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EFFECTS OF BODY FORCE ON THE PULSATING BLOOD FLOW IN ARTERIES

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The present investigation deals with the study of pulsating blood flow in single arteries in the presence of body force which usually arises unintentionally during travel in a road vehicle, an aircraft or a spacecraft. A blood vessel considered here is assumed to be rigid. The resulting equation which governs the flow field in the tube is one-dimensional and it is solved using the Fourier analysis. The results obtained in this analysis are the expressions for the local energy dissipation and the amplitude coefficients of mean velocity and wall shear stress. The numerical solutions of these results are shown graphically for better understanding of problem.

1. Introduction

The normal blood flow in the human circulatory system depends upon the pumping action of the heart. This pumping mechanism of heart produces a pressure gradient throughout the arterial and venous system. The pressure gradient consists of two components one of which is non-fluctuating and the other is fluctuating. Several workers have analysed blood flow under similar and other conditions. An excellent review of the pulsating blood flow is given in the literature of MILNOR [4].

Sometimes the human body is subjected to the whole-body acceleration (or vibrations) such as vibrations of a body in a vehicle, an aircraft or a spacecraft. This whole-body acceleration produces disturbances in the flow field of blood in arteries. Large amplitude body-acceleration causes fatal situations in the cardiovascular system resulting in headache, increase of pulse rate, abdominal pain, loss of vision, and haemorrhages in the face, neck, eyes, lungs and brain. Experimental results of the effect of the whole-body acceleration on blood flow in the arteries were reported by several workers (HIATT *et al.* [3], ARNTZENIUS *et al.* [1], VERDOUW *et al.* [7] and

BURTON *et al.* [2]. The details of body acceleration are given in the literature of SUD and SEKHON [5,6].

The present investigation deals with the theoretical study of pulsating flow blood in arteries in the presence of body acceleration. The theoretical results obtained in this analysis are the expressions for the local energy dissipation and the amplitude coefficients of mean velocity and wall shear stress.

2. Mathematical model

Pulsating blood flow through a long straight rigid blood vessel of constant radius R_0 is considered. The effect of body acceleration is taken into account in this analysis and it is acting in the direction of flow. The assumptions of constant fluid density and viscosity are also made.

Let u be the axial velocity component and F the body acceleration. Then the one-dimensional equation of motion governing the flow field in the tube is

$$\frac{R_0^2}{\nu}\frac{\partial u}{\partial t} = -\frac{R_0^2}{\nu}\frac{1}{\varrho}\frac{\partial P}{\partial x} + \left(\frac{\partial^2 u}{\partial y^2} + \frac{1}{y}\frac{\partial u}{\partial y}\right) + \frac{R_0^2}{\nu}F, \tag{2.1}$$

where $y = r/R_0$ and ν is the kinematic viscosity.

The boundary conditions are

$$u = 0 \qquad \text{at} \qquad y = 1 \tag{2.2}$$

and

$$\frac{\partial u}{\partial y} = 0 \qquad \text{at} \qquad y = 0. \tag{2.3}$$

3. Solution method

Pulsating flow of fluid is the superposition of periodic motion on the steady state laminar flow. Then the solution u of Eq. (2.1) consists of two parts, one of which is $u_s(y)$ for the steady state and the other is $u_p(y,t)$ for fluctuating motion. Therefore, the axial velocity component u satisfying Eq. (2.1) can be written as

$$u(y,t) = u_s(y) + u_p(y,t). \tag{3.1}$$

The corresponding pressure gradient $\partial P/\partial x$ is represented by

$$-\frac{1}{\varrho}\frac{\partial P}{\partial x} = K_0 + \frac{\partial p}{\partial x}, \tag{3.2}$$

where K_0 is the average pressure gradient for steady flow and $\partial p/\partial x$ is for fluctuating state. Substitution of Eqs. (3.1) and (3.2) into Eq. (2.1) gives two differential equations for u_s and u_p in the forms

$$\frac{1}{y}\frac{\partial}{\partial y}\left(y\frac{\partial u_s}{\partial y}\right) = -\frac{R_0^2}{\nu}K_0 \tag{3.3}$$

and

$$\frac{1}{y}\frac{\partial}{\partial y}\left(y\frac{\partial u_p}{\partial y}\right) - \frac{R_0^2}{\nu}\frac{\partial u_p}{\partial t} = -\frac{R_0^2}{\nu}\left(\frac{\partial p}{\partial x} + F\right). \tag{3.4}$$

The boundary conditions for steady flow are

$$u_s = 0 \qquad \text{at} \qquad y = 1 \tag{3.5}$$

and

$$\frac{\partial u_s}{\partial y} = 0 \qquad \text{at} \qquad y = 0, \tag{3.6}$$

whereas those for the periodic motion are

$$u_p = 0 \qquad \text{at} \qquad y = 1 \tag{3.7}$$

and

$$\frac{\partial u_p}{\partial y} = 0 \qquad \text{at} \qquad y = 0. \tag{3.8}$$

4. Solution

The problem of steady laminar flow is described by Eq. (3.3) together with boundary conditions (3.5) and (3.6). The resulting solution of this steady flow problem is

$$u_s = \frac{K_0 R_0^2}{4\nu}(1 - y^2). \tag{4.1}$$

The periodic problem is described by Eq. (3.4) subject to the boundary conditions (3.7) and (3.8). In order to solve this problem we set the solutions of $\partial p/\partial x$, $u_p(y,t)$ and F in the following forms:

$$\frac{\partial p}{\partial x} = Ke^{i\omega t}, \tag{4.2}$$

$$u_p(y,t) = \overline{u}_p(y)e^{i\omega t}, \tag{4.3}$$

$$F = a_0 e^{i\omega t}, \tag{4.4}$$

where $\overline{u}_p(y)$ is a function of y only, K is the amplitude of fluctuating pressure gradient, ω is the frequency of pressure pulse and a_0 is the amplitude of body acceleration. It is assumed that the frequency of periodic pressure gradient is equal to that of body acceleration. Substitution of Eqs. (4.2)–(4.4) into Eq. (3.4) gives an ordinary differential equation in $\overline{u}_p$ in the form

$$\frac{d^2\overline{u}_p}{dy^2} + \frac{1}{y}\frac{d\overline{u}_p}{dy} - i\beta^2\overline{u}_p = -\frac{R_0^2}{\nu}(K + a_0), \tag{4.5}$$

where β is the Wamersley parameter defined by $\beta^2 = R_0^2\omega/\nu$.

The boundary conditions (3.7) and (3.8) take the forms

$$\overline{u}_p = 0 \quad \text{at} \quad y = 1 \tag{4.6}$$

and

$$\frac{d\overline{u}_p}{dy} = 0 \quad \text{at} \quad y = 0. \tag{4.7}$$

The solution of Eq. (4.5) under the boundary conditions (4.6) and (4.7) is

$$\overline{u}_p = \frac{1}{i\beta^2}\frac{R_0^2}{\nu}(K + a_0)\left[1 - \frac{J_0(i^{3/2}\beta y)}{J_0(i^{3/2}\beta)}\right], \tag{4.8}$$

where J_0 is the Bessel function of order zero with complex argument. Then the resulting expression for $u_p(y,t)$ is

$$u_p(y,t) = \frac{R_0^2}{\nu}(K + a_0)\frac{1}{i\beta^2}\left[1 - \frac{J_0(i^{3/2}\beta y)}{J_0(i^{3/2}\beta)}\right]e^{i\omega t}. \tag{4.9}$$

Following the notations of McLachlan (1934) given by

$$J_n(i^{3/2}z) = M_n(z)e^{i\theta_n(z)}, \tag{4.10}$$

the relation (4.9) can be written as

$$u_p(y,t) = \frac{R_0^2}{\nu}\frac{M_0'}{\beta^2}(K + a_0)e^{i(\omega t - \varphi_0')}, \tag{4.11}$$

where

$$\begin{aligned} M_0' &= \left[1 + h_0^2 - 2h_0 \cos\varphi_0\right]^{1/2}, \\ h_c &= M_0(\beta y)/M_0(\beta), \\ \varphi_0 &= \theta(\beta y) - \theta_0(\beta), \\ \varphi_0' &= \frac{\pi}{2} + \tan^{-1}\left[h_0 \sin\varphi_0/(1 - h_0\cos\varphi_0)\right]. \end{aligned} \tag{4.12}$$

If the real part of the fluctuating pressure gradient is $K \cos\omega t$ then the expression for $u_p(y,t)$ is

$$u_p(y,t) = \frac{R_0^2}{\nu}\frac{M_0'}{\beta^2}(K + a_0)\cos(\omega t - \varphi_0'). \tag{4.13}$$

The two expressions (4.1) and (4.13) give the solutions of the problem.

5. Mean velocity, wall friction and energy distribution

Since the fluctuating pressure gradient produces periodic motion of fluid in the tube, therefore the instantaneous volumetric flow rate changes periodically and, accordingly, the mean velocity is also changed. The sectional mean velocity is defined by

$$u_m = 2\int_0^1 u(y,t)y\,dy \tag{5.1}$$

which, after substitution of the resulting expression for axial velocity component u, gives, on integration

$$u_m = \frac{K_0R_0^2}{8\nu} - \frac{K_0R_0^2}{\nu}\frac{F_1}{i\beta^2}\frac{J_2(i^{3/2}\beta)}{J_0(i^{3/2}\beta)}e^{i\omega t}, \tag{5.2}$$

where $F_1 = (K + a_0)/R_0$. The nondimensional form of u_m is

$$\overline{u}_m = u_m/(K_0R_0^2/8\nu) = 1 + \frac{8F_1}{i^3\beta^2}\frac{J_2(i^{3/2}\beta)}{J_0(i^{3/2}\beta)}e^{i\omega t}. \tag{5.3}$$

If the real part of the fluctuating pressure gradient is considered in this analysis, then the appropriate expression for nondimensional u_m is

$$(\overline{u}_m)_{re} = 1 + \frac{K}{K_0} \cdot 8h_{20}\left(1 + \frac{a_0}{K}\right)\cos(\omega t - \varepsilon_{20}), \tag{5.4}$$

where

$$\begin{aligned} \varepsilon_u &= 8h_{20}\left(1 + \frac{a_0}{K}\right)\frac{1}{\beta^2}, \\ \varepsilon_{20} &= \frac{3\pi}{2} + \theta_0(\beta) - \theta_2(\beta), \\ h_{20} &= M_2(\beta)/M_0(\beta). \end{aligned} \tag{5.5}$$

The expressions for ε_u and ε_{20} represent the coefficients of amplitude and phase lag to the wave of pressure gradient.

The instantaneous wall shear stress is defined by

$$\tau_w = -\frac{\mu}{R_0}\left(\frac{\partial u}{\partial y}\right)_{y=1}. \tag{5.6}$$

Substitution of u from Eq. (3.1) into Eq. (5.6) gives

$$\tau_w = -(\mu/R_0)\left[\left(\frac{\partial u_s}{\partial y}\right)_{y=1} + \left(\frac{\partial u_p}{\partial y}\right)_{y=1}\right] \tag{5.7}$$

which, with the help of Eqs. (4.1) and (4.9), takes the form

$$\tau_w = \frac{K_0\mu R_0}{2\nu}\left[1 + \frac{K}{K_0}\frac{2}{\beta i^{3/2}}\left(1 + \frac{a_0}{K}\right)\frac{J_1(i^{3/2}\beta)}{J_0(i^{3/2}\beta)}e^{i\omega t}\right]. \tag{5.8}$$

The nondimensional form τ of Eq. (5.8) with respect to the steady state in the absence of body force has the form

$$\tau = 1 + \frac{2K}{K_0}\left(1 + \frac{a_0}{K}\right) \cdot \frac{1}{\beta i^{3/2}}\frac{J_1(i^{3/2}\beta)}{J_0(i^{3/2}\beta)} \cdot e^{i\omega t} \tag{5.9}$$

whose real part is

$$\tau = 1 + \frac{K}{K_0}\varepsilon_\tau \cos(\omega t - \varepsilon_{10}), \tag{5.10}$$

where

$$\begin{aligned} \varepsilon_{10} &= \frac{3\pi}{4} + \theta_0(\beta) - \theta_1(\beta), \\ \varepsilon_\tau &= \frac{2h_{10}}{\beta}\left(1 + \frac{a_0}{K}\right), \\ h_{10} &= \frac{M_1(\beta)}{M_0(\beta)}. \end{aligned} \tag{5.11}$$

The expression for ε_τ represents the coefficient of amplitude of the shearing stress and the expression for ε_{10} gives the phase lag of it.

The local distribution of energy dissipation is defined by

$$D = \frac{1}{2}\mu\left(\frac{\partial u}{\partial r}\right)^2 . \tag{5.12}$$

The relation (5.12) takes, with the help of Eq. (3.1), the following form

$$(D)_{re} = \frac{K_0^2 R_0^2 \mu}{8\nu^2}\left[y + \frac{2}{K_0}(K + a_0)\frac{\overline{h}_{10}}{\beta}\cos(\omega t - \overline{\varepsilon}_{10})\right]^2 , \tag{5.13}$$

where

$$\begin{aligned} h_{10} &= M_1(\beta y)/M_0(\beta), \\ \overline{\varepsilon}_{10} &= \frac{3\pi}{4} + \theta_0(\beta) - \theta_1(\beta y). \end{aligned} \tag{5.14}$$

6. Discussion

The numerical solutions of the theoretical results, such as the dissipation of energy, the coefficients of mean velocity and wall shear stress are discussed for different amplitudes of body force. The nondimensional frequency parameter β plays an important role in this numerical analysis. The results for energy dissipation and the coefficients of amplitudes of mean velocity and wall shear stress are explained with the help of this parameter.

Figures 1 and 2 represent the variations of amplitude coefficients of mean velocity and wall shearing stress for different values of amplitude ratios a_0/K. It is observed that for $\beta = 0$ each curve in both Figures attains its maximum value and there is no remarkable change in it for all values of β less than unity. Beyond this value of β the coeficient changes significantly and approaches its asymptotic value very slowly as β increases. Thus, it is seen that the values of the frequency parameter β which plays an important role in the study of blood flow correspond to the steeply falling parts of the curves.

Again, it is also observed that for a particular value of β the amplitude coefficients increase as the amplitude ratio a_0/K increases. The deviation between any two consecutive solutions in the range is not appreciable; but beyond this range it changes remarkably for all values of β on the steeply falling parts of the curves.

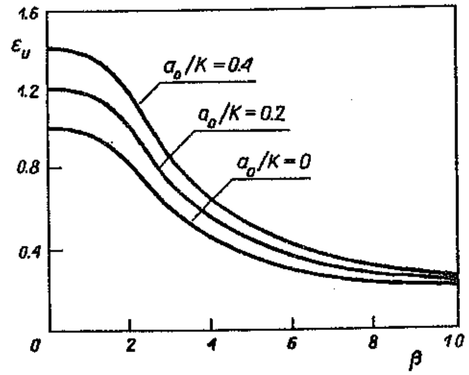

FIG. 1. Coefficients of amplitude of shearing stress.

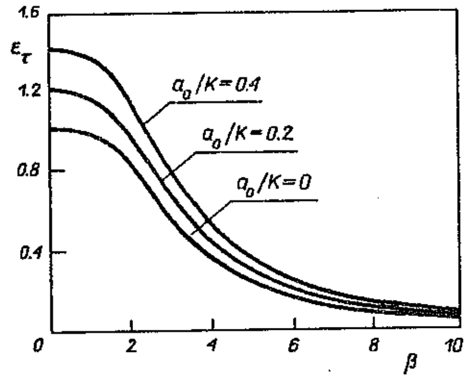

FIG. 2. Coefficients of amplitude of mean velocity

The variations of energy dissipation along the radius of the tube are shown in Figs. 3 and 4 for different values of a_0/K_0, keeping K/K_0 fixed. The dashed line in each of the figures represents the steady-state solution, while the continuous curves are for the present problem.

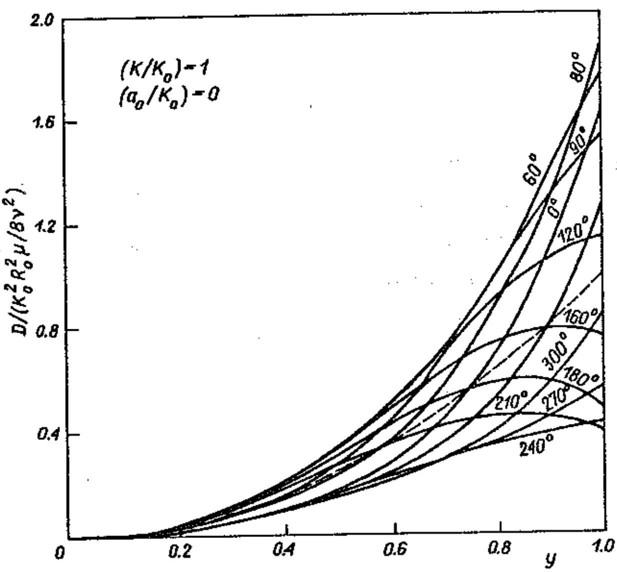

FIG. 3. Local energy distribution for $\frac{a_0}{K_0}=0$.

It is seen from the figures that the dissipations are fluctuated periodically about the steady flow. When the body force is not taken into account, the

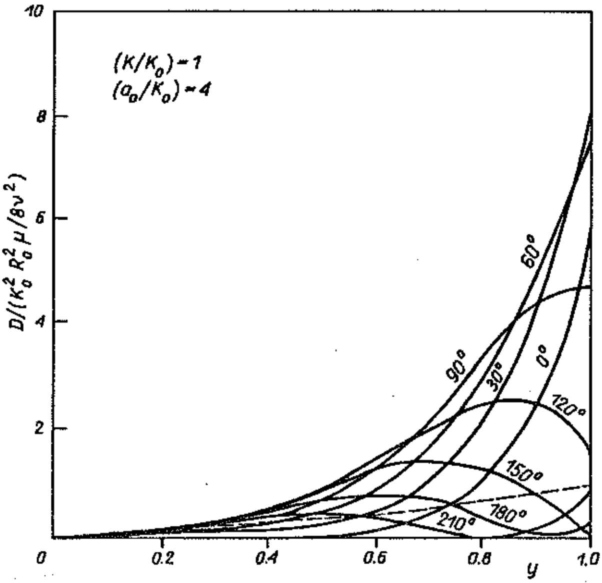

FIG. 4. Local energy distribution for $\frac{a_0}{K_0} = 4$.

maximum dissipation occurs at the tube wall. But the situation of this maximum dissipation enters into the flowing fluid when the effect of body force is considered and its distance from the vessel wall varies with time periodically. These phenomena will be more interesting if the situation of maximum dissipation is compared with the mechanism of generation of turbulence.

7. Conclusions

From the above numerical discussions of theoretical results the following conclusions can be drawn:

1. The effect of body force increases the amplitude coefficients of mean velocity and wall shearing stress; however, it does not affect these flow characteristics at higher frequency.

2. The situation of maximum energy dissipations occuring at the tube wall increases due to the effect of body force and spreads into the flowing fluid periodically with time.

References

1. A.C.Arntzenius, J.D.Laird, A.Noordergraff, P.D.Verdow and P.H.Huisman, *Body acceleration synchronous with the heartbeat*, Bilphy Cardiol., **29**, 1–5, 1972.
2. R.R.Burton, S.D.Leverett Jr and E.D.Michaelsow, *Man at high sustained* $+G_z$ *acceleration; a reviev*, Aerospace Med., **46**, 1251–1253, 1974.
3. E.P.Hiatt, J.P.Meechan and Galambos, *Reports on human acceleration*, Publication 901, NAS-NRC, Washington, D.C 1969.
4. W.R.Milnor, *Hemodynamics*, Williams and Williams, Baltimore 1982.
5. V.K.Sud and G.S.Sekhon, *Blood flow subjected to a single cycle of body acceleration*, Bull. Math. Biol., **46**, 937–949, 1984.
6. V.K.Sud and G.S.Sekhon, *Arterial flow under periodic body acceleration*, Bull. Math. Biol., **47**, 35–52, 1985.
7. P.D.Verdouw, A.C.Noordergraff, A.C.Artzenius and P.H.Huisman, *Relative movements between subject and support in body acceleration applied synchronously with the heartbeat (BASH)*, Bilphy Cardiol., **31**, 57–63, 1973.

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