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FUNDAMENTAL SOLUTIONS OF THE I-ST PLANE PROBLEM OF MICROPOLAR ELASTICITY WITH A HARMONICALLY VARYING DISTORTION FIELD

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The paper deals with the I-st plane problem of micropolar elasticity in the case of a distortion field varying harmonically in time. Differential equations of the problem, expressed in terms of displacements and rotations, are solved by direct integration with the help of the Fourier integral transforms. Fundamental solutions are obtained for the displacements and rotations induced by concentrated distortions acting in an infinite plane. All limit cases of the fundamental solutions are given which are relevant to derived theories.

1. Introduction

W. Nowacki [1, p.305] presents the basic theory of the distortion problem in a linear, homogeneous, isotropic, centro-symmetric micropolar medium. In the present paper we will consider the I-st plane strain problem with a distortion field varying harmonically in time. In a Cartesian frame Ox_1x_2 the problem is represented by a displacement vector $\mathbf{u}$ and a rotation vector $\boldsymbol{\varphi}$ of the form

$$\mathbf{u}(x_1,x_2,t) \equiv (u_1,u_2,0), \qquad \boldsymbol{\varphi}(x_1,x_2,t) \equiv (0,0,\varphi_3)\,. \tag{1.1}$$

The basic equations for the triple (u_1,u_2,φ_3) have the form

$$\begin{aligned} \Box_2 u_\alpha + (\lambda+\mu-\alpha)u_{\beta,\beta\alpha} + 2\alpha\epsilon_{\alpha\gamma}\varphi_{3,\gamma} &= R^0_\alpha\,, \\ \Box_4\varphi_3 + 2\alpha\epsilon_{\alpha\beta}u_{\beta,\alpha} &= M^0_3\,, \end{aligned} \tag{1.2}$$

where $\Box_2 = (\mu+\alpha)\nabla_1^2 - \rho\partial_t^2$, $\Box_4 = (\gamma+\varepsilon)\nabla_1^2 - 4\alpha - J\partial_t^2$, $\nabla_1^2(\ldots) = (\ldots)_{,\delta\delta}$, $\mu,\lambda,\gamma,\varepsilon,\alpha$ – elasticity constants, ρ – medium density, J – rotational inertia, and the symbol ∂_t denotes the derivative with respect to time t. The indicial notation with the summation convention is used. Partial derivatives with respect to the position variables (x_1, x_2) are denoted by a comma, Greek indices α, β, γ ... take values 1, 2, $\epsilon_{\alpha\beta}$ is the permutation symbol. No summation takes place if three indices are equal. Objects R^0_α, M^0_3 have the form

$$R^0_\alpha = \sigma^0_{\delta\alpha,\delta}\,, \qquad M^0_3 = \epsilon_{\delta\gamma}\sigma^0_{\delta\gamma} + \mu^0_{\varepsilon 3,\varepsilon}\,, \tag{1.3}$$

where

$$\begin{aligned} \sigma^0_{\alpha\beta} &= (\mu+\alpha)\gamma^0_{\alpha\beta} + (\mu-\alpha)\gamma^0_{\beta\alpha} + \lambda\gamma^0_{\gamma\gamma}\delta_{\alpha\beta}\,, \\ \mu^0_{\alpha 3} &= (\gamma+\varepsilon)\kappa^0_{\alpha 3} \end{aligned} \tag{1.4}$$

and $\delta_{\alpha\beta}$ is the Kronecker delta. Symbols $\gamma^0_{\alpha\beta}$, $\kappa^0_{\alpha 3}$ describe a field of given distortions. We will determine the fundamental solutions $(u^{\gamma_0}_{\alpha\gamma\varepsilon}, u^{\kappa_0}_{\alpha\gamma 3}, \varphi^{\gamma_0}_{3\gamma\varepsilon}, \varphi^{\gamma_0}_{3\gamma 3})$ which result from action of the distortion field in space $\mathbb{R}^2$ [14, p.60 and 202]:

$$\begin{aligned} \gamma^0_{\alpha\beta} &= \gamma_0^{\alpha\beta}\delta(x_1)\delta(x_2)e^{-i\omega t}\,, \\ \kappa^0_{\alpha 3} &= \kappa_0^{\alpha 3}\delta(x_1)\delta(x_2)e^{-i\omega t}\,, \end{aligned} \tag{1.5}$$

where $\delta(\ldots)$ – Dirac delta distribution symbol, ω – frequency of vibrations. A solution of the problem will be obtained by direct integration of the system of Eqs.(1.2). Functions (u_α, φ_3) satisfy the equations

$$\begin{aligned} \Box_1\left(\Box_2\Box_4 + 4\alpha^2\nabla_1^2\right)u_\alpha &= -\left[(\lambda+\mu-\alpha)\Box_4 - 4\alpha^2\right]R^0_{\beta,\beta\alpha} \\ &\quad - \Box_1\left(2\alpha\epsilon_{\alpha\beta}M^0_{3,\beta} - \Box_4 R^0_\alpha\right)\,, \\ \left(\Box_2\Box_4 + 4\alpha^2\nabla_1^2\right)\varphi_3 &= \Box_2 M^0_3 + 2\alpha\epsilon_{\delta\gamma}R^0_{\delta,\gamma}\,, \end{aligned} \tag{1.6}$$

where $\Box_1 = (\lambda+2\mu)\nabla_1^2 - \rho\partial_t^2$. We will solve the Eqs.(1.6) by using the Fourier transform of a function $f(\mathbf{x})$ [4]:

$$\begin{aligned} \tilde{f}(\boldsymbol{\xi}) &= (2\pi)^{-n/2}\int_{\mathbb{R}^n} f(\mathbf{x})\exp[i(\boldsymbol{\xi}\cdot\mathbf{x})]\,d\mathbf{x}\,, \\ f(\mathbf{x}) &= (2\pi)^{-n/2}\int_{\mathbb{R}^n} \tilde{f}(\boldsymbol{\xi})\exp[i(\boldsymbol{\xi}\cdot\mathbf{x})]\,d\mathbf{x}\,, \end{aligned} \tag{1.7}$$

where $\mathbf{x} \equiv (x_1,\ldots,x_n) \in \mathbb{R}^n$, $\boldsymbol{\xi} \equiv (\xi_1,\ldots,\xi_n) \in \mathbb{R}^n$, $(\boldsymbol{\xi}\cdot\mathbf{x}) = \xi_1 x_1 + \ldots + \xi_n x_n$. For elastodynamics we have $n = 3(x_1, x_2, t)$, for elastostatics $n = 2(x_1, x_2)$.

Now, let a distortion field $\gamma^0_{\delta\gamma}(x'_1,x'_2)$, $\kappa^0_{\beta 3}$ (x'_1,x'_2) harmonically varying in time be given in a domain $\widehat{\mathbb{R}}^2$ in space $\mathbb{R}^2$. Fields of displacements and rotations will be determined by using the Green function method:

$$
(1.8)\qquad
\begin{aligned}
u_\alpha(x_1,x_2) &= \int\limits_{\widehat{\mathbb{R}}^2} \Big[\gamma^0_{\delta\gamma}(x'_1,x'_2)U^{\gamma_0}_{\alpha\delta\gamma}(x_1,x_2;x'_1,x'_2) \\
&\qquad + \kappa^0_{\beta 3}(x'_1,x'_2)U^{\kappa_0}_{\alpha\beta 3}(x_1,x_2;x'_1,x'_2)\Big]\,dx'_1 dx'_2\,,\\
\varphi_3(x_1,x_2) &= \int\limits_{\widehat{\mathbb{R}}^2} \Big[\gamma^0_{\delta\gamma}(x'_1,x'_2)\Phi^{\gamma_0}_{3\delta\gamma}(x_1,x_2;x'_1,x'_2) \\
&\qquad + \kappa^0_{\beta 3}(x'_1,x'_2)\Phi^{\kappa_0}_{3\beta 3}(x_1,x_2;x'_1,x'_2)\Big]\,dx'_1 dx'_2\,.
\end{aligned}
$$

Green functions $U^{\gamma_0}_{\alpha\delta\gamma}$, $U^{\kappa_0}_{\alpha\beta 3}$, $\Phi^{\gamma_0}_{3\delta\gamma}$ and $\Phi^{\kappa_0}_{3\beta 3}$ are constructed on the basis of the fundamental solutions given in the paper.

In the micropolar theory of elasticity [1], independent fields of displacements and rotations and independent fields of force stresses and couple stresses are introduced. Both stress fields are represented by nonsymmetric second-order tensors which are linear functions of two nonsymmetric tensorial deformation fields defined with the help of six elasticity constants of Lamé type: $\mu,\lambda,\alpha,\beta,\gamma$ and ε. In the paper the attention is focussed on obtaining the fundamental solutions for (u_α,φ_3) within derived theories which represent certain reduced forms of the micropolar theory of elasticity. These are:

Theory of couple stresses [8] – in linear, isotropic, homogeneous elasticity a displacement field generates a rotation field, as in the classical theory of symmetric elasticity, and a symmetric stress field(1)is associated with a nonsymmetric field of couple stresses. Moreover, the stress is a linear function of a symmetric strain field as in the classical theory of elasticity, while the couple stress is a linear function of a nonsymmetric deformation field, defined such that the number of material constants of Lamé type is equal to four: μ,λ,η and l^*.

The displacement equations with a distortion field for the I-st plane strain problem take the form:

$$\overset{*}{\Box}_2\, u_\alpha + (\lambda+\mu)u_{\beta,\beta\alpha} + \mu l^{*2}\nabla^2_1\epsilon_{\alpha\gamma}\epsilon_{\delta\beta}u_{\beta,\delta\gamma} + F^0_\alpha = 0\,,$$

(1)The considerations deal with the so-called reduced approach in the theory of couple stresses where the symmetric part of the force stress tensor appears explicitly, and the antisymmetric part is eliminated (cf. M. SOKOŁOWSKI [8, p. 16 and 22]).

where $\overset{*}{\Box}_2 = \mu\nabla_1^2 - \rho\partial_t^2$, F_α^0 represents a field of given distortions $\kappa_{\alpha 3}^0$, $\varepsilon_{\alpha\beta}^0$.

Theory of classical symmetric elasticity [3] – the displacement equations with a distortion field $\varepsilon_{\alpha\beta}^0$ for the I-st plane strain problem are obtained from Eqs.$(1.2)_1$ by substituting $\alpha = 0$.

Theory of hypothetical medium [1, p. 33] – this is that reduction of the micropolar theory of elasticity where three from the six material constants vanish. In general there remain elasticity constants β, γ and ε, but in the I-st plane strain problem examined in the paper the constant β does not appear. The effect of microstructure of the hypothetical micropolar medium is described only by a rotation field and a nonsymmetric field of couple stresses which is a linear function of a nonsymmetric deformation field. The rotational equation with a distortion field $\kappa_{\alpha 3}^0$ is obtained from Eq.$(1.2)_2$ by substituting $\alpha = 0$.

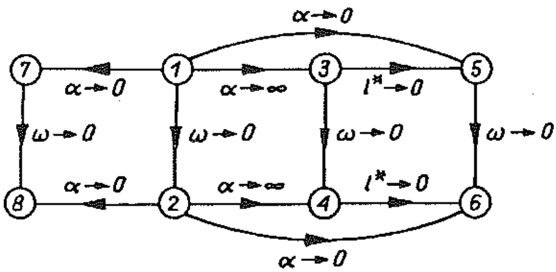

FIG. 1.

By constructing appropriate limit transitions, we will demonstrate full correspondence between the results for particular theories. In micropolar elastostatics we will obtain M. SUCHAR'S result [5]. The graphical diagram (Fig. 1) will help in illustrating the results to be presented. Numbers (1,2), (3,4), (5,6) and (7,8) denote: micropolar theory, couple-stress theory, classical theory and theory of hypothetical medium, respectively. Numbers (1,3,5,7) refer to elastodynamics, and (2,4,6,8) – to elastostatics. The symbols ($\alpha \to 0$, $\alpha \to \infty$, $l^* \to 0$, $\omega \to 0$) in the diagram denote necessary (not always sufficient) conditions for the respective limit transitions. When integrating the Eqs.(1.6), there appear the characteristic integrals (cf. [1, p. 181]):

$$(1.9) \qquad I_j = \text{p.f.}\frac{1}{2\pi}\int\limits_{\mathbb{R}^2} B_j \exp(-i\xi_\alpha x_\alpha)\, d\xi_1 d\xi_2$$

$$= \left[-(\mathbb{C} + \ln r), K_0(r/l), -\frac{r^2}{4}I_1\right],$$

where $j = 1,2,3$, $B_j \equiv \left[\xi^{-2}, (\xi^2 + l^{-2})^{-1}, \xi^{-4}\right]$, $\xi = (\xi_\alpha \xi_\alpha)^{1/2}$, $r = (x_\alpha x_\alpha)^{1/2}$, $\mathbb{C}$ – Euler constant, $K_0(\ldots)$ – modified Bessel function of third kind, $l = \left[\dfrac{(\gamma+\varepsilon)(\mu+\alpha)}{4\alpha\mu}\right]^{1/2}$. Symbol p.f. (the finite part) means that from the integrals divergent in the sense of the Cauchy principal value the so-called finite part I_j is extracted. This problem has been examined in detail by R. GANOWICZ [6, 7]. Note that in the sense of distributions the following equality holds

$$\frac{1}{2\pi} \int_{\mathbb{R}^1} \exp[it(\tau - \omega)] dt = \delta(\tau - \omega) . \tag{1.10}$$

The method of direct integration of Eqs.(1.2) used in the paper requires the assumption that the functions (u_α, φ_3) are of class $\mathbb{C}^6$ and $\mathbb{C}^4$, respectively (possibly except at the origin of the system Ox_1x_2 where the derivative is understood in the sense of distributions), and such that the formulae (1.7) and the properties of Fourier transforms could be applied to Eqs.(1.6). The paper is written in a concise form and presents in principle the final results. Detailed considerations about derivation of the applied equations and the presented results as well as the analysis of complex limit transitions are given in the papers [12] and [14].

The fundamental equation for stresses can be determined from the formulae

$$\begin{aligned} \sigma_{\alpha\beta} &= (\mu + \alpha) u_{\beta,\alpha} + (\mu - \alpha) u_{\alpha,\beta} - 2\alpha \epsilon_{\alpha\beta} \varphi_3 + \lambda u_{\gamma,\gamma} \delta_{\alpha\beta} , \\ u_{\alpha 3} &= (\gamma + \varepsilon) \varphi_{3,\alpha} , \end{aligned} \tag{1.11}$$

or by using the stress equations.

2. FIELD OF DISPLACEMENTS AND ROTATIONS INDUCED BY DISTORTION γ^0_{11}

From the graphical diagram (Fig.1) we obtain:

(1) *Micropolar elastodynamics.* We assume that the distortion field is represented by γ^0_{11}. The remaining distortions are equal to zero. By using (1.3)–(1.5), (1.7) and (1.9), from Eqs.(1.6) after essential transformations

we obtain the result

$$
\begin{aligned}
u_\alpha &= \frac{\gamma_0}{2\pi}e^{-i\omega t}\epsilon_{\alpha\beta}\left\{\frac{2\mu}{\mu+\alpha}\frac{1}{\sigma_2^2}[A_1K_0(-ik_1r)+A_2K_0(-ik_2r)\right.\\
&\qquad\left. - K_0(-i\sigma_1 r)]_{,12\beta} + a_{\alpha\beta}[K_0(-i\sigma_1 r)]_{,\alpha}\right\},\\
\varphi_3 &= \frac{\gamma_0}{2\pi}e^{-i\omega t}\frac{1}{l^2(k_1^2-k_2^2)}[K_0(-ik_1r)-K_0(-ik_2r)]_{,12},
\end{aligned}
\tag{2.1}
$$

where

$$
A_1=\frac{\sigma_2^2-k_2^2}{k_1^2-k_2^2},\qquad A_2=\frac{\sigma_2^2-k_1^2}{k_2^2-k_1^2},\qquad a_{12}=-1,\qquad a_{21}=\frac{\lambda}{2\mu+\lambda},
$$

$$
k_{1,2}^2=\frac{1}{2}\left(A\mp\sqrt{A^2-4\sigma_2^2k_4^2}\right),\quad A=\sigma_2^2+\sigma_4^2+\eta_0^2-\nu_0^2,\quad \sigma_1=\frac{\omega}{c_1},\quad \sigma_2=\frac{\omega}{c_2},
$$

$$
\sigma_4=\frac{\omega}{c_4},\quad c_1=\left(\frac{\lambda+2\mu}{\rho}\right)^{1/2},\quad c_2=\left(\frac{\mu+\alpha}{\rho}\right)^{1/2},\quad c_4=\left(\frac{\gamma+\varepsilon}{J}\right)^{1/2},
$$

$$
\nu_0^2=\frac{4\alpha}{\gamma+\varepsilon},\quad \eta_0^2=\frac{4\alpha^2}{(\mu+\alpha)(\gamma+\varepsilon)},\quad k_4^2=\frac{J\omega^2-4\alpha}{\gamma+\varepsilon},\quad \gamma_0^{11}=\gamma_0.
$$

On substituting Eqs.(2.1) and Eqs.(1.3)-(1.5) for γ_{11}^0 and taking

$$
\frac{1}{2\pi}\nabla_1^2(\ln r)=\delta(x_1)\delta(x_2)
\tag{2.2}
$$

the Eqs.(1.2) are fulfilled identically. Moreover, (u_α, φ_3) from Eqs.(2.1) satisfy the condition

$$
u_\alpha \longrightarrow 0,\quad \varphi_3\longrightarrow 0\quad \text{for}\quad r=(x_\alpha x_\alpha)^{1/2}\longrightarrow\infty.
\tag{2.3}
$$

The formulae (2.1) define the fundamental solution $(u_{\alpha 11}^{\gamma_0}, \varphi_{311}^{\gamma_0})$. We will consider the limit cases of the formulae (2.1). The results shall represent the fundamental solutions for the models (2)–(6). Respective basic equations within (2)–(6) shall be fulfilled identically and the condition (2.3) shall be satisfied; this will not be pointed out explicitly in the following.

(3) *Elastodynamics in the couple-stress theory.* We consider the case [(1) $\overset{\alpha\to\infty}{\longrightarrow}$ (3)] by assuming

$$
\left(\frac{\gamma+\varepsilon}{4\mu}\right)^{1/2}\longrightarrow l^*,\quad J\longrightarrow 0,\quad \gamma_0\longrightarrow\varepsilon_0,
\tag{2.4}
$$

where l^* is the elastic constant of the couple-stress theory. The result (2.1) reduces to the fundamental solution $(u^{\varepsilon_0}_{\alpha 11}, \varphi^{\varepsilon_0}_{311})$ for (3):

$$(2.5)\quad \begin{aligned} u_\alpha &= \frac{\varepsilon_0}{2\pi} e^{-i\omega t} \epsilon_{\alpha\beta} \left\{ \frac{2\mu}{\rho\omega^2} [A_1^* K_0(-ik_1^* r) + A_2^* K_0(-ik_2^* r) \right. \\ &\qquad \left. - K_0(-i\sigma_1 r)]_{,12\beta} + a_{\alpha\beta}[K_0(-i\sigma_1 r)]_{,\alpha} \right\}, \\ \varphi_3 &= \frac{\varepsilon_0}{2\pi} e^{-i\omega t} \frac{1}{l^{*2}(k_1^{*2} - k_2^{*2})} [K_0(-ik_1^* r) - K_0(-ik_2^* r)]_{,12}, \end{aligned}$$

where

$$A_1^* = \frac{k_2^{*2}}{k_2^{*2} - k_1^{*2}}, \qquad A_2^* = \frac{k_1^{*2}}{k_1^{*2} - k_2^{*2}}, \qquad \varepsilon_0^{11} = \varepsilon_0,$$

$$k_{1,2}^{*2} = \frac{1}{2(l^*)^2} \left(-1 \mp \sqrt{1 + \frac{4\rho\omega^2}{\mu}(l^*)^2} \right).$$

This can be justified as follows. The basic equations for (3) have the form

$$(2.6)\quad \begin{aligned} &\overset{*}{\Box}_2\, u_\alpha + (\lambda+\mu) u_{\beta,\beta\alpha} + 2\mu l^{*2} \nabla_1^2 \epsilon_{\alpha\gamma} \varphi_{3,\gamma} + F_\alpha^0 = 0, \\ &\varphi_3 = \frac{1}{2} \epsilon_{\alpha\beta} u_{\beta,\alpha}, \end{aligned}$$

where

$$\overset{*}{\Box}_2 = \mu \nabla_1^2 - \rho \partial_t^2, \qquad F_\alpha^0 = -\left(s^0_{\beta\alpha,\beta} + \frac{1}{2} \epsilon_{\alpha\delta} m^0_{\gamma 3,\gamma\delta} \right),$$

$$m^0_{\alpha 3} = 4\mu l^{*2} \kappa^0_{\alpha 3}, \qquad s^0_{\alpha\beta} = 2\mu \varepsilon^0_{\alpha\beta} + \lambda \varepsilon^0_{\gamma\gamma} \delta_{\alpha\beta}.$$

Symbols $\kappa^0_{\alpha 3}, \varepsilon^0_{\alpha\beta}$ denote the fields of given distortions. From Eq.(2.6) we obtain the equations

$$(2.7)\quad \begin{aligned} &\Box_1 \left[\nabla_1^2 (l^{*2} \nabla_1^2 - 1) + \hat{c}_2^{-2} \partial_t^2 \right] u_\alpha \\ &\qquad = \frac{1}{\mu} \Box_1 F_\alpha^0 + \left(\frac{1}{1-2\nu} + l^{*2} \nabla_1^2 \right) s^0_{\beta\gamma,\beta\gamma\alpha}, \\ &\left[\nabla_1^2 (l^{*2} \nabla_1^2 - 1) + \hat{c}_2^{-2} \partial_t^2 \right] \varphi_3 = \frac{1}{2\mu} \epsilon_{\delta\gamma} F^0_{\gamma,\delta}, \end{aligned}$$

where $\hat{c}_2 = (\mu/\rho)^{1/2}$, $\nu = \dfrac{\lambda}{2(\mu+\lambda)}$.

Equations (2.7) lead, on integrating and assuming

$$(2.8)\quad \varepsilon^0_{11} = \varepsilon_0^{11} e^{-i\omega t} \delta(x_1) \delta(x_2),$$

to the Eqs.(2.5), for which the formula $(2.6)_2$ holds.

(5) *Classical elastodynamics.* We will consider two variants:

VARIANT 1. It concerns the case $[(1) \xrightarrow{\alpha\to 0} (5)]$. By assuming Eqs.$(2.4)_{2,3}$ as well, the Eqs.$(2.1)_1$ reduces to the classical result $u^{\varepsilon_0}_{\alpha 11}$:

$$(2.9)\qquad u_\alpha = \frac{\varepsilon_0}{2\pi} e^{-i\omega t} \epsilon_{\alpha\beta} \left\{ \frac{2\mu}{\rho\omega^2} \left[K_0(-i\sigma_2^* r) - K_0(-i\sigma_1 r) \right]_{,12\beta} + a_{\alpha\beta} \left[K_0(-i\sigma_1 r) \right]_{,\alpha} \right\},$$

where $\sigma_2^* = \dfrac{\omega}{\hat{c}_2}$. To justify the Eq.(2.9), let us note the basic equations for (5):

$$(2.10)\qquad \overset{*}{\Box}_2\, u_\alpha + (\lambda+\mu) u_{\beta,\beta\alpha} = \sigma^0_{\beta\alpha,\beta},$$

where $\sigma^0_{\alpha\beta} = s^0_{\alpha\beta}$. From Eq.(2.10) we obtain the equations

$$(2.11)\qquad \Box_1\, \overset{*}{\Box}_2\, u_\alpha = -(\lambda+\mu)\sigma^0_{\gamma\delta,\gamma\delta\alpha} + \Box_1 \sigma^0_{\varepsilon\alpha,\varepsilon},$$

which, on taking Eq.(2.8) into account and integrating, lead to the Eq.(2.9). From the formula $(2.1)_2$ we do not recover the classical result $\varphi^{\varepsilon_0}_{311}$ since we have $\varphi_3 \xrightarrow{\alpha\to 0} 0$. Rotations φ_3 are obtained from Eqs.$(2.6)_2$ and (2.9), or from the equation

$$(2.12)\qquad \overset{*}{\Box}_2\, \varphi_3 = \frac{1}{2}\epsilon_{\alpha\beta}\sigma^0_{\gamma\beta,\gamma\alpha},$$

taking Eq.(2.8) into account:

$$(2.13)\qquad \varphi_3 = \frac{\varepsilon_0}{2\pi} e^{-i\omega t} \left[K_0(-i\sigma_2^* r) \right]_{,12};$$

VARIANT 2. It concerns the case $[(3) \xrightarrow{l^*\to 0} (5)]$. The result Eqs.(2.5) reduce to the classical Eqs.(2.9), (2.13).

(2) *Elastostatics in micropolar theory.* We consider the case $[(1) \xrightarrow{\omega\to 0} (2)]$. The limit transition is complex. Equations (2.1) reduce to the result for (2):

$$(2.14)\qquad \begin{aligned} u_1 &= -\frac{\gamma_0}{2\pi}\left[I_{1,1} + \frac{\gamma+\varepsilon}{2\mu}(I_1 - I_2)_{,122} - \frac{1}{1-\nu} I_{3,122} \right], \\ u_2 &= \frac{\gamma_0}{2\pi}\left[I_{1,2} + \frac{\gamma+\varepsilon}{2\mu}(I_1 - I_2)_{,112} + \frac{1}{1-\nu} I_{3,222} \right], \\ \varphi_3 &= \frac{\gamma_0}{2\pi}(I_1 - I_2)_{,12}. \end{aligned}$$

This is justified as follows. The basic equations for (2) have the form

$$(2.15)\qquad \begin{aligned} (\mu+\alpha)\nabla_1^2 u_\alpha + (\lambda+\mu-\alpha) u_{\beta,\beta\alpha} + 2\alpha\epsilon_{\alpha\gamma}\varphi_{3,\gamma} &= R^0_\alpha, \\ \left[(\gamma+\varepsilon)\nabla_1^2 - 4\alpha\right]\varphi_3 + 2\alpha\epsilon_{\alpha\beta}u_{\beta,\alpha} &= M^0_3. \end{aligned}$$

From Eqs.(2.15) we derive the equations

$$(2.16)\quad \begin{aligned} 2\mu\nabla_1^2\nabla_1^2(l^2\nabla_1^2-1)u_\alpha &= -\epsilon_{\alpha\beta}\nabla_1^2\left(M_3^0+\frac{\gamma+\varepsilon}{2\mu}\epsilon_{\delta\gamma}R^0_{\delta,\gamma}\right)_{,\beta} \\ &\quad + 2(l^2\nabla_1^2-1)\left(\nabla_1^2R^0_\alpha-\frac{\lambda+\mu}{2\mu+\lambda}R^0_{\beta,\beta\alpha}\right), \\ 2\mu\nabla_1^2(l^2\nabla_1^2-1)\varphi_3 &= \frac{\mu+\alpha}{2\alpha}\nabla_1^2M_3^0+\epsilon_{\delta\gamma}R^0_{\delta,\gamma}. \end{aligned}$$

Eqs.(2.16) imply, on substituting Eqs.(1.3), (1.4) and

$$(2.17)\quad \gamma^0_{11}=\gamma_0^{11}\delta(x_1)\delta(x_2),$$

the Eqs.(2.14).

(4) *Elastostatics in the couple-stress theory.* We will consider here two variants:

VARIANT 1. It concerns the case $[(3)\stackrel{\omega\to 0}{\longrightarrow}(4)]$. Equations (2.5) reduce to the formulae for (4):

$$(2.18)\quad \begin{aligned} u_1 &= -\frac{\varepsilon_0}{2\pi}\left[I_{1,1}+2l^{*2}(I_1-\overset{*}{I}_2)_{,122}-\frac{1}{1-\nu}I_{3,122}\right], \\ u_2 &= \frac{\varepsilon_0}{2\pi}\left[I_{1,2}+2l^{*2}(I_1-\overset{*}{I}_2)_{,112}+\frac{1}{1-\nu}I_{3,222}\right], \\ \varphi_3 &= \frac{\varepsilon_0}{2\pi}(I_1-\overset{*}{I}_2)_{,12}, \end{aligned}$$

where $\overset{*}{I}_2=K_0(r/l^*)$. Justification of Eqs.(2.18) is the following. The basic equations for (4) have the form Eq.$(2.6)_2$ and

$$(2.19)\quad \mu\nabla_1^2u_\alpha+(\lambda+\mu)u_{\beta,\beta\alpha}+\mu l^{*2}\nabla_1^2\epsilon_{\alpha\gamma}\varphi_{3,\gamma}+F^0_\alpha=0.$$

On separating the system of Eqs.$(2.6)_2$, (2.19) we obtain the equations

$$(2.20)\quad \begin{aligned} 2\mu\nabla_1^2\nabla_1^2(l^{*2}\nabla_1^2-1)u_\alpha &= -(1-\nu)^{-1}(l^{*2}\nabla_1^2-1)s^0_{\delta\gamma,\delta\gamma\alpha} \\ &\quad - 2l^{*2}\nabla_1^2\epsilon_{\alpha\beta}\epsilon_{\delta\gamma}F^0_{\gamma,\delta\beta}-2\nabla_1^2(l^{*2}\nabla_1^2-1)F^0_\alpha, \\ 2\mu\nabla_1^2(l^{*2}\nabla_1^2-1)\varphi_3 &= \epsilon_{\alpha\beta}F^0_{\beta,\alpha}, \end{aligned}$$

which lead, on substituting

$$(2.21)\quad \varepsilon^0_{11}=\varepsilon_0^{11}\delta(x_1)\delta(x_2),$$

to the Eqs.(2.18). For Eqs.(2.18) the relationship $(2.6)_2$ holds.

VARIANT 2. It concerns the case $[(2) \overset{\alpha\to\infty}{\longrightarrow} (4)]$. On assuming Eqs.$(2.4)_{1,3}$, the formulae (2.14) reduce to the formulae (2.18).

(6) *Classical elastostatics.* We will consider three variants:

VARIANT 1. It concerns the case $[(5) \overset{\omega\to 0}{\longrightarrow} (6)]$. The formulae (2.9), (2.13) reduce respectively to the formulae for (6):

$$
\begin{aligned}
u_1 &= -\frac{\varepsilon_0}{2\pi}\left(I_{1,1} - \frac{1}{1-\nu} I_{3,122}\right), \\
u_2 &= \frac{\varepsilon_0}{2\pi}\left(I_{1,2} + \frac{1}{1-\nu} I_{3,222}\right), \\
\varphi_3 &= \frac{\varepsilon_0}{2\pi} I_{1,12}.
\end{aligned}
\tag{2.22}
$$

Justification: the basic equations for (6) have the form

$$
\mu\nabla_1^2 u_\alpha + (\lambda+\mu) u_{\beta,\beta\alpha} = \overset{0}{\sigma}_{\beta\alpha,\beta}\,, \tag{2.23}
$$

where $\overset{0}{\sigma}_{\alpha\beta} = \overset{0}{s}_{\alpha\beta}$. From Eq.(2.23) by using Eq.$(2.6)_2$ we obtain the equations

$$
\begin{aligned}
2\mu\nabla_1^2\nabla_1^2 u_\alpha &= 2\nabla_1^2 \overset{0}{\sigma}_{\varepsilon\alpha,\varepsilon} - \frac{1}{1-\nu}\overset{0}{\sigma}_{\gamma\delta,\gamma\delta\alpha}\,, \\
2\mu\nabla_1^2\varphi_3 &= \epsilon_{\alpha\beta}\overset{0}{\sigma}_{\gamma\beta,\gamma\alpha}.
\end{aligned}
\tag{2.24}
$$

On integrating and taking Eq.(2.21) into account, the Eqs.(2.24) reduce to the formulae (2.22) for which Eq.$(2.6)_2$ holds.

VARIANT 2. It concerns the case $[(4) \overset{l^*\to 0}{\longrightarrow} (6)]$. The formulae (2.18) reduce to Eqs.(2.22).

VARIANT 3. It concerns the case $[(2) \overset{\alpha\to 0}{\longrightarrow} (6)]$. On assuming $(2.4)_3$, the formulae $(2.14)_{1,2}$ reduce to Eqs.$(2.22)_{1,2}$. Rotations φ_3 are determined from Eq.$(2.6)_2$ since the result Eq.$(2.14)_3$ does not lead to Eq.$(2.22)_3$ on account of: $\varphi_3 \overset{\alpha\to 0}{\longrightarrow} 0$.

Below we will put together the results for (u_α, φ_3) induced by distortions $\overset{0}{\gamma}_{22}, \overset{0}{\gamma}_{12}, \overset{0}{\gamma}_{21}, \overset{0}{\kappa}_{\alpha 3}$ within the models (1)–(8) (Fig. 1). Certain departures from the rules given in Point 2 for the limit transitions will be given in the form of propositions. They shall concern the distortions $\overset{0}{\gamma}_{12}$, $\overset{0}{\gamma}_{21}$ and $\overset{0}{\kappa}_{\alpha 3}$.

3. Distortion γ^0_{22}

The results for (1):

$$u_\alpha = \frac{\gamma_0}{2\pi} e^{-i\omega t} \epsilon_{\beta\alpha} \left\{ \frac{2\mu}{\mu+\alpha} \frac{1}{\sigma_2^2} [A_1 K_0(-ik_1 r) + A_2 K_0(-ik_2 r) - K_0(-i\sigma_1 r)]_{,12\beta} + a_{\beta\alpha}[K_0(-i\sigma_1 r)]_{,\alpha} \right\},$$

(3.1)

$$\varphi_3 = -\frac{\gamma_0}{2\pi} e^{-i\omega t} \frac{1}{l^2(k_1^2 - k_2^2)} [K_0(-ik_1 r) - K_0(-ik_2 r)]_{,12},$$

where $\gamma_0^{22} = \gamma_0$.

The results for (3):

$$u_\alpha = \frac{\varepsilon_0}{2\pi} e^{-i\omega t} \epsilon_{\beta\alpha} \left\{ \frac{2\mu}{\rho\omega^2} [A_1^* K_0(-ik_1^* r) + A_2^* K_0(-ik_2^* r) - K_0(-i\sigma_1 r)]_{,12\beta} + a_{\beta\alpha}[K_0(-i\sigma_1 r)]_{,\alpha} \right\},$$

(3.2)

$$\varphi_3 = -\frac{\varepsilon_0}{2\pi} e^{-i\omega t} \frac{1}{l^{*2}(k_1^{*2} - k_2^{*2})} [K_0(-ik_1^* r) - K_0(-ik_2^* r)]_{,12},$$

where $\varepsilon_0^{22} = \varepsilon_0$.

The results for (5):

$$u_\alpha = \frac{\varepsilon_0}{2\pi} e^{-i\omega t} \epsilon_{\beta\alpha} \left\{ \frac{2\mu}{\rho\omega^2} [K_0(-i\sigma_2^* r) - K_0(-i\sigma_1 r)]_{,12\beta} + a_{\beta\alpha}[K_0(-i\sigma_1 r)]_{,\alpha} \right\},$$

(3.3)

$$\varphi_3 = -\frac{\varepsilon_0}{2\pi} e^{-i\omega t} [K_0(-i\sigma_2^* r)]_{,12}.$$

The results for (2):

(3.4)

$$\begin{aligned} u_1 &= \frac{\gamma_0}{2\pi} \left[I_{1,1} + \frac{\gamma+\varepsilon}{2\mu}(I_1 - I_2)_{,122} + \frac{1}{1-\nu} I_{3,111} \right], \\ u_2 &= -\frac{\gamma_0}{2\pi} \left[I_{1,2} + \frac{\gamma+\varepsilon}{2\mu}(I_1 - I_2)_{,112} - \frac{1}{1-\nu} I_{3,112} \right], \\ \varphi_3 &= -\frac{\gamma_0}{2\pi}(I_1 - I_2)_{,12}. \end{aligned}$$

The results for (4):

$$
(3.5)\qquad
\begin{aligned}
u_1 &= \frac{\varepsilon_0}{2\pi}\left[I_{1,1} + 2l^{*2}(I_1 - \overset{*}{I}_2)_{,122} + \frac{1}{1-\nu} I_{3,111}\right],\\
u_2 &= -\frac{\varepsilon_0}{2\pi}\left[I_{1,2} + 2l^{*2}(I_1 - \overset{*}{I}_2)_{,112} - \frac{1}{1-\nu} I_{3,112}\right],\\
\varphi_3 &= -\frac{\varepsilon_0}{2\pi}(I_1 - \overset{*}{I}_2)_{,12}\,.
\end{aligned}
$$

The results for (6):

$$
(3.6)\qquad
\begin{aligned}
u_1 &= \frac{\varepsilon_0}{2\pi}\left(I_{1,1} + \frac{1}{1-\nu} I_{3,111}\right),\\
u_2 &= -\frac{\varepsilon_0}{2\pi}\left(I_{1,2} - \frac{1}{1-\nu} I_{3,112}\right),\\
\varphi_3 &= -\frac{\varepsilon_0}{2\pi} I_{1,12}\,.
\end{aligned}
$$

For the models (1)–(6) the following equality holds:

$$
\varphi_{311}^{\gamma_0} = -\varphi_{322}^{\gamma_0}\,.
$$

4. Distortions γ_{12}^0, γ_{21}^0

The results for (1):

$$
(4.1)\qquad
\begin{aligned}
u_{\alpha 12} = \frac{\gamma_0}{2\pi} e^{-i\omega t} \Bigg\|\Bigg[\epsilon_{\alpha\beta} \Bigg\{ &\frac{2\mu}{\mu+\alpha}\frac{1}{\sigma_2^2}[A_1 K_0(-ik_1 r)\\
&+ A_2 K_0(-ik_2 r) - K_0(-i\sigma_1 r)]_{,22} + [A_1 K_0(-ik_1 r)\\
&+ A_2 K_0(-ik_2 r)]\Bigg\}_{,\beta} - \frac{2\mu}{2\mu+\lambda}[K_0(-i\sigma_1 r)]_{,2}\delta_{\alpha 1}\Bigg]\Bigg\|,\\
\varphi_{312} = \frac{\gamma_0}{2\pi} e^{-i\omega t} \frac{1}{l^2(k_1^2 - k_2^2)} \Bigg\{ &[K_0(-ik_1 r) - K_0(-ik_2 r)]_{,22}\\
&+ \frac{\rho\omega^2}{2\mu}[K_0(-ik_1 r) - K_0(-ik_2 r)]\Bigg\},
\end{aligned}
$$

where $\gamma_0^{12} = \gamma_0$, and

$$
\begin{aligned}
u_{\alpha 21} = \frac{\gamma_0}{2\pi} e^{-i\omega t} \Bigg\| \Bigg[\epsilon_{\beta\alpha} \Bigg\{ \frac{2\mu}{\mu+\alpha} \frac{1}{\sigma_2^2} [A_1 K_0(-ik_1 r) \\
+ A_2 K_0(-ik_2 r) - K_0(-i\sigma_1 r)]_{,11} + [A_1 K_0(-ik_1 r) \\
+ A_2 K_0(-ik_2 r)] \Bigg\}_{,\beta} - \frac{2\mu}{2\mu+\lambda} [K_0(-i\sigma_1 r)]_{,1} \delta_{\alpha 2} \Bigg] \Bigg\|, \\
\varphi_{321} = -\frac{\gamma_0}{2\pi} e^{-i\omega t} \frac{1}{l^2(k_1^2 - k_2^2)} \Bigg\{ [K_0(-ik_1 r) - K_0(-ik_2 r)]_{,11} \\
+ \frac{\rho\omega^2}{2\mu} [K_0(-ik_1 r) - K_0(-ik_2 r)] \Bigg\},
\end{aligned} \tag{4.2}
$$

where $\gamma_0^{21} = \gamma_0$.

The results for (3):

$$
\begin{aligned}
u_\alpha = \frac{\varepsilon_0}{2\pi} e^{-i\omega t} \Bigg\| \Bigg[\epsilon_{\alpha\beta} \Bigg\{ \frac{2\mu}{\rho\omega^2} [A_1^* K_0(-ik_1^* r) + A_2^* K_0(-ik_2^* r) \\
- K_0(-i\sigma_1 r)]_{,22} - \frac{2\mu}{\rho\omega^2} [A_1^* K_0(-ik_1^* r) + A_2^* K_0(-ik_2^* r) \\
- K_0(-i\sigma_1 r)]_{,11} \Bigg\}_{,\beta} + a_{\alpha\gamma} [K_0(-i\sigma_1 r)]_{,\gamma} \Bigg] \Bigg\|, \\
\varphi_3 = \frac{\varepsilon_0}{2\pi} e^{-i\omega t} \frac{1}{l^{*2}(k_1^{*2} - k_2^{*2})} \Bigg\{ [K_0(-ik_1^* r) - K_0(-ik_2^* r)]_{,22} \\
- [K_0(-ik_1^* r) - K_0(-ik_2^* r)]_{11} \Bigg\},
\end{aligned} \tag{4.3}
$$

where $a_{11} = a_{22} = 0$, $a_{12} = a_{21} = -\dfrac{2\mu}{2\mu+\lambda}$, $\varepsilon_0^{12} = \varepsilon_0^{21} = \varepsilon_0$.

The results for (5):

$$
\begin{aligned}
u_\alpha = \frac{\varepsilon_0}{2\pi} e^{-i\omega t} \Bigg\{ \frac{4\mu}{\rho\omega^2} \Big[K_0(-i\sigma_2^* r) \\
- K_0(-i\sigma_1 r) \Big]_{,12\alpha} + b_{\alpha\beta} \Big[K_0(-i\sigma_2^* r) \Big]_{,\beta} \Bigg\}, \\
\varphi_3 = \frac{\varepsilon_0}{2\pi} e^{-i\omega t} \Big[K_0(-i\sigma_2^* r)_{,22} - K_0(-i\sigma_2^* r)_{,11} \Big],
\end{aligned} \tag{4.4}
$$

where $b_{11} = b_{22} = 0$, $b_{12} = b_{21} = 2$.

The results for (2):

$$
\begin{aligned}
u_{112} &= -\frac{\gamma_0}{2\pi}\left[I_1 + \frac{\gamma+\varepsilon}{2\mu}(I_1 - I_2)_{,22} + \frac{1}{1-\nu}I_{3,11}\right]_{,2}, \\
u_{212} &= -\frac{\gamma_0}{2\pi}\left[I_1 - \frac{\gamma+\varepsilon}{2\mu}(I_1 - I_2)_{,22} + \frac{1}{1-\nu}I_{3,22}\right]_{,1}, \qquad (4.5)\\
\varphi_{312} &= \frac{\gamma_0}{2\pi}(I_1 - I_2)_{,22}
\end{aligned}
$$

and

$$
\begin{aligned}
u_{121} &= -\frac{\gamma_0}{2\pi}\left[I_1 - \frac{\gamma+\varepsilon}{2\mu}(I_1 - I_2)_{,11} + \frac{1}{1-\nu}I_{3,11}\right]_{,2}, \\
u_{221} &= -\frac{\gamma_0}{2\pi}\left[I_1 + \frac{\gamma+\varepsilon}{2\mu}(I_1 - I_2)_{,11} + \frac{1}{1-\nu}I_{3,22}\right]_{,1}, \qquad (4.6)\\
\varphi_{321} &= -\frac{\gamma_0}{2\pi}(I_1 - I_2)_{,11}.
\end{aligned}
$$

The results for (4):

$$
\begin{aligned}
u_1 &= -\frac{\varepsilon_0}{\pi}\left[I_1 - l^{*2}(I_1 - \overset{*}{I}_2)_{,11} + l^{*2}(I_1 - \overset{*}{I}_2)_{,22} + \frac{1}{1-\nu}I_{3,11}\right]_{,2}, \\
u_2 &= -\frac{\varepsilon_0}{\pi}\left[I_1 - l^{*2}(I_1 - \overset{*}{I}_2)_{,22} + l^{*2}(I_1 - \overset{*}{I}_2)_{,11} + \frac{1}{1-\nu}I_{3,22}\right]_{,1}, \qquad (4.7)\\
\varphi_3 &= \frac{\varepsilon_0}{2\pi}\left[(I_1 - \overset{*}{I}_2)_{,22} - (I_1 - \overset{*}{I}_2)_{,11}\right].
\end{aligned}
$$

The results for (6):

$$
\begin{aligned}
u_1 &= -\frac{\varepsilon_0}{\pi}\left(I_1 + \frac{1}{1-\nu}I_{3,11}\right)_{,2}, \\
u_2 &= -\frac{\varepsilon_0}{\pi}\left(I_1 + \frac{1}{1-\nu}I_{3,22}\right)_{,1}, \qquad (4.8)\\
\varphi_3 &= \frac{\varepsilon_0}{2\pi}(I_{1,22} - I_{1,11}).
\end{aligned}
$$

PROPOSITION 1. For the following limit transitions: $[(1) \overset{\alpha\to\infty}{\longrightarrow} (3)]$, $[(2) \overset{\alpha\to\infty}{\longrightarrow} (4)]$, $[(1) \overset{\alpha\to 0}{\longrightarrow} (5)]$, and $[(2) \overset{\alpha\to 0}{\longrightarrow} (6)]$, within the models (1), (2) of the micropolar theory the expressions $(u^{\gamma_0}_{\alpha 12} + u^{\gamma_0}_{\alpha 21})$, $(\varphi^{\gamma_0}_{312} + \varphi^{\gamma_0}_{321})$, should be constructed and for them the respective limit transition should be made [13]. In the couple-stress theory (3), (4) and in the classical theory of elasticity (5), (6), we have

$$
u^{\varepsilon_0}_{\alpha 12} = u^{\varepsilon_0}_{\alpha 21}, \qquad \varphi^{\varepsilon_0}_{312} = \varphi^{\varepsilon_0}_{321}.
$$

5. Fields of displacements and rotations induced by distortions $\kappa^0_{\gamma 3}$

The results for (1):

$$
(5.1)\quad
\begin{aligned}
u_{\alpha\gamma 3} &= -\frac{\kappa_0}{2\pi} e^{-i\omega t} \frac{2\mu}{\mu+\alpha} \frac{1}{k_1^2 - k_2^2} \epsilon_{\alpha\beta} \left[K_0(-ik_1 r) - K_0(-ik_2 r) \right]_{,\gamma\beta} ,\\
\varphi_{3\gamma 3} &= \frac{\kappa_0}{2\pi} e^{-i\omega t} \frac{1}{k_1^2 - k_2^2} \left(\nabla_1^2 + \frac{\rho\omega^2}{\mu+\alpha} \right) \left[K_0(-ik_1 r) - K_0(-ik_2 r) \right]_{,\gamma} .
\end{aligned}
$$

PROPOSITION 2. For $\alpha \to 0$ (substituting $\rho = 0$) from Eq.$(5.1)_1$ we conclude that $u_\alpha \to 0$. We have the case $[(1) \overset{\alpha\to 0}{\longrightarrow} (7)]$. The Eq.$(5.1)_2$ reduces to

$$
(5.2)\qquad \varphi_{3\gamma 3} = -\frac{\kappa_0}{2\pi} e^{-i\omega t} \left[K_0(-ik_4^* r) \right]_{,\gamma} ,
$$

where $k_4^* = \dfrac{J\omega^2}{\gamma+\varepsilon}$, $\kappa_0^{\gamma 3} = \kappa_0$. This is the solution of the equation for (7)

$$
(5.3)\qquad \overset{*}{\Box}_4\, \varphi_3 = \mu^0_{\delta 3,\delta} ,
$$

where $\overset{*}{\Box}_4 = (\gamma+\varepsilon)\nabla_1^2 - J\partial_t^2$.

The results for (2):

$$
(5.4)\quad
\begin{aligned}
u_{\alpha\gamma 3} &= -\frac{\kappa_0}{2\pi} \frac{\gamma+\varepsilon}{2\mu} \epsilon_{\alpha\beta} (I_1 - I_2)_{,\gamma\beta} ,\\
\varphi_{3\gamma 3} &= -\frac{\kappa_0}{2\pi} I_{2,\gamma} .
\end{aligned}
$$

We have the cases $[(2) \overset{\alpha\to 0}{\longrightarrow} (8)]$ and $[(7) \overset{\omega\to 0}{\longrightarrow} (8)]$.

The result for (8) has the form

$$
(5.5)\qquad \varphi_{3\gamma 3} = -\frac{\kappa_0}{2\pi} I_{1,\gamma} .
$$

This is the solution of the equation

$$
(5.6)\qquad (\gamma+\varepsilon)\nabla_1^2 \varphi_3 = \mu^0_{\delta 3,\delta} .
$$

The results for (3):

$$
(5.7)\quad
\begin{aligned}
u_{\alpha\gamma 3} &= -\frac{\kappa_0}{\pi} e^{-i\omega t} \frac{1}{k_1^{*2} - k_2^{*2}} \epsilon_{\alpha\beta} \left[K_0(-ik_1^* r) - K_0(-ik_2^* r) \right]_{,\gamma\beta} ,\\
\varphi_{3\gamma 3} &= \frac{\kappa_0}{2\pi} e^{-i\omega t} \frac{1}{k_1^{*2} - k_2^{*2}} \left[K_0(-ik_1^* r) - K_0(-ik_2^* r) \right]_{,\gamma\alpha\alpha} .
\end{aligned}
$$

The results for (4):

$$(5.8)\qquad \begin{aligned} u_{\alpha\gamma 3} &= -\frac{\kappa_0}{\pi} l^{*2} \epsilon_{\alpha\beta}(I_1 - \overset{*}{I}_2)_{,\gamma\beta}\,, \\ \varphi_{3\gamma 3} &= -\frac{\kappa_0}{2\pi} \overset{*}{I}_{2,\gamma}\,. \end{aligned}$$

PROPOSITION 3. If $l^* \to 0$ then from Eqs.(5.7) and (5.8) we obtain that $u_\alpha \to 0$ and $\varphi_3 \to 0$.

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