P. Wpływ P, G i m na pole prędkości u_r oraz grubość warstwy δ w płynie lepkosprężystym są takie same jak w cieczy niutonowskiej. Przy wzroście k wzrastają również u_r i δ; odwrotne zjawisko stwierdza się w pewnej odległości od ścianki. Oszacowano siły tarcia τ_r na powierzchni ścianki dla różnych wartości P, G, m, k.

Резюме

НЕУСТАНОВИВШЕЕСЯ МАГНЕТОГИДРОДИНАМИЧЕСКОЕ СВОБОДНОЕ ТЕЧЕНИЕ ВЯЗКОУПРУГОЙ ЖИДКОСТИ ВДОЛЬ ВЕРТИКАЛЬНОЙ СТЕНКИ

Проанализовано свободное неустановившееся конвекционное течение несжимаемой электропроводящей жидкости вдоль колеблющейся вертикальной стенки в присутствии поперечного магнитного поля. Течение охарактеризовано безразмерным числом P (Прандтля), G (Грашофа), m (магнитный параметр), ω (частота) и k (вязкоупругий параметр). Констатировано, что профиль температуры можно сравнить с затухающей гармонической волной, движущейся в направлении перпендикулярном к стенке. Так безразмерная температура, как и толщина пограничного слоя δ_T возрастают при падении значения P. Влияние P, G и m на поле скорости u_r и толщину слоя δ в вязкоупругой жидкости аналогично как в ньютоновской жидкости. При росте k возрастают тоже u_r и δ; обратное явление констатируется на некотором расстоянии от стенки. Оценены силы трения τ_r на поверхности стенки для разных значений P, G, m, k.

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