ENGINEERING TRANSACTIONS • Enging. Trans. • 40, 1, 145-155, 1992
Polish Academy of Sciences • Institute of Fundamental Technological Research

BRIEF NOTES

THE EFFECT OF MANUFACTURING ERRORS ON THE STIFFNESS OF ROLLING CONTACT BEARINGS

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The paper presents an analysis of the effect of manufacturing errors on the stiffness of rolling contact bearings. The analysis is confined to considerations of the unevenness of raceways and the variable dimensions and variable elasticity of rolling elements. Unevenness of the rollers in roller bearings can also be easily incorporated into the analysis.

1. INTRODUCTION

The elastic characteristics of rolling contact bearings and their variation in rotation, resulting in parametric excitation, are among the basic causes of the vibrational phenomena that occur during the rotation of a precision rotor in a rolling contact bearing, along with kinematic excitation. It is very difficult to study these characteristics because the rolling contact bearing is a complex mechanical system.

A number of studies wholly or partly devoted to the study of stiffness in rolling contact bearings have been undertaken [1, 2, 3,]. NOVIKOV [1] derived expressions in a linearized form for axial and radial stiffness of a radial thrust bearing taking into account the nonlinearity of elastic characteristics. KHARLAMOV [2] analysed the case of static equilibrium of a radial thrust bearing when the axial load appreciably exceeds the radial load. Simple formulas for the radial, axial and transverse stiffness of the bearing were obtained for this case by linearizing the equations. The formulas obtained for radial and axial stiffness were compared with those obtained by Novikov.

In the theoretical analysis carried out by SZUCKI [3] a procedure is given for computation of the elasticity of a ball bearing. The load is represented in the form of radial force, axial force and the torque in the plane of these forces. The assumption was made that in a mounted bearing, clearance and preload are absent.

In all the above mentioned studies the dependence of stiffness on angular rotation of the cage due to changes in the position of the rolling bodies and errors in components have not been taken into account. The importance of this problem was stressed in the theoretical work of TAMURA snd SHIMIZU [4] and that of NEUBERT [5] but has not been solved.

Although the components of precision rolling contact bearings can be manufactured with high accuracy, certain errors in the geometric shape and dimensions and deviation in the properties of the material are unavoidable. These errors basically have an effect only on the periodic component of stiffness. It is known that even in ideal bearings the amplitude of the periodic component of stiffness is considerable in the case of initial clearance. Hence the effect of errors is considered only for the case of preloaded bearings when the change in stiffness caused by imperfections of geometric shape and the characteristics and sizes of components outdo the change in stiffness in ideal bearings.

This paper presents an analysis which is confined to consideration of the unevenness of raceways and the variable dimensions and variable elasticity of rolling elements. Unevenness of the rollers of roller bearings can also be taken into account without any special difficulty. As far as errors in the geometric shape of the balls are concerned, it is rather difficult to include them in the analysis because the axis of rotation of a ball changes during operation of the bearing.

2. RADIAL STIFFNESS OF A ROLLING CONTACT BEARING AS A FUNCTION OF ERRORS IN COMPONENTS

During the rotation of a bearing the rolling elements roll along the raceways of the outer and inner rings and, hence, their contact points continuously change. In the case of an ideal bearing this condition has no real significance. In an actual bearing, the rolling elements are either climbing over bumps or entering troughs while rolling along the raceway and the mutual position of the unevenness of the outer and inner rings changes continuously.

At the initial moment of time, let the ball or roller be in contact with point 1 (Fig.1) of the stationary outer ring and with point 2 of the inner ring rotating in an anticlockwise direction. After a time t point 2 moves away from the stationary point 1 by an angle $\omega_{\text{in}}t$, where ω_{in} is the angular

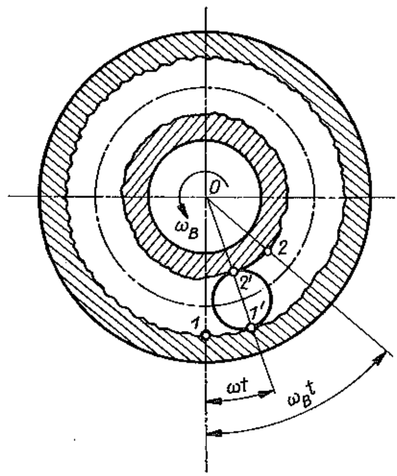

FIG. 1. Displacement of contact points of outer and inner rings of bearing with same rolling element during rotation of inner ring.

velocity of the inner ring. The ball is displaced during this time in the same direction by an angle ωt, where $\omega t < \omega_{\text{in}}t$ since the angular velocity of the cage ω is less than ω_{in}. Then the rings make contact with the ball at points $1'$ and $2'$. This means that the point of contact of the ball with the outer ring moved along the profile of the ring in an anticlockwise direction through an angular distance ωt, and the point of contact of the ball with the inner ring moved in a clockwise direction through a distance $\omega_{\text{in}}t - \omega t = (\omega_{\text{in}} - \omega)t$, or $q\omega t$, if

$$q = \frac{\omega_{\text{in}}}{\omega} - 1\,.$$

So, during the rotation of the cage through an angle $\omega t = \varphi$, the contact point between the stationary ring and the ball moves through an angular distance φ and the contact point of the inner ring with the ball through a distance $q\varphi$. For the i-th rolling element these angular distances can be written respectively in the form $\varphi + i\gamma$ and $q\varphi + i\gamma$, where $\gamma = (2\pi)/m$ is the angular distance between the rolling elements, m is the number of rolling elements in the bearing and $i = 0, 1, 2, ..., m-1$. It is possible to consider the raceway as a circle with a variable radius. Since its graph is a closed curve, this function is periodic and can be expanded in a Fourier series. Taking the above into account, the profile of the raceway of the outer ring can be written in the form:

$$R_0 + \sum_l \Delta R_l \sin\{l(\varphi + i\gamma) + \alpha_l\} \tag{2.1}$$

and the profile of the raceway of the inner ring, along which the ball rolls simultaneously, in the form:

$$r_0 + \sum_p \Delta r_p \sin\{p\,(q\varphi + i\gamma) + \alpha_p\}\,. \tag{2.2}$$

Here, R_0 and r_0 are the constant components of the radii of the raceways of the outer and inner rings, respectively; l and p are the order of the harmonic of unevenness in the raceways of the outer and inner rings, respectively; ΔR_l and Δr_p are the amplitudes of these harmonics; α_l and α_p are the phase angles.

Let us determine now the limits of the summation of the series given by Eqs. (2.1) and (2.2). Obviously it is meaningful to take the upper limit as infinity, since for sufficiently large values of l and p unevenness of raceways is covered by the contact area and stops affecting periodic component of stiffness. Besides, the upper limit of the quantities l and p, which will be called s, cannot be determined rigorously and uniquely since it depends on the value of preload, mutual displacement of rings, mechanical properties of the materials of the components and other factors. The lower limit in Eq. (2.2) is equal to unity since, without exception, all lower harmonics of errors in raceways of a rotating ring affect the change in contact deformation during rotation under the condition of x = constant. It is impossible to say this about the stationary outer ring. The first harmonic of unevenness is caused by eccentricity which does not affect the change in contact deformation since the bearing is automatically centered in the inner ring. If, due to design or some other factor, the center of the outer ring still does not coincide with the center of rotation of the inner ring, this misalignment is taken as the value of x in Eq. (2.3) given below. Thus the lower limit of summation in Eq. (2.1) or minimum value of l should be taken equal to two but the minimum value of p in Eq. (2.2) as equal to unity.

Let the value of radial displacement x of the inner ring with respect to the outer remain constant during the rotation of the bearing. Then the total contact deformation at places where the rings rub against the rolling elements is determined by the expression:

$$\delta_i = [g_i + (x_1 + x_0)\cos(\varphi + i\gamma)) + (y_1 + y_0)\sin(\varphi + i\gamma) + \sum_{l=2}^{s} \Delta R_l \sin\{l(\varphi + i\gamma) + \alpha_l\} + \sum_{p=1} \Delta r_p \sin\{p(q\varphi + i\gamma) + \alpha_p\}], \tag{2.3}$$

where $g_i = g + \Delta D_i$, ΔD_i is the deviation of the actual diameter of the i-th rolling element from the nominal diameter, x_1 – the mutual displacement of rings in the direction x under the action of radial load, x_0 – the mutual displacement of rings in the direction x for zero radial load, y_1 – the mutual displacement of rings in the direction y under the action of radial load in an ideal bearing, y_0 – the mutual displacement of rings in the direction y for zero radial load $x_0 = x - x_1$.

Substituting the expression for total contact deformation, (Eq. (2.3)), into the equation giving the applied radial load on the bearing,

$$P_i = K\delta_i^n \cos\eta_i$$

where n is the index of power for ball bearings ($n = 3/2$) and for roller bearings ($n = 10/9$), η_i is the angle between lines of action of radial load (direction of displacement of the moving ring) and the radius passing through the center of the i-th rolling element; summing with respect to i, the formula for radial elastic force for the entire bearing is obtained:

$$P = \sum_{l=0}^{m-1} K_i[g_i + (x_1 + x_0)\cos(\varphi + i\gamma) + y_0 \sin(\varphi + i\gamma) + \sum_{l=2}^{s} \Delta R_l \sin\{l(\varphi + i\gamma) + \alpha_i\} + \sum_{p=1}^{s} \Delta r_p \sin\{p(q\varphi + i\gamma) + \alpha_p\}]^n \cos(\varphi + i\gamma), \tag{2.4}$$

where K_i is the coefficient of proportionality. Its value, except for a number of geometric parameters, depends on the mechanical properties of the material of the rolling elements and the rings of the bearing, namely, on the modulus of elasticity and Poisson's ratio. It can be therefore said that value of K for the contact of each rolling element with the rings depends on the mechanical properties of the material of a particular rolling element and can be used to describe the differences in the mechanical properties of the materials. In Eq. (2.4) usually $y_1 = 0$, because only a bearing with preload is being considered here, when the values of y_1 are small and only slightly affect the stiffness.

Values of x_0 and y_0 are determined from the condition that the projections of the elastic force of the complete bearing on the axes x and y are equal to zero (in the absence of radial load):

$$P = 0, \qquad P_y = 0.$$

In this case, $x_1 = 0$, $y_1 = 0$ and the condition written above takes the form:

$$P_y = \sum_{l=0}^{m-1} K_i[g_i + x_0 \cos(\varphi + i\gamma) + y_0 \sin(\varphi + i\gamma) + \sum_{l=2}^{s} \Delta R_l \sin\{l(\varphi + i\gamma) + \alpha_l\} + \sum_{p=1}^{s} \Delta r_p \sin\{p(q\varphi + i\gamma) + \alpha_p\}]^n \sin(\varphi + i\gamma), \tag{2.5}$$

$$P = \sum_{i=0}^{m-1} K_i[g_i + x_0 \cos(\varphi + i\gamma) + y_0 \sin(\varphi + i\gamma) + \sum_{l=2}^{s} \Delta R_l \sin\{l(\varphi + i\gamma) + \alpha_l\} + \sum_{p=1}^{s} \Delta r_p \sin\{p(q\varphi + i\gamma) + \alpha_p]^n \cos(\varphi + i\gamma). \tag{2.6}$$

By expanding the above expressions, (Eqs. (2.5), (2.6)), in a McLaurin series in powers of x_0 and y_0 and limiting the terms containing x_0 and y_0 to the zeroth and first order, the following is obtained:

$$\begin{aligned} A + Bny_0 + Cnx_0 &= 0, \\ B + Cny_0 + Enx_0 &= 0, \end{aligned} \tag{2.7}$$

where

$$\begin{aligned} A &= \sum_{i=0}^{m-1} K_i \left[g_i + \sum_l + \sum_p\right]^n \sin\eta_i, \\ B &= \sum_{i=0}^{m-1} K_i \left[g_i + \sum_l + \sum_p\right]^n \sin^2\eta_i, \\ C &= \sum_{i=0}^{m-1} K_i \left[g_i + \sum_l + \sum_p\right]^{n-1} \sin\eta_i \cos\eta_i, \end{aligned}$$

$$
\begin{aligned}
D &= \sum_{i=0}^{m-1} K_i \left[g_i + \sum_l + \sum_p\right]^n \cos\eta_i\,, \\
E &= \sum_{i=0}^{m-1} K_i \left[g_i + \sum_l + \sum_p\right]^{n-1} \cos^2\eta_i\,, \\
\sum_l &= \sum_{l=2}^{s} \Delta R_l \sin\{l(\varphi + i\gamma) + \alpha_l\}\,, \\
\sum_p &= \sum_{p=1}^{s} \Delta r_p \sin\{p(q\varphi + i\gamma) + \alpha_p\}\,.
\end{aligned}
$$

Solving the system of equations given above with respect to x_0 and y_0 gives:

$$
x = \frac{AE - CD}{n(C^2 - BE)}\,, \qquad y = \frac{BD - AC}{n(C^2 - BE)}\,. \tag{2.8}
$$

After substituting these expressions into Eq. (2.4) and differentiating with respect to x_1, the formula for radial stiffness of a radial bearing is obtained which incorporates the errors in the components:

$$
\begin{aligned}
k = n \sum_{i=0}^{m-1} K_i \Bigg[& g_i + x_1 \cos(\varphi + i\gamma) + \frac{AE - CD}{n(C^2 - BE)} \cos(\varphi + i\gamma) \\
& + \frac{BD - AC}{n(C^2 - BE)} \sin(\varphi + i\gamma) + \sum_l + \sum_p \Bigg]^{n-1} \cos^2(\varphi + i\gamma)\,.
\end{aligned} \tag{2.9}
$$

3. The effect of variable dimensions and variable stiffness of rolling elements on bearing stiffness

In order to bring out the effect of variable dimensions and variable elasticity of rolling elements on the radial stiffness of the bearing it is assumed that:

(i) there are no other errors,

(ii) the bearing has an even number of rolling elements,

(iii) there are two identical "defective" rolling elements placed opposite each other.

Under such conditions,

$$
y_1 = y_0 = x_0 = 0
$$

and the formula for radial stiffness (Eq. (2.9)) is considerably simplified,

$$k = n \sum_{i=0}^{m-1} K_i \left[g_i + x\cos(\varphi + i\gamma)\right]^{n-1} \cos^2(\varphi + i\gamma)\,. \tag{3.1}$$

The maximum allowable value of variable dimensions of the rolling elements depends on the class of accuracy and the dimensions of the bearing. The coefficient K_i takes into account the difference in the mechanical properties of the rolling elements, as already mentioned. Any difference in this coefficient as that between the rolling elements included in one assembly is due to the prevailing manufacturing technology. The assembly of bearings does not guarantee that rolling elements made from steel of absolutely identical mechanical properties will be fitted in a particular bearing. Nor does it exclude the possibility of a difference in mechanical properties arising during heat treatment. The deviation in the value K_i in Eq. (3.1) is equal to the deviation in the contact rigidity of the ball, since

$$k_i = nK_i\delta_i^{n-1}\,. \tag{3.2}$$

In Fig. 2 the function $k(\varphi)$ is graphically shown using Eq. (3.1) for $m = 6$

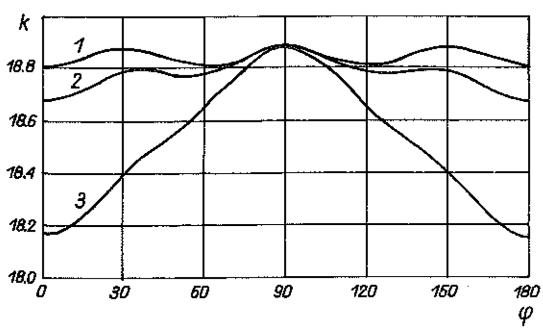

FIG. 2. Variation in radial stiffness during rotation of bearing with six balls: 1 – ideal bearing; 2 – zeroth and fourth balls smaller than rest; 3 – stiffness of zeroth and fourth balls less than stiffness of other balls.

and $n = 3/2$. The graphs correspond to the following values of K_i and g_i:

(i) $K_i = 1$ and $g_i = 20\ \mu$m for all values of i (ideal bearing),

(ii) $K_i = 1$ for all values of i, $g_0 = g_3 = 19.7\ \mu$m, $g_1 = g_2 = g_4 = g_5 = 20\ \mu$m (stiffness of all balls is identical but deformation, due to preload, of two balls is less than the deformation of the remainder by 1.5 % because of negative deviation in their diameters),

(iii) $K_0 = K_3 = 0.985$, $K_1 = K_2 = K_4 = K_5 = 1$, $g_i = 20$ μm for all values of i (stiffness of two balls less than the rigidity of the remainder by 1.5 %, diameters of all balls identical).

A comparison of the graphs shows that classification of the rolling elements of bearings, especially precision bearings with preload, based only on dimensional-geometric parameters is not enough. It is necessary to verify the rigidity of the rolling elements too. Besides, it is not rational to manufacture precision bearings with a small number of rolling elements since the change in their rigidity during rotation is considerable even in the case of absolute identity of the rolling elements.

4. The effect of waviness of raceways on the stiffness of bearings

Assuming that the unevenness of the raceway of the outer and inner rings of the bearing comprises only harmonics whose order is equal to m, or less or more than that by a multiple of two, and that phase angles of all harmonics are equal (then $x_0 = y_0 = 0$), it is possible to simplify equation (2.9) appreciably:

$$k = n \sum_{i=0}^{m-1} K_i \left[g_i + x\cos(\varphi + i\gamma) + \sum_{l=2}^{s} \Delta R_l \sin\{l(\varphi + i\gamma) + \alpha_i\} + \sum_{p=1}^{s} \Delta r_p \sin\{p(q\varphi + i\gamma) + \alpha_p\} \right]^{n-1} \cos^2(\varphi + i\gamma). \tag{4.1}$$

Graphs shown in Fig. 3 were plotted using Eq. (4.1) for $m = 6$, $x = 0$, $K_i = K = 1$, $g_i = g = 5$, $q = 1.64$ and $\alpha_l = \alpha_p = 0$, corresponding to three cases:

(i) the inner ring is ideal, raceway of outer ring has three harmonics of unevenness whose amplitudes are $\Delta R_3 = 1$, $\Delta R_6 = 0.5$ and $\Delta R_{12} = 0.3$,

(ii) the outer ring is ideal but inner ring has the same type of unevenness,

(iii) both rings have the above mentioned unevenness simultaneously.

As a result of superposing the periodic component due to the unevenness of the raceways of the inner ring on the component due to the unevenness of the outer ring, and in the general case also due to variable dimensions, variable elasticity and change in the position of rolling elements during rotation (when $x \neq 0$), the function $k(\varphi)$ becomes almost periodic because

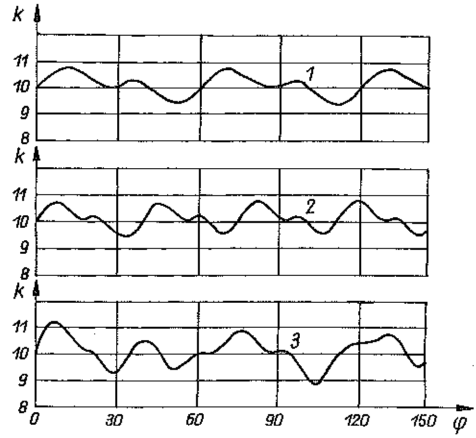

FIG. 3. Variation in radial stiffness of bearing with six balls due to waviness of stationary: (1) outer ring; (2) inner ring; (3) both rings.

the rotational speed of the moving ring and the cage are, in general, not repeated.

The repetition of the rotational speeds is characterized by the quantity q, which is determined by the relation

$$q = \frac{\omega_{\text{in}}}{\omega} - 1 . \tag{4.2}$$

However,

$$\frac{n_{\text{in}}}{n_c} = \frac{2D_0}{D_0 - d_r \cos\beta} , \tag{4.3}$$

where n_{in} and n_c are the rotational speeds of the inner ring and cage, respectively,

$$\frac{n_{\text{in}}}{n_c} = \frac{\omega_{\text{in}}}{\omega} .$$

D_0 is the diameter of the circle passing through the centers of the rotating elements, d_r is the diameter of the rolling elements.

By substituting the Eq. (4.3) in Eq. (4.2) the following is obtained:

$$q = \frac{2D_0}{D_0 - d_r \cos\beta} . \tag{4.4}$$

Thus, the quantity q depends not only on the dimensional geometric parameters of the bearing (D_0, d_r, β) but also on the value of preload. With changes in this, the effective values of diameters D_0 and d_r and constant angle β also change. Consequently, the quantity q not only cannot be computed with great accuracy but also does not remain a constant during the operation of the bearing.

5. Closing remarks

In this paper the influence of nonuniform sizes, unequal elasticity of the rolling bodies and errors in the geometric shape of raceways on the periodic component of radial stiffness of bearings with preload was analysed.

It was shown that the variable stiffness of the rolling bodies in the assembly of a precision bearing can affect the periodic component of stiffness more than their unequal dimensions. In the light of this it is proposed that the rolling bodies of precision bearings are sorted on the basis of stiffness.

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Received April 18, 1991.