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B R I E F   N O T E S

SIMPLE FORMULA FOR CRITICAL TENSION OF TRUNCATED STRUCTURALLY ORTHOTROPIC SHELLS OF REVOLUTION

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A structurally orthotropic shell of revolution is loaded by tensible forces applied to the ends of the shell. Stability of the structure is analyzed by means of the asymptotic method and the Galerkin procedure.

Buckling of truncated isotropic hemispherical shell under axial tension was studied for the first time by YAO [1] and was also considered by other authors (e.g. see [2–5]). The technique of analysis was mostly numerical.

Using asymptotic method and the Galerkin procedure, a simple analytical formula for critical tension is proposed in this note. This formula may be used for structurally orthotropic and (in the limit case) isotropic shells of revolution.

We discuss here structurally orthotropic shells of revolution. Initial axisymmetric tensile stress resultants N are applied at the ends of the shell. It was shown in [2–5] that the pre-buckling nonlinearity and bending effects are small in this case and can be neglected. Hence, it is possible to assume here the pre-buckling state to be homogeneous and free of bending. Then the pre-buckling axial membrane stress resultant in the shell N^0 may be calculated in the form

$$N^0 = -R_0 N R_1^{-1} \cos^2\theta (1+\gamma)^{-1/2} .$$

Here R_0 – the parameter which characterizes the averaged value of radius of curvature; θ – the meridional coordinate on the surface of revolution; $0 \le \alpha_0 \le \theta \le \alpha_1$, N – the edge stress resultant, parameter γ defining the deviation of the shell from the spherical form ($\gamma = 0$ for a sphere, $\gamma = -1$ for a paraboloid, $\gamma > -1$ for an ellipsoid, $\gamma < -1$ for a hyperboloid), R_1 – the radius of the meridional principal curvature, $R_1 = R_0\left(1 + \vartheta \cos^2 \theta^{-3/2}\right)$.

Now we pass to the stability equations. It was found in [5] that buckling patterns of the problem reveal one meridional wave and many circumferential waves, what gives us the possibility to use the simple approximate "semimomentous" [6] shell theory in order to describe the above mentioned phenomenon. The essence of this simplified theory consists in neglecting all terms containing derivatives with respect to the meridional coordinate θ in comparison with the terms containing derivatives with respect to circumferential coordinate φ, because in this case $\partial/\partial\varphi \gg \partial/\partial\theta$. Then the stability equations of the "semimomentous" theory for shells of revolution may be written in the following form [6]:

$$
\begin{aligned}
D_2 w_{\varphi\varphi\varphi\varphi} + K_2 \Phi_{\varphi\varphi\varphi\varphi} - R_1 R_2^6 \cos^6\theta \Phi_{\varphi\varphi} + N^0 R_1^{-1} R_2^2 w_{\varphi\varphi} &= 0, \\
A_{11}\Phi_{\varphi\varphi\varphi\varphi} - K_2 w_{\varphi\varphi\varphi\varphi} + R_1^{-1} R_2^2 \cos^2\theta w_{\varphi\varphi} &= 0.
\end{aligned} \tag{1}
$$

Here $A_{11} = B_{22}(B_{11}B_{22} - B_{12})$; B_{11}, B_{22} – the membrane rigidities in the meridional and circumferential directions; B_{12} – Poisson's ratio, D_2 – the circumferential bending stiffness, K_2 – the coefficient, which takes into account eccentricity of rings, w – the radial displacement, Φ – Airy's stress function, R_2 – the principal radius of curvature, $R_2 = R_0(1 + \vartheta\cos^2\theta)^{1/2}$.

Let us choose the bundary conditions associated with the stability equations to be (clamped edges):

$$
w = w_\theta = \Phi = \Phi_\theta = 0 \quad \text{for} \quad \theta = \alpha_0, \alpha_1. \tag{2}
$$

Then the functions w, Φ may be assumed as follows:

$$
w = AF(\theta)\sin n\varphi\,, \qquad \Phi = BF(\theta)\sin n\varphi\,,
$$

here $F(\theta) = (\sin\theta - \sin\alpha_0)^2(\sin\theta - \sin\alpha_1)^2$ is the function satisfying boudary conditions (2).

Using Galerkin's procedure, from Eqs. (1) one obtains the set of two algebraic homogeneous linear equations for constants A, B. Determnation of eigenvalue N_c (critical tension) follows from minimization of their determinant with respect to n, what yields

$$
N_c^0 = \frac{2(1+\gamma)^{3/2}}{A_{11}R_0}\left[\left(A_{11}D_2 + K_2^2\right)^{1/2} + K_2\right] P\,.
$$

Here

$$P = \frac{\int_{\alpha_0}^{\alpha_1} (1 + \gamma \cos^2 \theta)^2 \sec^4 \theta \, F(\theta) \, d\theta}{\int_{\alpha_0}^{\alpha_1} (1 + \gamma \cos^2 \theta)^{5/2} \sec^4 \theta \, F(\theta) \, d\theta}.$$

Observing the results of numerical calculations of P we can conclude that $P \cong 1$ in the large range of values of θ ($P = 1$ for a sphere). Assuming $P = 1$, simple formula for the critical value of axial tension is

$$N_c^0 = \frac{2(1+\gamma)^{3/2}}{A_{11} R_0} \left[\left(A_{11} D_2 + K_2^2 \right)^{1/2} + K_2 \right].$$

In the case of an isotropic shell the critical axial tension is reduced to

$$N_c^0 = \frac{E h^2 (1+\gamma)^{1/2}}{R_0 (3(1-\nu^2))^{1/2}}, \tag{3}$$

where E – Young's modulus, ν – Poisson's ratio, h – the shell thickness.

Comparison of the critical value N_c^0 calculated according to Eq. (3) with the known numerical results N_n given in [2, 5] (that is the ratio N_c/N_m) is shown in Table 1.

Table 1.

$\gamma \backslash l$	20	30	40	50	60
−0.5	1.12	1.04	1.02	1.01	1.00
0	1.02	1.01	1.00	1.00	1.00
0.5	1.01	1.00	1.00	1.00	1.00

The following values of parameters have been used in the analysis:

$$\nu = 0.3, \quad R_0 h^{-1} (1+\gamma)^{-1/2} = 500.$$

In Table 1 the following notation is used:

$$l = l_s (R_0 h)^{-1/2} (1+\gamma)^{1/2} (12(1-\nu^2))^{-1/4},$$

where l_s is the length of the shell in the meridional direction.

For higher values of ϑ the difference between N_c^0 and N_n vanishes. For $\vartheta < 0.5$ the error of N_c^0 calculated from (3) increases. It means that in this case the terms neglected in the simplified stability equations (1) become essential.

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