ENGINEERING TRANSACTIONS • *Engng. Trans.* • **42**, 3, 299–313, 1994
Polish Academy of Sciences • Institute of Fundamental Technological Research

TORSIONAL VIBRATION OF A SYSTEM WITH A FRICTION CLUTCH UNDER RANDOM TORQUE EXCITATION

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The paper presents a theoretical study of torsional vibration in a two-mass model of a transmission system with a multi-disc flexible friction clutch under random excitation. Energy is dissipated through micro-slip processes between discs, characterized by a hysteresis loop. It is assumed that the nominal driving torque is disturbed by a stationary Gaussian random excitation with known moments. Spectral density as well as the mean value and standard deviation of both the relative torsional vibration and the driven part vibration are derived using a stochastic linearization technique applicable in hysteretic systems. The role of system parameters is shown by examples in which a band model of random excitation is assumed.

1. Introduction

The relation between external load and relative angular disc displacements is of fundamental importance for the design of friction clutches and their proper selection for particular engine-machine systems. Friction clutches of usual design, including single and multi-disc systems, have an important property of damping torsional vibrations as a result of microslip between torsionally flexible discs. This phenomenon is well-known and referred to as a structural hysteresis loop (see GOODMAN and KLAMP [5] or PIAN [11] for early studies). In the Polish literature an overview of structural friction problems with applications can be found in the works by OSIŃSKI [10] and GIERGIEL [4]. Structural friction is a natural source of damping present in every real device.

In friction clutches the magnitude of dissipation can be controlled in such a way that the best dynamic properties of the entire transmission system are obtained. Nominal driving or resistance torques of such systems are usually disturbed by additional forces of periodic or random nature. Torque disturbances cause relative torsional vibrations of rotating elements, which in turn result in wear, noise, energy consumption, damages, etc. From the point of view of clutch design it is important to establish a relation between the external driving load and the corresponding torsional motion of the transmission system. Therefore, a dynamical analysis based on more advanced models is necessary.

Structural hysteresis loop of relative motion between the circumferentially flexible discs has been studied for a long time. The early results by GOODMAN and KLAMP [5] and PIAN [11] are summarized in the book by OSIŃSKI [10].

During the past two decades attention was mainly focused on dynamical analysis of systems with structural friction, using relatively simple models of both the stick-slip process and the mechanical system. A number of papers devoted to various dynamical problems of friction clutches was presented by Skup [13–16], who developed an analytic description of the dynamic friction torque in a multi-disc clutch with torsionally flexible discs and shafts, and applied this result to solve vibration problems in transmission systems related to various excitation loads.

Recently, structural friction attracts more and more attention, and it is considered as an effective damping factor which can be involved in active vibration control (BEARDS [1], FRISCHGESELL and SZOLC, [3]). Therefore, more advanced stick-slip models are developed based mainly on finite elements (see PIETRZAKOWSKI [12], GRUDZIŃSKI *et al.* [6] and OSIŃSKI *et al.* (1993)).

An important feature of the structural hysteresis loop is that it explicitly depends on the displacement amplitude, and a direct numerical integration of the dynamical equations of motion is difficult. Therefore, approximate analytical techniques are developed and applied in such systems (CAUGHEY [2]). The present paper is concerned with theoretical analysis of torsional vibration of a transmission system with friction clutch driven by a torque of random nature. An approximate correlation method presented by KURNIK *et al.* [7] is applied, in which the Hilbert transformation is used to reduce the nonlinear equation of motion to two recurrent linear equations for auto- and cross-correlation functions of the input (torque) and output (angular displacement) processes. It is assumed that the nominally constant driving torque is disturbed by a stationary non-white Guassian excitation with a given spectral density. Nonlinear relation between mean values and standard deviations of the random driving torque and the corresponding relative torsional vibrations of the system are derived and presented in examples.

2. Vibrating system and governing equations

We shall consider a two-mass discrete model of a transmission system (engine-machine) shown in Fig. 1. Equations of motion of this system can be written down as follows

$$
\begin{aligned}
I_1\,\ddot{\varphi}_1 &= -M(\varphi, a, \dot{\varphi}) + M_n + M_d(t),\\
I_2\,\ddot{\varphi}_2 &= M(\varphi, a, \dot{\varphi}) + M_r,
\end{aligned}
\tag{2.1}
$$

where
- φ_1, φ_2 angular coordinates of the driving and driven parts, respectively,
- φ, a relative angular displacement and its amplitude ($\varphi = \varphi_1 - \varphi_2$),
- I_1, I_2 mass moments of inertia of the driving and driven part,
- M_n nominal driving torque,
- M_r resistance torque,
- $M_d(t)$ random disturbance of the driving torque.

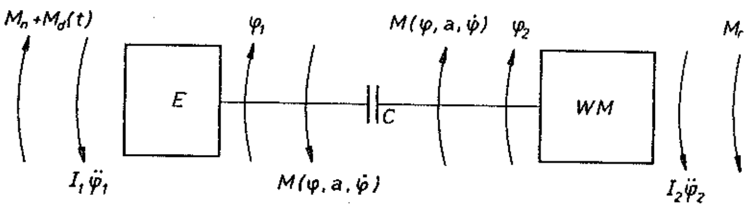

FIG. 1. Considered model of the transmission system.

Since we are interested in steady motions of the considered system we shall assume $M_n = M_r$ which provides a uniform rotation of the undisturbed system. Equations (2.1) can be reduced to a single nonlinear equation describing the relative displacement between the driving and driven parts as functions of time

$$
\ddot{\varphi} + \theta(a, \varphi, \dot{\varphi}) = G_0 + G(t),
\tag{2.2}
$$

where

$$
I_z = \frac{I_1 I_2}{I_1 + I_2}, \qquad \theta(\varphi, a, \dot{\varphi}) = \frac{M(a, \varphi, \operatorname{sgn}\,\dot{\varphi})}{I_z},
$$
$$
G_0 = \frac{M_n}{I_z}, \qquad G(t) = \frac{M_d(t)}{I_1}.
$$

To determine the clutch moment M we shall make use of the well-known expressions describing structural hysteresis in a friction clutch with flexible discs (see OSIŃSKI [10] and SKUP [16]). In a multi-disc clutch with n flexible disc pairs schematically shown in Fig. 2, the relative angular displacement between the clutch input and output in steady motion can be described in the following hysteretic form

$$\varphi^- = \kappa\left\{\gamma M - 3\left[1+(\gamma M_{\max}+1)^{1/3}\right] + 6\left[\frac{\gamma}{2}(M_{\max}-M)+1\right]^{1/3}\right\},$$

(2.3)
$$\varphi^+ = \kappa\left\{\gamma M + 3\left[1-(\gamma M_{\max}+1)^{1/3}\right] + 6\left[\frac{\gamma}{2}(M_{\max}-M)+1\right]^{1/3} - 6\left[\frac{\gamma}{2}(M-M_{\min})+1\right]^{1/3}\right\},$$

where φ^-, φ^+ – relative displacements for the lower and upper branches of the hysteresis loop, respectively, $M_{\min}$, $M_{\max}$ – minimum and maximum clutch moment in a cycle,

$$\gamma = \frac{3}{2\pi\mu p r^3}, \qquad \kappa = \frac{n\mu p r(k_1+k_2)}{6k_1k_2}, \qquad k_1 = G_1h_1, \qquad k_2 = G_2h_2 .$$

The last notations are: p – pressure per unit area, μ – friction coefficient, r – inner radius of the discs, G_1, G_2, h_1, h_2 – shear moduli and thicknesses of the discs forming friction pairs.

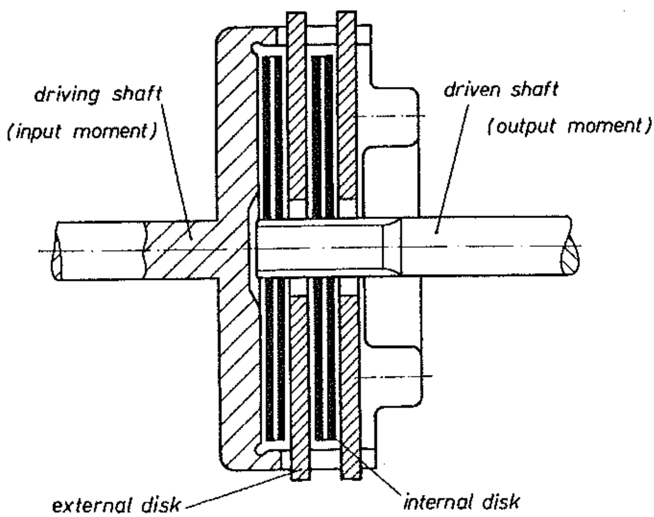

FIG. 2. Scheme of a multidisc friction clutch.

Since in real friction clutches we have $\gamma(M_{\max}-M_{\min}) \ll 1$, expressions with powers 1/3 in Eqs. (2.3) can be expanded into power series. Neglecting the terms of orders higher than 2, we obtain

$$(2.4)\qquad \begin{aligned} \varphi^- &= \eta\left\{(M_{\max}+M)^2-2M^2\right\},\\ \varphi^+ &= \eta\left\{(M-M_{\min})^2+(M_{\max}+M_{\min})^2-2M_{\min}^2\right\}, \end{aligned}$$

where $\eta = \kappa\gamma^2/6$.

Expressions (2.4) can now be easily reversed to obtain the hysteresis loop in terms of the clutch moment as a function of the relative displacement.

$$(2.5)\qquad \begin{aligned} M^- &= \frac{1}{\sqrt{\eta}}\left(\sqrt{0.5\varphi_{\max}}-\sqrt{\varphi_{\max}-\varphi}\right),\\ M^+ &= \frac{1}{\sqrt{\eta}}\left(\sqrt{0.5\varphi_{\max}}+\sqrt{\varphi-\varphi_{\min}}-\sqrt{\varphi_{\max}-\varphi_{\min}}\right), \end{aligned}$$

where $\varphi_{\min}$ and $\varphi_{\max}$ denote the minimum and maximum relative displacement in a cycle, respectively (Fig. 3).

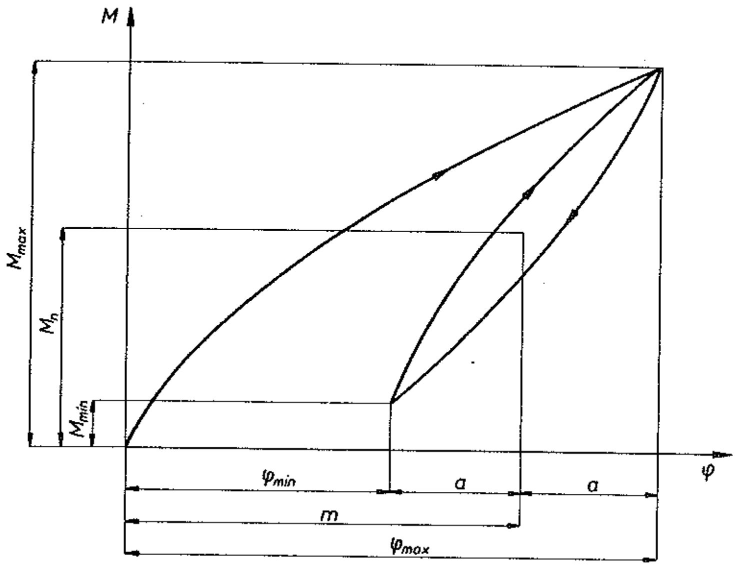

FIG. 3. Hysteresis loop of the structural friction in the friction clutch.

Introducing mean value m of the displacement cycle, amplitude a and the new centered coordinate x as follows

$$(2.6)\qquad a=\frac{\varphi_{\max}-\varphi_{\min}}{2},\qquad m=\frac{\varphi_{\max}+\varphi_{\min}}{2},\qquad x=\varphi-m,$$

we finally obtain the nonlinear term in Eq. (2.7)

$$(2.7)\qquad \theta = \begin{cases} \theta^{+} = \alpha\left(\dfrac{1}{\sqrt{2}}\sqrt{m+a} - \sqrt{2a} + \sqrt{a+x}\right) & \text{for} \quad \dot{x} < 0, \\ \theta^{-} = \alpha\left(\dfrac{1}{\sqrt{2}}\sqrt{m+a} - \sqrt{a-x}\right) & \text{for} \quad \dot{x} > 0, \end{cases}$$

where

$$\alpha = \frac{1}{I_z\sqrt{\eta}}.$$

Thus, the nonlinear equation of motion (2.2) with a stationary Guassian right-hand side process is now fully determined.

3. Analysis of relative vibration

We assume that the spectral properties of the right-hand side process $G(t)$ are known and determined by the spectral density $S_{GG}(\omega)$ or by the corresponding correlation function $K_{GG}(t)$. Our aim in this section is to derive the relation between the input and output characteristics – mean values G_0 and m and standard deviations σ_G and σ, respectively ($\sigma_G = \sigma_M/I_1$, where σ_M is the standard deviation of the random torque). Kurnik *et al.* [7] presented an approximate correlation method which is a normal closure technique applicable to hysteretic systems. The method consists in searching for a solution of a nonlinear hysteretic equation with a stationary Gaussian excitation in the form of another Gaussian process of unknown variance. Respecting this assumption and applying the Hilbert transformation (see Titchmarsh, [17]), we can reduce the problem to the system of two recurrent equations for the auto- and cross-correlation functions of the input (G) and output (φ) processes. The equations can be solved using the Fourier transformation, and the resulting spectral density integrated over the infinite frequency interval leads to the above mentioned relation between the mean values of excitation and displacement and the standard deviations. Since the nonlinear term in Eq. (2.2) explicitly depends on the vibration amplitude a, it is convenient to introduce the envelope and phase processes a and ψ as well as the adjoint process y as follows:

$$(3.1)\qquad \begin{aligned} x(t) &= a(t)\cos\psi(t), \\ y(t) &= a(t)\sin\psi(t). \end{aligned}$$

The processes $x(t)$ and $y(t)$ are one-to-one Hilbert transforms and the Hilbert transformation is determined as follows:

$$y(t) = H\{x(t)\} = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(\tau)}{\tau - t} d\tau, \tag{3.2}$$

where the integral is understood in the sense of Cauchy's principal value. Processes $x(t)$ and $y(t)$ are both Gaussian (if $x(t)$ is), they have identical autocorrelation and spectral density functions and their cross-correlation function $K_{yx}(t)$ is a Hilbert transform of $K_{xx}(t)$. This property will be used in the further analysis.

Averaging both sides of Eq. (2.2) we obtain

$$G_0 = P(m, \sigma), \tag{3.3}$$

where

$$P(m,\sigma) = \frac{1}{2\pi\sigma^2} \iint_{-\infty}^{+\infty} \theta\left(x + m, \sqrt{x^2+y^2}, \operatorname{sgn} \dot{x}\right) \times \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) dx dy, \tag{3.4}$$

where in turn θ is represented by two branches described by (2.7). Expression (3.3) is the first of two nonlinear relations between four quantities: G_0, σ_G, m and σ. In order to obtain the other one, we shall follow the procedure presented by KURNIK *et al.* [7]. Equations for the auto- and cross-correlation functions of processes x, y and G can be obtained by multiplying both sides of Eq. (2.2) first by $x(t_1)$, then by $G(t_1)$, and by averaging

$$\begin{aligned} \ddot{K}_{xx}(\tau) + E\left[\theta\left(x(t)+m, a, \operatorname{sgn} \dot{x}\right) x(t_1)\right] &= K_{Gx}(\tau), \\ \ddot{K}_{xG}(\tau) + E\left[\theta\left(x(t)+m, a, \operatorname{sgn} \dot{x}\right) G(t_1)\right] &= K_{GG}(\tau), \end{aligned} \tag{3.5}$$

where $\tau = t - t_1$.

Applying the following property of arbitrary Gaussian random variables X, Y, Z and function f (Malakhov [9])

$$E\left[Z f(X,Y)\right] = E\left[\frac{\partial f}{\partial X}\right] E[ZX] + E\left[\frac{\partial f}{\partial Y}\right] E[ZY] \tag{3.6}$$

and respecting (3.1), we obtain

$$\begin{aligned} E\left[\theta\left(x(t)+m, a, \operatorname{sgn} \dot{x}\right) x(t_1)\right] &= E\left[\frac{\partial \theta}{\partial X}\right] K_{xx}(\tau) + E\left[\frac{\partial \theta}{\partial Y}\right] K_{yx}(\tau), \\ E\left[\theta\left(x(t)+m, a, \operatorname{sgn} \dot{x}\right) G(t_1)\right] &= E\left[\frac{\partial \theta}{\partial X}\right] K_{xG}(\tau) + E\left[\frac{\partial \theta}{\partial Y}\right] K_{yG}(\tau), \end{aligned} \tag{3.7}$$

Denoting $E[\partial\theta/\partial x] = Q_1$ and $E[\partial\theta/\partial y] = Q_2$ and applying (3.7) we can express Eqs. (3.5) in the form

$$
\begin{aligned}
\ddot{K}_{xx} + Q_1 K_{xx} + Q_2 K_{yx} &= K_{Gx}(t), \\
\ddot{K}_{xG} + Q_1 K_{xG} + Q_2 K_{yG} &= K_{GG}(t),
\end{aligned}
\tag{3.8}
$$

where

$$
\begin{aligned}
Q_1 &= \frac{1}{2\pi\sigma^4} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} x\theta\left(x + m, \sqrt{x^2 + y^2}, \operatorname{sgn}\dot{x}\right) \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) dx\,dy, \\
Q_2 &= \frac{1}{2\pi\sigma^4} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} y\theta\left(x + m, \sqrt{x^2 + y^2}, \operatorname{sgn}\dot{x}\right) \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) dx\,dy.
\end{aligned}
\tag{3.9}
$$

Taking into account that $K_{yx} = H\{K_{xx}\}$, $K_{yG} = H\{K_{xG}\}$ and making use of the following properties (see LEVIN, [8]):

$$
\begin{aligned}
S_{yx}(i\omega) &= -i \operatorname{sgn}\omega S_{xx}(\omega), \\
S_{Gx}(i\omega) &= S_{xG}(-i\omega),
\end{aligned}
\tag{3.10}
$$

where S_{xx}, S_{yx} and S_{xG} are spectral density functions being Fourier transforms of the correlation functions K_{xx}, K_{yx} and K_{xG}, respectively, we obtain, after applying the Fourier transformation to Eqs. (3.8),

$$
S_{xx}(\omega) = \frac{S_{GG}(\omega)}{[Q_1(m,\sigma) - \omega^2]^2 + [Q_2(m,\sigma)]^2} \,. \tag{3.11}
$$

Integration of the spectral density (3.11) leads to the second nonlinear relation between G_0, σ_G, m and σ (σ_G is included in $S_{GG}(\omega)$)

$$
\sigma^2 = \int\limits_{-\infty}^{\infty} S_{xx}(\omega)\, d\omega, \tag{3.12}
$$

which, together with (3.3), forms a system of two nonlinear equations for m and σ.

$$
\begin{aligned}
g(m, \sigma, G_0, \sigma_G; \mathbf{p}) &= 0, \\
h(m, \sigma, G_0, \sigma_G; \mathbf{p}) &= 0,
\end{aligned}
\tag{3.13}
$$

where $\mathbf{p} = [p, \mu, r, n, h_1, G_1, h_2, G_2, I_1, I_2]$ is a vector of the system parameters (see Sec. 2).

In the last part of this section we shall calculate the quantities P, Q_1 and Q_2 defined in expressions (3.4) and (3.9).

$$P(m,\sigma) = \frac{\alpha}{2^{1/2}\sigma^2}\int_0^\infty \left((m+a)^{1/2} - a^{1/2}\right) a\exp\left(-\frac{a^2}{2\sigma^2}\right) da,$$

$$(3.14)\quad Q_1(m,\sigma) = \frac{2^{3/2}\alpha}{3\pi\sigma^4}\int_0^\infty a^{5/2}\exp\left(-\frac{a^2}{2\sigma^2}\right) da,$$

$$Q_2 = -\frac{1}{2}Q_1.$$

In the examples the integrals in (3.14) are calculated by means of the quadrature method. Note that Q_1 and Q_2 do not depend on m, so that one of Eqs. (3.13) does not include m and the equations can be solved separately.

4. Random vibration of the driven part

Having the characteristics m and σ of the relative vibration $\varphi = \varphi_1 - \varphi_2$, we can determine the spectral properties and the standard deviation of the random disturbance of motion of the driven part. Addition of Eqs. (2.1) under the assumption that $M_n = M_r$ leads to the following equation:

$$(4.1)\qquad \ddot{\varphi}_1 + \frac{I_2}{I_1}\ddot{\varphi}_2 = G(t),$$

which can be rewritten in the form

$$(4.2)\qquad \ddot{Z} = -G(t)$$

after introducing the variable $Z = \varphi_1 + I_2\varphi_2/I_1$. Solving equations

$$(4.3)\qquad \begin{aligned} \varphi &= \varphi_1 - \varphi_2, \\ Z &= \varphi_1 + \frac{I_2}{I_1}\varphi_2 \end{aligned}$$

with respect to φ_2, we obtain

$$(4.4)\qquad \varphi_2 = \lambda(Z - \varphi),$$

where $\lambda = I_1/(I_1 + I_2)$.

It follows from (4.2), (4.3) and (4.4) that the angle φ_2 can be decomposed into two parts: a uniform rotation and a random vibration with zero mean value

$$(4.5)\qquad \varphi_2 = C + Dt + \phi(t),$$

where

$$\phi = \lambda(z - x), \tag{4.6}$$

and $z(t)$ is a stationary zero-mean solution of Eq. (4.2). Spectral density of the stationary part $z(t)$ of the solution of Eq. (4.2) can be expressed as follows:

$$S_{zz}(i\omega) = \frac{1}{(i\omega)^4} S_{GG}(i\omega). \tag{4.7}$$

Now we can determine spectral properties of the stationary disturbance of motion of the driven part $\phi(t)$. First we derive its autocorrelation function,

$$\begin{aligned} K_{\phi\phi}(t_1, t_2) &= \lambda^2 E\left\{[z(t_1) - x(t_1)]\,[z(t_2) - x(t_2)]\right\} \\ &= \lambda^2 \left\{K_{zz}(t_1,t_2) - K_{zx}(t_1,t_2) - K_{xz}(t_1,t_2) + K_{xx}(t_1,t_2)\right\}. \end{aligned} \tag{4.8}$$

For stationary processes z, x and ϕ, introducing $t = t_2 - t_1$, we have

$$K_{\phi\phi}(t) = \lambda^2 \left\{K_{zz}(t) - K_{xz}(t) - K_{xz}(-t) + K_{xx}(t)\right\}. \tag{4.9}$$

Applying Fourier's transformation $F\{\cdot\}$ to both sides of expression (4.9) and taking into account that

$$F\left\{K_{xz}(t) + K_{xz}(-t)\right\} = 2\mathrm{Re}\, F\left\{K_{xz}(t)\right\} = 2\mathrm{Re}\left\{S_{xz}(i\omega)\right\} \tag{4.10}$$

and

$$S_{xz}(i\omega) = \frac{1}{(i\omega)^2} S_{xG}(i\omega), \tag{4.11}$$

we get

$$S_{\phi\phi}(i\omega) = \lambda^2 \left\{\frac{1}{\omega^4} S_{GG}(i\omega) + \frac{2}{\omega^2}\mathrm{Re}\left[S_{xG}(i\omega)\right] + S_{xx}(i\omega)\right\}. \tag{4.12}$$

In the above expression the Fourier transform S_{xG} of the cross-correlation function $K_{xG}(t)$ is still unknown. It can be determined by applying Fourier's transformation to the second of Eqs. (3.8) and making use of the properties (3.10), what yields

$$S_{xG}(i\omega) = S_{GG}(i\omega) \frac{Q_1 - \omega^2 + i\,\mathrm{sgn}\,(\omega) Q_2}{(Q_1 - \omega^2)^2 + Q_2^2}. \tag{4.13}$$

Taking into account that S_{GG} is a real function of ω, we finally obtain the spectral density of the driven part motion $\phi(t)$

$$S_{\phi\phi}(\omega) = \lambda^2 S_{GG}(\omega) \left\{\frac{1}{\omega^4} + \frac{2Q_1/\omega^2 - 1}{(Q_1 - \omega^2)^2 + Q_2^2}\right\}. \tag{4.14}$$

Expressions (4.14) and (3.11) fully determine the probabilistic properties of stationary Gaussian processes of disturbance $\phi(t)$ of the driven part and relative torsional vibration $x(t)$.

5. Examples

The numerical calculations have been made for the following set of data:

$$h_1 = 0.00125\,[\mathrm{m}], \quad h_2 = 0.00103\,[\mathrm{m}], \quad I_1 = 0.05\,[\mathrm{kgm}^2],$$
$$I_2 = 0.07\,[\mathrm{kgm}^2], \quad G = 8.2 \times 10^{10}\,[\mathrm{N/m}^2], \quad \mu = 0.25.$$

Since the driving torque disturbances in many types of engines have meaningful values of the spectral density in a limited middle-range band of frequencies, we assume the one-sided spectral density function of the torque in a polynomial form with non-zero values in an interval (ω_1, ω_2).

$$S_{MM}(\omega) = \sigma_M^2 \sum_{n=1}^{N} c_n \omega^n \left[H(\omega - \omega_1) - H(\omega - \omega_2)\right], \tag{5.1}$$

where $H(\cdot)$ denotes Heaviside's step function and the integral of the sum in Eq. (5.1) equals 1. In the numerical calculations we assumed $N = 3$ (quadratic form of), $S_{MM}(\omega_1) = S_{MM}(\omega_2) = 0$ and $(\omega_1, \omega_2) = (300, 1300)$ [1/s].

The resulting spectral density functions of the relative torsional vibration as well as vibration of the driven part are presented in Fig. 4 for various

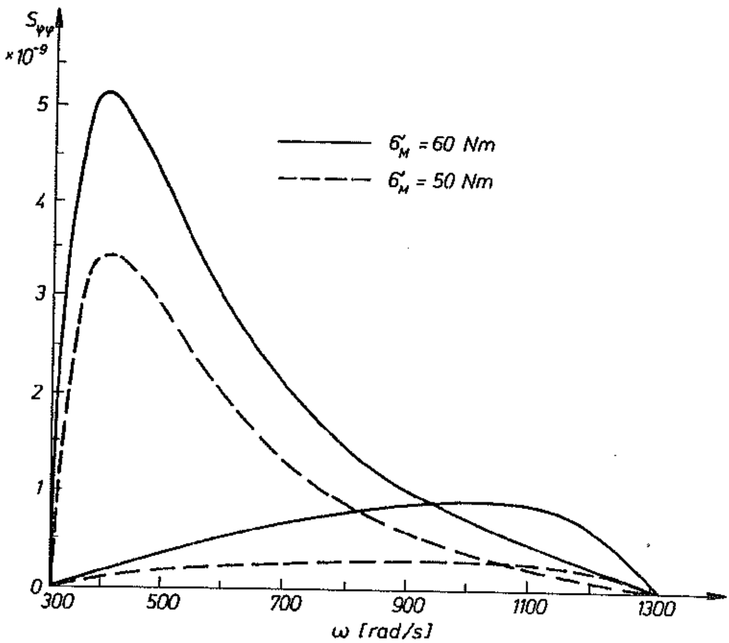

FIG. 4. Spectral density function of torsional vibration; a) relative vibration between the driving and driven parts; b) driven part vibration.

driving torque standard deviations. The curves corresponding to vibration of the driven part have the maximum zone strongly shifted into the lower range of frequencies. The process $z(t)$ especially contributes to this effect (expressions (4.2) and (4.14)). It should be noted that broad-band spectral densities $S_{MM}(\omega)$ may lead to infinite variances of vibration of the driven part.

Figures 5 and 6 present the mean value (m) and the standard deviation (σ) of the relative driving/driven part vibration as functions of the nominal driving torque (M_n) and its standard deviation (σ_M), respectively. Note that σ does not depend on M_n but m depends on σ_M. Functions $\sigma = \sigma(\sigma_M)$ are progressively nonlinear. This is a consequence of the degressive static characteristic of the coupling (see Fig. 2) resulting from the structural friction.

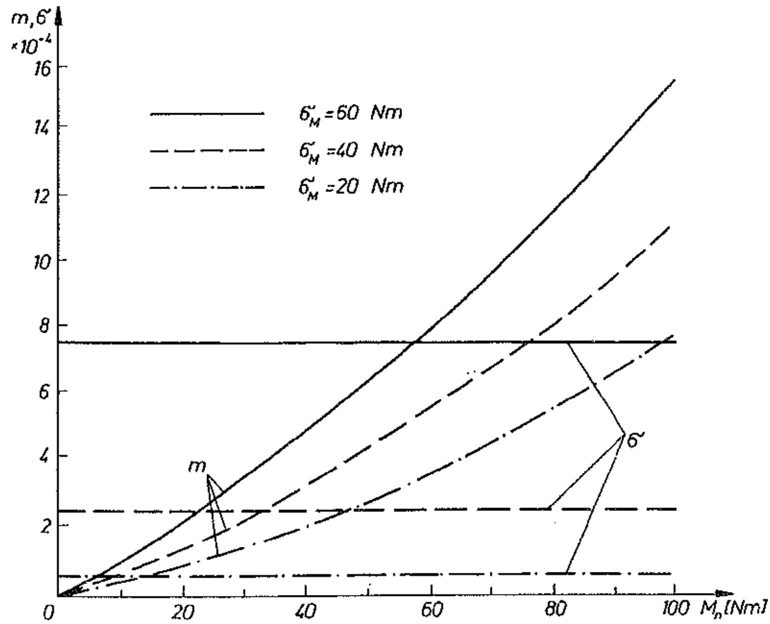

FIG. 5. Mean value and standard deviation of relative torsional vibration vs. nominal (mean) driving torque.

The role of the disc pressure in its wide range from decoupling to the practically rigid connection between the driving and driven parts is shown in Fig. 7. The standard deviations of both the relative and driven part vibrations exhibit extremum values with respect to pressure. For very high disc pressure the system practically moves like a rigid body, so that the relative vibration vanishes and the vibration of the driven part is that of a single-degree-of-freedom system with the moment of inertia equal to $I_1 + I_2$.

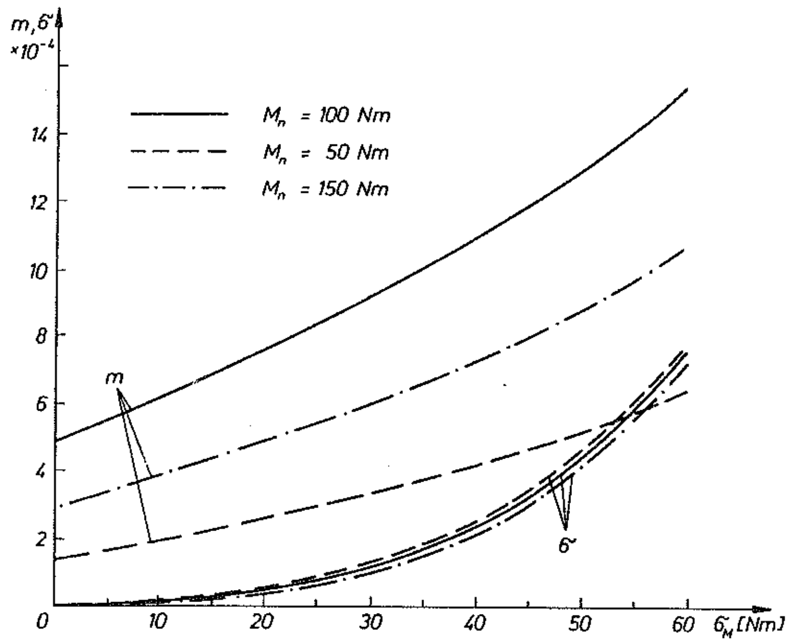

FIG. 6. Mean value and standard deviation of relative torsional vibration vs. standard deviation of the driving torque.

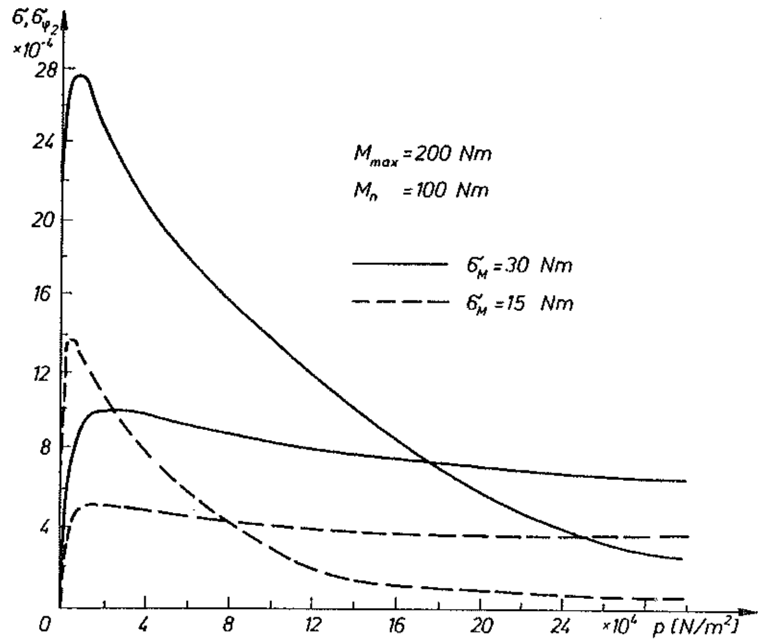

FIG. 7. Torsional vibration standard deviations as functions of the pressure for assumed maximum clutch torque; a) relative vibration, b) driven part vibration.

6. Concluding remarks

The normal closure technique employing the Hilbert transformation is applicable and efficient in dynamical analysis of systems with hysteresis loop resulting from the structural friction.

Moments of random processes of both relative engine/machine torsional vibrations can be determined under the assumption of the driving torque disturbance in the form of a stationary Gaussian stochastic process with given spectral properties.

The mean of the relative torsional vibration depends not only on the mean of the driving torque but also on its standard deviation. Increase in the driving torque variance leads to an increase in the mean relative torsion, what is a result of a wider microslip region.

Analysing a two-degrees-of-freedom model of the transmission system we can show variations of the intensity of energy dissipation due to the structural friction, but the model is too simple to display the role of the friction clutch in reducing dynamical stress in the elements of the system. This practically important problem cannot be solved unless a more appropriate model of the transmission system is admitted and the method of analysis (normal closure technique) is extended to be applicable to multi-degree-of-freedom systems. This task is planned for the nearest future.

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Received April 28, 1994.