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ANALYSIS OF IDENTIFICATION METHODS FOR THE VISCOELASTIC PROPERTIES OF MATERIALS

L. DIETRICH and K. TURSKI (WARSZAWA)

A complete solution of the constitutive equations for a viscoelastic material described by Burgers model is presented under various initial conditions. Possibilities to determine the viscoelastic properties of materials for various loading patterns of a specimen are pointed out. Test results on asphalt mixture obtained under various loads and strain rates are discussed. Creep, cyclic deformations and constant strain rate loading are found to enable the viscoelastic properties to be determined in different ranges of strain rates. The obtained results of these types of tests provide a consistent spectrum of these properties as functions of the initial strain rates.

1. Introduction

Experimental determination of constants or functions to characterize materials exhibiting viscosity is a very serious problem, even in the framework of viscoelasticity. The reason is a white noise of measurements on one hand and the problem of identification of a model suitably describing the material behaviour on the other. Approximate nature of an assumed model together with measurement errors, closely associated with the assumed loading patterns of specimens often lead to the unacceptable values of material constants. Various loading patterns and types of testpieces have been used to determine the characteristic functions for viscoelastic materials. A survey of the methods in use can be found, among others, in [8]. Such types of investigations have been employed as creep test [1], loading with constant stress rate [9], natural vibrations [10, 11, 13] and forced vibrations [14]. In many situations certain advanced numerical techniques are used to optimize the selection of tested material constants by assuming as objective functions departures of theoretical description from an experimental curve in a considered range [12, 13]. In spite of this procedure, the test results obtained with different loading patterns are not comparable. They are often represented in a manner characteristic for a given experimental procedure, as is, for instance, the case with cyclic loading where the real and imaginary parts of the complex modulus and an angle of phase between stress and strain are determined, while the data to determine the model constants are lacking. Sometimes the discrepancy of results obtained for various loading patterns are too large – differ by an order of magnitude – that they can only be assiociated with given test conditions and any comparisons with other test data are pointless.

In the paper the tests are presented for the same material – a certain type of asphaltic concrete – performed for three loading patterns: creep under constant load, monotonic increase of load at constant strain rate and cyclic tension-compression straining at a prescribed amplitude and various frequencies.

A solution of the differential equation for an assumed four-parameter Burgers model is given for various test arrangements with particular emphasis on their suitability to identify characteristic material functions. The test results for various loading patterns and strain rates are compared. It is found out that the results of creep, cyclic straining and loading with constant strain rate make it possible to determine the characteristic functions of viscoelastic material at various overlapping ranges of strain rates. The obtained results for different loading patterns give a consistent picture of relationships between Young's moduli, viscosity coefficients and initial strain rate, which was the maximum strain rate in the performed test.

2. Analysis of loading patterns

To analyse and assess the suitability of various loading patterns a four-parameter Burgers model is chosen to describe the behaviour of the considered material. The Burgers model is an in-series combination of Maxwell's and Kelvin's models and makes it possible to describe an instantaneous elasticity, viscous flow and viscous internal friction that causes an effect of elastic lag. Respective moduli have subscripts K and M – for Kelvin and Maxwell elements, respectively.

The constitutive Burgers equation [1] has the form

$$\sigma + A\dot{\sigma} + B\ddot{\sigma} = C\dot{\varepsilon} + D\ddot{\varepsilon}. \tag{2.1}$$

where

$$A = \frac{\eta_M}{E_M} + \frac{\eta_M}{E_K} + \frac{\eta_K}{E_K}, \tag{2.2}$$

$$B = \frac{\eta_K \eta_M}{E_K E_M}, \tag{2.3}$$

$$C = \eta_M, \tag{2.4}$$

$$D = \frac{\eta_K \eta_M}{E_K}, \tag{2.5}$$

inverse relationships are:

$$\eta_M = C, \tag{2.6}$$

$$E_M = D/B, \tag{2.7}$$

$$\eta_K = \frac{D}{A - \frac{BC}{D} - \frac{D}{C}}, \tag{2.8}$$

$$E_K = \frac{C}{A - \frac{BC}{D} - \frac{D}{C}}. \tag{2.9}$$

The solution of the Eq.(2.1) introduces two unknown constants to be determined from the boundary conditions for $t = 0$.

Tables 1 and 2 contain eight examples of constitutive equations obtained for various cases of controlling the stress or the strain function, resulting from the solution of Eq.(2.1). For each case the equation $\{\sigma(t)$ or $\varepsilon(t)\}$ of the function forcing the process is given (underlined in Tables), initial condition at the commencement of the process for $t = 0$ and a response of the material. Expressions for τ, J_1, J_2 and r_1, r_2, E_1, E_2, referring respectively to the loading control and strain control are shown at the end of each set of equations.

So far as the experimental identification of viscoelastic properties is concerned, only such loading patterns will be meaningful for which initial conditions can be met for a given solution and maintenance of the required controlling magnitudes during measurements can be ensured as well in such patterns for which the response function of the model material can be rearranged to reach the form enabling calculations of all the four coefficients of the Eq.(2.1) on the basis of acquired experimental data.

In Table 1 the solutions are presented for the four used loading patterns in which the stress or the force (provided the cross-section changes of the specimens can be ignored) are the control parameters.

Table 1. Relations between $\varepsilon(t)$ for assumed forms of $\sigma(t)$ and initial conditions for $t=0$.

1. $\underline{\sigma=\sigma_0}$; $t=0$: $\varepsilon=\varepsilon_0$, $\dot\varepsilon=\dot\varepsilon_p$. $\varepsilon=\sigma_0\left[\frac{B}{D}+\frac{t}{C}+\left(\frac{A}{C}-\frac{D}{C^2}-\frac{B}{D}\right)\left(1-e^{-bt}\right)\right]$		
2. $\underline{\sigma=at}$; $t=0$: $\varepsilon_0=0$, $\dot\varepsilon=\dot\varepsilon_p$. $\varepsilon=a\left[\left(\frac{A}{C}+\frac{D}{C^2}\right)t+\frac{1}{2C}t^2+\left(\frac{B}{C}-\frac{D^2}{C^3}-\frac{AD}{C^2}\right)\left(1-e^{-bt}\right)\right]$		
3. $\underline{\sigma=\sigma_a\sin\omega t}$; $t=0$: $\varepsilon=0$, $\dot\varepsilon=\dot\varepsilon_p$. $\varepsilon=\sigma_a\left[J_1\sin\omega t-J_2\cos\omega t+\frac{1}{\omega C}-\left(\frac{1}{\omega C}-J_2\right)e^{-bt}\right]$		
4. $\underline{\sigma=\sigma_a\cos\omega t}$; $t=0$: $\varepsilon=0$, $\dot\varepsilon=\dot\varepsilon_p$. $\varepsilon=\sigma_a\left[J_2\sin\omega t+J_1\cos\omega t+\left(\frac{B}{D}-J_1\right)e^{-bt}\right]$		
a. $J_1=\frac{AC-D+BD\omega^2}{C^2+D^2\omega^2}$;	b. $J_2=\frac{C+(AD-BC)\omega^2}{\omega(C^2+D^2\omega^2)}$.	
c. $b=\frac{1}{\tau}=\frac{C}{D}$,	d. $\varepsilon_a=\sigma_a\lvert J^*\rvert$,	e. $\lvert J^*\rvert=\sqrt{J_1^2+J_2^2}$.

The first pattern corresponds to creep of specimen under constant stress σ_0. This pattern is relatively easy to realize and therefore frequently employed to determine the necessary constants for viscoelastic materials. One of the ways [1] to calculate the four material constants on the basis of creep curve is the determination of four characteristic values of the functions $\varepsilon(t)$ for a given stress σ_0 from the formula

$$\varepsilon=\sigma_0\left\{\frac{1}{E_M}+\frac{1}{\eta_M}+\frac{1}{E_K}+\left[1-\exp(t/\tau)\right]\right\}=\sigma_0\,J(t)\,. \tag{2.10}$$

For $t=0$, $\varepsilon=\varepsilon_0$, $\dot\varepsilon=\dot\varepsilon_p$, we get

$$\varepsilon_0=\frac{\sigma_0}{E_M}\,,\qquad \mathrm{tg}\alpha=\dot\varepsilon_p=\sigma_0\left(\frac{1}{\eta_K}+\frac{1}{\eta_M}\right). \tag{2.11}$$

The strain rate when $t\to\infty$ tends asymptotically to the value

$$\dot\varepsilon_\infty=\mathrm{tg}\beta=\frac{\sigma_0}{\eta_M}\,. \tag{2.12}$$

Table 2. Relations of $\sigma(t)$ for assumed forms of strains $\varepsilon(t)$ and initial conditions for $t = 0$.

5. $\underline{\varepsilon = \varepsilon_m;}$ $\quad t = 0: \quad \sigma = \sigma_0\,, \quad \dot{\sigma} = \dot{\sigma}_p\,.$ $\sigma = \dfrac{D\varepsilon_m}{B(r_1 - r_2)}\left[-\left(r_2 + \dfrac{A}{B} - \dfrac{C}{D}\right)e^{r_1 t} + \left(r_1 + \dfrac{A}{B} - \dfrac{C}{D}\right)e^{r_2 t}\right]$	
6. $\underline{\varepsilon = vt;}$ $\quad t = 0: \quad \sigma_0 = 0\,, \quad \dot{\sigma} = \dot{\sigma}_p\,.$ $\sigma = v\left\{C + \dfrac{1}{r_1 - r_2}\left[\left(\dfrac{D}{B} + Cr_2\right)e^{r_1 t} - \left(\dfrac{D}{B} + Cr_1\right)e^{r_2 t}\right]\right\}$	
7. $\underline{\varepsilon = \varepsilon_a \sin\omega t;}$ $\quad t = 0: \quad \sigma_0 = 0\,, \quad \dot{\sigma} = \dot{\sigma}_p\,.$ $\sigma = \dfrac{\varepsilon_a}{r_1 - r_2}\left[\left(\dfrac{D}{B}\omega - E_1\omega + E_2 r_2\right)e^{r_1 t} + \left(-\dfrac{D}{B}\omega + E_1\omega - E_2 r_1\right)e^{r_2 t}\right]$ $+\varepsilon_a(E_1 \sin\omega t + E_2 \cos\omega t)$	
8. $\underline{\varepsilon = \varepsilon_a \cos\omega t;}$ $\quad t = 0: \quad \sigma = \sigma_0\,, \quad \dot{\sigma} = \dot{\sigma}_p\,.$ $\sigma = \dfrac{\varepsilon_a}{r_1 - r_2}\left\{\left[\left(\dfrac{C}{B} - \dfrac{AD}{B^2}\right) + \left(E_1 - \dfrac{D}{B}\right)r_2 + E_2\omega\right]e^{r_1 t}\right.$ $\left. - \left[\left(\dfrac{C}{B} - \dfrac{AD}{B^2}\right) + \left(E_1 - \dfrac{D}{B}\right)r_1 + E_2\omega\right]e^{r_1 t}\right\} + (-E_2 \sin\omega t + E_1 \cos\omega t)\varepsilon_a$	
e. $E_1 = \omega^2 \dfrac{AC - D + BD\omega^2}{A^2\omega^2 + (\omega^2 B - 1)^2}\,;$	f. $E_2 = \omega \dfrac{C + (AD - BC)\omega^2}{A^2\omega^2 + (\omega^2 B - 1)^2}\,.$
g. $r_{1,2} = -\dfrac{A}{2B} \pm \sqrt{\left(\dfrac{A}{2B}\right)^2 - \dfrac{1}{B}}\,;$	h. $\sigma_a = \varepsilon_a \lvert E^* \rvert;$ i. $\lvert E^* \rvert = \sqrt{E_1^2 + E_2^2}\,;$

The asymptote is located at the height whose ordinate is denoted by ε_K,

$$\varepsilon_K = \sigma_0 \left(\frac{1}{E_K} + \frac{1}{E_M}\right). \tag{2.13}$$

These four characteristic values of the creep diagram for $t = 0$ and $t \to \infty$ are seen in Figs.1 and 2 where the calculated functions $\varepsilon(t)$ and $\dot{\varepsilon}(t)$ for the Burgers model are also shown by assuming certain values of the coefficients A, B, C, D, marked in the figure caption.

From the Eqs. (2.11)–(2.13) it follows

$$E_M = \frac{\sigma_0}{\varepsilon_0}, \tag{2.14}$$

$$\eta_M = \frac{\sigma_0}{\mathrm{tg}\beta}, \tag{2.15}$$

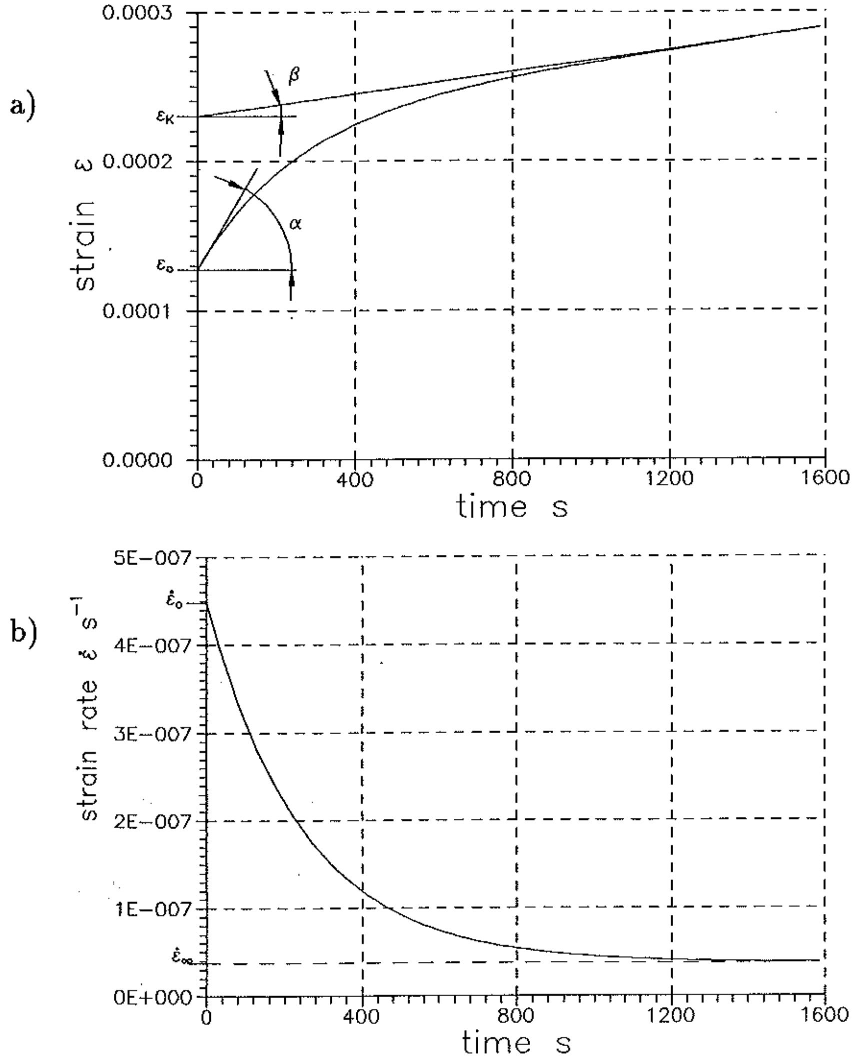


FIG. 1. Creep at constant stress as a function of time a. Strain (Equation 1, Table 1), $\varepsilon_0 = 0.0001274$, $\varepsilon_K = 0.0002296$; b. Strain rate: $\dot{\varepsilon}_0 = 0.44811 \cdot 10^{-8}$, $\dot{\varepsilon}_\infty = 3.749 \cdot 10^{-8}$. Data: $\sigma_0 = 1.076\,\mathrm{kPa}$, $A = 6374\,\mathrm{s}$, $B = 846000\,\mathrm{s}^2$, $C = 28.7\,\mathrm{GPa\,s}$, $D = 7142\,\mathrm{GPa\,s}^2$.

$$E_K = \frac{\sigma_0}{\varepsilon_K - \varepsilon_0}, \tag{2.16}$$

$$\eta_K = \frac{\sigma_0}{\mathrm{tg}\alpha - \mathrm{tg}\beta}. \tag{2.17}$$

Acceptability of thus determined constants depends on the accuracy of determination of the characteristic values of the diagram $\varepsilon(t)$ referring to the measurements at an instant zero and an infinitely long time.

In the tests on real materials it is usually very difficult to assess the instantaneous elastic strains (for $t = 0$) since the stress in the specimen increases gradually to a certain value σ_0 at a finite rate even when a dead-weight is used for applying the load. Similar difficulties are encountered to determine an asymptote of creep curve; its accuracy is closely associated with the duration of creep test. From Figs.1a, 1b it can be seen that the example calculated from the creep equation of the Burgers model by using coeficients A, B, C, D as shown in the figure requires the creep test to be continued for at least 1500 s to determine the ordinates of the curve $\varepsilon(t)$ and its assymptote for $t \to \infty$ in a satisfactory manner. In another test (Fig.6) a creep rate became constant not earlier than after 20000 s.

Accuracy of the initial portion of the diagram is vital for the determination of E_M and η_K, both depending on the elastic strain ε_0 and angle α. The values E_K and η_M are interconnected through the rate $\dot{\varepsilon}_\infty$ the calculation of which is rather insensitive to some imperfections of the $\varepsilon(t)$ curve.

The calculations of constants consist in a curve-fitting procedure and various methods are here employed including the multiparameter optimization procedure [13].

Another effective method to calculate the four material constants from the creep curve is to read four ordinates of the $\varepsilon(t)$ diagram at four evenly spaced instants of time t_n such that $t_n - t_{n-1} = \text{const}$. In this case the four relationships obtained from the Eq.(2.10) can be uncoupled and the values E_K, E_M, η_K, η_M calculated. However, to ensure the same accuracy of calculations as in the previous method, the ordinates of $\varepsilon(t_n)$ have to be measured much more accurately which is only possible with the help of digital registration of test results. Moreover, the calculated material constants depend on the selected range of measurements.

The second pattern in Table 1 corresponds to a monotonic loading of a specimen with constant stress rate, corresponding to $\sigma = at$. The strain rate is then equal to the creep function (see Eq.(2.10))

$$\dot{\varepsilon} = J(t) . \tag{2.18}$$

The procedure of calculations the coefficients remains the same as in the case of creep except that the function (2.18) must first be determined by differentiating the experimental function $\varepsilon(t)$ which leads to additional difficulties and errors in the identification of material constants. This procedure is suitable only when a testing machine is available which enables sufficiently accurate measurements to be registered in order to differentiate the test data.

The two next patterns refer to forced vibrations. The stress changes according to the function $\sigma = \sigma_0 \sin \omega t$ result in an asymmetric strain cycle whose mean value $\varepsilon_m = \sigma_a / \omega \eta_M$ decreases as the frequency of loading grows. The knowledge of the mean strain makes it possible to measure directly the coefficient η_M for a given frequency ω. The value of mean strain decreases as the frequency of loading is getting larger and tends to infinity when $\omega \to \infty$.

When the forcing stresses follow the function $\sigma = \sigma_0 \cos \omega t$, the strains, conversely to the previous case for $t = \infty$, develop according to a symmetric sine curve independently of the frequency ω. Experimental realization of this type of loading will be only an approximation of the initial conditions at $t = 0$ assumed in the theoretical solution.

The solutions for four loading patterns in which the strains in the specimens are the control parameters are presented in Table 2. The first pattern that consists in an abrupt generation of a constant value of strain and its maintenance in time does not allow to determine the material properties, since the response equation for $\sigma(t)$ contains a combination of exponential functions in which the constants are involved. This pattern is difficult to realize and of no value as far as the identification of material constants is concerned.

Constant strain rate yields a linear strain increase in time and a stress that is described by an equation in Table 2, row no.6. The $\sigma(t)$ and $\dot{\sigma}(t)$ curves are shown in Figs. 2a, 2b. For the selected values of the coefficients in the function $\sigma(t)$ a long duration of the test is necessary (more than 8 hrs) to reach a small stress rate indicating that the maximum stress is near enough. Since the stress is proportional to the strain rate, the latter affects only the value of load without any change in the location of its extremum value. Thus a shift of the extremum as a function of the strain rate means that the material properties are influenced by the strain rates.

For $t \to \infty$ the stress tends to a constant value

$$\sigma_\infty = vC = v\eta_M \,, \tag{2.19}$$

which means that an assiociated asymptote is horizontal.

For $t = 0$ we have

$$\dot{\sigma}_p = v\frac{D}{B} = vE_M \,. \tag{2.20}$$

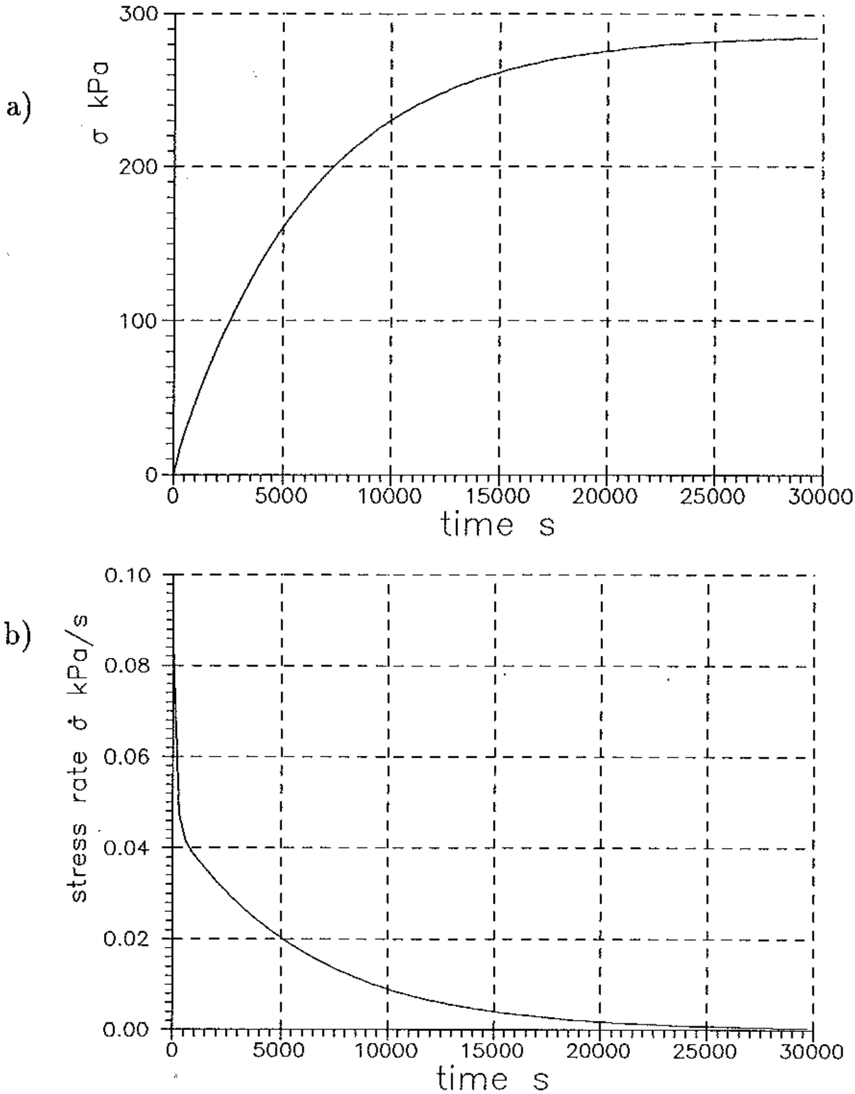


FIG. 2. Loading at constant strain rate as a function of time a. Stress (Equation 2, Table 2); b. Stress rate. Data: $v = 0.00001\,1/\mathrm{s}$, $A = 6374\,\mathrm{s}$, $B = 846000\,\mathrm{s}^2$, $C = 28.7\,\mathrm{GPa\,s}$, $D = 7142\,\mathrm{GPa\,s}^2$.

These two equations make it possible to calculate two constants of the Burgers model:

$$\eta_M = \frac{1}{v}\sigma_\infty \,, \tag{2.21}$$

$$E_M = \frac{1}{v}\dot{\sigma}_p \,. \tag{2.22}$$

The remaining constants are involved in the exponents $r_{1,2}$ (Tables 1, 2) which means that the constants cannot be found from experiments in a direct manner. For $t = 0$ a sudden drop of the stress rate $\dot{\sigma}$ can be observed in Fig.2b, so the value of E_M can suffer a large error. It is worth emphasizing that this test programme is an only method that enables E_M and η_M to be measured experimentally at a constant strain rate.

The strain changes according to the function $\varepsilon = \varepsilon_a \sin \omega t$ result in the stress variations described by the equation in Table 2, row no.7. For $t \to \infty$ a symmetric stress cycle (Fig.3a) is observed as a response to the strain sinusoid

$$\varepsilon = \varepsilon_a \sin \omega t, \qquad \sigma_\infty = \varepsilon_a \left(E_1 \sin \omega t + E_2 \cos \omega t \right) , \tag{2.23}$$

where E_1, E_2 are given in Table 2.

An influence of the remaining parameters of the equation for $\sigma(t)$ practically ceases to be felt after a lapse of some scores of seconds (Fig.3b) and the stress cycle can be considered to be symmetric.

Direct calculations of the constants of the Burgers model from experiments appear to be impossible due to a combined form of the function of material response. Results of such test serve to calculate the complex modulus and an angle of phase shift that can be used to determine the Burgers model constants [5] in the following way.

The values E_1, E_2 are obtained from tests and then the compliances J_1, J_2 are calculated from the formulae:

$$J_1 = \frac{E_1}{|E^*|^2} , \qquad J_2 = \frac{E_2}{|E^*|^2} . \tag{2.24}$$

The formulae for the compliances J_1, J_2 (Table 1) can be rearranged to take the form

$$J_1 = \frac{1}{E_M} + \frac{h}{E_K} , \tag{2.25}$$

$$\omega J_2 = \frac{1}{\eta_M} + \frac{1}{\eta_K}(1-h) , \tag{2.26}$$

where $h = \dfrac{1}{1+(\omega\tau)^2}$, $\tau = \dfrac{\eta_K}{E_K}$.

After eliminating h a straight line equation is arrived at in the J_1, ωJ_2 – plane:

$$\frac{1}{\tau} J_1 + \omega J_2 = \frac{1}{\tau E_M} + \frac{1}{\eta_M} + \frac{1}{\eta_K} . \tag{2.27}$$

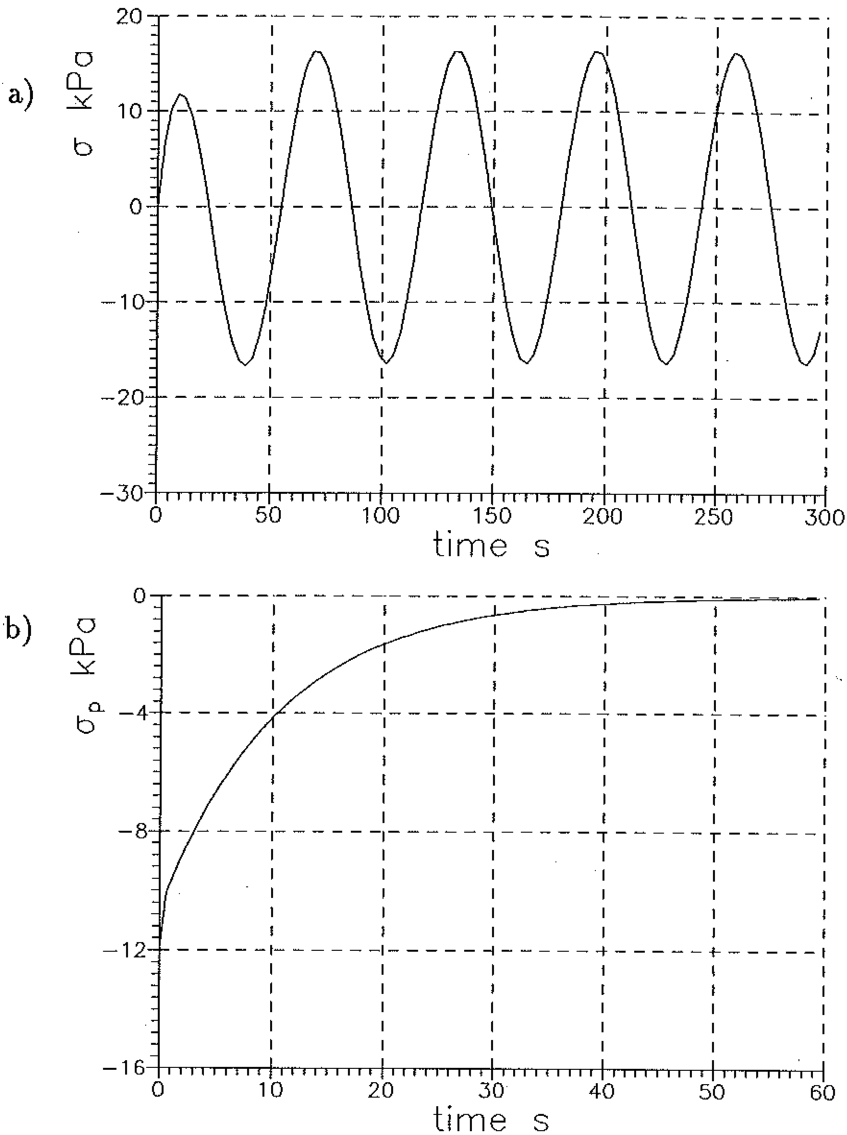


FIG. 3. Cyclic straining a. Steady symmetric stress cycle for $t > 60\,\mathrm{s}$; b. Vanishing of an initial asymmetry of stress; Data: $\varepsilon_a = 0.00005$, $\omega = 0.1\,1/\mathrm{s}$, $A = 10.8\,\mathrm{s}$, $B = 1.58\,\mathrm{s}^2$, $C = 4.8\,\mathrm{GPa\,s}$, $D = 3.43\,\mathrm{GPa\,s}^2$.

Tests curves of J_1, J_2 in the coordinate system J_1, ωJ_2 enable the time τ to be found followed by the calculation of h and disclosing the relationships $J_1(h)$ and $\omega J_2(h)$. What remains to be done is to work out a procedure for the identification of material linear spring constants and coefficients of viscosity.

In this paper we deal with a material having a very high viscosity coefficient η_M, being 10 to 100 times as large as the viscosity coefficient η_K. The method described in [5] had it that the value of η_M was determined as the last one of the four constants; this circumstance led to large errors and even to the negative, physically unacceptable values. That is why η_M is assumed as calculated from the equation for $\eta_M(\dot{\varepsilon})$ describing the creep test results with constant strain rate and next η_K is calculated from the Eq.(2.26), E_K from the formula for τ and E_M from the formula (2.25). This procedure automatically ensures consistency of the calculated values J_1, J_2 with those obtained from experiments.

When the cyclic strains are forced according to the function $\varepsilon = \varepsilon_a \cos \omega t$, it is the realization of initial conditions at $t = 0$ that creates the basic difficulty. This pattern is not employed in experimental investigations.

Experimental identification of the characteristics of viscoelastic material is particularly convenient when creep test is performed under controlled initial strain rate, when monotonic load is applied at constant strain rate (executed in standard strength testing machines of the kinematic type) and when cyclic load is applied at constant strain amplitude. The last experiments can be performed in hydraulic strength testing machines controlled by a feedback loop; as a controlling parameter the deformation of the specimen can be taken and even, for the considered confined range of strains, the displacements of a traverse of the machine. Each of the above patterns is associated with different ranges of strain rates of the specimens. These three testing procedures are reported in this paper whose main objective was to compare the experimental results obtained in various procedures and under different ranges of strain rates.

3. Tests

The presented tests were made on specimens of a modified asphaltic concrete whose composition and specimens preparation were described in [15] together with the results of mesurements of material characteristics in the temperatures ranging from +40°C to −20°C. The material served only as an example to illustrate the behaviour of a viscoelastic material and to evaluate different methods of identification of the characteristic material functions. Three loading patterns were assumed. In the first pattern the creep of specimen was caused by time-independent compressive load. The load values were chosen in such a way as not to cause the strains of specimens greater than $5 \cdot 10^{-4}$. Special testing device had to be constructed as schematically shown in Fig.4. A cylindrical specimen with the diameter of 50 mm had both faces machined to make then perfectly parallel. Length of the cylinder was 82 mm. Both faces were glued with epidian to the metal platens 5 mm thick. These platens slightly protruded outside the cylindrical surface of the specimen. At the distance of 30 mm from the centre of cylinder two induction gauges were fastened, each having the measuring base ±0.2 mm. These gauges were connected through the conditioning units (4), digital voltmeters (5) and a suitable interface to the Commodore 128 computer (7). The computer was programmed to control the course of readings and their registration on magnetic tape, synchronized with the measurements of the two induction gauges.

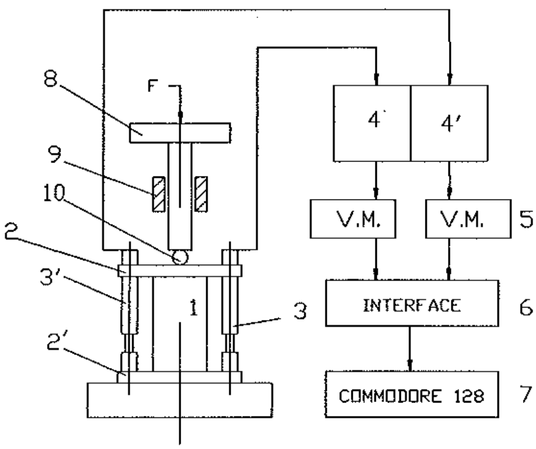


FIG. 4. Arrangement of the apparatus for testing creep under compression.

The cylinder was loaded by means of a dead-weight lying on the plate (8) which rested on a pilot bar capable of sliding inside the beering sleeve (9) whose lower end was supported on the centre of the upper platen (2) by means of a ball (10).

The second testing pattern consisted in the compression of the specimen at constant strain rate. Identical specimens with similar platens were used as in the creep test. Universal kinematically controlled testing machine FPZ 100 was used; the velocity of cross-head changed from 1 to 10 mm/min and the load range was 10 KN. Strains in the specimen were determined by observing the displacements of the cross-head; strain rates were calculated with allowance for the rigidity of loading system.

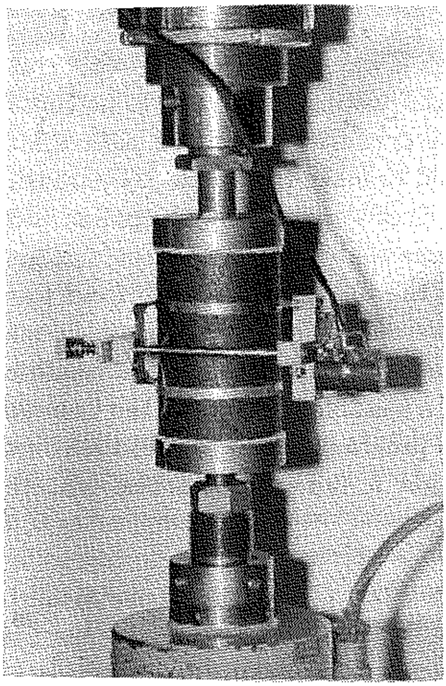


FIG. 5. View of a specimen and extensometer during cycling strain test.

The third pattern consisted in cyclic loading of cylindrical specimens with the diameter 72 mm and 120 mm long. As in the first pattern, the faces of cylinders were machined to make them parallel, special steel grips were glued with epidian adhesive. Such a testpiece was placed in the hydraulic testing machine Instron 1251 which was controlled in the feedback loop. In Fig.5 a specimen is shown together with the Denison extensometer having the base 50 mm. Its fixing teeth rested on two metal bands enclosing the cylinder tested, thus avoiding the extensometer teeth to be driven into a soft material of the specimen. Measurement of strains is considered correct in the central part of the cylinder, at a distance from the end disturbances. Cyclic loading was realized with the frequencies ranging from 0.1 Hz to 50 Hz and constant value of the strain amplitude amounting to $5 \cdot 10^{-5}$. Cyclic loading test were performed in the following manner:

- a function generator was activated for a given frequency at both amplitude and the mean level equal to zero,
- strain amplitude was gradually increased and the changes in forces and strains were observed on a two-channel oscilloscope up to the strain value $5 \cdot 10^{-5}$,

• a number of consecutive steady stress cycles and sinusoidal forced strains of the amplitude $5 \cdot 10^{-5}$ were registered in the digital memory of the oscilloscope.

4. Test results

Creep strains were registered during 20000 s (Fig.6) since such a long period of time appeared necessary to stabilize the strain rate. Initial creep rate, observed on a number of specimens, was $3.3 \div 4.1 \cdot 10^{-6}$ 1/s. For instance, the calculations showed that an increase in initial rate by 25 % resulted in an increase in strain by 55 % during 150 s. The whole curve $\varepsilon(t)$ increased accordingly. However, it is impossible to conclude from a single test of this type whether the growth of strains is a result of an increase in the initial strain rate, or this rate and values of strains are simply generated in an accidentally easier deformable specimen. Suitable explanation of this issue can only be obtained by making some additional creep tests controlled by the strain rate in the initial period of creep; special experimental arrangement would have been necessary for the purpose.

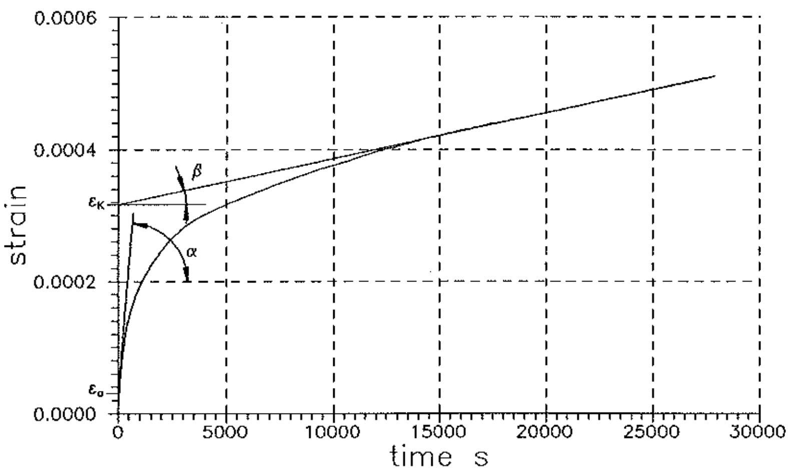


FIG. 6. Changes in strain under long-term creep. Stress $\sigma_0 = 4.9$ kPA. $\varepsilon_0 = 0.304 \cdot 10^{-4}$, $\varepsilon_K = 3.165 \cdot 10^{-4}$, $\mathrm{tg}\alpha = 3.9 \cdot 10^{-4}$, $\mathrm{tg}\beta = 0.697 \cdot 10^{-8}$.

Loading with steady strain rate was registered as a stress σ-time t curve for various velocities of the piston of the testing machine. The diagram shown in Fig.7 serves as an example for the strain rate $50 \cdot 10^{-5}$ 1/s which corresponds to the velocity of compression 2.4 mm/min. The diagram was curvilinear up to the maximum force which was accompanied by barrelling and appearance of deep cracks on the surface.

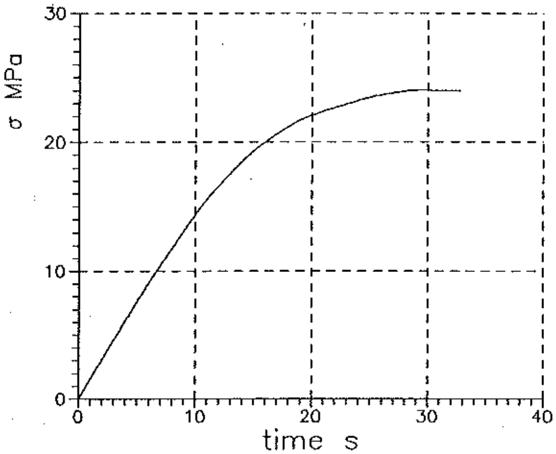


FIG. 7. Changes in stress at constant strain rate 0.0005 1/s.

Cyclic loading was applied while the amplitude of the strain sinusoid was kept constant irrespective of the vibration frequencies. In practice, a small

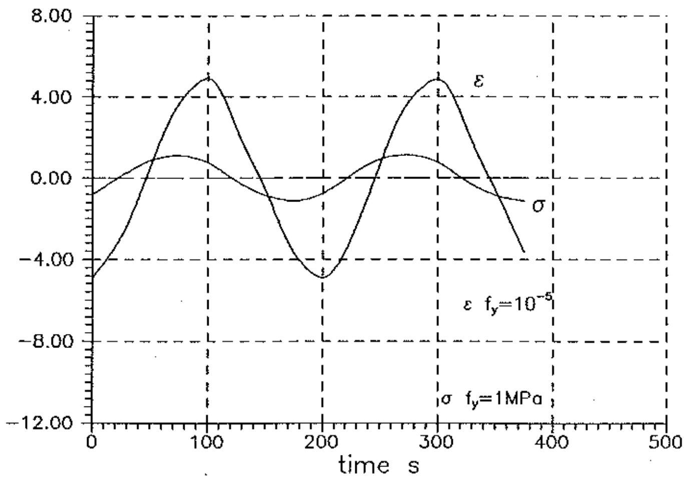


FIG. 8. Cyclic straining with the amplitude $\varepsilon_a = 4.9 \cdot 10^{-5}$ and frequency 5.076 Hz and a steady stress cycle.

amplitude of vibrations was set first and next, observing the sinusoid on the oscilloscope screen, the amplitude was increased up to the assumed value $\sim 5 \cdot 10^{-5}$ and, after a while, the constant stress amplitude was registered (Fig.8). Vibration frequency was changed from 0.1 to 50 Hz. The shown diagrams correspond to $f = 5.076$ Hz. The stress sinusoid is slightly concave when the stress drops while the strain sinusoid has an undistorted shape. After a series of measurements it is possible to prepare the diagrams as functions of vibration frequency for a complex modulus $|E^*|$ and an angle of phase φ (Fig.9) [1].

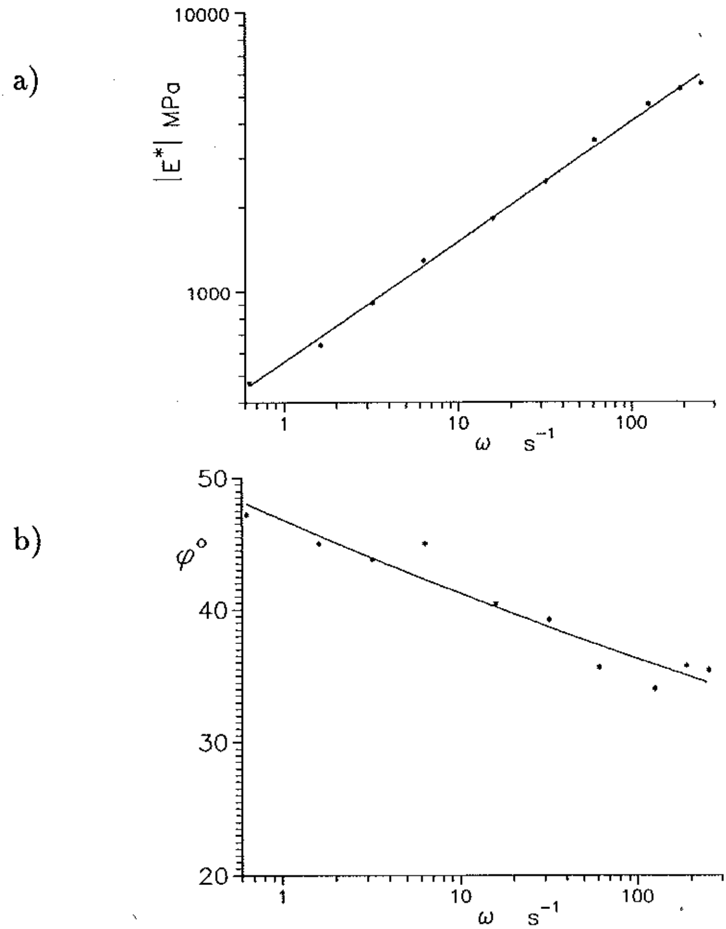


FIG. 9. Test results of cyclic straining at $\varepsilon_a \simeq 0.00005$. a. Complex modulus. Mean line $|E^*| = 558.621\omega^{0.42991}$; b. An angle of phase shift between the strain and stress cycles. Mean line $\varphi = 46.8299\omega^{-0.0555393}$.

5. Identification of moduli and their relationships with the strain rates

The experiments performed for various programmes made it possible to determine the character of changes of Burgers model constants in a wide range of strain rates. The initial strain rate appeared to be a significant factor in the behaviour of asphalt during its loading. That is why the mechanical properties of the material are diagrammatically presented as functions of this initial rate. A specific asphaltic mixture was used to prepare specimens. However, qualitatively, an influence of strain rates on the linear spring and viscoelastic properties can be valid also for other materials of the same type.

The experiments were performed for the programmes no. 1, 6, 7 (Tables 1 and 2). Designations of experimental points are given in the figures.

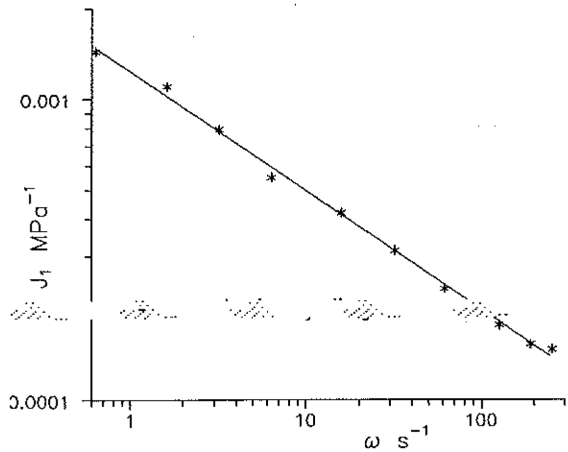


FIG. 10. Values of J_1 as functions of angular velocity ω.

For the cyclic loading the calculated material constants [12] had a particularly great scatter in spite of insignificant local departures of test data from the mean curves (Fig. 10, 11). The procedure is now improved by assuming initial data as ordinates calculated from the mean line equation for the rate ω registered in the tests. The mean curve equations have the form:

$$J_1 = 0.123444\omega^{-0.396035}\,[\mathrm{MPa}],$$
$$\omega J_2 = 0.0208291\omega^{0.524778}\,[\mathrm{MPa\,s}].$$

Constants of the Burgers model were determined for two neighbouring cal-

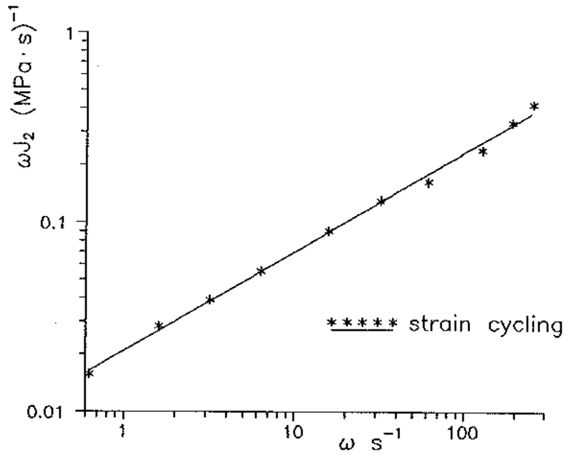


FIG. 11. Values of ωJ_1 as functions of angular velocity ω.

culated values J_1, ωJ_2 with the use of the method described in Section 2.

E_M and η_M, η_K – curves are shown in Figs.12a and 13 in the logarithmic scale and the E_K – curve (Fig.12b) in the semi-logarithmic scale. All the test points for the elasticity modulus E_M (Fig.12a) lie along one line for the three test programmes. The test results lead to the equation

$$E_M = 47223.2(\dot{\varepsilon})^{0.403114}\,[\mathrm{MPa}]. \tag{5.1}$$

Viscosity coefficient η_M was determined for the three test programmes (Fig.13): creep, loading with constant strain rate and cyclic straining. For the last one the coordinates of points were calculated from the mean line equation resulting from the first two programmes. Creep tests for η_M were continued up to the attainment of steady strain rate which took several hours at the strain not larger than $5 \cdot 10^{-4}$ and creep rate of the order of magnitude $(4 \div 7) \cdot 10^{-9}$ 1/s.

The constant strain rate tests showed a large drop of viscosity η_M accompanying increasing strain rates. The test results can be expressed analytically in the form

$$\eta_M = 354.07(\dot{\varepsilon})^{-0.663537}\,[\mathrm{MPa\,s}]. \tag{5.2}$$

The elasticity modulus E_K and viscosity coefficient η_K were calculated for creep and cyclic load test only. In the former test the values of E_K are calculated from the formula (2.16) in which the value ε_K can be large in

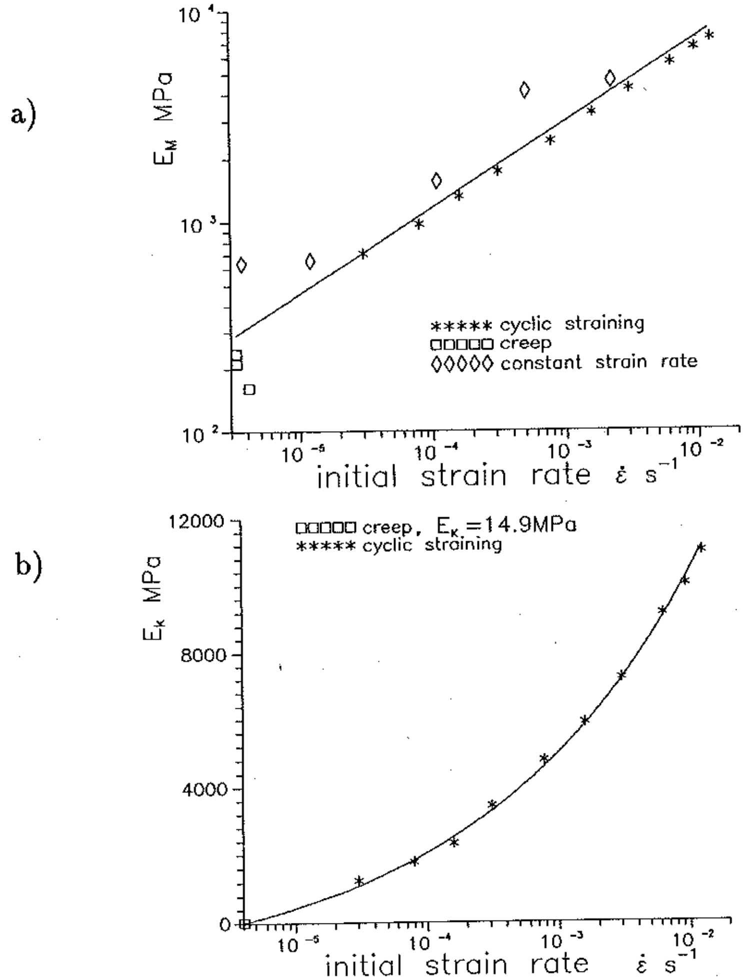


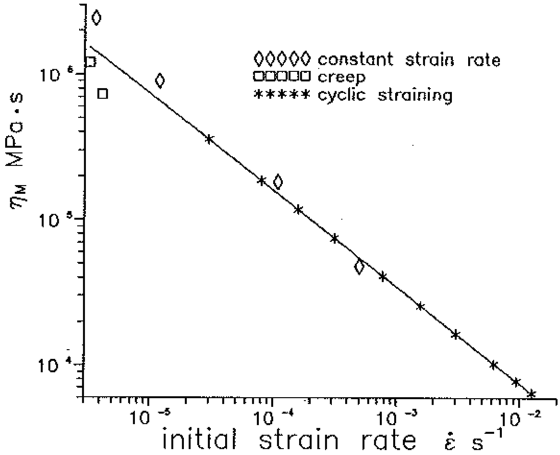
FIG. 12. Comparison of Young's moduli as functions of initial strain rate for Burgers model, obtained in various test patterns; a. Elastic Maxwell element; b. Elastic Kelvin element.

comparison with ε_0. Similarly for η_K in the formula (2.17) value tgα can be large in comparison with tgβ. Both formulae can be simplified to become:

$$E_K = \frac{\sigma_0}{\varepsilon_K}, \qquad \eta_K = \frac{\sigma_0}{\mathrm{tg}\alpha}.$$

Comparison of the above formulae shows that the values E_M and η_K are determined from the initial shape of the $\varepsilon(t)$ – curve while η_M and E_K depend on the shape of the $\varepsilon(t)$ – curve after a long period of creep. Nevertheless, all the moduli and coefficients are functions of the initial strain rate.

a)



b)

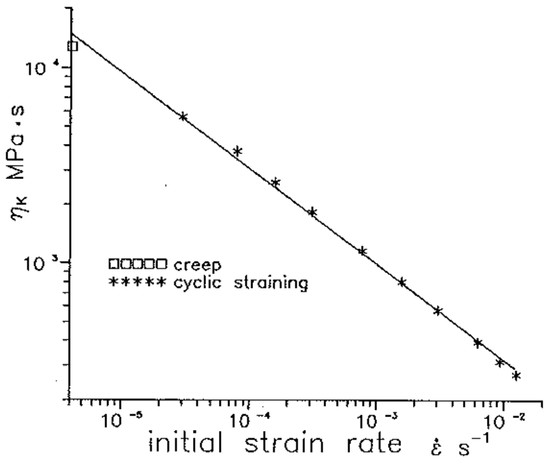


FIG. 13. Comparison of viscosity coefficients as functions of initial strain rate for Burgers model, obtained in various test patterns; a. Viscous Maxwell element; b. Viscous Kelvin element.

Variations of the modulus E_K and the viscosity coefficient η_K can be described by the equations:

$$(5.3)\qquad E_K = -1500 + 40051.4(\dot{\varepsilon})^{0.26246}\,[\mathrm{MPa}],$$

$$(5.4)\qquad \eta_K = 33.446(\dot{\varepsilon})^{-0.490405}\,[\mathrm{MPa\,s}].$$

The presented analysis demonstrates that the applied test programmes are capable of determining an influence of the initial strain rate on the Burgers model constants. Particular programmes enable to collect experimental data in various intervals of the strain rates. The found moduli and coefficients can be considered constant only at steady strain rate.

6. Conclusions

Such testing programmes are sought that make it possible to identify the material properties to within a certain prescribed accuracy. Direct measurements of those properties are impossible so appropriate loading patterns and calculation algorithms must be selected to ensure this accuracy.

Two test programmes, listed in Tables 1 and 2 appeared useless:

• loading with constant stress rate according to the formula $\sigma = at$ (programme No 2) is not suitable since constant stress is too difficult to be maintained during sufficiently long period of time,

• sudden sustained strain ε_m (programme No 5) is useless since not all of the model constants can be determined for an arbitrary time t.

Three test programmes are applicable:

• creep at constant stress σ_0 (programme No 1),

• loading at constant strain rate according to the formula $\varepsilon = vt$ (programme No 6); this procedure is particularly convenient for the identification of the constants of the Maxwell model,

• cyclic strain controlled by the function $\varepsilon = \varepsilon_a \sin \omega t$ (programme No 7).

For these three programmes there exist procedures of identification of material properties. Comparison of the results obtained from the three procedures is useful in assessing an influence of strain rates on the values of linear spring constants and coefficients of viscosity. The initial strain rate should be measured very carefully since it is a decisive factor in the determination of material characteristics as functions of time. Accidental differences in the initial strain rates may lead to errors and scatters of measured data.

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POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH, WARSZAWA.

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